A factor analysis of volatility across the term structure: the Spanish case

Sonia Benito
Alfonso Novales

Departamento de Economía Cuantitativa
Univerisdad Complutense
Somosaguas
28223 Madrid
Spain

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Abstract

We show how the term structure of volatilities for zero-cupon interest rates from the Spanish secondary debt market can be explained by a reduced number of factors. This factor representation can be used to produce time series volatilities across the whole term structure. As an alternative, volatilities can also be derived from a factor model for interest rates themselves. We find evidence contrary to the hypothesis that these two procedures lead to statistically equivalent time series, so that choosing the right model to estimate volatility is far from trivial. The volatility factor model fits univariate EGARCH volatility time series much better than the interest rate factor model does. However, observed differences seem to be of little consequence for VaR estimation on zero coupon bonds.

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Email address: soniabm@ccee.ucm.es and anovales@ccee.ucm.es

Introduction

Searching for a sensible factor representation of the term structure of interest rates has been object of study for some time. If interest rates at any given maturity could be written, to a reasonable approximation, as a linear combination of a small number of factors, then fluctuations of the yield curve could be characterized by just analyzing the behavior of the chosen factors. These could either be rates of return for specific maturities, like the one month rate, simple linear combinations of them, like the spread between a long- and a short-term rate, or more complicated linear combinations of interest rates at different maturities. In particular, interest rate forecasts for every maturity could be derived from forecasts for the factors.

With some differences across a variety of international fixed income markets, this type of analysis concludes in a positive note, by characterizing a small number of factors able to represent, to a large extent, the behavior of the term structure of interest rates [Stock and Watson (1988), Elton, Gruber and Michaely (1990), Litterman and Scheinkman(1990), Hall, Anderson and Granger(1992), Zhang (1993), Engsted and Tanggaard(1994), Navarro and Nave(1997), Domínguez and Novales(2000), Benito(2001)]. Even though this line of research was originally proposed to reduce the dimensionality of a usually large vector of interest rates by obtaining a simple linear representation of the term structure, there is some sense in which representing interest rates levels by a small number of factors also leads to a simple representation of interest rates fluctuations. This is why sometimes a reference is made to the fact that the factor representation is a representation of interest rates as well as volatilities across the term structure.

On the other hand, if we have a set of time series for estimated volatilities for each of a large set of maturities across the term structure, we can directly search for a factor representation of the set of volatility time series. We show in this paper that, maybe contrary to a simple intuition, the volatility series estimated from a factor model for interest rates are not equal to those obtained from a factor model for volatilities. This observation may have significant consequences for many issues related to risk management in fixed income markets, like Value at Risk (VaR) analysis, in which a numerical estimate of the future evolution of risk over the term structure is needed, to be compared with that obtained from similar markets.

The rest of the paper is organized as follows. In section 2 we describe the data used in the paper and present estimates of the conditional volatility of interest rates. In section 3, we provide evidence on reducing the dimensionality of the vector of volatilities for the set of maturities chosen to summarize the term structure. In section 4, we evaluate the ability of the factor model for volatilities to account for the volatility of the term structure of interest rates (TSIR). In section 5 we use a factor model for daily interest rates changes to produce volatility series of each interest rate considered. These are compared with those obtained from a factor model for interest rate volatilities. We present the main conclusions in section 6.

2. The data

We have used daily prices from the secondary market for Spanish goverment debt. Using quoted closing prices for bonds, we estimated the Nelson-Siegel model every day, from which zero-coupon rates can be inferred for any maturity. We focus on 1-, 3-, 6-, 8-, and 10-month rates, together with 1-, 3-, 5-, 6-, 7-, 8-, 9-, and 10 year rates. Our sample runs from September 1st, 1995 to December 31st, 2002.

Since January 1999, when the European Monetary Union was created, the European Central Bank, together with the individual central banks have been in charge of implementing monetary policy in all country members, among them Spain. Before that, the Spanish Central Bank was the single offical organism in charge of monetary police in Spain. Over the sample period considered, not only the institution in charge of monetary police, but also the way how policy is implemented, have changed. It is then almost mandatory to perform the common factor study in two different subsamples. The first sample covers from September 1st, 1995 to December 31st, 1998, the pre-monetary union period, while the second sample runs from January 4th, 1999 to December 31st, 2002.

An EGARCH(1,1) model can be shown to adequately represent the conditional volatility in both subsamples, with parameter estimates being shown in Table 1. Table 2 presents sample correlations between any two volatilities. The conditional volatility of the one month rate shows a high correlation with the volatilities of the 3-, 6-, 8-, 10-month and 1-year interest rates, while the conditional volatility of the 10-year rate displays a large correlation with the volatility of the 3-, 5-, 6-, 7-, 8-, 9-year rates of interest.

Correlations among the conditional volatility of short term interest rates were higher in the second than in the first subsample. On the contrary, correlations among the conditional volatility of the longer term interest rates were higher in the first than in the second subsample. It looks as if there is substantial volatility transmission across adjacent maturities, whereas transmision of volatility between the two extremes of the term structure is much less obvious. In adition, the central region, represented by the one year maturity, seems to display some specific properties. This preliminary evidence suggests that it might be hard to obtain a good representation of volatility across the term structure with just two factors, and that al least three factors might be needed. Exploring that possibility is the object of the next sections.

3. A Principal Component Analysis of Volatilities along the Term Structure

In an attempt to reduce the dimensionality of the vector of 13 time series of conditional volatilities, we compute their principal components. The first five eigenvalues of the variance-covariance matrix of conditional volatilities are 24,85, 10,73, 2,29, 0,41 and 0,30 in the first sample, with a percent cumulative explained variance of 63,53%, 90,98%, 96,84%, 97,89% and 99,35%. This is consistent with observations in the previous section, since three principal components would be enough to capture 95% of the time variation in the conditional volatilities, while up to five principal components would be needed to capture 99% of the time variation. The explanatory ability increases somewhat in the second sample, in which cumulative explained percent variance is: 82,99%, 92,86%, 97,46%, 98,98 and 99,54%. In this case, the first four factors capture 99% of the time variation in volatility, although again, three of them would be enough to capture 95% of the variation in the whole set of volatility time series.

Table 3 shows that, for the first sample, the coefficients defining the first principal component are quite similar over the whole term structure, so that this component can be interpreted as the general level of volatility. The second component is represented with coefficients of the same sign over the short-end of the term structure (1-month to 1-year), and coefficients of opposite sign over the 3- to 10- year maturity range. Even though the coefficients **change somewhat** for the different maturities, this component can be interpreted as representing the difference between the levels of volatility between the two ends of the term structure. In that respect, it is worthwhile noting that the volatility of the 1-year interest rate does not have any presence in this second component.

The loadings of the long term volatilities in the composition of the third principal component are almost zero, so that this component is represented as a linear combination of volatilities in the shorter end of the term structure. Because of the signs of the different coefficients, changes in this third component would imply changes of different sign in the volatilities of the 1-, 3-, 6- month rates, relative to changes in the volatility of the 8-, 10-, month and 1 year rates. This third component could again be interpreted as representing changes in the curvature of the term structure of volatilities.

Results in the second subsample are very similar regarding the third component, while there are some significant differences for the first and second principal components. The loadings of long term volatilities in the composition of the first principal component are now almost zero so that, in the second subsample, this component can be seen to represent the general level of volatility in the 1-month to 1-year range, since all coefficients share the same sign. The loadings of short term volatilities in the composition of the second principal component are almost zero while the longer maturities enter with the same sign, so that this second component can now be seen as representing the general level of volatility in the 3- to 10-year range.

4. Explaining volatilities with the principal components

We now proceed to evaluating the ability of the first three principal components to account for the conditional volatility at each of the 13 maturities considered. To that extent, we use the three components as explanatory variables in a system of regression equations having alternatively the volatility at each maturity as the dependent variable. We will refer to this system as the factor model for interest rate volatilities.

Figures 1(a) to 12(a) present the conditional volatility obtained from an univariate EGARCH(1,1) model estimated for each of the 13 maturities considered in the first sample (except for the 10-month maturity)[1], together with the volatility obtained for each maturity from the estimated factor model. The conditional volatility obtained from a factor model seems to exhibit a very similar behavior to the volatility estimated with the univariate EGARCH(1,1) model. The major differences between both series are observed in the 1-, 3- and 10-year interest rates. This is best seen in figures 1(b) to 12(b), were we present a scatter graph of both series at each maturity. This observation **seems to also arise** in the second subsample (see figures 13(a) and 13(b) to 24(a) and 24(b)).

In the first subsample, the regresion R-square is very high in all cases, being above 95% for most maturities (table 4). The ability of the first three components to explain the volatility of the 10 month, 1- and 3 years is a little lower. The fit in the second subsample is very similar, although the explanatory power for the 3-, 9- and 10 year interest rates is now somewhat lower.

Mean Absolute Errors for the linear projections of volatility on the first three components is very low in each of the two subsamples and for each of the 13 maturities considered, being below 0.3 **basic points** in all cases (table 4). With only a few exceptions, Root Mean Square Errors (RMSE) in Table 4 are below 5% in the first sample, reflecting the fact that the three first principal components explain, on average, 95% of the fluctuation in volatility over the term structure. RMSE values increase up to almost 10% for the 10 month, 1- and 3 year maturities. RMSE are a bit larger in the second sample, but they remain below 10% in all cases.

So far, we have shown that a relatively simple representation can account for the time behavior of volatility over the term structure of interest rates. As a

[1] Which we have excluded for reasons of space.

consequence, we can obtain volatility forecasts for a large set of interest rates at different maturities by forecasting just three variables, the first three principal components. Volatility forecasts are central for many applications in risk management, so the relevance of our analysis is that it allows us to measure portfolio risk with a minimum effort.

5. Do factor models for interest rate volatility and for interest rate changes lead to the same volatility forecasts?

If we have a good model to account for the term structure of interest rates, it is natural to think that this model should also be able to account in a good way for the behavior of interest rate volatility. Following this view, we have used a factor model created to explain interest rate fluctuations, to estimate the variance and covariance matrix of a large set of interest rates.

Alexander (2000) obtained the variance and covariance matrix of a large set of interest rates by just estimating the variance of the first three principal components of interest rate changes. Gento (2000) estimated the variance and convariance matrix of a large set of interest rates from the secondary Spanish public debt market by just estimating the variance of two variables: the 4-month rate and the spread between the 7-year and the 4-month rate. Abad and Benito (2004) use the Nelson and Siegel model, which represents the zero-coupon curve through four parameters, to generate the variance-covariance matrix for a large set of interest rates by just estimating the variance of daily time series of estimated parameters.

An alternative way to estimate the volatility of all interest rates in the term structure with a minimum cost is to use the volatility factor model we describe in the previous section. But then, the question is whether the volatility representation that emerges from a factor model for interest rate changes will be the same as the one we get from a factor model for volatility. To provide an answer to this question we have computed the first three principal components of daily interest rate changes.

5.1 A principal component analysis for interest rate changes

The percentage variance explained by the first principal component in the first subsample is of 51.04%. The second and third components explain, respectively, a 40.54% and 5.80% of the variance, so that the first three components together explain more than 95% of daily changes in the variance along the term structure of interest rates. At a difference of results obtained with the levels of interest rates in Section 2, now both subsamples produce very similar results. The percent variance explained by the first component is 58.8%, while the percentage of variance explained by the second and third components is 35.2% and 4.16%, respectively. The percent cumulative variance explained by the first three components is in this case of 98.1%.

