Path multimodality in a Feedforward SNN module, using LIF with Latency model

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Abstract–In this paper, simulation results of a feedforward SNN affected by synchronous stimuli are discussed. The network transmission properties are investigated in function of both neuron and network parameters; in particular, the connection probability and the synaptic strengths are considered. By means of an event-driven method, all simulations are conducted using the LIF with Latency (LIFL) model. Typical cases are discussed, in which a network section (module) is able to process the input information, introducing a particular behavior, called path multimodality. Through this phenomenon, the output layer of the network can generate a number of temporally spaced groups of synchronous spikes. The multimodality feature can be applied for various purposes, for instance in coding or else transmission issues.

Keywords: feedforward SNN; LIF with latency model; synchrony; firing rate; path multimodality.

INTRODUCTION

Spiking Neural Networks (SNNs) represent a class of biological inspired neural networks that are widely implemented in many research fields, such as engineering, neurophysiology, neuroscience. These kind of networks seem very useful, in particular, for the elaboration of both sensory and cognitive information [1], [2], [3], [4], [5]. For example, the transmission of the spiking activity through a neural network appears of great interest and many computational models have been proposed to approach this problem, such as feedforward models with convergent-divergent connections [6], [7], [8], [9], [10], [11], [12], [13]. These studies involve *rate codes* or else *temporal codes* [1], [14], depending on the network activity which can be asynchronous, or else synchronous, respectively. This difference is due to the necessity of saving the information in subsequent stages of a sensory pathway. In these studies, Leaky Integrate-and-Fire (LIF) models are typically used. Their low computational cost allows investigating the properties of large spiking networks, maintaining some important biological neuronal features (i.e., subthreshold decay, synaptic integration and spike threshold) [15], [16].

The information propagation depends on the network parameters. In fact, it was shown that feedforward networks with low probability of shared connections and strong synaptic strengths permit a faithful transmission of asynchronous firing rate. In the opposite case, a transmission of synchrony is allowed [17].

In the paper, simulations results of a feedforward SNN affected by synchronous stimuli are discussed. The network transmission properties are investigated in function of the connection probabilities and the synaptic strengths, in particular, in terms of the neuron parameters and the network features. Furthermore, all simulations are conducted on a *LIF with Latency* (LIFL) model, simulated through an event-driven method. The latency phenomenon introduces a nonlinearity in the suprathreshold neuron behavior, allowing the information propagation in a *multimodal* manner through consecutive layers of a feedforward SNN. It will be shown how the latency property affects synchronous inputs to a neuronal layer, and causes temporal clusters for the spike generated by the layer itself. Then, the typical behaviors that occur under certain hypothesis will be classified, and will be explained how the temporal distance of the neuronal groups depends on the network parameters. With the aim to distinguish this kind of multimodality from those already defined in literature [18], [19], this property will hereinafter be called *path multimodality*. Taking into account that synchronous spikes are provided to the network input, the path multimodality represent the capability of a network section (*module*) of generating in the output network a number of temporally spaced groups of synchronous spikes.

NEURON MODEL, LATENCY EFFECT, PATH MULTIMODALITY

A. Neuron Model

In this work, a simple spiking neuron model with features similar to those of the classic LIF is introduced [15]. The main difference from the LIF model is the presence of an expression, called *firing equation* (1), which qualitatively describes the neuron behavior in the suprathreshold region: when the membrane potential S reaches the spike threshold, the firing is not instantaneous, but it occurs after a variable continuous time delay t_f, called latency [20].

$$t_f = 1 / (S - 1) \quad (1)$$

In (1), the relationship between the time delay t_f and the membrane potential S is a branch of rectangular hyperbola (Fig. 1) [21]. This is an approximation of the behavior of the real biological phenomenon and it has been obtained by simulating a patch of neuronal membrane stimulated with short current pulses, solving the Hodgkin-Huxley equations [22] by means of the NEURON simulator [23].

