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Belief-Based Model of Career Dropout Under Monopsonistic Employment and Noisy Evaluation

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Abstract

This paper develops a belief-based dynamic optimisation framework to explain career continuation decisions in settings characterised by monopsonistic employment and asymmetric performance evaluation. Extending Holmström's career concerns model, we consider agents who must decide whether to continue or exit their vocation based on subjective beliefs updated from noisy signals. Unlike the original framework, our model assumes a single institutional employer and limited feedback transparency, turning the agent's decision into an optimal stopping problem governed by evolving belief thresholds. Analytical results demonstrate how greater signal noise, higher effort costs, and more attractive outside options raise the probability of exit. To validate the framework, we confront belief-based dropout decisions using original survey data from over 8000 football referees in Europe, showing that threats, unmet development expectations, and perceived stagnation significantly predict dropout. The results offer practical insights for institutions, such as sports federations, academic bodies, and civil services, on how to improve retention through increased transparency and better support structures. This study contributes to the literature by integrating optimal stopping theory and dynamic labor models in a novel context of constrained career environments.

Keywords: career concerns; optimal stopping; asymmetric information; belief updating; dynamic optimisation

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1. Introduction

In institutional and vocational careers such as sports officiating, academia, and social service, individuals often face the challenge of deciding whether to continue or exit their profession under significant uncertainty. These environments are typically characterized by two major constraints: (i) monopsonistic employment structures, where a single entity controls career opportunities (e.g., a national federation or civil service body), and (ii) asymmetric or opaque evaluation systems, where individuals receive little or noisy feedback about their own performance or prospects. In such settings, exit is not merely a response to failure but a decision made under subjective belief dynamics formed from incomplete signals about the evolution of their future career.

Despite the extensive literature on dynamic incentives, few formal models capture this belief-driven career persistence. Most notably, Holmström's influential model of career concerns formalized how individuals exert effort to build a reputation over time when wages depend on employer beliefs [1]. Although Holmström does not explicitly characterize the labor market structure, his model of dynamic incentives assumes a frictionless environment, with no mention of job mobility constraints or employer power. However, the focus on internal feedback and career trajectories makes it especially suitable for application in monopsonistic settings, such as those explored in this article. Holmström's assumptions limited its applicability to real-world systems where workers face no external job alternatives and must act on imperfect or noisy feedback.

This study enriches Holmström's model in two key directions. First, it presents a monopsony employer, where the agent has no access to an external labor market and must compare the expected value of staying with a fixed outside option. Second, asymmetric information via noisy performance signals is incorporated: the agent cannot observe their true evaluation but instead updates subjective beliefs based on imperfect cues. The resulting framework is a belief-based dynamic optimisation problem with a stay/exit (optimal stopping) structure, suitable for analysing dropout under uncertainty.

Our central research question is:

How do individuals decide whether to continue or exit a career when employed by a monopolistic institution and evaluated through noisy, asymmetric performance feedback?

To answer this, career continuation is modeled as a discrete-time decision problem in which the agent maximizes expected utility over time. At each period, the agent observes a signal about their likely standing and must decide whether to continue or exit, comparing the expected future utility of staying with an outside option. The analytical conditions are derived for optimal dropout and explore how signal precision (transparency) influences belief evolution and exit thresholds.

To complement the theoretical framework, belief-based decision paths are confronted using parameter values informed by original survey data on football referees in Europe. This case illustrates the model's applicability to vocational contexts where career persistence is affected by evaluation opacity and institutional monopoly. Our simulation shows how improved signal precision can reduce unnecessary dropout, offering insights into retention strategies in federated or hierarchical systems.

The paper proceeds as follows. Section 2 reviews the related literature starting from the Holmström's Career Concerns Model. Section 3 presents the enriched theoretical model defined by our contributions. Section 4 describes the simulation framework and empirical setting. Section 5 presents the simulation results by means of the most extensive study in Europe about football referees' careers. Section 6 discusses the theoretical and practical implications. Section 7 outlines possible institutional policy responses. Finally, Section 8 presents the main conclusions.

2. Background and Related Literature

Understanding the foundations of career continuation decisions in environments of asymmetric feedback and limited mobility requires synthesising insights from dynamic incentive theory, optimal stopping processes, and institutional labor economics. This section reviews the core literatures that inform the theoretical model developed in this study.