The differences found in principal component analysis of interest rates and interest rate changes suggest that, at least for the second sample, the volatility representation obtained from both approaches might differ significantly.

Table 5 shows that, for the first sample, the coefficients defining the first principal component are again quite similar over the whole term structure, so that this component can be seen to represent daily global shifts across the whole term structure of interest rates. The second component is characterized by coefficients of opposite sign at both ends of the term structure, so that this component can be interpreted as a slope component of interest rate changes. Finally, the third component can be interpreted as a curvature component. These results are similar to those presented in Section 2. Results for the second sample are also quite similar to those in Section 2 and admit the same interpretation as in the first subsample, with only some minor diferences.

As we did in Section 4, we evaluate the ability of the first three principal components to explain daily changes in interest rates. We do so, by estimating:

$$\boldsymbol{dr_t^j} = \sum_{\boldsymbol{i}=1}^{3} f^j{}_i \boldsymbol{df_{i,t}} + e_{i,t} \qquad (1)$$

where $\boldsymbol{dr_t^j}$ represents daily changes in interest rate at the j –th maturity, for $j = 1$, 3-, 6-, 12-month, 1-, 3-, 5-, 6- and 10-years, and $\boldsymbol{df_{i,t}}$, $i = 1, 2, 3$, represents the first three principal components of daily changes in interest rates. The R-squared of the regression is quite high in all cases, being generally above 95% in both subsamples. The Mean Absolute Error is below 1 **basic point** in all maturities, in each of the two samples considered (Table 6).

Following Alexander (2000), we estimate the variance and covariance matrix of the vector of interest rates using the expression:

$$\boldsymbol{Var(dr_t) = AVar(df_t)A^T} \qquad (2)$$

where $\boldsymbol{Var(df_t)}$ is a diagonal matrix with the conditional variance of the first three principal components along the diagonal, and $\boldsymbol{A}$ is a 13 by 3 matrix having in each row the estimated coefficient from regression (1). $\boldsymbol{Var(dr_t)}$, is a 13 by 13 matrix representing the conditional variance-covariance matrix of interest rates.

Coefficients in matrix $\boldsymbol{A}$ are the loadings of each interest rate in each principal component so that, in fact, it is not necessary to estimate equation (1) to get the variance-covariance matrix of a large set of interest rates, once the principal component analysis has been done. An EGARCH(1,1) model can be used to represent the conditional volatility of the first three components in each subsample. Once we have the conditional volatility of the first three components, we use expresion (2) to get the conditional volatility of the 13 interest rates considered.

As we did in the previos section, we compare in Figures 25(a) to 36(b) the conditional variance estimates obtained from this procedure, together with those obtained from estimating an EGARCH(1,1) specification for each interest rate. The behavior of the conditional variance obtained from the factor model for interest rate changes is similar to the bahavior of the conditional variance estimated from an EGARCH specification for each individual interest rate. However, it is important to notice that the variance obtained from a factor model for interest rate volatilities fits the variance obtained from an EGARCH specification better than the volatility obtained from a factor model for interest rate levels **(see figures 1(a)-12(a))**, as can be seen by comparing Figures 1(b)-12(b) with Figures 25(b)-36(b).

Table 7 presents the Root Mean Square Error for both factor models: the one for interest rate volatilities, and the one for interest rate changes. In the first subsample, the Root Mean Square Error for the factor model of volatilities remains below 5% for most maturities. The Root Mean Square Error for the factor model for interest rate changes is above 16% in all cases. In the second subsample, the Root Mean Square Error for the factor model of volatilities falls between 2.3% for the 8-month interest rate and 8.1% for the 3-month interest rate. For most maturities, the Root Mean Square Error for the factor model for interest rate changes is more than three times the Root Mean Square Error for the factor model for volatilities. Table 8 presents the Mean Absolute Error for both models. In each of the two samples and for all maturities the Mean Absolute Error is higher for the factor model for changes in interest rates.

These results suggest that the volatility representation obtained from a factor model for interest rate volatilities is different from the volatilities obtained from a factor model for interest rate changes. To further test that hypothesis, we proceed as follows.

5.2 A comparative analysis of volatility.

Mann-Whitney and Kruskal-Wallis statistics to test for whether both volatility series have the same mean, are shown in Table 9. In the first subsample, both statistics offer evidence against the null hypothesis of homogeneity for the shorter maturities, up to 3-years. For the longer maturities, we do not find evidence against such hypothesis. In the second subsample, we find evidence against the null hypothesis of equal means for most maturities. The Siegel-Tukey statistic to test for whether the conditional volatilities produced by the two models have the same variance, is presented in the same table. For most maturities, this statistic offers strong evidence against the null hypothesis.

Finally, we test for whether the volatility representation we get from a factor model for interest rate volatilities is similar to that obtained from a factor model for interest rate changes. To do so, we present in Table 10 the values of the Wilcoxon and the Kolmogorov-Smirnov statistics. In both subsamples and for all maturities, the two statistics offer evidence against the null hypothesis of homogeneity of the volatility series generated by the two factor models.

Summarizing, the analysis in this section has shown that the conditional volatility estimates produced by the factor model for interest rate volatilities and the factor model for interest rate changes display statistically significant differences being the first approach, as it should be expected, the one that fits better the volatilities that emerge from a univariate specification. This evidence might be taken to suggest that in order to estimate the volatility of the term structure, we are better off by using a volatility factor model than an interest rate factor model. The point is that we do not know whether the volatility series we obtain through univariant specifications constitute a better estimate of risk than the volatility series we get from an interest rate factor model.

However, good volatility estimation is usually not an end in itself but rather, an interesting property likely to lead to good risk management. We now turn in the next section to analyzing the ability of both factor models for risk evaluation of fixed-income assets.

6. Approximating bond price changes through the factor model.

In general, a risk manager will be directly interested on the performance of a given volatility model in estimating a risk indicator like Value at Risk (VaR) so, as an extension of the analysis we have presented in the previous sections, it would be interesting to compare the ability of both models to estimate the VaR of a given fixed-income portfolio. However, that is a somewhat complex exercise that requires specification and estimation of the conditional variance-covariance matrix for the vector of interest rates, so we focus in this section on estimating VaR for a set of individual zero coupon bonds, paying attention to the performance of both factor models at short-, medium- and long-term maturities.

Using continuous discount factors, the theoretical price for the zero coupon bonds can be written,

$$p_t(i) = N \exp^{-t_{i \times r_t(i)}} \qquad (4)$$

for i = 1-, 3-, 6-, 8-, 10-months, 1-, 3-, 5-, 6-, 7-, 8-, 9-, 10-years, where N denotes the face value of the bonds, which we take to be one, t_i denotes time to maturity for the *i*-th bond, and $r_t(i)$ is the zero coupon rate of interest at maturity *i*. From (4), we can approxímate price changes through,

$$dp_t\,(i) \approx -D\,d(r_t\,(i)) \qquad (5)$$

where *D* denotes duration: $D = t_i \exp^{-t_i \times r_t(i)}$.

Expression (5) can be used to approxímate the variance of bond price changes as,

$Var(\,dp_t\,(i)) = D^2\; Var(d(r_t\,(i)))$, so that the standard deviation of price changes can then be approximated by,

$$s_{\,dp(i)} = D s_{\,dr(i)} \qquad (6)$$

where $s_{\,dp(i)}$, $s_{\,dr(i)}$ denote the conditional standard deviation for the *i*-th bond price changes and for interest rates at maturity *i* years.

Once we have the conditional mean and standard deviation for bond price changes, their VaR can be obtained,

$$VaR(a\,\%) = m_{dp(i)} + s_{\,dp(i)}\,k_{a\,\%} \qquad (7)$$

In previous sections we have used two different methods to estimate the conditional variance of interest rate changes: a factor model for the conditional variances of a vector of interest rates, and a factor model for interest rate changes themselves. Here, we use both approaches to produce two different approximations to the variance of price changes in zero coupon bonds. Each approximation can be used in (7), in turn, to compute the VaR for each zero coupon bond under consideration.

We perform this exercise at a 5% confidence level and a one-day horizon, for each of the zero coupon bonds considered. We then examine actual daily price changes in the theoretical zero coupon bonds, as implied by daily fluctuations in zero coupon interest rates, and compare them with the 5% VaR. If the estimation of the theoretical VaR is appropriate, we should expect about 5% of daily price changes to be below thats thresholds. Our first sample being of size 813 data points, that amounts to 41 daily price changes below the 5% VaR. The size ot the second sample considered is 992, suggesting that about 50 daily price changes should be below the 5% VaR.

The results for the first sample are shown in table 11. The absolute and relative frequencies of price changes below the 5% VaR is shown for each bond. Fort short-term bonds, between 1- and 10-month maturity, we observe between 14 and 24 daily price changes fulfilling that condition with the factor model for volatilities, and between 20 and 27 daily price changes for the factor model for interest rate changes. This amounts to a percentage between 1,7% and 2,9% under the first factor model, and between 2,5% and 3,3% under the second model. VaR estimates seem to perform somewhat better for medium- and longer-term bonds, approaching the 5% theoretical confidence level. Nevertheless, the relative frequency of daily price changes below 5% is for almost all bonds lower then the theoretical level, so both models seem to overestimate the level of risk, at least according to the VaR measure. No model seems to do better than the other in estimating VaR.

Results for the second sample are displayed in table 12. The number of daily price changes below the 5% VaR for short-term bonds falls now between 45 and 51, with a relative frequency between 4,5% and 5,1%. Relative frequencies obtained under the factor model for interest rates fall between 3,7% for the 3-month bond and 5,2% for the 10-month bond. Being lower that the theoretical 5% confidence level, we obtain again evidence of risk overestimation. Differences between both models are minor, although the factor model for volatilities seem to produce relative frequencies closer to the 5% level. In medium and long-term bonds, we have a relative frequency of daily price changes below 5% between 4,0% and 4,9% under the factor model for volatilities, and between 4,7% and 5,5% under the factor model for interest rate changes. In this second sample, the factor model for interest rate changes seems to produce a slight underestimation of risk.

Summarizing these results, no model seems to produce a better VaR estimate than the other. Both models produce reasonable VaR estimates for medium- and long-term bonds, while overestimating risk for short-term bonds. Differences in VaR estimation between both factor models are very small, so both should be considered essentially equivalent from the point of view of risk evaluation through a VaR measure.

Tabla 11. September 1995 to December 1998

	Factor model volatility												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below VaR(5%)	24	20	17	14	24	38	38	35	37	36	37	36	35
Relative frequencies VaR(5%)	2.9	2.5	2.1	1.7	2.9	4.7	4.7	4.3	4.5	4.4	4.5	4.4	4.3

	Factor model interest rate												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below VaR(5%)	21	20	21	21	27	45	42	38	34	35	38	40	42
Relative frequencies VaR(5%)	2.6	2.5	2.6	2.6	3.3	5.5	5.1	4.7	4.2	4.3	4.7	4.9	5.1

Table 12: January 1999 to December 2002

	Factor model volatility												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below:													
VaR(5%)	49	47	51	50	45	40	46	48	47	49	47	48	48
Relative frequencies													
VaR(5%)	4.9	4.7	5.1	5.0	4.5	4.0	4.6	4.8	4.7	4.9	4.7	4.8	4.8

	Factor model interest rate												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below													
VaR(5%)	39	37	42	51	52	52	55	51	50	50	47	52	55
Relative frequencies													
VaR(5%)	3.9	3.7	4.2	5.1	5.2	5.2	5.5	5.1	5.0	5.0	4.7	5.2	5.5

7. Conclusions

Searching for a sensible factor representation of the term structure of interest rates has been object of study for some time. If interest rates at any given maturity could be written, to a reasonable approximation, as a linear combination of a small number of factors, then fluctuations of the yield curve could be characterized by just analyzing the behavior of the chosen factors.

