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Global and local technical changes: A new decomposition of the Malmquist productivity index using virtual units

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ABSTRACT

In estimating productivity change over time, technical change is frequently miscalculated as the geometric average of technological changes between two periods based on firm-specific information in the dataset. However, the frontier shift over time is a global phenomenon linked to relative technological progress or regress across the entire frontiers. In this paper, we fill this gap by determining the technical change using synthetic observations generated at random within a unit hypercube and calculating the distances between them and the two frontiers being evaluated. Accordingly, we propose a decomposition of the Malmquist index's traditional technical change into two components: average global technical change, which is shared by all production units, and local technical change, which captures how each firm experiences global technical change. In this way, our approach establishes a new research avenue in production economics based on using randomly generated virtual points to assess overall phenomena.

1. Introduction

The Malmquist Productivity Index (MI) is a well-known measure of productivity change used extensively in economics, operational research, and management science. Malmquist (1953) first proposed the concept, which was later formalized and popularized by Caves et al. (1982). Later, Färe et al. (1992) showed how to implement and decompose MI using data envelopment analysis (DEA). This decomposition consists of the catching-up effect (efficiency change (EC)) and the frontier-shift effect (technical change (TC)). The MI has a significant advantage over other classical alternatives such as the Törnqvist, Laspeyres, Paasche, and Fisher indices in that it is a nonprice dependent approach to determining the productivity change of a set of decision-making units (DMUs) over time. This property fits well to the benchmarking of public services, and for this reason, the MI has been extensively applied in sectors like education (De Witte and López-Torres, 2017; Arbona et al., 2022), health (Hollingsworth, 2008; Pourmahmoud and Bagheri, 2023), energy (Huang et al., 2017), microfinance industry (Soltane Bassem, 2014), the judicial system (Falavigna et al., 2018; Giacalone et al., 2020), or long-term care (Salinas-Jiménez et al., 2003; Ozbugday et al., 2020). Other recent applications of the MI include green productivity change (Ge et al., 2023; Zhou et al., 2023), the digital economy (Guo et al., 2023), primary stakeholder management (Ben Lahouel et al., 2022), banks (Alexakis et al., 2019), and productivity growth in cities (Wei et al., 2020).

The MI decomposition assumes that a productivity change can be caused by both an EC and a TC. For example, a positive (negative) EC allows us to quantify the portion of productivity change caused by technical efficiency improvements (deterioration). Additionally, the TC component captures the extent to which the production frontier has shifted over time due to technological advancements or deteriorations. A positive TC indicates that the production frontier has shifted outwards (i.e., in the output orientation, more output can be produced with the same inputs), whereas a negative TC indicates that the production

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frontier has shifted inwards. This information is useful for policymakers and firm managers who want to identify the specific factors driving productivity growth to develop strategies for capitalizing on additional improvements.

According to this framework, the "business as usual" premise for calculating the overall TC between two periods in the sector under study is to average the individual TC scores of all DMUs included in the empirical sample. Practitioners use this overall measure to summarize the industry's ability to shift the production technology. However, some authors argue that the frontier shift over time should be considered as a global phenomenon affecting the entire technologies in the two periods for the corresponding sector, rather than the average TCs of the specific DMUs under evaluation (Balk and Althin, 1996; Asmild and Tam, 2007; Otsuki, 2013).

Nevertheless, TC has generally been miscalculated in the literature as the geometric average of technology changes between two periods concerning firm-specific information in the data set, rather than as an industry-wide movement. In this paper, we fill a gap in the literature by calculating a measure of global technical change (GTC) using thousands of synthetic observations generated at random in the input–output space and evaluating the distances between them and the two empirical frontiers under consideration. This allows us to scan the technological improvements and deteriorations across the entire frontiers rather than focusing solely on the observations in our sample.

To accomplish this, we propose a numerical analysis randomly generating a grid of virtual DMUs within a unit hypercube containing a transformed version of the original data cloud, with all dimensions, inputs, and outputs previously normalized between zero and one. Furthermore, using this new information on the differences between the two frontiers in the two evaluation periods, we propose for the first time a decomposition of the traditional TC into an average GTC, which is shared by all production units, and a local technical change (LTC), which captures how each firm individually experiences GTC. After calculating the distance between each synthetic point and the two frontiers, the GTC measure will be the average distance between the two frontiers calculated across all synthetic virtual units. LTC will be calculated as the ratio of the traditional notion of TC in the MI for a DMU to the GTC.

These two new terms of the traditional decomposition of productivity change in the MI are important for practitioners because (1), in this way, we can obtain more accurate and realistic estimations of overall TC and its distribution in the sector under evaluation, thus avoiding potential misleading results, and (2), it allows insights into how each firm experienced TC over time (over or under the global tendency). Furthermore, our approach opens a new research line in production economics by utilizing randomly generated virtual points to globally evaluate production phenomena.

The paper is organized as follows. Section 2 briefly introduces the MI and provides context for measuring GTC. Section 3 defines the new decomposition of TC into a GTC and LTC, as well as the steps for implementing this methodology in practice, which are illustrated with a simple numerical example. Section 4 applies the new decomposition to a group of 42 Swedish pharmacies that operated from 1980 to 1989, as previously described in Färe et al.'s (1992) seminal paper. Finally, Section 5 summarizes the conclusions and suggests some future research directions.

2. Background

2.1. The malmquist index

In this section, we briefly introduce the MI and the idea of GTC and LTC. Let us assume that we have a panel of $j = 1,\dots,J$ DMUs and, at least, two different time periods, t and $t+1$. In this context, x_{ji}^{τ} denotes the amount of the i-th input, $i = 1,\dots,m$, used by the j-th DMU, in the period τ, $\tau = t, t+1$. In addition, y_{jr}^{τ} denotes the amount of the r-th output, $r = 1,\dots,n$, produced by the j-th DMU. In vector notation, we use (x_j^{τ}, y_j^{τ}) to denote the input–output bundle corresponding to the j-th DMU in period τ. Additionally, we define the technology capable of transform a vector of inputs $x = (x_1,\dots,x_m) \in R_+^m$ into a vector of outputs $y = (y_1,\dots,y_n) \in R_+^n$ as $T^{\tau} = \{(x^{\tau}, y^{\tau}) : x^{\tau} \textit{ can produce } y^{\tau}\}$.