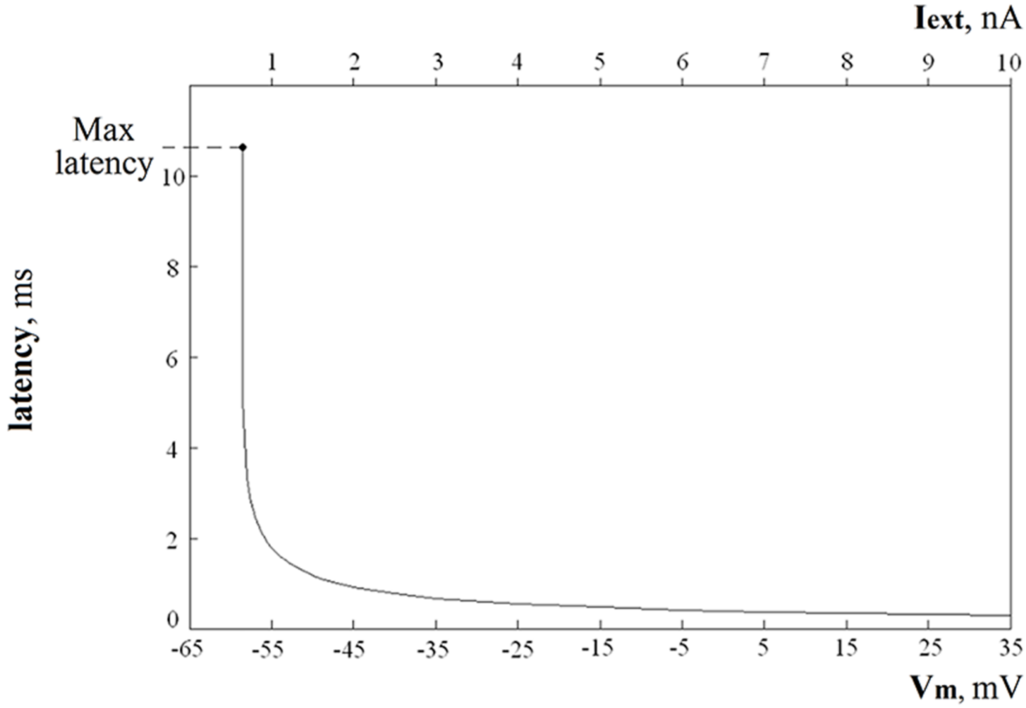

Figure 1. Latency as a function of the membrane potential (or else of the current amplitude *Iext* , equivalently).

In this model, normalized real and non-negative quantities are considered. Thus, the variable S indicates the inner state of the neuron, and the variable t_f, called *time-to-fire*, can be linked to the spike latency. In addition, the *firing threshold* $S_0 = 1+d$ is introduced, where d denotes the *threshold constant*. The latter quantity has been chosen in order to have a finite maximum value for the time-to-fire, $t_{f,max} = 1/d$. When $S = 0$ the neuron is in its resting state. Taking into account that each neuron receives synaptic inputs from a large number of synapses (synaptic convergence) and propagates spikes to a high number of neurons (synaptic divergence), the quantity P_r, *presynaptic weight*, denotes the signal transmitted (i.e., single pulses) from one neuron to a number of the other neurons. Finally, the quantity P_w, *postsynaptic weight*, is associated to each connection, indicating the strength between a couple of neurons. For the sake of convenience, the weights are chosen to be constant, then not subjected to synaptic plasticity [24], and arbitrarily assigned between 0 and a maximum value. Moreover, if P_w is equal to 0, the related connection is not present. The following equations describe the rules for the state updating, in passive mode (*subthreshold region*) and in active mode (*suprathreshold region*), respectively (Fig. 2).