An alternative way to represent volatility across the Term Structure of Interest Rates is by means of a factor model for interest rate volatilities. Even though it might seem as if volatility estimates for a given maturity obtained from a volatility factor model ought to be similar to those obtained from an interest rate factor model, the proporsition requires being tested, specialy because estimating volatilities through a factor model for interest rate changes is a standard procedure. The purpose of this paper has been, in fact, to test whether both volatility representations are statistically equivalent.

We have used zero-coupon rates from the Spanish secundary public debt market for 1-, 3-, 6-, 8-, and 10-month, 1-, 3-, 5-, 6-, 7-, 8-, 9-, 10- year maturities, and to analyze robustness, we have splitted the sample in two. The first subsample runs from September 1st, 1995 to December 31st, 1998, (the pre-Monetary Union period), while the second covers the January 4st, 1999 to December 31st, 2002 period.

As a first, more standard approach, we have constructed the first three principal components of daily interest rate changes, which explain more than 95% of the variability in the term structure, and we have used an EGARCH(1,1) model to estimate their conditional variance. The projection of daily changes in each individual interest rate on the three principal components is used to to estimate the conditional volatility for each of the 13 interest rates considered from the conditional variance time series for the components.

An alternative approach uses an EGARCH(1,1) specification to estimate the conditional volatility of each single interest rate considered. The first three principal components of the set of conditional variances explain more than 95% of the variability in the term structure of volatilities. We then estimate the volatility along the term structure using the linear projections of the volatility at each of the 13 maturities considered on the three volatility principal components.

To test if the volatility series estimated by both models are statistically equivalent we implement a variety of formal non-parametric tests. By and large, the evidence is contrary to such hypothesis, so that the election of the model used to estimate conditional volatilities across the term structure of interest rates is, in general, far from irrelevant. In addition, we compare the volatility series we get from an EGARCH univariate model with those we get from both factor models. The factor model for interest rate volatilities fits the set of univariate EGARCH volatilities much better than the factor model for daily interest rate changes. So, this analysis suggests that there is not the same information regarding volatility in the volatility of a vector of interest rates than in interest rates themselves. It might also be the case that some information on second order moments is lost when computing a small set of principal components for interest rate changes.

This evidence might lead us to think that if we want to estimate conditional volatility across the term structure, it might be better to use a volatility factor model than an interest rate factor model. However, as the volatility of each individual interest rate is not observable, the statement weakens somewhat. In fact, the result just tells us that the volatility factor model fits the univariate volatility of interest rates better than the interest rate factor model does. What we do not know is whether the volatility series we obtain through univariate modelling constitutes a better risk estimate than the volatility series we get from an interest rate factor model. It is for this reason that we cannot assert that the volatility factor model is superior to the interest rate factor model.

Precissely because of that, we have also examined whether statistically significant differences in volatility estimation are relevant for risk estimation. As a first analysis, we have just considered individual zero coupon bonds of different maturities, leaving the analysis of portfolio risk for further research, since it requires a more laborious sspecification of conditional covariances over the term structure. Our results suggest that, at least for this specific set of assets, differences in estimated volatilities do not lead to noticeable differences in Value at Risk estimation, so a risk manager might be indifferent between the two factor model approaches, in spite of their statistical differences.

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Table 1 (a): Estimations of the univariate model (Conditional, Mean and Variance)

Sample: September 1995 to December 1998

	Autorregresive model								EGARCH(1,1) model			
	α	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	σ	ω	θ	β	γ
1 m.	0.003	-0.184	-0.059	-0.065	-0.053			0.106	-0.573	0.230	-0.045	0.909
	(0.003)	(0.036)	(0.043)	(0.045)	(0.040)				(0.081)	(0.026)	(0.018)	(0.014)
3 m.	-0.001	-0.204	-0.047					0.082	-0.521	0.236	-0.021	0.929
	(0.002)	(0.034)	(0.045)						(0.057)	(0.022)	(0.017)	(0.009)
6 m.	-0.004	-0.189	-0.092	0.057	0.044	0.140	0.045	0.057	-0.268	0.216	-0.047	0.980
	(0.001)	(0.031)	(0.037)	(0.044)	(0.041)	(0.042)	(0.039)		(0.024)	(0.018)	(0.017)	(0.003)
8 m.	-0.006	-0.169	-0.036	0.062	0.106	0.091	0.075	0.049	-0.292	0.2050	-0.033	0.975
	(0.001)	(0.033)	(0.037)	(0.040)	(0.040)	(0.037)	(0.036)		(0.024)	(0.024)	(0.016)	(0.003)
10 m.	-0.008	-0.124	-0.025	0.051	0.069	0.039		0.044	-0.723	0.242	-0.046	0.911
	(0.001)	(0.036)	(0.039)	(0.038)	(0.047)	(0.044)			(0.086)	(0.030)	(0.021)	(0.012)
1 y.	-0.009	-0.084	-0.028	0.042				0.045	-3.424	0.373	-0.011	0.494
	(0.001)	(0.043)	(0.039)	(0.032)					(0.662)	(0.055)	(0.036)	(0.101)
3 y.	-0.007	0.022	-0.009					0.054	-0.171	0.132	-0.011	0.988
	(0.001)	(0.035)	(0.037)						(0.043)	(0.025)	(0.012)	(0.004)
5 y.	-0.007	0.029	-0.054					0.058	-0.161	0.113	-0.028	0.987
	(0.002)	(0.037)	(0.036)						(0.031)	(0.018)	(0.010)	(0.003)
6 y.	-0.007	0.030	-0.075					0.059	-0.161	0.102	-0.028	0.985
	(0.002)	(0.037)	(0.036)						(0.034)	(0.018)	(0.009)	(0.004)
7 y.	-0.007	0.023	-0.081					0.059	-0.169	0.102	-0.028	0.984
	(0.002)	(0.037)	(0.036)						(0.038)	(0.019)	(0.010)	(0.005)
8 y.	-0.008	0.011	-0.081					0.059	-0.169	0.103	-0.028	0.983
	(0.002)	(0.038)	(0.037)						(0.038)	(0.019)	(0.010)	(0.005)
9 y.	-0.008	-0.005	-0.081					0.059	-0.204	0.116	-0.031	0.979
	(0.002)	(0.040)	(0.037)						(0.046)	(0.021)	(0.012)	(0.006)
10 y.	-0.007	-0.016	-0.088					0.059	-0.204	0.116	-0.031	0.979
	(0.002)	(0.040)	(0.038)						(0.046)	(0.021)	(0.012)	(0.006)

The models used to estimate the conditional mean and variance are respectively:

$$\nabla r_t = a + \sum_{i=1}^{p} d_i \nabla r_{t-i} + e_t \qquad (1)$$

$$\log h_t^2 = v + q_1\left(\frac{e_t}{h_t}\right) + b_1\left(\left|\frac{e_t}{h_t}\right| - \left(\frac{2}{p}\right)^{1/2}\right) + g_1 \log h_{t-1}^2 \qquad (2)$$

s is the standard deviation.

Table 1 (a): Estimations of the univariate model (Conditional, Mean and Variance)

Sample: January 1999 to December 2002

	Autorregresive model							EGARCH(1,1) model			
	α	δ_1	δ_2	δ_3	δ_4	δ_5	σ	ω	θ	β	γ
1 m.	-0.004	-0.168	-0.115	0.073	-0.050		0.079	-1.202	0.429	-0.179	0.833
	(0.001)	(0.036)	(0.036)	(0.031)	(0.024)			(0.076)	(0.027)	(0.019)	(0.011
3 m.	-0.003	-0.127	-0.044	0.074	-0.055	-0.014	0.071	-1.102	0.527	-0.113	0.870
	(0.001)	(0.035)	(0.033)	(0.035)	(0.031)	(0.022)		(0.064)	(0.028)	(0.021)	(0.009)
6 m.	-0.001	-0.043	-0.059	0.044			0.064	-1.001	0.449	-0.111	0.882
	(0.001)	(0.037)	(0.037)	(0.034)				(0.054)	(0.023	(0.021	(0.008
8 m.	0.000	-0.096	-0.051	0.051			0.059	-1.133	0.399	-0.122	0.855
	(0.001)	(0.037)	(0.036)	(0.037)				(0.069)	(0.020)	(0.019)	(0.011)
10 m.	0.000	-0.109	-0.056				0.057	-1.395	0.385	-0.137	0.810
	(0.001)	(0.038)	(0.036)					(0.095)	(0.019)	(0.018)	(0.015)
1 y.	-0.001	-0.113	0.000				0.055	-1.756	0.376	-0.147	0.749
	(0.001)	(0.042)	(0.035)					(0.132)	(0.018)	(0.017)	(0.021)
3 y.	-0.001						0.053	-0.301	0.139	-0.036	0.966
	(0.001)							(0.050)	(0.017)	(0.011)	(0.008)
5 y.	-0.001						0.054	-0.333	0.157	-0.038	0.963
	(0.002)							(0.067)	(0.021)	(0.014)	(0.010)
6 y.	-0.000	-0.031					0.052	-0.359	0.164	-0.034	0.961
	(0.001)	(0.034)						(0.085)	(0.025)	(0.016)	(0.012)
7 y.	-0.000	-0.032					0.050	-0.366	0.164	-0.030	0.960
	(0.001)	(0.033)						(0.101)	(0.027)	(0.016)	(0.015)
8 y.	-0.000	-0.029					0.047	-0.352	0.158	-0.023	0.963
	(0.001)	(0.032)						(0.010	(0.028)	(0.017)	(0.016)
9 y.	-0.000						0.046	-0.349	0.151	-0.018	0.962
	(0.001)							(0.104)	(0.028)	(0.015)	(0.015
10 y.	-0.000						0.044	-0.303	0.137	-0.014	0.969
	(0.001)							(0.095)	(0.027)	(0.014)	(0.013

Note: see note to Table 1(a).