In this setting, it is worth defining the Shephard output distance function (Shephard, 1953) that is widely used in production economics. For the DMU j observed in period s, $s = t, t+1$, the Shephard output distance function with respect to the technology in period τ is defined as $D_O^{\tau}(x_o^s, y_o^s) = \min_{\theta}\{\theta > 0 : (x_o^s, y_o^s/\theta) \in T^{\tau}\}$, which measures the distance of DMU_o from the reference technology T^{τ}. The output distance function assumes that inputs are provided and expands the output vectors as far as possible to achieve the benchmark technology. We use the nonparametric DEA methodology to estimate the technology T^{τ} on a sample of J DMUs. In particular, constant return-to-scale (CRS) technology (Charnes et al., 1978), T^{τ} is estimated in DEA as.

$$T^{\tau} = \left\{ \begin{array}{l} (x^{\tau}, y^{\tau}) \in R_+^{m+n} : y_r^{\tau} \le \sum_{j=1}^{J} \lambda_j y_{jr}^{\tau}, \forall r = 1,\dots,n, \\ x_i^{\tau} \ge \sum_{j=1}^{J} \lambda_j x_{ji}^{\tau}, \forall i = 1,\dots,m, \lambda_j \ge 0, \forall j = 1,\dots,J \end{array} \right\}$$

In DEA, the Shephard output distance function $D^{\tau}(x_o^s, y_o^s)$ is estimated, assuming CRS, as the inverse of the optimal value for φ_o of the following linear programming model (known as the output-oriented radial model in the DEA literature):

$$\begin{aligned} &\left(D^{\tau}\left(x_o^s, y_o^s\right)\right)^{-1} = \max_{\lambda,\varphi_o} \varphi_o \\ &s.t. \quad \sum_{j=1}^{J} \lambda_j x_{ji}^{\tau} \le x_{oi}^s, i = 1,\dots,m \\ &\sum_{j=1}^{J} \lambda_j y_{jr}^{\tau} \ge \varphi_o y_{or}^s, r = 1,\dots,n \\ &\lambda_j \ge 0, j = 1,\dots,J \end{aligned}$$

The output-oriented MI measures the productivity change for a DMU observed in two periods t, $t+1$ as follows:

$$M\left(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}\right) = \left[\frac{D^t\left(x_o^{t+1}, y_o^{t+1}\right)}{D^t\left(x_o^t, y_o^t\right)} \cdot \frac{D^{t+1}\left(x_o^{t+1}, y_o^{t+1}\right)}{D^{t+1}\left(x_o^t, y_o^t\right)}\right]^{1/2}$$

Equation was originally decomposed by Färe et al. (1992) into a EC and a TC as follows:

$$M\left(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}\right) = EC_o^{t,t+1} \cdot TC_o^{t,t+1} = \underbrace{\frac{D^{t+1}\left(x_o^{t+1}, y_o^{t+1}\right)}{D^t\left(x_o^t, y_o^t\right)}}_{Efficiency\ \ Change} \cdot \underbrace{\left[\frac{D^t\left(x_o^t, y_o^t\right)}{D^{t+1}\left(x_o^t, y_o^t\right)} \cdot \frac{D^t\left(x_o^{t+1}, y_o^{t+1}\right)}{D^{t+1}\left(x_o^{t+1}, y_o^{t+1}\right)}\right]^{1/2}}_{Technical\ \ Change}$$

A MI higher (lower) than 1 implies productivity improvements (losses) from period t to period $t+1$. Furthermore, Eq. includes two components. The first ratio shows the change in technical efficiency between the two periods. Therefore, $EC_o^{t,t+1} > 1$ ($EC_o^{t,t+1} < 1$) denotes an efficiency gain (deterioration), whereas EC $= 1$ indicates no changes in technical efficiency. The second measure in square brackets is the TC in, denoted as $TC_o^{t,t+1}$, and is the geometric mean of two TCs: the one experienced by (x_o^t, y_o^t) with respect to technologies in t and $t+1$ and the one experienced in the next period by (x_o^{t+1}, y_o^{t+1}) with respect to technologies in t and $t+1$. Its value is interpreted in the same way to $EC_o^{t,t+1}$, where $TC_o^{t,t+1} > 1$ ($TC_o^{t,t+1} < 1$) now means technical progress (regress).

It is worth underlining that in what follows the terms "technical progress" and "technical regress" should be understood as "relative measures" on how the empirical production frontier defined by the best performers within a sample of DMUs evolves concerning the previous period. In production economics, beginning with Solow (1957), the concept of technical progress (regress) has long been associated with outward (inward) shifts of production frontiers (Tulkens and Vanden Eeckaut, 1995).

However, as Fried et al. (2008) and O'Donnell (2018) state, there is no (absolute) technical regress, understood as the loss of existing techniques, in the belief that the best technologies methods and systems, once known, are not forgotten or burned down and remain available for adoption. However, it is common to observe "relative technical regress" in some real-life sectors, especially in the public sector. This happens, for example, when public infrastructures, such as railways or airports, deteriorate their performance because the intermediate costs are higher while the service provided is the same (Murillo-Melchor, 1999) or when regulation increases salaries without a direct link to productivity (Baumol and Bowen, 1965; Vassdal and Sørensen Holst, 2011). Another example occurs when processes, internal management, or regulation for producing some public services become more complex or cumbersome (Bjurek and Hjalmarsson, 1995) or when the demand falls but the public service needs to maintain their input resources (Mattsson et al., 2018). By analogy, we also measure the "relative technical progress" of our specific sample belonging to a sector within a country, regardless of the existence of a significant technical gap in this technology with respect to the most productive real technology.