$$S = S_a + P_r P_w - Ld \cdot \Delta t, \qquad \text{for} \qquad S < S_0 \quad (2)$$

$$S = S_a + P_r P_w, \qquad \text{for} \qquad S \geq S_0 \quad (3)$$

In (2) e (3), S_a represents the previous state, Ld the subthreshold linear decay, and Δt is the temporal difference between two consecutive incoming spikes. When $S \geq S_0$ (suprathreshold region), the neuron becomes active and it is ready to fire, but it remains still sensitive to incoming spikes. In this region, for each new state the bijective relationship (1) is evaluated. Thus, as times advances and new spikes arrive, the state S is properly increased (decreased), due to the excitatory (inhibitory) effect, and t_f is evaluated [25]. When the last computed t_f expires, a spike is generated, the inner state becomes $S = 0$ (reset) and the neuron remains insensitive for a time called the *absolute refractory period*, like in the case of biological neurons.

Due to the necessity of simulating the latency effect, the use of an event-driven approach for the simulation of the spiking neural network is required. Moreover, this simulation technique let to investigate large network properties with high precision, requiring a low computational cost [26], [27], [28].

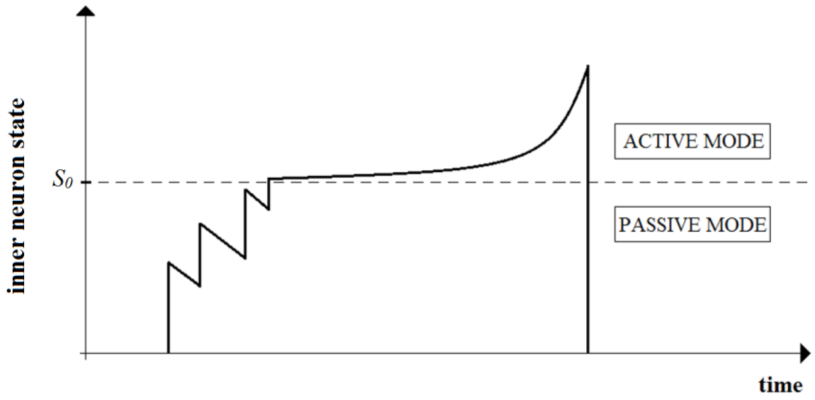

Figure 2. Passive and Active Mode. S_0 indicates the Firing Threshold.

B. Latency Effect on the behavior of a FFN module

Defining a *module* as a network section that is able to introduce particular delays, here it is considered a feedforward module without axonal delays and affected by a synchronous vertical input train (Fig. 3). At each simulation, the incoming spikes have the same amplitude (i.e., P_r is constant, P_w is fixed by (4)). Thanks to the presence of the latency in the neuron model, the information through the network can be transmitted in an asynchronous manner to the output (Figure 4). The particular behavior of the transmission mainly depends on both postsynaptic weights and connection probability. As an example, in the case of a sparsely connected network with high postsynaptic weights values, an asynchronous pulse train is generated (Figure 3).

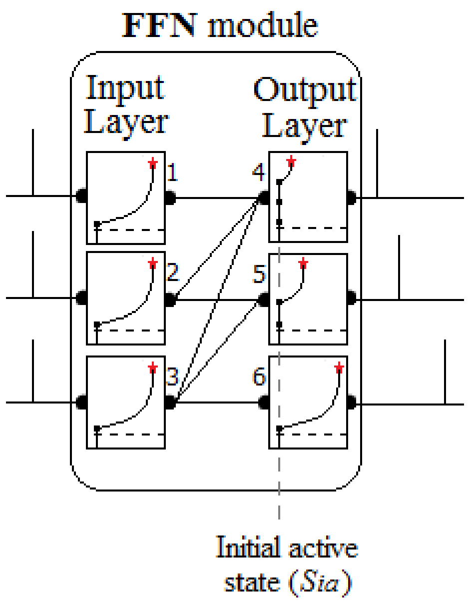

Figure 3. An example of a Feedforward Module Topology (FMT) composed by 6 LIFL neurons. It is shown a case of sparsely connected network; it is possible to evaluate the *initial active state* (S_{ia}) reached by each output neuron, and then the related time to fire.