2.1. Dynamic Incentives and Holmström's Career Concerns Model

Rust introduced a computational framework for estimating dynamic discrete choice models through his study of bus engine replacement decisions [2]. Although the context was mechanical maintenance, not labor, the methodological innovation, solving and esti-

mating decision rules under uncertainty with serial correlation in state variables, paved the way for career-oriented models in economics. His nested fixed-point algorithm remains a benchmark for solving such models and continues to inform applications in labor, education, and health economics.

Building on this framework, Keane and Wolpin offered one of the earliest and most influential structural models of career decisions [3]. They modelled young men's choices among working, attending school, joining the military, or remaining inactive, while accounting for the evolution of wages, skills, and preferences over time. Their model integrated both human capital accumulation and labor market uncertainty, showing how individuals respond dynamically to both observed and unobserved changes in the environment.

The key contribution of this strand of literature is its ability to test counterfactual policy interventions and estimate deep structural parameters, such as preference heterogeneity, discount factors, and learning dynamics, rather than relying solely on reduced-form associations. This study draws from these techniques to model the career continuation decisions of referees, adapting the structural logic of dynamic discrete choice to environments characterized by a monopsony employer, asymmetric feedback, and uncertain performance evaluation.

Holmström established the foundational model for dynamic reputational incentives in career decision-making. In his framework, individuals choose effort over time not because of contemporaneous rewards but due to how current performance affects future wages through evolving employer beliefs. The model highlights how output, which is affected by unobservable effort and by noise, becomes a proxy for ability, and thus effort is indirectly incentivised. A central insight is that effort is front-loaded, declining over time as uncertainty about ability diminishes [1].

These mechanisms are not confined to abstract modeling; empirical evidence confirms that employer concentration and limited outside options shape career outcomes in practice. For instance, Staiger et al. show, using a natural experiment in the U.S. healthcare sector, that nurses' wages are heavily influenced by hospital monopsony power, underscoring how restricted labor mobility interacts with incentive dynamics [4].

Subsequent work has extended Holmström's insights. Sannikov developed a continuous-time principal-agent model where agent continuation utility becomes a key state variable, and belief updating about ability shapes contract design and the agent's decision to persist or quit [5]. Heinsalu provided important corrections to Holmström's equilibrium logic, clarifying the mathematical consistency of belief formation over time and the effort-wage link [6].

Related work by Dewatripont et al., Gibbons and Murphy, and Meyer and Vickers has formalised incentive problems in dynamic environments, though often without explicitly modelling career exit or belief-based thresholds [7–9].

Recent structural approaches, such as those surveyed by Aguirregabiria and Mira, provide formal methods for estimating dynamic decision models with belief updating and forward-looking behavior, reinforcing the relevance of career concern mechanisms under uncertainty [10].

2.2. Optimal Stopping Theory and Belief-Driven Decision Processes

Agents assess whether the expected utility of continuing a career exceeds that of exiting, thus comparing future value against an outside option. Optimal stopping theory offers powerful tools to formalize such threshold-based decision rules.

Belief updating in noisy environments has also been studied in the context of optimal timing and signal interpretation. Milgrom and Roberts showed how agents adjust expectations and strategies when future opportunities depend on imperfect or strategic signals [11].

These mechanisms are echoed in Bénabou and Tirole's treatment of self-confidence and motivation, where individuals interpret feedback through internal belief systems that affect long-term effort and engagement [12].

Recent literature such as Entwistle et al. and Liu and Mu provides comprehensive reviews and asymptotic results for optimal stopping in stochastic environments [13,14]. They show how individuals form expectations over future outcomes based on sequential signals and adjust stopping times accordingly. This framework has been applied extensively in finance but is adaptable to labor settings where performance information arrives over time. The career continuation problem modeled here can also be framed as an optimal stopping problem, in which agents assess whether the expected utility of continuing exceeds that of exiting, given their current belief and external conditions. This formal structure has been widely used in decision theory and applied economics to capture threshold-based behavior under uncertainty [14].

Hunt and Caliendo further develop these ideas by surveying dynamic optimisation under timing risk, including bounded and unbounded uncertainty, flow and level shocks, and time-consistent decision rules [15]. Their classification is directly relevant to modelling when an agent decides to exit a career under evolving and uncertain evaluations.