Table 2: Sample correlations

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
1 m.	1.00	0.94	0.81	0.69	0.60	0.61	0.46	0.45	0.43	0.42	0.41	0.40	0.39
3 m.		1.00	0.89	0.77	0.68	0.67	0.39	0.37	0.35	0.34	0.33	0.33	0.32
6 m.			1.00	0.96	0.83	0.80	0.29	0.27	0.25	0.23	0.22	0.21	0.18
8 m.				1.00	0.90	0.83	0.26	0.22	0.20	0.19	0.18	0.16	0.13
10 m.					1.00	0.82	0.31	0.26	0.25	0.23	0.22	0.20	0.19
1 y.						1.00	0.50	0.49	0.48	0.46	0.45	0.43	0.40
3 y.							1.00	0.96	0.94	0.92	0.91	0.89	0.88
5 y.								1.00	1.00	0.99	0.98	0.97	0.95
6 y.									1.00	1.00	0.99	0.98	0.97
7 y.										1.00	1.00	0.99	0.98
8 y.											1.00	1.00	0.99
9 y.												1.00	1.00
10 y.													1.00

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
1 m.	1.00	0.95	0.88	0.85	0.81	0.74	0.13	0.16	0.21	0.25	0.26	0.25	0.22
3 m.		1.00	0.97	0.93	0.87	0.80	0.09	0.12	0.18	0.24	0.26	0.26	0.23
6 m.			1.00	0.99	0.94	0.88	0.14	0.15	0.22	0.28	0.30	0.29	0.25
8 m.				1.00	0.98	0.94	0.20	0.20	0.26	0.31	0.32	0.30	0.25
10 m.					1.00	0.98	0.25	0.24	0.29	0.32	0.32	0.30	0.24
1 y.						1.00	0.28	0.26	0.30	0.32	0.32	0.29	0.23
3 y.							1.00	0.94	0.91	0.86	0.80	0.74	0.69
5 y.								1.00	0.99	0.95	0.88	0.81	0.73
6 y.									1.00	0.98	0.94	0.88	0.80
7 y.										1.00	0.98	0.94	0.87
8 y.											1.00	0.98	0.94
9 y.												1.00	0.98
10 y.													1.00

Table 3: Representation of the principal components

Sample: September 1995 to December 1998	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.44	0.38	0.27	0.17	0.15	0.22	0.22	0.20	0.18	0.18	0.18	0.17
Second principal component	-0.20	-0.25	-0.32	-0.25	-0.13	-0.03	0.32	0.34	0.32	0.31	0.31	0.31	0.32
Third principal component	0.58	0.28	-0.32	-0.49	-0.37	-0.30	-0.08	-0.06	-0.05	-0.05	-0.04	-0.02	0.00

Note: the table shows the coefficients of each volatility in each principal component.

Sample: January 1999 to December 2002	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.55	0.43	0.33	0.27	0.22	0.03	0.03	0.04	0.04	0.04	0.04	0.03
Second principal component	-0.12	-0.14	-0.02	0.04	0.08	0.09	0.43	0.46	0.41	0.36	0.32	0.29	0.27
Third principal component	0.64	0.16	-0.29	-0.36	-0.40	-0.41	0.03	0.08	0.08	0.07	0.07	0.07	0.08

Note: the table shows the coefficients of each volatility in each principal component.

Table 4: The accurate ajust of the principal component model

Sample: September 1995 to December 1998	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Determination coefficient (R^2)	0.99	0.97	0.98	0.99	0.86	0.84	0.90	0.99	0.99	0.99	0.99	0.98	0.96
Mean Absolute Error	0.2	0.2	0.2	0.1	0.3	0.3	0.4	0.1	0.1	0.1	0.1	0.2	0.2
Roots Mean Square Error	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7

Note: the table shows the coefficients of each volatility in each principal component.

Sample: January 1999 to December 2002	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Determination coefficient (R^2)	0.99	0.99	0.98	1.00	0.98	0.93	0.87	0.95	0.98	0.97	0.93	0.85	0.75
Mean Absolute Error *	0.3	0.3	0.2	0.1	0.2	0.3	0.3	0.2	0.1	0.1	0.2	0.2	0.3
Roots Mean Square Error ++	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7

(*) basic point
(++) percentage

Table 5: Composition of the principal components
(interest rate dairy variation)

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.44	0.37	0.29	0.25	0.21	0.17	0.21	0.25	0.26	0.26	0.26	0.26	0.26
Second principal component	-0.52	-0.39	-0.22	-0.13	-0.04	0.03	0.25	0.27	0.27	0.27	0.27	0.27	0.27
Third principal component	0.40	0.09	-0.25	-0.39	-0.46	-0.50	-0.25	0.06	0.12	0.14	0.15	0.14	0.12

Note: the table shows the weight of each interest rate in the composition of the first three principal component

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.46	0.43	0.40	0.37	0.35	0.33	0.17	0.11	0.09	0.08	0.08	0.07	0.07
Second principal component	-0.20	-0.17	-0.11	-0.07	-0.03	0.01	0.32	0.40	0.40	0.38	0.36	0.34	0.32
Third principal component	0.55	0.26	-0.05	-0.21	-0.33	-0.42	-0.40	-0.05	0.06	0.13	0.18	0.20	0.22

Note: the table shows the weight of each interest rate in the composition of the first three principal component

Table 6: The accurate ajust of the principal component model
(interest rate dairy variation)

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Determination coefficient (R^2)	0.99	0.98	0.99	0.99	0.95	0.88	0.91	0.97	0.99	1.00	0.99	0.98	0.96
Mean Absolute Error *	0.4	0.2	0.3	0.3	0.3	0.4	1.1	0.7	0.4	0.2	0.3	0.6	0.8

(*) basic point

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Determination coefficient (R^2)	0.99	1.00	1.00	0.99	0.99	0.99	0.95	0.96	0.98	1.00	0.99	0.95	0.89
Mean Absolute Error *	0.5	0.1	0.2	0.3	0.3	0.3	0.6	0.7	0.4	0.1	0.3	0.6	0.9

(*) basic point

Table 7: Roots Mean Square Error for each volatility

	Sample: September 1995 to December 1998												
	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Factor model volatility	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7
Factor model interest rate	17.7	20.0	34.2	30.4	18.0	16.5	21.0	18.8	18.4	18.5	18.6	18.8	19.4

Note: the error are written en pertentage

	Sample: January 1999 to December 2002												
	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Factor model volatility	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7
Factor model interest rate	31.2	34.2	15.9	11.6	15.1	19.0	22.4	15.2	12.3	10.6	10.7	12.2	14.9

Note: the error are written en pertentage

Table 8: Mean Absolute Error

	Sample: September 1995 to December 1998												
	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Factor model volatility	0.2	0.2	0.2	0.1	0.3	0.3	0.4	0.1	0.1	0.1	0.1	0.2	0.2
Factor model interest rate	1.3	1.1	1.2	1.0	0.6	0.5	0.8	0.8	0.8	0.8	0.8	0.8	0.8

Note: the error are written in basic point

	Sample: January 1999 to December 2002												
	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Factor model volatility	0.5	0.1	0.2	0.3	0.3	0.3	0.6	0.7	0.4	0.1	0.3	0.6	0.9
Factor model interest rate	1.4	1.1	0.5	0.5	0.6	0.8	0.8	0.6	0.4	0.4	0.4	0.4	0.5

Note: the error are written in basic point

Table 9. Means and variances equality test

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	2.56	2.54	5.31	5.91	1.99	4.14	2.03	0.52	0.06	0.28	0.06	0.55	1.48
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Kruskal-Wallis	6.53	2.43	28.16	34.98	3.96	17.15	4.12	0.27	0.00	0.08	0.00	0.31	2.18
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Siegel-Tukey	0.90	2.24	8.88	10.03	4.98	3.40	7.67	5.51	3.20	2.25	2.00	2.16	2.66
	(0.37)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.05)	(0.66)	(0.01)

Note: the number in parenthesis is the p-value

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	9.90	9.83	4.65	0.60	0.60	3.07	6.39	3.74	2.75	2.49	3.33	5.57	9.23
	(0.00)	(0.00)	(0.00)	(0.55)	(0.55)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Kruskal-Wallis	97.95	96.53	21.63	0.36	0.36	9.47	40.84	13.97	7.58	6.19	11.09	31.01	85.21
	(0.00)	(0.00)	(0.00)	(0.55)	(0.55)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Siegel-Tukey	6.51	7.23	1.16	3.42	3.42	6.83	10.35	4.18	5.17	6.28	7.63	8.88	9.95
	(0.37)	(0.00)	(0.24)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: the number in parenthesis is the p-value

Table 10: Homogenity samples test

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Wilcoxon	3.2	1.6	5.7	6.0	0.1	-11.6	-5.3	-3.7	-3.3	-3.2	-3.8	-5.0	-6.7
Kolmogorov-Smirnov(*)	0.072	0.083	0.201	0.207	0.100	0.100	0.116	0.081	0.075	0.072	0.065	0.072	0.083

(*) the critical value for Kolmogorov test are 0,055, 0,067 y 0,081 at a confidence level of 90%, 95% y 99%.

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Wilcoxon	17.3	19.3	16.9	6.0	-5.7	-10.3	-14.0	-13.5	-11.5	-10.1	-11.3	-15.3	-19.7
Kolmogorov-Smirnov(**)	0.246	0.236	0.100	0.047	0.112	0.206	0.152	0.103	0.091	0.102	0.119	0.156	0.239

(**) the critical value for Kolmogorov test are 0,050, 0,061 y 0,073 at a confidence level of 90%, 95% y 99%.

The critical value for wilcoxon test are 1,29, 1,65 y 2,3 at a confidence level of 90%, 95% y 99%.

September 1, 1995 to december 31, 1998. Figures 1 to 6.