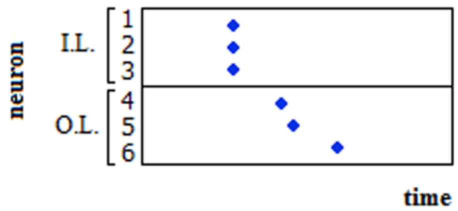

Figure 4. Raster plot of a simulation made using the module shown in Fig. 3. It is reported the temporal behavior of the FMT. On the vertical axis the neuron number is denoted (I.L. stands for Input Layer, whereas, O.L. stands for Output Layer). On the horizontal axis the spike timings are indicated, expressed as normalized unit of time.

In the present work, a double layer feedforward network composed by 50 neurons per layer is used to carry out considerations about the multimodality phenomenon (Fig. 5). To this purpose, the connections characterization, between the two layers, is realized by means of the two parameters P_w and *CF*. The simulation procedure is shown in the *Simulation Results* section.

C. Path Multimodality

As mentioned in the introduction, the path multimodality is the capability of a network path (a feedforward module, in this particular study) of generating in the output more groups of synchronous spikes temporally spaced, when it is stimulated by a synchronous input train. If the number of neurons in a FMT grows up, the phenomenon described above is more evident. Indeed, under particular conditions, this behavior is replicated, so that, in the output of a module, some temporal groups composed by synchronous spikes can be obtained. Depending on the network parameters, these spike timing groups intervals can present regular or else irregular distributions. The purpose of the present work is to investigate this phenomenon and the behaviors that could be obtained varying some network parameters.