2.2. Global technical change

Some authors argue that when decomposing the MI, the shift of the frontier over time should be considered globally, taking into account the entire movement of technology from period t to period $t+1$. According to Balk and Althin (1996), using we have as many measures of TC as DMUs. However, the typical procedure to capturing an industry's TC in a single index is to average it.

Asmild and Tam (2007) propose an interesting and novel approximation to GTC when the MI is calculated over more than two periods to address this issue. In this case, if there are $t = 1, \ldots, T$ periods and a panel of J production units, they propose computing the GTC by calculating the TC for all observed $J \times T$ DMUs in relation to each time period. As a result of this approach, the traditional technical change in calculated using the information of $J \times 2$ DMUs is expanded to $J \times T$ DMUs, yielding a better estimation of the GTC. However, one disadvantage of this approach is sample dependence, which means that adding DMUs or time periods to a data set will alter GTC results for previous periods of time. Furthermore, while using $J \times T$ observations instead of $J \times 2$ reduces the potential uneven distribution of DMU projections to production frontiers, it does not eliminate the issue of incorporating a potential bias when calculating the GTC.

Otsuki (2013) proposes a different strategy for estimating the GTC that relies on a set of directional vectors. In the output orientation, directional output vectors are generated independently of the actual sample's output levels, whereas directional input vectors are fixed at specific levels, such as the overall sample mean or a specific period. The mean of the TC measure calculated using the directional vector approach is independent and unaffected by the uneven distribution of sampled data points because the directional vectors are evenly distributed in the output–input space. According to Otsuki (2013), the limitation of this approach is that the input vector is fixed to represent the entire sample, implying that the effect of a nonproportional change in outputs on the output set is unpredictable.

This paper proposes a new decomposition of $TC_o^{t,t+1}$ in using a more elaborate procedure, building upon previous work. In our opinion, the TC experienced by a firm is determined by two drivers, which should be identified separately: a GTC that summarizes the technology behavior for the entire industry, which is common to all production units, and a new LTC that captures how each company locally experiences GTC over the two considered periods.

3. Methodology

3.1. Measuring global technical change and local technical change

Asmild and Tam (2007) discuss the possibility of extending their approach to include a set of artificial DMUs. Following the idea of creating synthetic DMUs, we propose using a hypercube. In mathematics, a hypercube is a p-dimensional generalization of a square ($p = 2$), a cube ($p = 3$), or a tesseract ($p = 4$). This geometric figure is consistently compact and convex. It is a regular polytope with mutually perpendicular sides, equal sides, and right angles. A hypercube's vertices are evenly spaced apart. The number of vertices in a hypercube is $2{\cdot}n$, where n represents the number of dimensions. Each vertex connects to exactly n other vertices (Coxeter, 1973, for a more detailed description). In particular, we will use the unit hypercube, a hypercube whose line segments between two connected vertices are one unit long (Fig. 1).

MI is commonly used in empirical problems in which the inputs and outputs used by DMUs are quantitative variables measured in different units and orders of magnitude, and we do not have information about market prices. Taking advantage of the unit invariance property of radial models in DEA (Lovell and Pastor, 1995), we propose normalizing all DMU inputs and outputs by the corresponding maximum observed values of DMUs in the two evaluation periods. As a result, all DMUs' variables will be bounded between 0 and 1 and free of units of measurement. This ensures that the MI results and traditional decomposition remain unchanged, whereas all DMUs and technologies from the two evaluation periods are naturally embedded within a unit hypercube.

Once the variables are normalized by the maximum value, ranging in the interval (0,1], the procedure for calculating the GTC is as follows. First, the dimensions of the unit-hypercube H must be defined following the empirical problem, namely the sum of inputs and outputs ($m + n$). The second step is to generate a set of K uniformly distributed random values for each dimension, ranging from zero to one. Finally, we combine the generated values from the various variables to produce a uniformly distributed set of synthetic DMUs within the unit hypercube.

Let us assume that $\aleph = \{(x_k^H, y_k^H)\}_{k=1}^{K^{m+n}}$ is the set of K artificial production units uniformly distributed inside a unit-hypercube H. Our proposal for further decomposing the TC in is

$$M\left(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1}\right) = EC_o^{t,t+1} \cdot TC_o^{t,t+1} = EC_o^{t,t+1} \cdot GTC_H^{t,t+1} {\cdot} LTC_o^{t,t+1}$$

where

$$GTC_H^{t,t+1} = \left(\prod_{k=1}^{K^{m+n}} \frac{D_c^t\left(x_k^H, y_k^H\right)}{D_c^{t+1}\left(x_k^H, y_k^H\right)} \right)^{\frac{1}{K^{m+n}}} \text{ and } LTC_o^{t,t+1} = \frac{TC_o^{t,t+1}}{GTC_H^{t,t+1}}.$$

In, $M(x_o^t, y_o^t, x_o^{t+1}, y_o^{t+1})$, $EC_o^{t,t+1}$ and $TC_o^{t,t+1}$ follow the same definitions as in. However, the traditional technical change $TC_o^{t,t+1}$ has been decomposed into two new components: a global technical change $GTC_H^{t,t+1}$ and

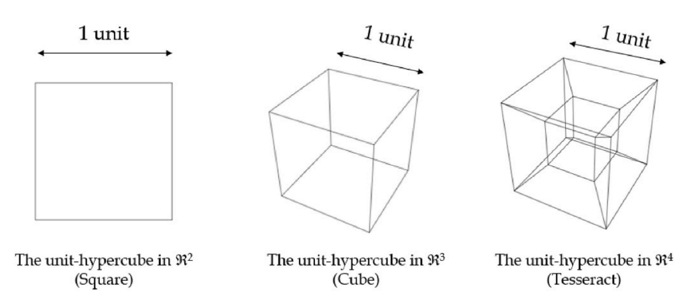

Fig. 1. Examples of unit hypercubes.

a local technical change $LTC_o^{t,t+1}$. The component $GTC_H^{t,t+1}$ captures the overall frontier shift between the two periods, avoiding any potential bias that may arise when DMUs are not uniformly distributed across the production set but are concentrated in specific regions. To achieve this, $GTC_H^{t,t+1}$ averages the global frontier shift between *t* and *t* + *1* evaluated for each synthetic point (x_k^H, y_k^H), $k = 1, ..., K^{m+n}$, and then projected against the technologies in *t* and *t* + *1*. The new local technical change $LTC_o^{t,t+1}$ in, calculated as a residual, measures the relative position of each real DMU at *t* and *t* + *1* in relation to the synthetic average $GTC_H^{t,t+1}$.