Figure 1(a). Conditional Standars Desviation

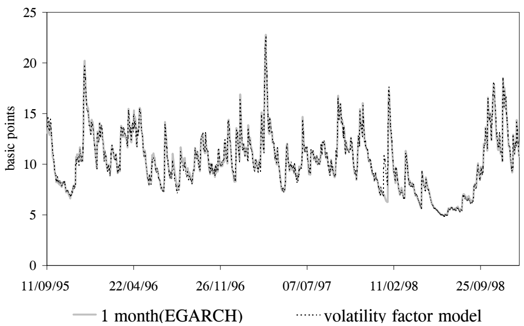


Figura 1(b). 1 month

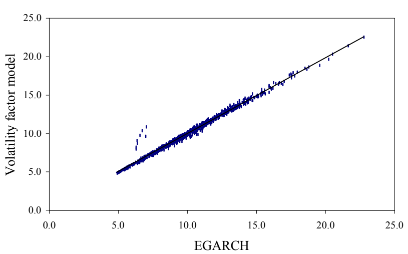


Figure 4(a). Conditional Standard Desviation

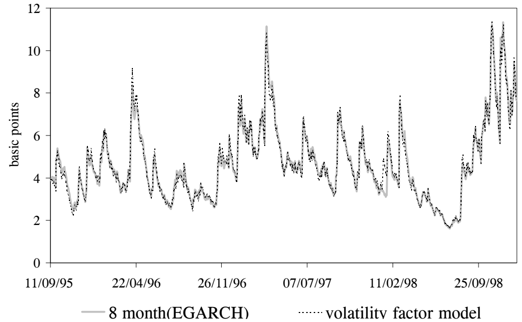


Figure 4(b). 8 meses

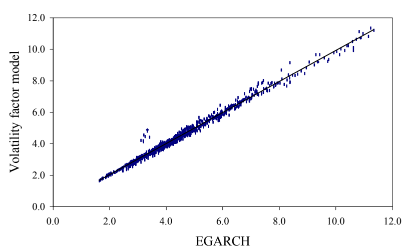


Figure 2(a). Conditional Standard Desviation

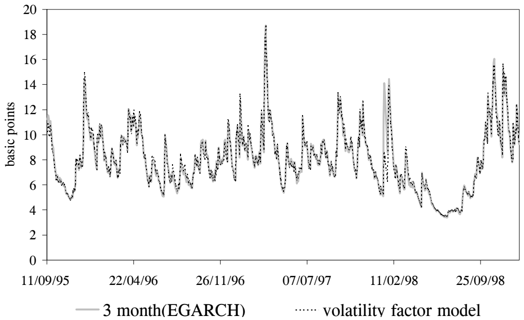


Figure 2(b). 3 month

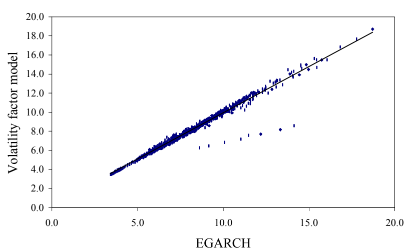


Figure 6(a). Conditional Standard Desviation

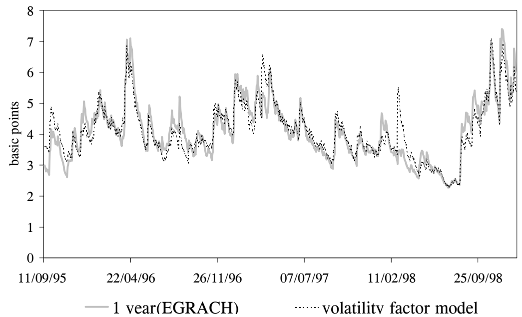


Figure 6(b). 1 year

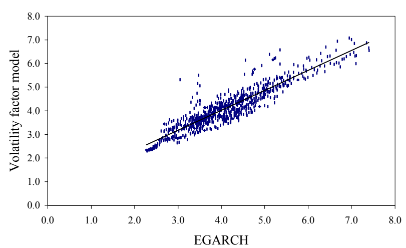


Figure 3(a). Conditional Standard Desviation

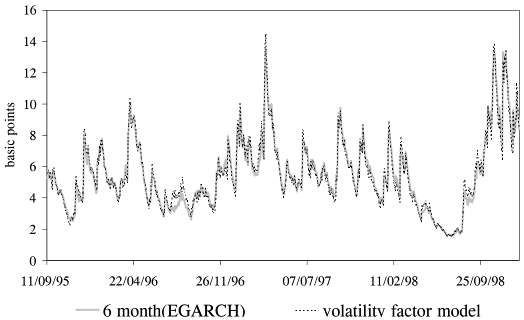


Figure 3(b). 6 meses

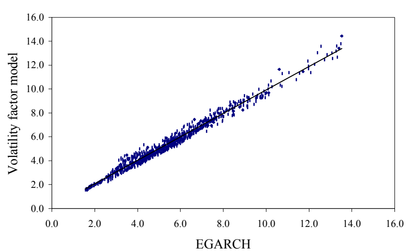


Figure 7(a). Conditional Standard Desviation

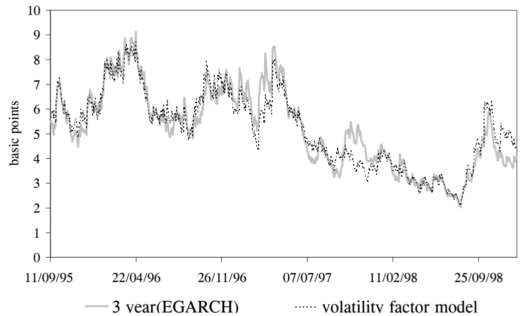
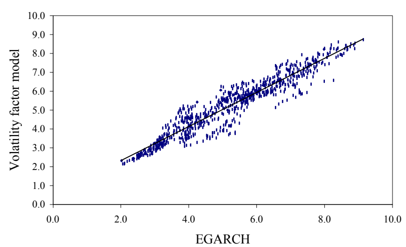


Figure 6(b). 3 años



September 1, 1995 to december 31, 1998. Figures 7 to 12.

Figure 8(a). Conditional Standard Desviation

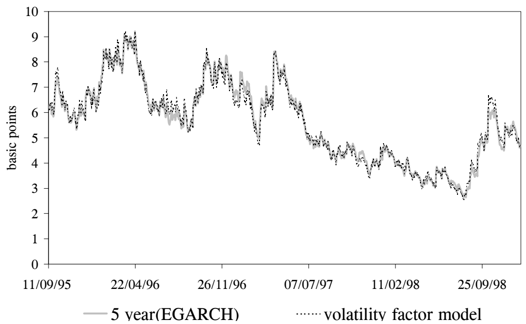


Figure 8(b). 5 year

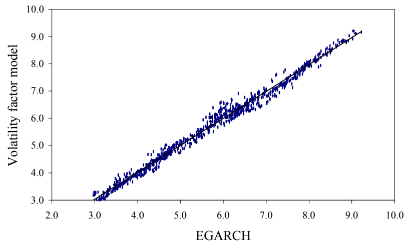


Figure 11(a). Conditional Standard Desviation

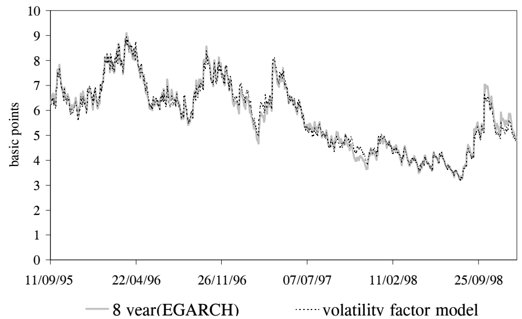


Figure 11(b). 8 year

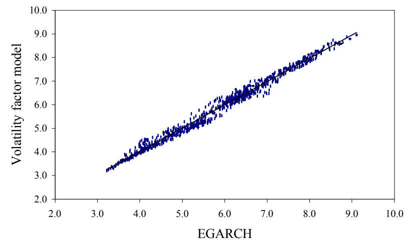


Figure 9(a). Conditional Standard Desviation

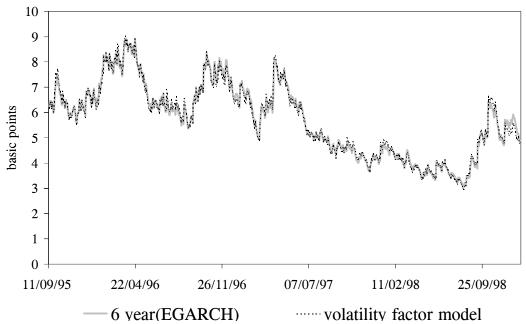


Figure 9(b). 6 year

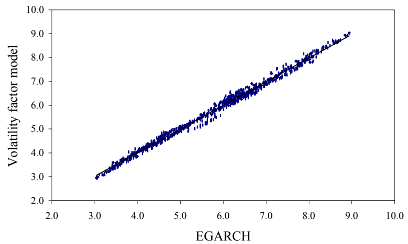


Figure 12(a). Conditional Standard Desviation

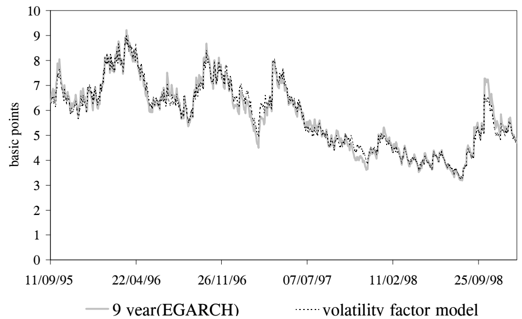


Figure 12(b). 9 year

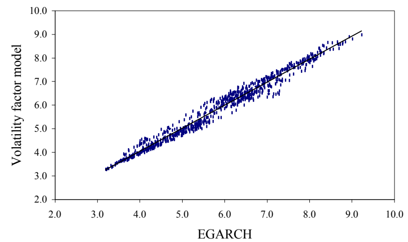


Figure 10(a). Conditional Standard Desviation

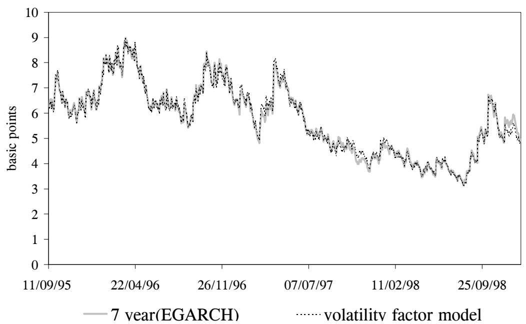


Figure 10(b). 7 year

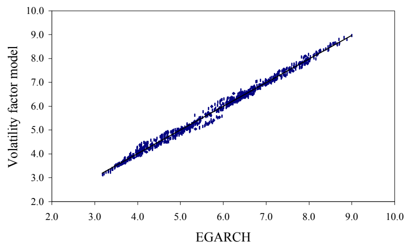


Figure 13(a). Conditional Standard Desviation

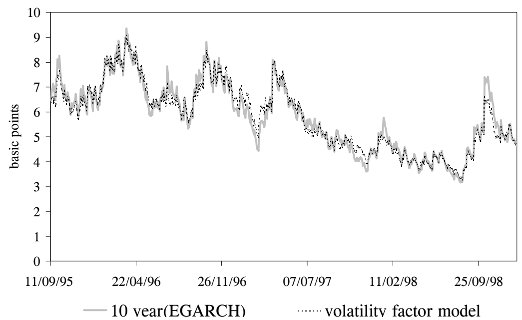
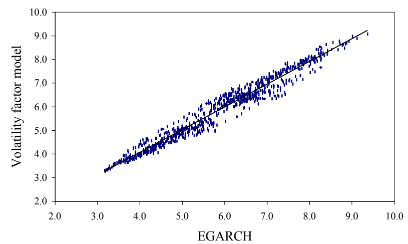


Figure 12(b). 10 year



January 4, 1999 to December 31, 2002. Figures 13 to 18.