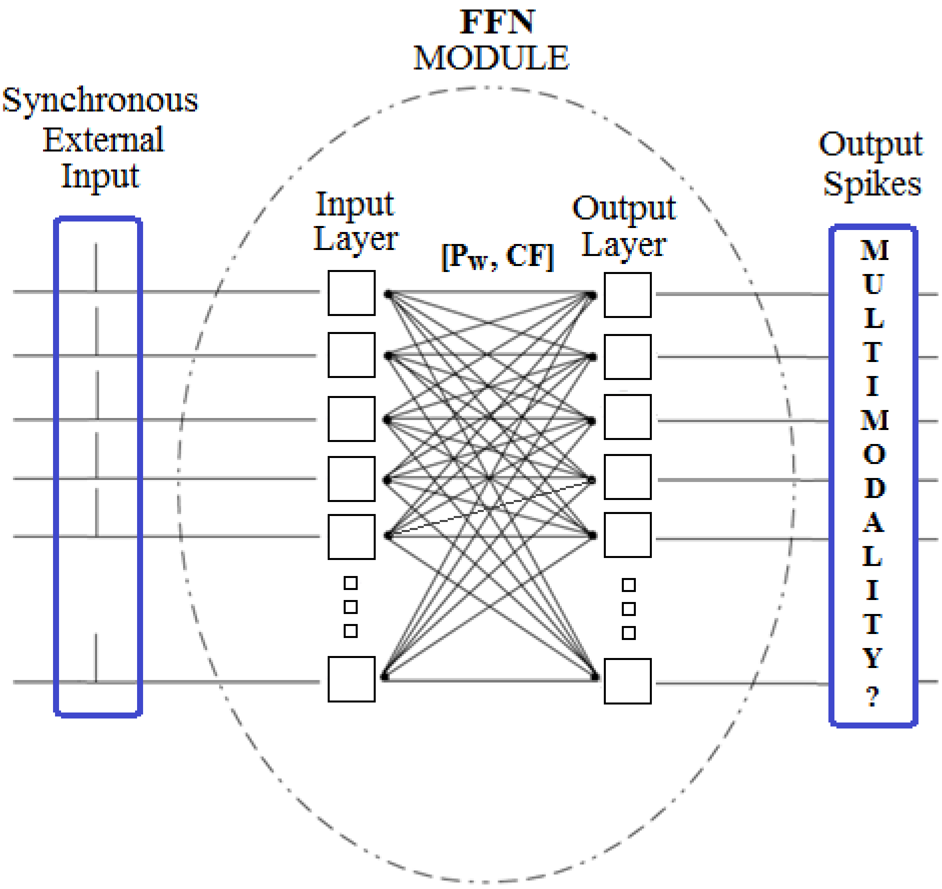

Figure 5. Input: external sources that generate synchronous spikes. Input Layer and Output Layer represent the feedforward network module; the connections between the layers could be both fully and sparsely connected. The output behavior is analyzed and the kind of multimodality is classified.

In order to reduce the freedom degrees in the simulation process, constant and identical P_w values are assigned for the connections between the two layers, as shown in the next section, so that most of the output spikes are generated.

In this way, defining the neuron *fan-in* as the number of the incoming connections, when the module receives only synchronous inputs, the output spike timing distribution is uniquely related to the fan-in configuration of the output layer neurons.

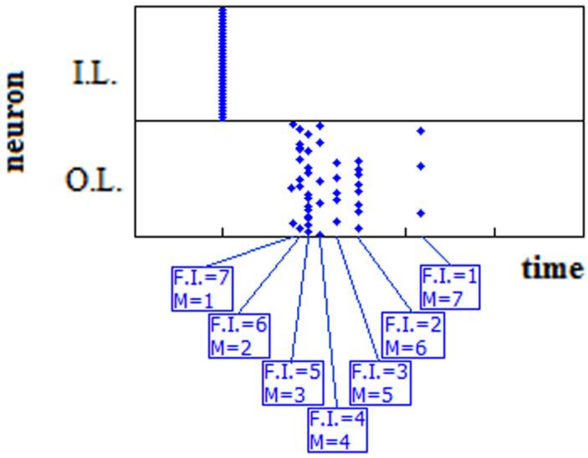

Figure 6. Raster plot of a typical simulation. The seven spike timings groups emerged after the stimulation are indicated with the related *fan-in* (F.I.) and the corresponding *mode* (M).

On the basis of the above described conditions, the number of the possible modes of a FMT is equal to the number of the different fan-in values concerning the output layer (Figure 6).
Taking into account the above described network conditions and neuron features, it seems quite easy to imagine how a neuron model, that does not implement the latency phenomenon, does not allow generating different spike timings intervals through the network. Then, the introduction of the latency in the model allows generating output spikes with intervals related to the network parameters.
In general, depending to which input layer neuron is excited, it is easy to understand how the output can be characterized by some *modes* rather than other ones. In addition, the discussion becomes more complex if a real scenario is considered, in which other conditions could be present: asynchronous input spikes, variable input spike amplitude, variable connection weights P_w and axonal delays. In future works, these conditions will be introduced in order to extend the present discussion.

SIMULATION RESULTS

In this section, in order to study the behavior of the network under proper conditions, both model and network parameters are introduced. Also, the simulation results are presented. There are two basic parameters: postsynaptic weight and connection factor. The fist one is defined by the following relationship:

$$P_w = S_0/(N \cdot R) \qquad (4)$$

Where S_0 represent the firing threshold (conventionally fixed to 1.04), N is the number of the active connections and R is the neuronal threshold ratio. The latter can be expressed by the relationship:

$$R = S_0/S_{ia} \qquad (5)$$

in which S_{ia} is the initial active state (as indicated in Figure 3) reached by an output neuron, thanks to the overall incoming spikes. The values of R have been chosen in the range [0.1÷0.9] with step equal to 0.1. The expressions (4)–(5) have been obtained on the basis of similar relationships discussed in Burkitt and Clark (1999).
In addition, the connection factor is defined as:

$$CF = N/N_{tot} \qquad (6)$$

As mentioned above, N is the number of the active connections and N_{tot} represents the total number of the connections. In the case of fully connected network, CF is equal to 1. Note that, in order to obtain values of CF in the range [0.1÷1], the connections are chosen in a pseudorandom manner. The results, shown in the following raster plots, have been obtained by simulating a network composed of 2 layers: the first one, input layer, provides synchronized inputs. The second one, output layer, is connected to the input layer with connections having a certain connection probability. Therefore, the network can be sparse (i.e., $0 < CF < 1$) or else fully (i.e., $CF = 1$) connected.
The simulations have been performed by varying P_w (i.e., R and N) and CF (i.e., N).

On the basis of the nonlinearity of the latency curve (Figure 1), it can be possible to notice substantially different output spike timing distribution behaviors. Indeed, depending on the portion of the curve affected by the combination P_w-CF, they can be essentially grouped in three typical trends:

- *Nonlinear behavior*. It is characterized by an irregular increasing of the spike timings distance between the synchronous groups, due to the fan-in decreasing of the target neuron (i.e., the decreasing of the number of the incoming connections). In this case, the spike timing groups are irregularly distributed.

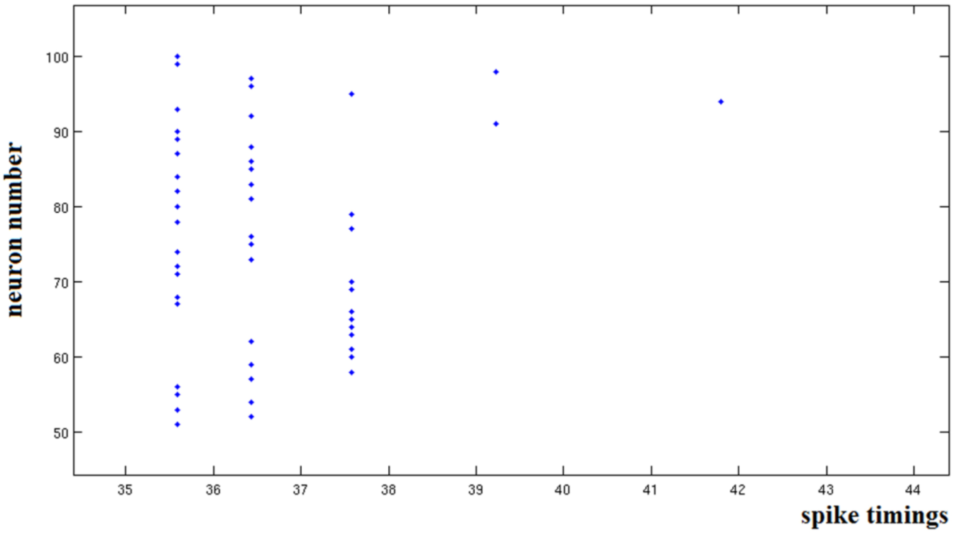

Figure 7. Raster plot that shows *nonlinear* behavior. In this simulation have been noticed fan-in is equal to 50, 49, 48, 47, 46 for the output neurons.

- *Linear behavior*. It is characterized by similar intervals among synchronous groups. In this case, the spike timing groups are quite regularly distributed.