3.2. Numerical example

We provide a simple numerical example to illustrate how to calculate[1] the $GTC_H^{t,t+1}$ and $LTC_o^{t,t+1}$ defined in. Assume an industry made up of eleven DMUs, *A, B, C, D, E, F, G, H, I, J*, and *K* that use CRS technology to produce two outputs (y_2 and y_1) from a single input (x) over two time periods (*t* and *t* + *1*). The first column of Table 1, "Inputs and outputs," lists the outputs and input data for the two periods and the 11 DMUs.

Fig. 2 shows the performance of the DMUs and the production frontiers for the two periods using normalized data from Table 1's third column, "Ratios normalized by the maximum values of ratios." The piecewise linear form of the nonparametric frontier in a DEA model under CRS shows that *A, B, C, D*, and *E* are efficient in period *t*, but only *A, C, D*, and *E* remain fully efficient in period *t* + 1. It is also worth noting that DMUs *C* and *I* have identical output–input ratio values in both periods. Fig. 2 clearly shows that DMUs are more specialized in producing more output 1 than output 2 concerning the consumed input. Regarding TC, Fig. 2 shows how some stretches showing technical progress and others showing technical regress, despite the overall TC between *t* and *t* + *1* appearing to be positive. However, whether the mean TC measured for the observed DMUs using the MI resulted in technical progress in the industry remains unclear.

Fig. 3 shows our approach to obtaining a new decomposition of TC in. The GTC is calculated by averaging the individual TCs calculated after projecting 1,000,000 synthetic DMUs uniformly generated inside the hypercube, which in this 2-dimensional example is a square with a line segment of length one, against the estimated empirical production frontiers in *t* and *t* + 1. This tool aims to use synthetic DMUs to "scan" the distance between the two production frontiers in *t* and *t* + *1* globally.

For example, in Fig. 3, we highlight the two distances for the synthetic DMU *L* where $D_c^t(x_L^H, y_L^H) = OL/OP$, while $D_c^{t+1}(x_L^H, y_L^H) = OL/OQ$, being the TC for this specific DMU *L* positive (progress) and equal to $D_c^t(x_L^H, y_L^H)/D_c^{t+1}(x_L^H, y_L^H) = OQ/OP$.

The $GTC_H^{t,t+1}$ of this industry, computed as the average TC of all synthetic DMUs, can be graphically illustrated through the dashed production frontier in Fig. 3, which is represented as a Hicks-neutral technical change (Aparicio et al., 2018) concerning the production technology in *t*. Table 2 shows the traditional MI decomposition and the two new components resulting from the TC.

First, note that the geometric mean of individual TCs in Table 2 is 0.9756. Therefore, a mean TC of less than 1 indicates that this industry experienced a technical regress between the two periods. However, the new decomposition of the TC yields a more valuable result, obtaining an $GTC_H^{t,t+1} = 1.0254$ equal value for all firms because it represents the GTC of the entire industry. This result indicates that the production technology of this industry at $t + 1$ experienced a 2.54% parallel shift upward in global terms relative to the production frontier at *t*. This result confirms that, in general, the mean TC value in the MI may be skewed because the empirical sample sizes are insufficient to cover the entire surface of the two production technologies, and the observed DMUs are not evenly distributed across the production set.

Second, the LTC is obtained as $LTC_o^{t,t+1} = TC_o^{t,t+1}/GTC_H^{t,t+1}$ and may be interpreted as the TC experienced by each DMU once the $GTC_H^{t,t+1}$, common for the entire industry, is discounted. For example, Fig. 2 reveals that DMU C is located at the exact point where the two production frontiers intersect and, in this case, the traditional TC indicates, $TC_o^{t,t+1} = 1$. However, this value decomposes in two terms, a $GTC_H^{t,t+1} = 1.0254$ and a $LTC_o^{t,t+1} = 0.9752$ pointing out that the TC experienced by firm C is below the average or GTC for the whole industry. Alternatively, the TC of firm *E* is $TC_o^{t,t+1} = 1.3960$. This change is decomposed in the average $GTC_H^{t,t+1} = 1.0254$ and a $LTC_o^{t,t+1} = 1.3614$, pointing out that DMU *E* managed to shift the production technology above the average *GTC* experienced by the industry.

The simulation with synthetic production units extends beyond the calculus of an average *GTC* by allowing us to explore how the TC of 1,000,000 synthetic DMUs is distributed. Fig. 4 displays a kernel density estimate plot that shows the distribution of synthetic TCs.

Fig. 4 shows the kernel plot, where the distribution of scores is informative and allows one to observe that TC is a non-neutral complex theoretical concept in which technical progress and regression can occur concurrently. We also see two bumps in the distribution that observe to the two stretches in Fig. 3 where the distance between the two technologies is very similar, resulting in common distance measures for all synthetic DMUs projected against these two zones, under technical regress and technical progress, respectively, of the two production frontiers.