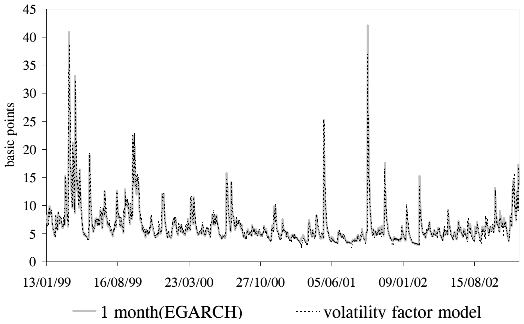


Figure 1(a). Conditional Standard Desviation

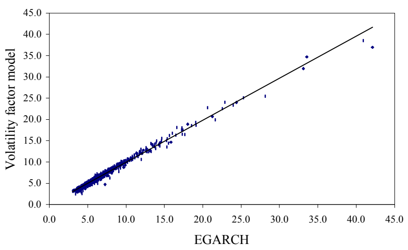


Figura 1(b). 1 month

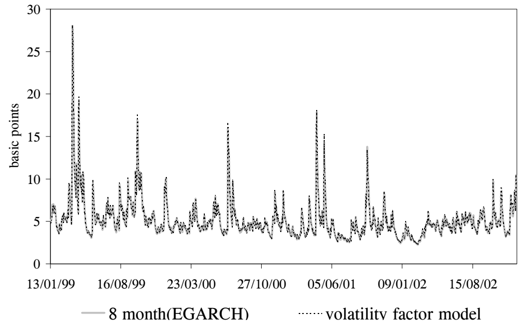


Figure 4(a). Conditional Standard Desviation

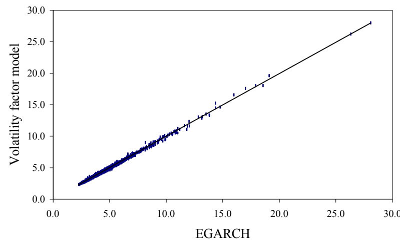


Figure 4(b). 8 meses

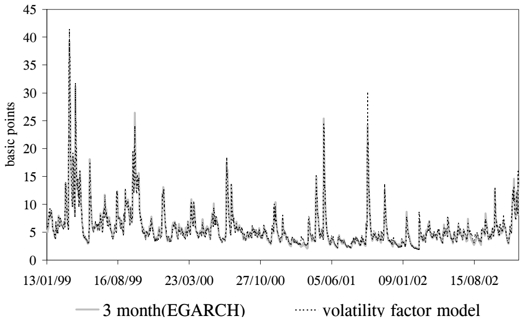


Figure 2(a). Conditional Standard Desviation

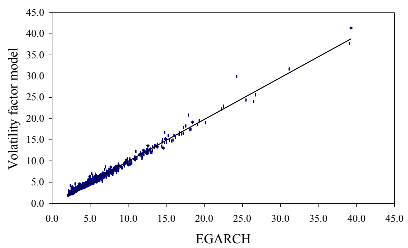


Figure 2(b). 3 month

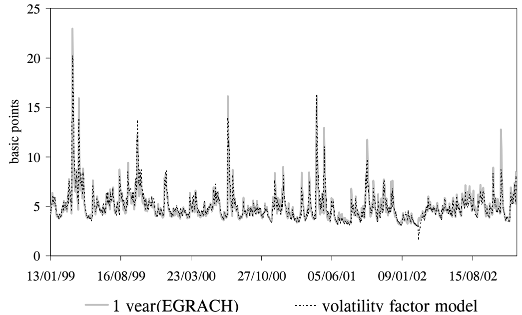


Figure 6(a). Conditional Standard Desviation

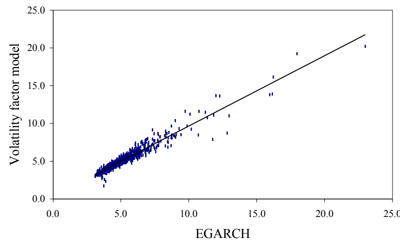


Figure 6(b). 1 year

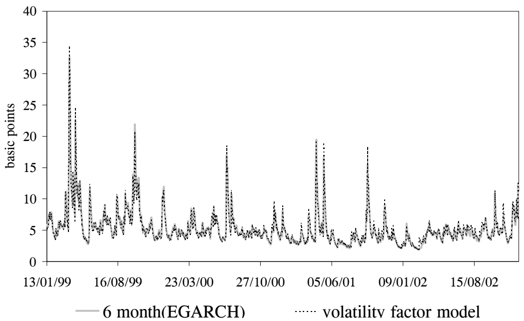


Figure 3(a). Conditional Standard Desviation

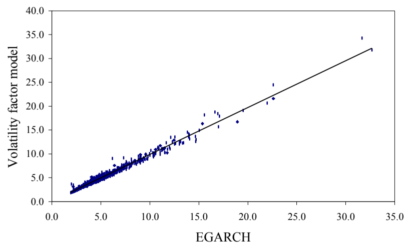


Figure 3(b). 6 meses

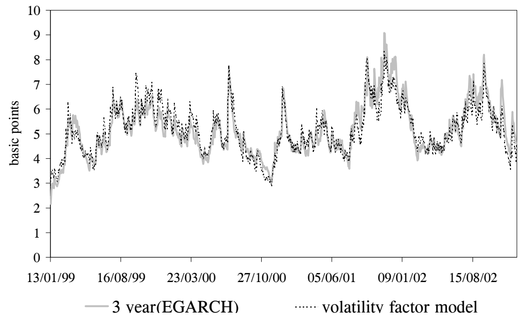


Figure 7(a). Conditional Standard Desviation

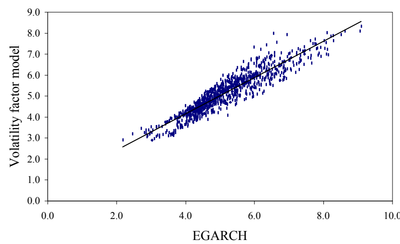


Figure 6(b). 3 años

January 4, 1999 to December 31, 2002. Figures 19 to 24.

Figure 8(a). Conditional Standard Desviation

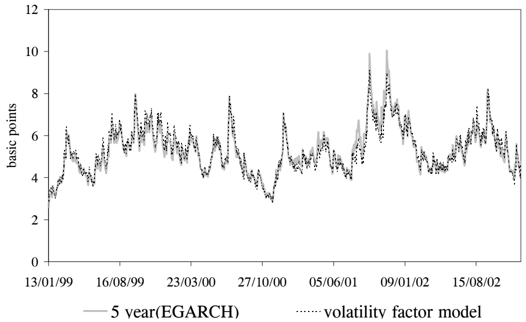


Figure 8(b). 5 year

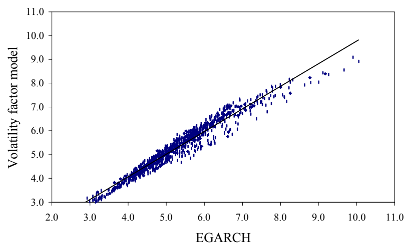


Figure 11(a). Conditional Standard Desviation

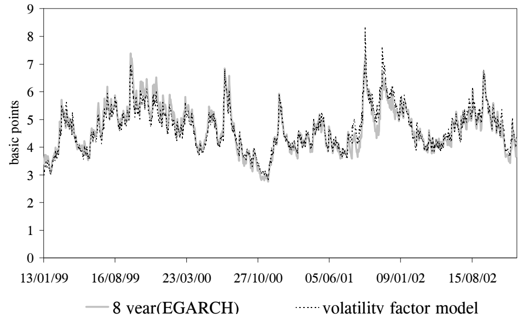


Figure 11(b). 8 year

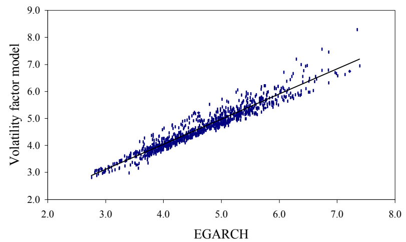


Figure 9(a). Conditional Standard Desviation

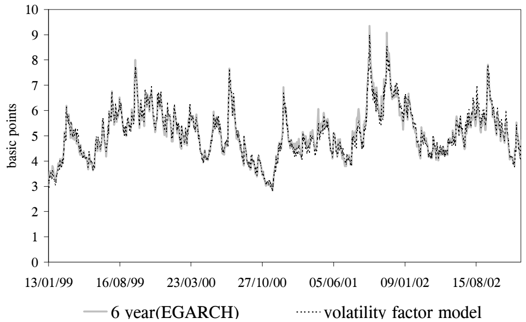


Figure 9(b). 6 year

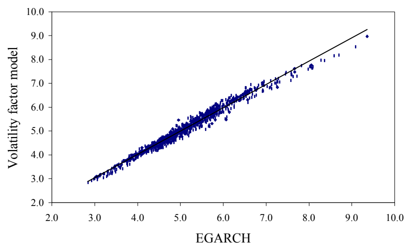


Figure 12(a). Conditional Standard Desviation

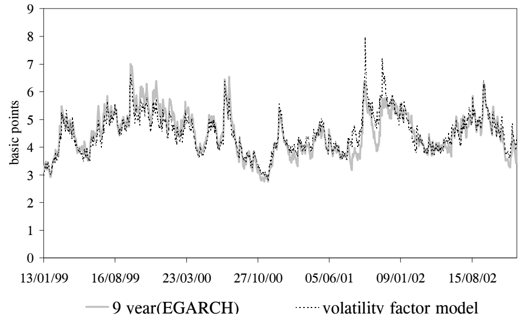


Figure 12(b). 9 year

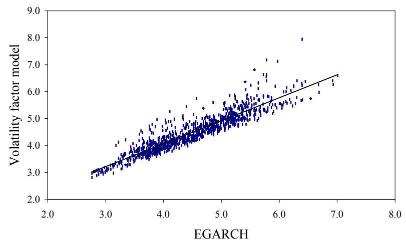


Figure 10(a). Conditional Standard Desviation

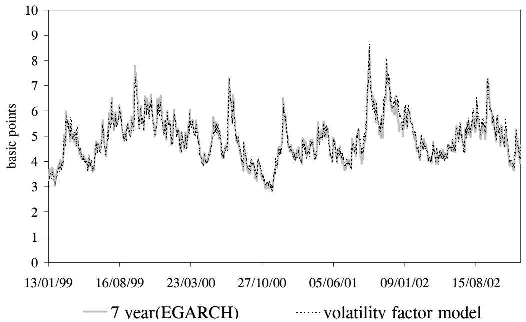


Figure 10(b). 7 year

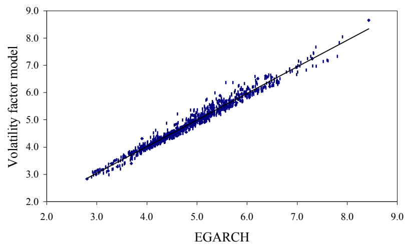


Figure 13(a). Conditional Standard Desviation

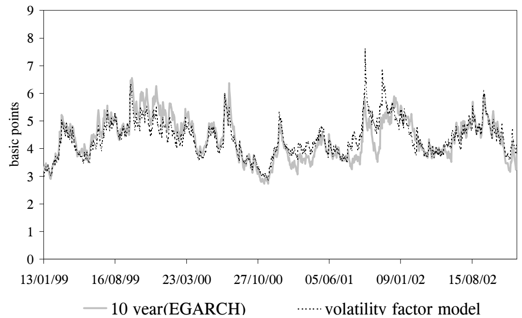
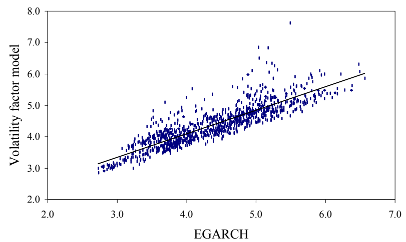


Figure 12(b). 10 year



September 1, 1995 to december 31, 1998. Figures 25 to 30.