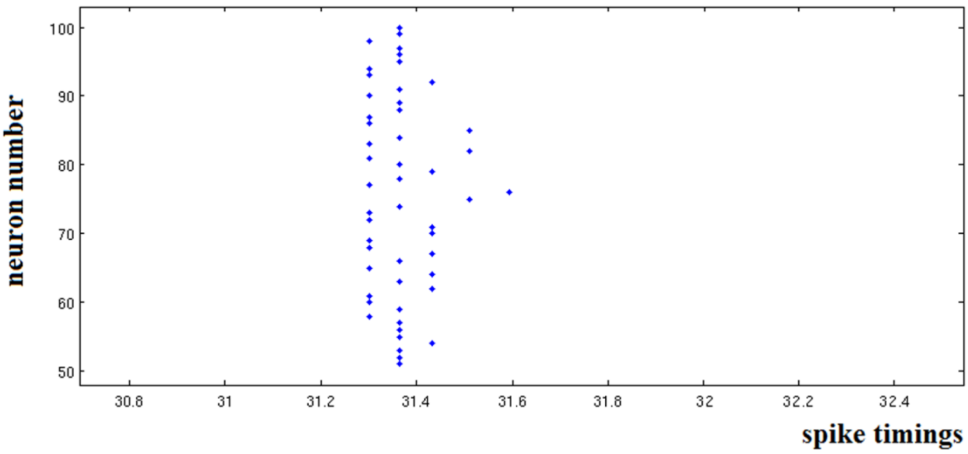

Figure 8. Raster plot that shows *linear* behavior. In this simulation have been noticed fan-in is equal to 50, 49, 48, 47, 46 for the output neurons.

- *Perfect-Synchrony behavior*. It is characterized by a single spike timing group. This emerges only in the case of CF = 1.

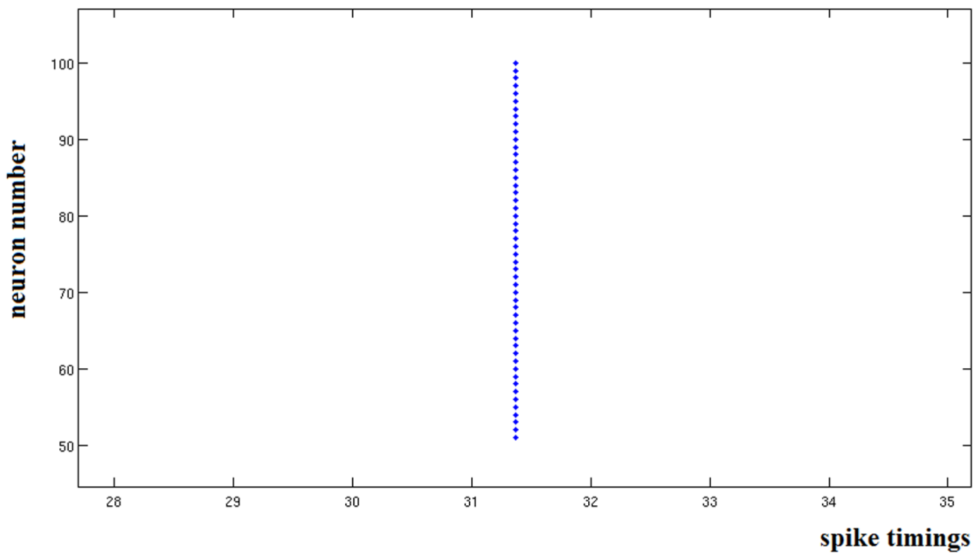

Figure 9. Raster plot that shows *perfect-synchrony* behavior. The neurons fan-in is equal to 50 for each neuron.

In all the cases, thanks to the latency effect, it is possible to identify several activated synchronous groups. Depending on the neuronal and network parameters, these groups can be temporally distributed in linear, or nonlinear manner. Then, depending on the network parameters, it is possible to generate pulse trains with both irregular or else regular spike timing group distribution, even in the case of synchronous input spikes. Note that, only in the case of $CF = 1$, the firings of the output layer are generated in a perfect synchrony manner. Furthermore, more the weights and *CF* are high (i.e., high P_w and low R values), more the spike timing groups appear close in time.
Finally, for *CF* increasing, the number of synchronous groups will decrease, up to the limit value of $CF = 1$. In this case, only a synchronous group is present (as shown in Figure 9).
The behaviors above illustrated are reported in Table I, in which each couple of *CF*-*R* values represents the mean over 100 simulations.
Note that, for each simulation result the following procedure is applied:

- Determination of the temporal distances among the spike groups (i.e., Δ).
- Determination of consecutive Δ ratios (i.e., Δ_{n+1}/Δ_n with $n \neq 0$).
- Calculation of the mean over all the ratios (labeled with "*rm*").
- Define a maximum error allowed (*toll*).
- If and only if $\Delta \neq 0$ then:
  - If $(1 - toll) < rm < (1 + toll)$, then the behavior is classified as Linear (L).
  - Else, the behavior is classified as Nonlinear (NL).
- If $\Delta = 0$, then the behavior is classified as Perfect-Synchrony (PS).