In this simulation, we arbitrarily set $K = 1,000,000$ as the number of synthetic DMUs required to obtain the GTC. To provide some guidelines for setting *K* to calculate the GTC, we recalculate the GTC as well as some statistics on its distribution for different numbers of *K* ranging from 100 to 10,000,000. Table 3 shows that the results remain fairly stable as DMUs increase beyond 10,000 synthetic units.

To summarize, this new decomposition is highly useful for avoiding misleading conclusions from averaging the observed individual TCs that, in this case, leads us to conclude that this industry had a technical regress. The new results suggest that the complex non-neutral TC between the two available production technologies in the two periods can be decomposed into two terms. On the one hand, the GTC captures the average Hicks-neutral TC made by the industry globally in period *t* + *1* with respect to period *t*, leading to the conclusion that in this example there was, on average, a technical progress. On the other hand, the LTC measures how each DMU performed with respect to this GTC and is useful for concluding that, most firms in our example were producing closer to the region where the technology was experiencing a technical regress. We would also conclude that the results of DMUs *D* and *E* experienced technical progress beyond the average GTC. This result might be interpreted in real life in the sense that the two leader firms of this industry, *D* and *E*, were able to introduce new managerial or technology innovations that allowed them to improve their productivity by shifting up, on average, the production frontier of the whole analyzed sector.

4. Empirical application

In this empirical application, we use the well-known database of

[1] To replicate this numerical example, we provide the R code used for obtaining the results of this section in Annex A.

[2] These steps aim to draw this simple numerical example to facilitate its understanding. However, most empirical problems cannot be plotted, so in those cases each output and input will be directly normalized by the maximum value of each variable found, considering the two evaluated periods. The R code provided in Annex A for replicating this numerical example works in this more general way.

Table 1
Production data for eleven DMUs in two periods.

DMU	*Inputs and Outputs*						*Outputs divided by the Input (ratios)*				*Ratios normalized by the maximum values of ratios*			
	t			*t + 1*			*t*		*t + 1*		*t*		*t + 1*	
	y_2	y_1	x	y_2	y_1	x	y_2/x	y_1/x	y_2/x	y_1/x	$(y_2/x)^a$	$(y_1/x)^a$	$(y_2/x)^a$	$(y_1/x)^a$
A	1	6.5	1	3	16.5	3	1	6.5	1	5.5	0.1111	1	0.1111	0.8462
B	9	18	3	2	5	1	3	6	2	5	0.3333	0.9231	0.2222	0.7692
C	5	5	1	5	5	1	5	5	5	5	0.5556	0.7692	0.5556	0.7692
D	12	6	2	15	8	2	6	3	7.5	4	0.6667	0.4615	0.8333	0.6154
E	13	2	2	9	2	1	6.5	1	9	2	0.7222	0.1538	1	0.3077
F	6	16	4	1.5	5	1	1.5	4	1.5	5	0.1667	0.6154	0.1667	0.7692
G	10	15	5	5	10	5	2	3	1	2	0.2222	0.4615	0.1111	0.3077
H	8	8	2	7	9	2	4	4	3.5	4.5	0.4444	0.6154	0.3889	0.6923
I	9	12	3	12	16	4	3	4	3	4	0.3333	0.6154	0.3333	0.6154
J	3	13.5	3	3	15	4	1	4.5	0.75	3.75	0.1111	0.6923	0.0833	0.5769
K	2	2	1	3	6	2	2	2	1.5	3	0.2222	0.3077	0.1667	0.4615
Max[a]	13	18	5	15	16.5	5	6.5	**6.5**	**9**	5.5				

[a] These values are obtained by dividing the data in the column "Outputs divided by the Input (ratios)" by the maximum values found in these variables in the two periods (in bold). The second column of Table 1 shows the two outputs divided by the input in each period. This is an intermediate step toward plotting the DMU and production frontiers for both periods. Finally, once the maximum values for each ratio in the two periods have been found, the information in the second column in Table 1 is normalized by these maximum values[2] to define all variables in the (0,1] interval.

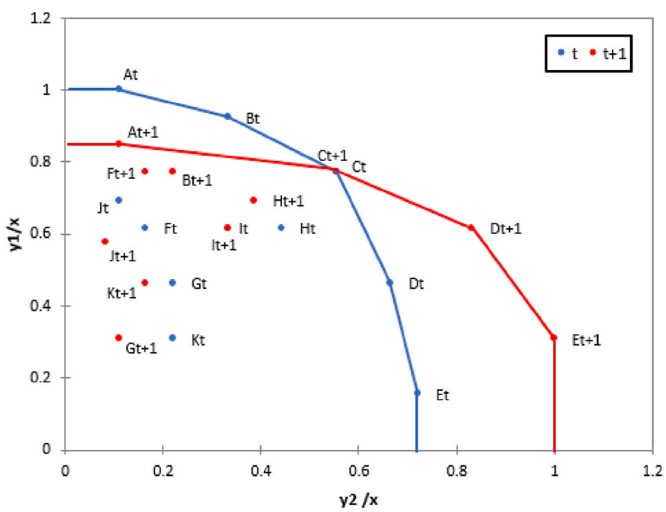

Fig. 2. Production frontiers for 11 DMUs in two time periods.

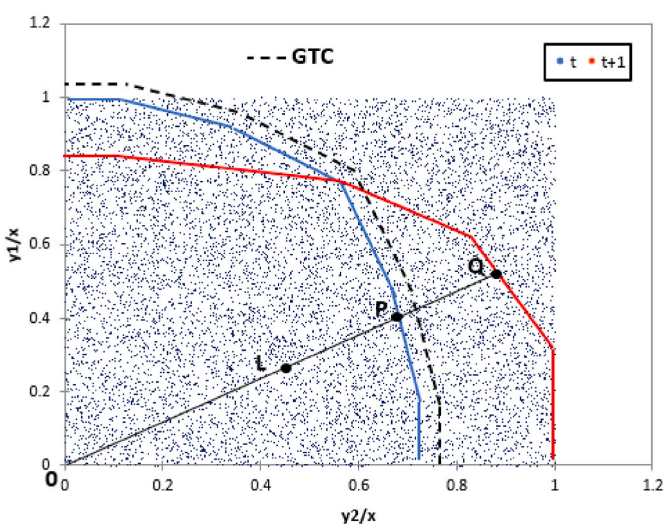

Fig. 3. The Global Technical Change (GTC) estimated by projecting 1,000,000 synthetic DMUs against production frontiers in *t* and *t* + *1*.