2.3. Asymmetric Information and Belief in Labor Decisions

In labor market models, asymmetric information typically appears in the form of hidden effort, imperfect monitoring, or opaque evaluations. Belief updating occurs when the framework through which agents update beliefs about their standing or performance based on received signals.

The institutional implications of asymmetric feedback are further magnified when individuals rely heavily on internal signals due to the lack of external validation mechanisms. In such cases, Bayesian learning processes may lead to either premature exit or excessive persistence, depending on the perceived credibility and interpretability of feedback over time.

Ederer and Manso show how exploration incentives and delayed feedback can affect persistence and disengagement in innovation settings, insights that parallel vocational contexts where performance is difficult to evaluate [16].

Recent empirical research provides robust evidence that monopsonistic structures are not exceptional but a general feature of labor markets. Sokolova and Sorensen, through a comprehensive meta-analysis of firm-level labor supply elasticities, demonstrate that employer wage-setting power is widespread, establishing monopsony as an empirical baseline across diverse contexts [17]. Complementing this, Azar et al. use detailed U.S. vacancy data to show that higher employer concentration in local labor markets, measured by the Herfindahl–Hirschman Index, is systematically associated with lower posted wages [18]. Similarly, Marinescu et al., using French matched worker–firm data, provide instrumental-variable evidence that labor market concentration directly reduces both hiring and wages [19]. Taken together, these contributions reinforce the relevance of analyzing career continuation decisions within monopsonistic environments, as in the case of refereeing.

Further supporting this view, Schubert et al. show that outside options play a critical role in conditioning the wage effects of employer concentration [20]. Their findings emphasize that the power of monopsonistic employers is not only a function of internal evaluation systems but also of the credibility of external opportunities, a mechanism that resonates with our framework where referees' dropout decisions depend on both noisy feedback and the perceived availability of alternatives.

In the context of Holmström's model, beliefs are typically held by the employer [1]; this paper reverses that assumption, placing the belief update process in the agent's hands. Using noisy feedback, such as promotion decisions or appointment levels, the agent forms and updates beliefs over time. This learning process ultimately governs the optimal stay/exit decision, turning performance signals into a subjective, probabilistic calculus of future rewards.

2.4. Applications in Vocational and Monopolistic Settings

The theoretical concerns discussed above are especially salient in institutional and vocational contexts, such as refereeing, teaching, religious vocations, social care, and early academic careers, where employment is monopsonistic and evaluative feedback is often opaque. Aliende et al. underscores the cumulative effect of organisational decisions on career longevity in a monopolistic setting [21]. Their empirical insights provide a complementary backdrop to the belief-based framework developed in the present study, particularly regarding how institutional context and career bottlenecks shape exit probabilities.

Whitford and Hustinx provide sociological and behavioral perspectives on why individuals stay or leave in such constrained environments, showing that lack of transparency, perceived injustice, and institutional closure drive disengagement [22,23]. Jade Xu investigates career outcomes from STEM and non-STEM pathways, revealing the role of institutional environments and feedback systems in long-term persistence [24]. Mazerolle et al. explore how burnout and lack of institutional support contribute to attrition among athletic trainers, reinforcing the role of opaque feedback and high emotional costs in early career dropout [25].

In labor markets with monopsonistic characteristics, such as national federations, academia, or public service, constrained mobility limits external opportunities and magnifies the psychological effects of internal evaluation systems. Manning emphasizes how limited outside options increase employer power and shape wage dynamics [26], while Frey and Osterloh argue that under conditions of incomplete information, fairness and intrinsic motivation can be more effective drivers of retention than incentive-based contracts [27]. These insights suggest that institutional design and feedback systems play a crucial role in career persistence.

2.5. Behavioral Insights on Career Decision-Making

Beyond rational assessments of performance and external opportunities, behavioral economics offers crucial insights into how individuals make career continuation decisions under uncertainty and institutional constraint. Notably, Camerer and Lovallo argue that overconfidence and self-selection biases can significantly distort career-entry and continuation decisions [28]. Their experimental evidence shows that individuals often enter competitive environments even when objective chances of success are low, driven by an inflated belief in their relative ability. This mechanism can plausibly extend to career persistence: workers may remain in a role longer than rational models predict, due to an overestimation of future success or internal rank, especially in opaque evaluative environments.