Figure 25(a). Conditional Standard Desviation

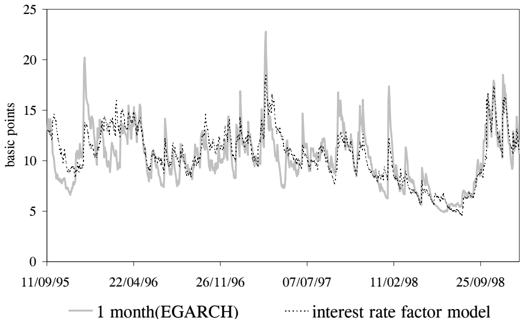


Figura 25(b). 1 month

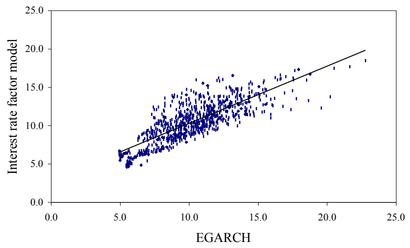


Figure 28(a). Conditional Standard Desviation

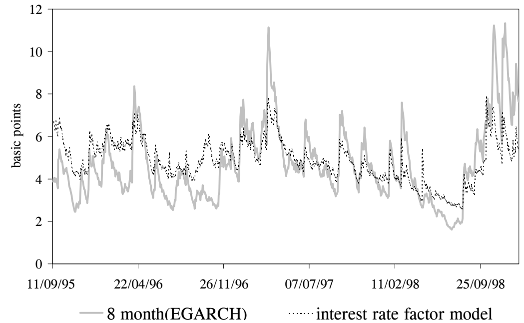


Figure 28(b). 8 meses

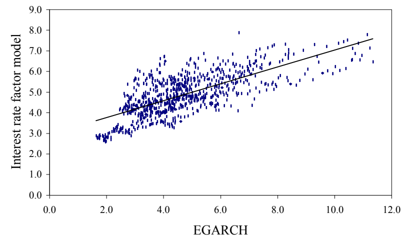


Figure 26(a). Conditional Standard Desviation

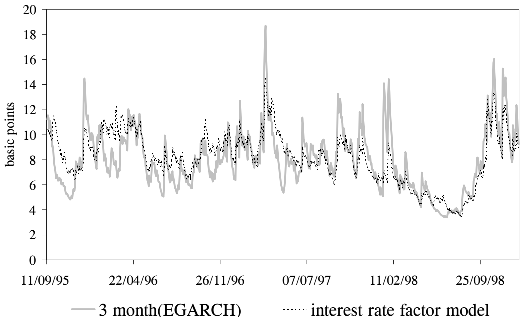


Figure 26(b). 3 month

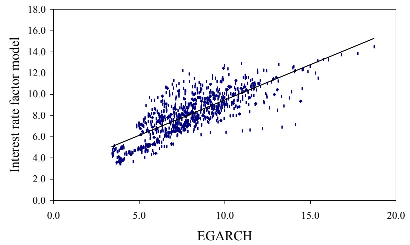


Figure 29(a). Conditional Standard Desviation

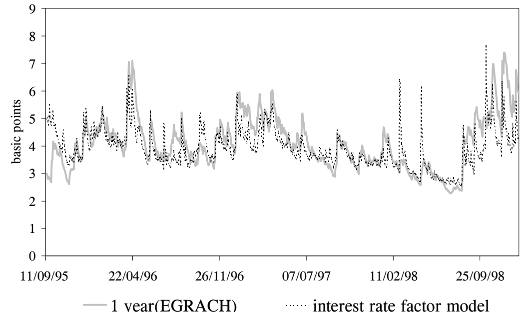


Figure 29(b). 1 year

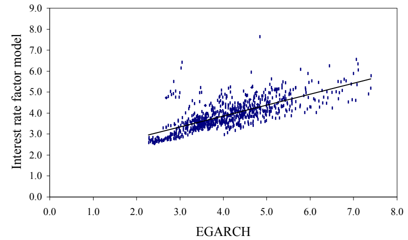


Figure 27(a). Conditional Standard Desviation

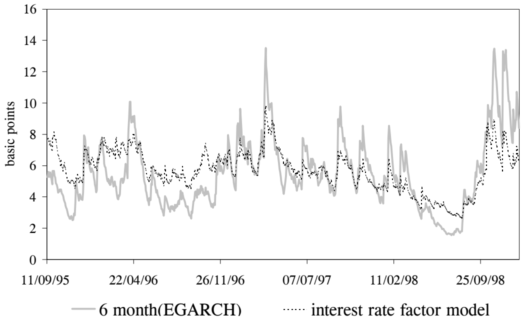


Figure 27(b). 6 meses

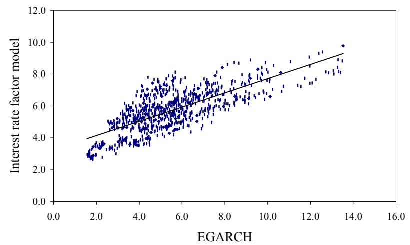


Figure 30(a). Conditional Standard Desviation

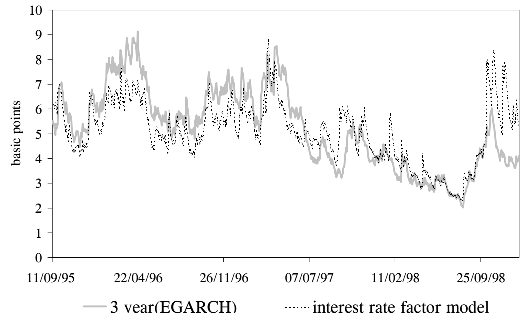
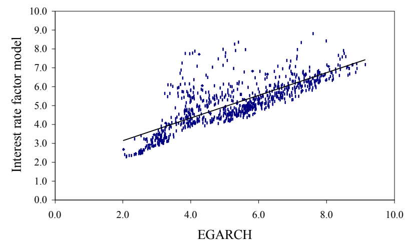


Figure 30(b). 3 años



September 1, 1995 to december 31, 1998. Figures 31 to 36.

Figure 31(a). Conditional Standard Desviation

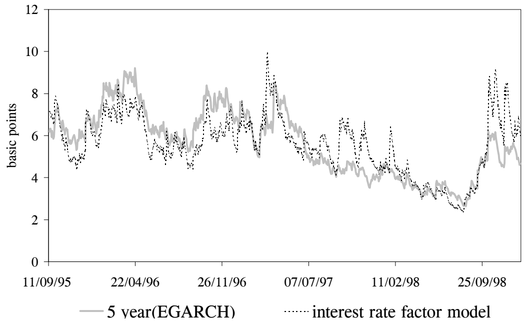


Figure 31(b). 5 year

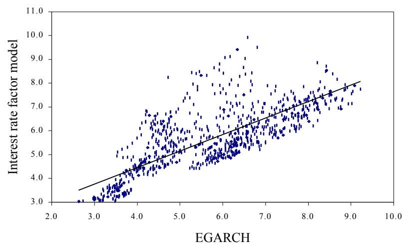


Figure 32(a). Conditional Standard Desviation

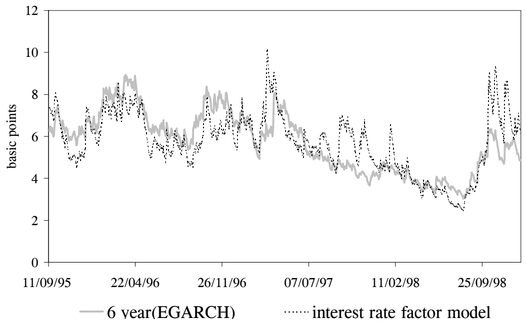


Figure 32(b). 6 year

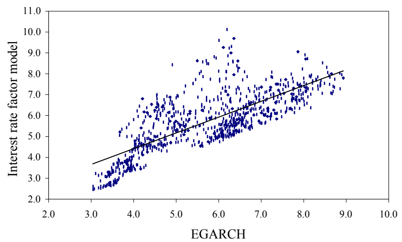


Figure 33(a). Conditional Standard Desviation

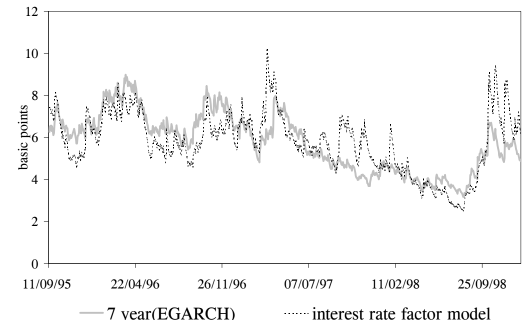


Figure 33(b). 7 year

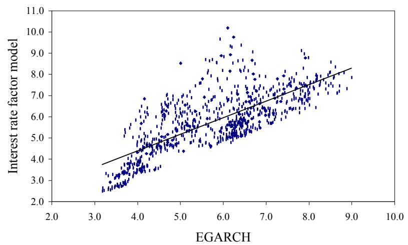


Figure 34(a). Conditional Standard Desviation

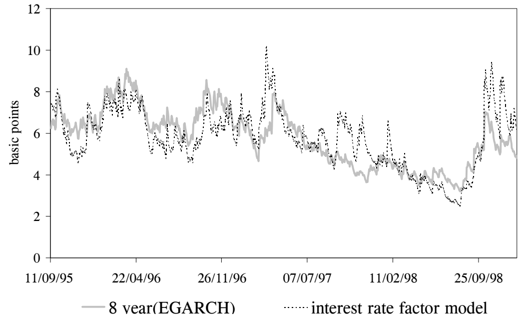


Figure 34(b). 8 year

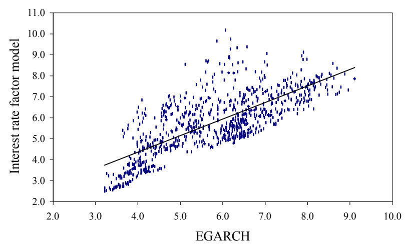


Figure 35(a). Conditional Standard Desviation

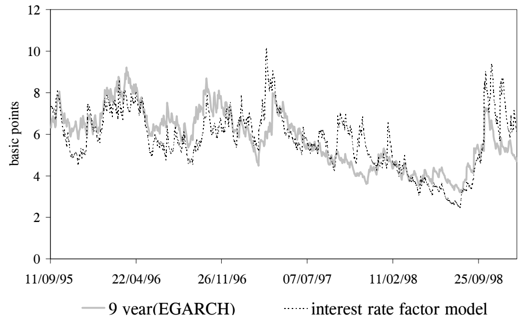


Figure 35(b). 9 year

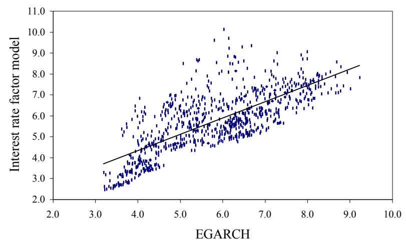


Figure 36(a). Conditional Standard Desviation

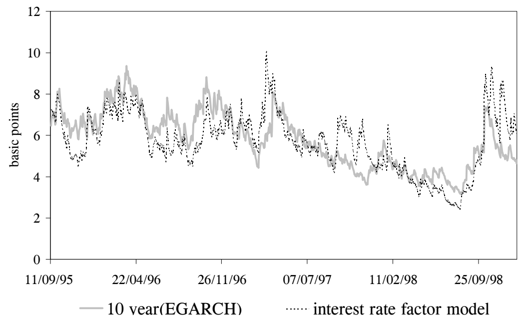
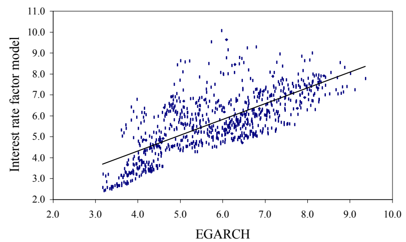


Figure 36(b). 10 year



January 4, 1999 to December 31, 2002. Figures 37 to 42.