Swedish pharmacies[3] previously analyzed in the seminal papers of Färe et al. (1992), Balk and Althin (1996), and Althin (2001). This set of 42 Swedish pharmacies produced four outputs from four inputs between 1980 and 1989. The four outputs are "Drug deliveries to hospitals" (O1); "Prescription drugs for outpatient care" (O2); "Medical appliances for the handicapped" (O3) and "Over-the-counter goods" (O4). The first three outputs (O1, O2, and O3) are measured in terms of number of times, while the fourth (O4) is measured using, 1980 prices. The four inputs are "Labor input for pharmacist" (X1); "Labor input for technical staff" (X2); "Equipment services" (X3) and "Building services" (X4). Both labor inputs (X1 and X2) are measured in hours per year, X3 is calculated using the annual depreciation of pharmacy equipment at 1980 prices, and X4 is assumed to be proportional to available floor space (measured in square meters).

Table 4 shows the MI and its decomposition, consistent with those Färe et al. (1992) reported. Furthermore, Table 4 presents a new breakdown of TCs in GTC and LTC. All individual results have been averaged using the geometric mean.

To interpret the results,[4] note first that the average GTC over two years is a common value for all DMUs, as opposed to the traditional TC measure. In Table 4, the LTC is the geometric average of the corresponding individual LTC (Table B2), obtained by dividing the TC (Table B1) by the GTC (Table 4). As expected, the traditional average TC that summarizes the global shift in frontier technologies between two periods does not correspond to the GTC. Differences range from 1.58 (82–83) to 7.96 (80–81) percentage points; in some cases, the results may lead to incorrect interpretations. For example, from 1980 to 1981, the TC concluded that there was a technical regress in the sector (0.9698), whereas the GTC shows overall technical progress (1.0494). This result shows that measuring industry-wide TC solely based on the individual TCs observed in our sample can produce misleading results, as seen in our previous simulation.

Second, the LTC is a measurement of each DMU's average TC over the two periods analyzed, considering its relative position at *t* and *t* + 1 in relation to the GTC. Table 4 shows that, in general, the GTC and LTC do not follow the same direction and must be interpreted differently. For

[3] We only provide new results. Färe et al. (1992) provided all details about the database, the variables, descriptive statistics and disaggregated Malmquist index and its decomposition over time for the set of 42 pharmacies.

[4] For simplicity, we do not show the complete list of individual results for all DMUs and periods available in Färe et al. (1992). With the goal of following the explanation, we only provide the individual scores for TC in the Annex B (Table B1) and LTC (Table B2).

Table 2
Numerical example: the Malmquist index and its components.

DMU	Malmquist index and its components			New TC decomposition	
	Malmquist index (MI)	Efficiency Change (EC)	Technical Change (TC)	Global Technical Change (GTC)	Local Technical Change (LTC)
A	0.8490	1	0.8490	1.0254	0.8280
B	0.8192	0.9333	0.8777	1.0254	0.8559
C	1	1	1	1.0254	0.9752
D	1.2697	1	1.2697	1.0254	1.2382
E	1.3960	1	1.3960	1.0254	1.3614
F	1.2337	1.4229	0.8670	1.0254	0.8456
G	0.6393	0.7083	0.9025	1.0254	0.8801
H	1.0691	1.0972	0.9744	1.0254	0.9502
I	1	1.0606	0.9429	1.0254	0.9195
J	0.8300	0.9711	0.8548	1.0254	0.8336
K	1.3307	1.4167	0.9393	1.0254	0.9161
G. Mean	1.0126	1.0379	0.9756	1.0254	0.9515

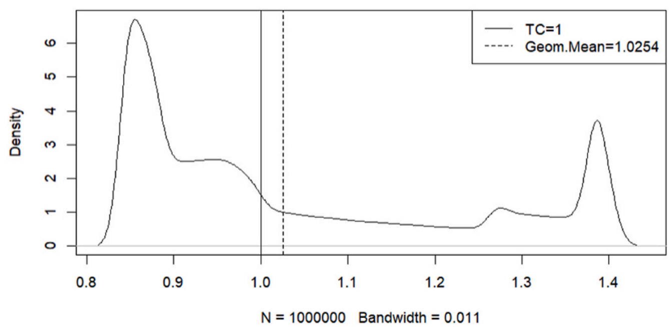

Fig. 4. Kernel density plot of the distribution of the technical change estimated for $K = 1{,}000{,}000$ synthetic DMUs in the numerical example.

example, from 1983 to 1984, the average TC was 0.9544, the lowest value analyzed during the study period. However, according to the GTC = 0.8980, the situation was even worse for this period. Despite the high technical regress, 36 of the 42 pharmacies (Table B2) outperformed the industry average, increasing the value of the geometric mean LTC (LTC = 1.0627).

Following Otsuki (2013), it is clear that the GTC should be summarized by providing a single value, such as the geometric mean TC in Table 4, for each pair of periods, or by using a set of TC measures to fully characterize the distribution. Another obvious result, as demonstrated in the previous numerical example, is that our procedure allows the distribution of results for the 1,000,000 synthetic DMUs used in the analysis to be explored. Table 5 shows how the average GTC shown in Table 4 is distributed across each pair of periods.