The following table summarizes the behaviors obtained varying both network and neuron parameters, with $toll = 0.2$:

R\CF	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	NL	NL	NL/L	NL/L	NL/L	NL/L	L	L	L	PS
0.2	NL	NL	NL/L	NL/L	NL/L	NL/L	L	L	L	PS
0.3	NL	NL	NL	NL/L	NL/L	NL/L	NL/L	L	L	PS
0.4	NL	NL	NL	NL/L	NL/L	NL/L	NL/L	NL/L	L	PS
0.5	NL	NL	NL	NL	NL	NL/L	NL/L	NL/L	NL/L	PS
0.6	NL	NL	NL	NL	NL	NL	NL/L	NL/L	NL/L	PS
0.7	NL	NL	NL	NL	NL	NL	NL	NL/L	NL/L	PS
0.8	NL	NL	NL	NL	NL	NL	NL	NL	NL	PS
0.9	NL	NL	NL	NL	NL	NL	NL	NL	NL	PS

Table I. Network behaviors obtained varying the parameters *R* (neuron parameter, see (5)) and *CF* (i.e., *N* representing a network parameter). NL: nonlinear; L: linear; PS: perfect-synchrony.

As mentioned above, each item in Table I summarizes the behaviors obtained with a set of 100 simulations, so that the case of mixed behaviors has been pointed out. As it can be observed from the previous figures, the behaviors in terms of *multimodality* of the module reflect the latency curve portion (see Figure 1) of the output layer neurons involved in the generation of the output spikes.

CONCLUSION

In this paper, the case of feedforward spiking neural networks stimulated by synchronous inputs is analyzed. Under proper working conditions, the chance of the generation of groups of synchronous spikes is investigated.

In order to obtain quite biological plausible dynamics, a Leaky Integrate-and-Fire with Latency model is considered. On the basis of the nonlinearity introduction in the neuron model, new kind of behaviors can emerge in a simple spiking FNN module, in which the generation of some synchronous spike timing groups is possible.

The implementation of a simple module is due to the necessity of analyzing the transmission of the information through the module itself, avoiding the superposition of other effects. As mentioned in the introduction, there are conditions that allow asynchronous propagation and other conditions that allow synchronous propagation. In this work, we focused our study on the simple case of synchronous inputs, showing how the latency phenomenon can affect the behavior of the network for the generation of the output spike sequence. For this purpose, typical cases are investigated by varying the parameters of the model, showing how a network section is able to process the input information, highlighting a particular phenomenon, called *path multimodality*. Depending on the distribution of spike timing groups, different trends have been classified: nonlinear, linear, and perfect-synchrony behaviors. Each case has been related to combinations of both network and neuron parameters, as shown in the summarizing table. Note that, the response time behavior of the neuron depends basically on the amount of stimulation (see equations (1), (2), (3)), and then on the portion of latency curve affected by the stimuli. Finally, a mapping of the behaviors varying the model parameters has been provided.

Moreover, taking into account that a feedforward network is composed by several modules, the multimodality can be exploited for several purposes, for instance in coding or else transmission issues, and all possible related applications.

In future works, more plausible scenarios will be considered, in which further conditions can be present, such as asynchronous input spikes, variable input spike amplitude, variable connection weights P_w and axonal delays. Moreover, the study can be enhanced combining the modules in recurrent configurations and finally introducing inhibitory neurons and synaptic plasticity in the process.

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