Figure 37(a). Conditional Standard Desviation

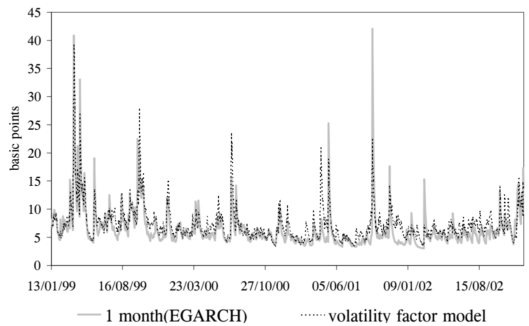


Figura 37(b). 1 month

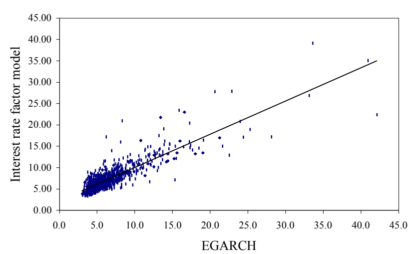


Figure 38(a). Conditional Standard Desviation

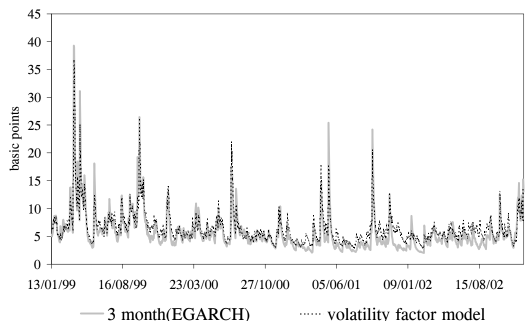


Figure 38(b). 3 month

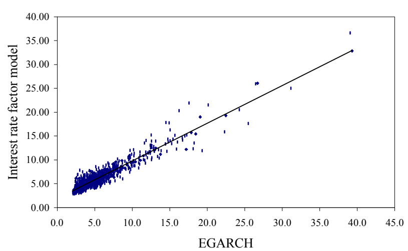


Figure 39(a). Conditional Standard Desviation

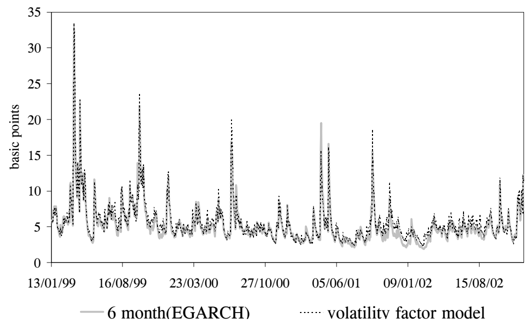


Figure 39(b). 6 meses

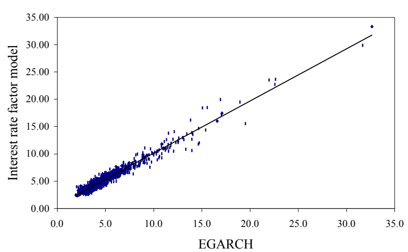


Figure 40(a). Conditional Standard Desviation

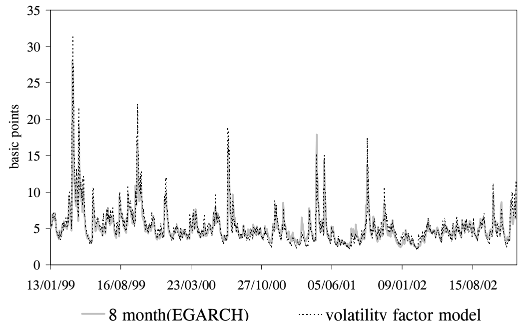


Figure 40(b). 8 meses

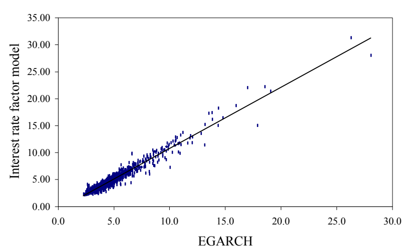


Figure 41(a). Conditional Standard Desviation

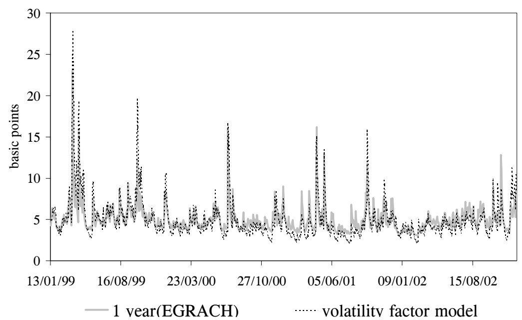


Figure 41(b). 1 year

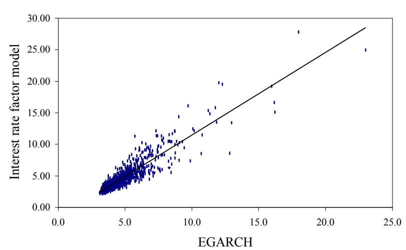


Figure 42(a). Conditional Standard Desviation

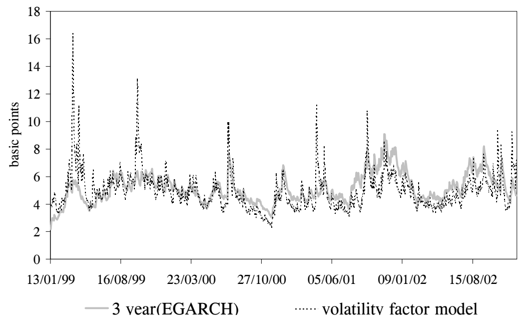
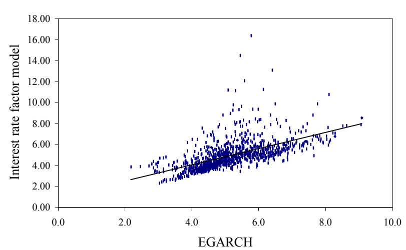


Figure 42(b). 3 años



January 4, 1999 to December 31, 2002. Figures 43 to 48.

Figure 43(a). Conditional Standard Desviation

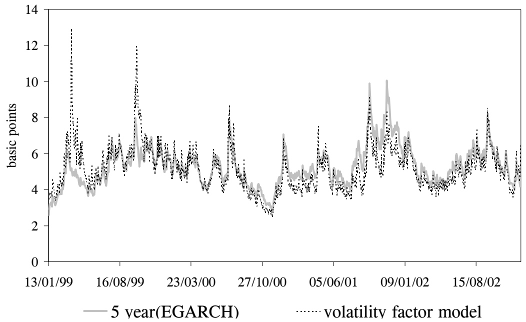


Figure 43(b). 5 year

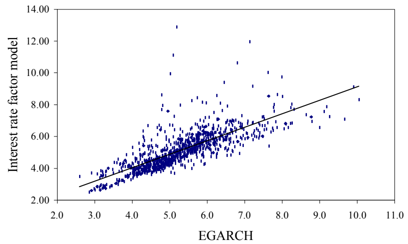


Figure 44(a). Conditional Standard Desviation

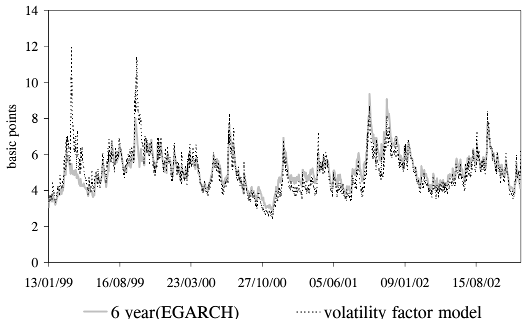


Figure 44(b). 6 year

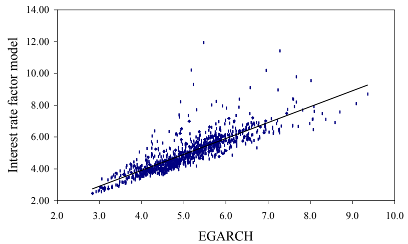


Figure 45(a). Conditional Standard Desviation

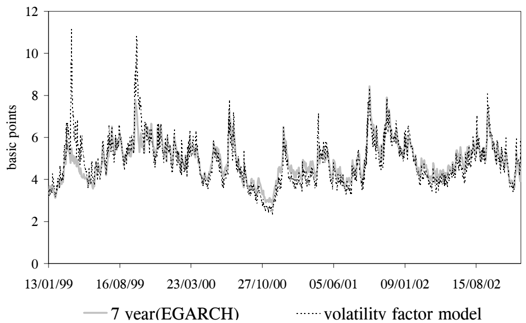


Figure 45(b). 7 year

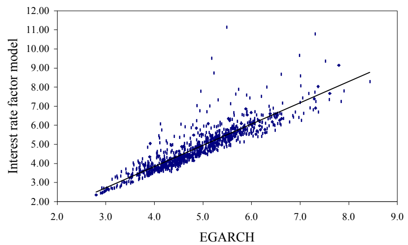


Figure 46(a). Conditional Standard Desviation

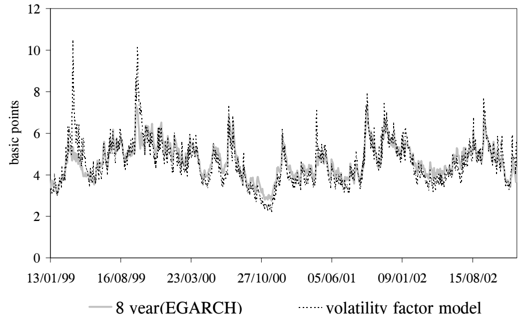


Figure 46(b). 8 year

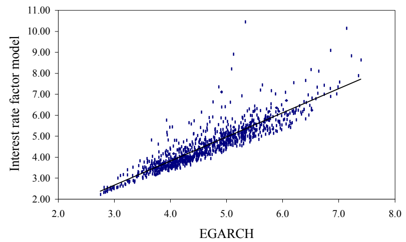


Figure 47(a). Conditional Standard Desviation

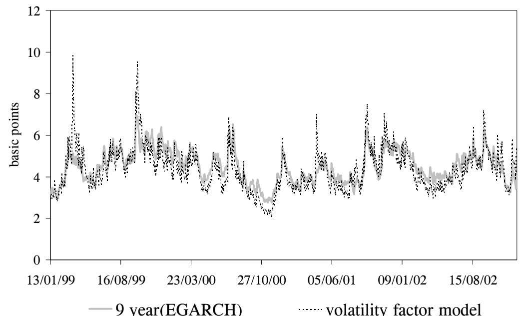


Figure 47(b). 9 year

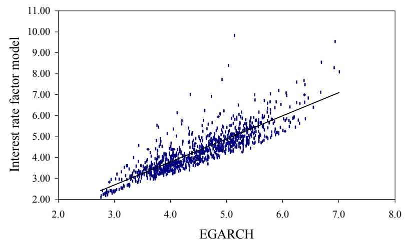


Figure 48(a). Conditional Standard Desviation

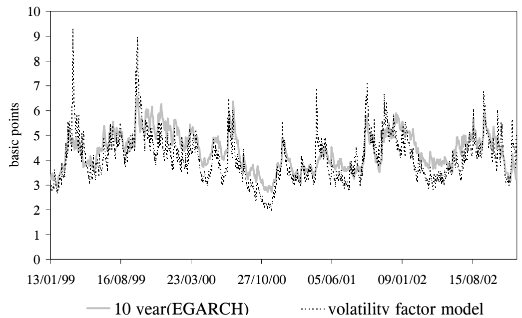


Figure 48(b). 10 year