The score distribution is informative and allows us to conclude that, in general, the shapes of technologies in two periods intersect, indicating that there are areas where technical progress or regress occurred simultaneously. For example, the GTC between 1981 and 1982, as well as1988 and 1989, was positive for more than 75% of projected synthetic DMUs, whereas a technical regress for more than this amount of DMUs occurred during the 1983–1984 period. Fig. 5 also depicts kernel density plots to better explore the shape of the distribution of GTC values in our data set of 1,000,000 synthetic DMUs, using a single continuous curve.

Fig. 5 shows that, in general, the shape of GTC value distributions is not smooth and frequently multimodal. The range of the distributions also varies significantly, and, as previously stated, the percentage of synthetic units with TC assessments that are above or below 1 varies from period to period.

5. Conclusion

This paper introduces a new way to decompose TC in the MI as GTC and LTC. To do so, we use the concept of a hypercube, which is used in mathematics to better understand complex systems in higher dimensions. Balk and Althin (1996), Asmild and Tam (2007), and Otsuki (2013) proposed measuring a GTC, in which the TC component is suggested as a global phenomenon affecting the sector's frontier shift rather than the geometric average of TCs measured in the empirical sample. Unlike the previous authors, our proposal is based on two subcomponents: a GTC across the industry and how each firm locally experiences the GTC over time, depending on its position concerning

Table 4
Malmquist index and its decomposition in Färe et al. (1992) and the new decomposition in GTC and LTC for 42 Swedish pharmacies between 1980 and 1989 (Geometric Means).

Year	Färe et al. (1992)			New decomposition	
	Malmquist Index (MI)	Efficiency Change (EC)	Technical Change (TC)	Global Technical Change (GTC)	Local Technical Change (LTC)
8081	0.9911	1.0220	0.9698	1.0494	0.9241
8182	1.0740	0.9353	1.1483	1.1736	0.9785
8283	1.0246	1.0548	0.9713	0.9555	1.0166
8384	0.9424	0.9875	0.9544	0.8980	1.0627
8485	1.0435	1.0039	1.0395	1.0135	1.0256
8586	1.0189	0.9868	1.0325	1.0146	1.0177
8687	1.0665	1.0015	1.0649	1.0229	1.0411
8788	1.0435	1.0133	1.0298	1.0007	1.0291
8889	1.0513	1.0056	1.0455	1.0193	1.0257
G. Mean	1.0277	1.0007	1.0269	1.0140	1.0127
Accum.	1.2786	1.0065	1.2703	1.1337	1.1205

Table 3
Global Technical Change estimated with different K numbers.

K	Geom. Mean	Min	1Q	Median	Av. Mean	3Q	Max
100	1.02898	0.8462	0.8789	0.9682	1.0456	1.2586	1.4032
1,000	1.02669	0.8462	0.8732	0.9623	1.0433	1.2085	1.4073
10,000	1.02505	0.8462	0.8748	0.9633	1.0417	1.2130	1.4074
100,000	1.02529	0.8462	0.8747	0.9630	1.0421	1.2135	1.4074
1,000,000	1.02540	0.8462	0.8746	0.9622	1.0422	1.2146	1.4074
10,000,000	1.02518	0.8462	0.8746	0.9623	1.0419	1.2133	1.4074

Table 5
Global Technical Change distribution for 1,000,000 synthetic DMUs over the 1980 and 1989 periods.

Period	Min.	1st Q	G. Mean	Median	A. Mean	3rd Q	Max.
8081	0.4672	0.9802	1.0494	1.0460	1.0578	1.1172	1.6857
8182	0.6513	1.0324	1.1736	1.1110	1.1929	1.2783	2.5390
8283	0.4570	0.8835	0.9555	1.0212	0.9659	1.0575	1.2746
8384	0.5281	0.8273	0.8980	0.9185	0.9078	0.9792	1.3163
8485	0.8613	0.9788	1.0135	1.0179	1.0148	1.0441	1.3506
8586	0.6309	0.9789	1.0146	1.0262	1.0173	1.0632	1.2248
8687	0.7740	0.9642	1.0229	1.0342	1.0251	1.0735	1.3223
8788	0.8371	0.9612	1.0007	0.9976	1.0024	1.0523	1.1854
8889	0.8052	1.0006	1.0193	1.0322	1.0212	1.0619	1.1989

1st Q: First quartile; G. Mean: Geometric mean; A. Mean: Arithmetic Mean; 3rd Q: Third quartile.

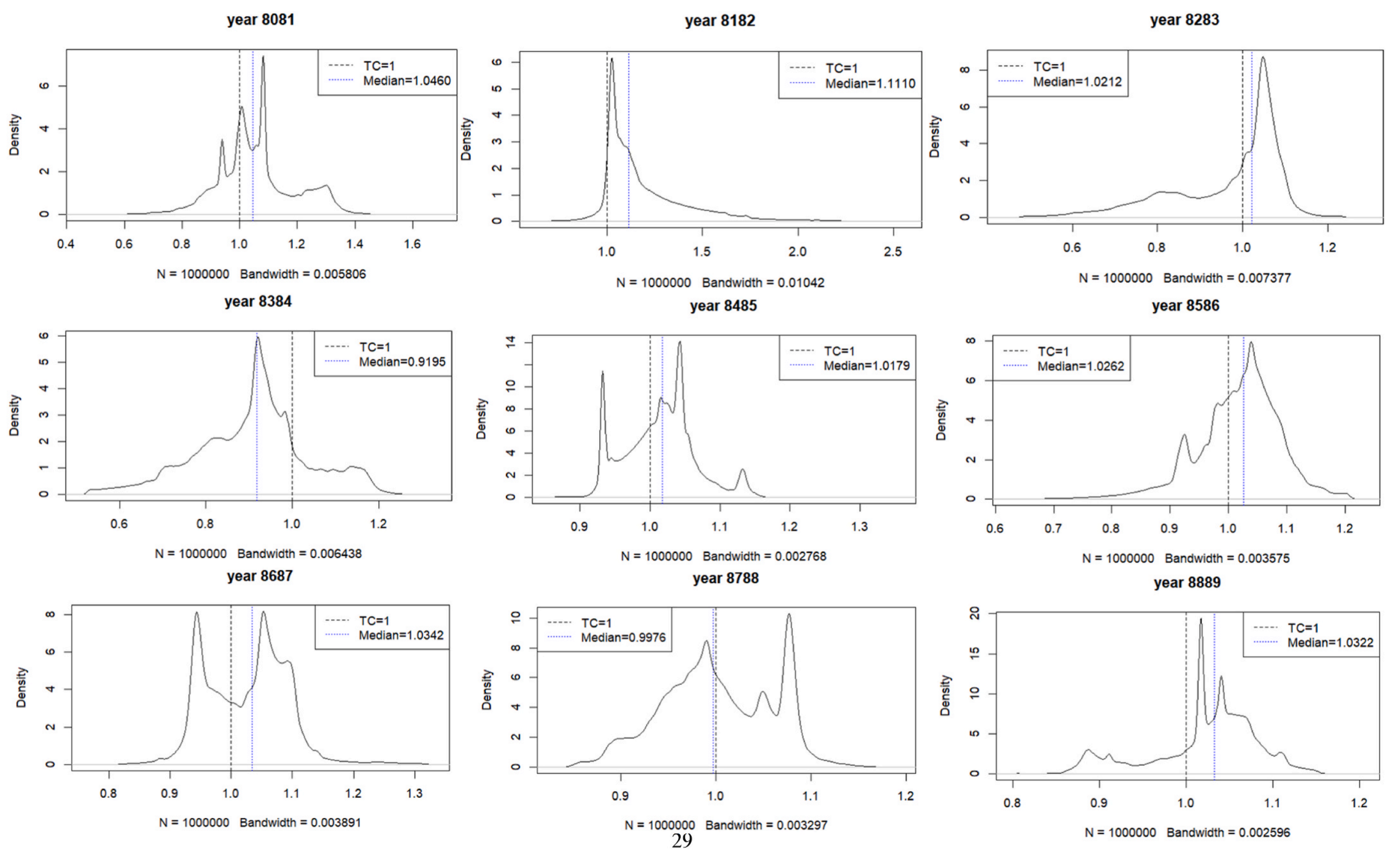

Fig. 5. Kernel density plots of the global technical changes in the Swedish pharmacy industry.

technologies at t and $t + 1$.

In our approach, we can identify both drivers of productivity change, GTC and LTC, by embedding the database inside a unit hypercube and using the unit invariance property of radial models in DEA. Generating a large number of synthetic DMUs uniformly distributed within the unit hypercube allows the surfaces of both technologies in the two periods to be scanned, yielding a set of distances that fully characterize the GTC. Creating a synthetic grid of points to be used as a reference for evaluating technical progress or regress is an advantage over previous approaches. In particular, Asmild and Tam (2007) used actual observations to measure global technological change. According to these authors, one disadvantage of global indices is that they are calculated from all observations in the sample (from all periods), making them sensitive to adding additional data (DMUs or periods) to the data set. On the contrary, our approach does not suffer from this flaw. Our GTC component, determined for previous periods, remains unchanged when new data from new periods is observed. This seems to be consistent with the idea that new data cannot affect the past.

In our empirical application, we use the same panel data of 42 Swedish pharmacies between 1980 and 1989 that were previously analyzed in the seminal papers by Färe et al. (1992), Balk and Althin (1996), and Althin (2001). GTC results provide new information about this industry and allow for the visualization of GTC distribution using kernel density curves.

This paper should open up several new research opportunities in the future. First, we analyze the GTC using 1,000,000 synthetic DMUs. Although our numerical example shows that estimations are fairly stable after 10,000 synthetic DMUs, more research is needed to determine a suitable K number for dealing with the trade-off between number of dimensions, computation time, and precision. Second, in this paper, we assume CRS technologies; another challenge is to extend the decomposition of GTC and LTC for production technologies with variable returns to scale (VRS) over time. A few decomposition approaches in the literature assume VRS. Färe et al. (1994) were the first authors to break down the MI under VRS into three categories: technical efficiency change, TC, and scale efficiency change. Ray and Desli (1997) criticized this approach because Färe et al.'s (1994) TC component may exaggerate or undervalue actual technological change on best practice technologies in periods t and $t+1$. Gilbert and Wilson (1998), Simar and Wilson (1998), Zofio and Lovell (1998), and Wheelock and Wilson (1999) expanded on this research. They proposed a four-part decomposition: technical efficiency change, TC, scale efficiency change, and scale bias of TC (see also Lovell (2003) as a survey of different decompositions). Unfortunately, it is common in such decompositions to encounter infeasibilities when calculating the mixed period components. Our approach may mitigate the problem in this context because we use thousands of synthetic units generated at random, some of which present infeasibility issues while others do not. We could even generate additional virtual points at random until we reach the desired number of feasible values. However, the possible decomposition of the MI under our approach assuming VRS warrants further elaboration and research, and it stands out as an intriguing research avenue for the future.

Furthermore, the concept of a GTC can be extended to other indices, such as the Camanho and Dyson (2006) index to address productivity gaps between DMU groups, and its time-based version proposed by Aparicio and Santín (2018). Another possibility is to develop alternatives to the MI. One of these alternatives is the Luenberger productivity index (Chambers et al., 1996; Chambers and Pope, 1996), which is based on the directional distance function and decomposed additively. In this case, to apply our approach, a unit-invariant version of the directional distance function is required (Aparicio et al., 2016).

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CRediT authorship contribution statement

Juan Aparicio: Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Visualization, Funding acquisition. **Daniel Santín:** Conceptualization, Methodology, Software, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization.

Declaration of competing interest

None

Data availability

Replication package: Mendeley Data, V1, doi: 10.17632/xznv43bvjp.1.

pharmacy (Original data) (Mendeley Data)

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Appendix ASupplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2024.106674.

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