Complementing this, Kahneman and Tversky's Prospect Theory highlights that individuals evaluate outcomes relative to reference points and are typically loss averse—they weigh losses more heavily than equivalent gains. In career terms, this implies that negative signals (e.g., poor evaluations, lack of promotion) may trigger disproportionate emotional and cognitive responses, particularly if perceived as losses relative to prior expectations [29]. This can lead to early exits even in cases where long-term prospects might be stable or improving. Moreover, ambiguity in performance signals (i.e., noisy or inconsistent feed-

back) can trigger ambiguity aversion, prompting individuals to avoid or exit environments where outcome predictability is low.

Recent advances in decision theory also emphasize that rationality can be context-sensitive and shaped by salience effects. Giarlotta et al. demonstrate that when signals are noisy, the salience of information may alter perceived payoffs without violating rational choice principles. This perspective reinforces the idea that referees' continuation decisions can be influenced by how performance feedback is framed and perceived [30].

This paper focuses on football referees, a case where evaluations are internal, feedback is limited, and alternative employers are non-existent. The model proposed here is particularly well-suited to this context, though it generalizes to a wide range of vocational paths governed by asymmetric information and constrained opportunity structures.

3. Theoretical Model

This section presents the theoretical framework that builds on Holmström's foundational model of dynamic career incentives. Holmström's model is based on the idea that individuals supply effort over time to shape others' beliefs about their ability. Wages in each period depend on the expected output, which is itself a function of perceived ability. The agent's effort, being unobservable, affects current performance and thus indirectly influences future wage trajectories through a learning process by the employer or the market.

In the base model, the labor market is competitive and the employer has full access to performance-related outputs. The agent exerts effort to influence the market's evolving posterior belief about their fixed but unknown ability, based on noisy output. The agent maximizes lifetime expected utility over an infinite horizon, trading off effort cost against reputational gains that increase expected future wages. As uncertainty about ability decreases over time, so does the marginal return to effort, producing a declining effort profile and potential dynamic inefficiency [1].

This paper extends Holmström's model in two directions. First, the competitive market assumption is replaced with a monopsony employer. The agent faces a single institution (e.g., a federation) that controls appointments and promotions and provides no alternative employment pathways. This structural constraint alters the agent's optimisation problem: they now compare the expected utility of continuing under a fixed or uncertain internal career path with the utility of exiting to a predefined outside option, which may reflect a less-preferred but more predictable alternative (e.g., quitting refereeing to pursue a non-sporting career).

Second, Holmström's focus is shifted from employer-held beliefs to agent-held beliefs. In this enriched model, the agent is uncertain about how they are evaluated and does not observe performance assessments directly. Instead, they receive noisy signals that are probabilistically related to their underlying performance. These signals, such as match assignments, informal feedback, or promotion delays, are interpreted subjectively and used in a belief updating process to revise the agent's belief, which represents their subjective probability of being retained or promoted. The decision to continue or exit is now governed by a value function that evolves over time based on belief dynamics.

Agent's expected utility is defined recursively. It denotes the value of continuing at a given time belief. The agent compares this with the fixed outside option and decides where the utility is from wage or reward at time, is the cost of effort, and is the discount factor. Belief updating occurs with the precision of the signal determining how volatile the belief process is. Lower transparency (i.e., higher signal noise) increases uncertainty and may accelerate exits.

The model generates an endogenous exit threshold: if the expected future utility of continuing falls below, the agent rationally exits. The decision rule is thus an optimal stopping rule based on evolving beliefs, signal quality, and institutional constraints.

This enriched framework allows us to explore how opaque feedback and monopolistic control interact to shape retention outcomes in vocational careers. The next section develops simulations to illustrate these dynamics using referee career data.

3.1. Holmström Model

The starting point in Holmström's model is the estimation of how worker effort evolves over time when future rewards depend on the firm's beliefs about their abilities, formed under noisy signals [1].

Being:

- θ the ability value ϵ R~ N (μ_0, σ_0^2)
- a_t the worker's effort
- ε_t the noise

Then: $Y_t = \theta + a_t + \varepsilon_t$

The employer updates its beliefs about θ according to Bayes' rule, which modifies μ_t affecting future wages:

- $W_t = E[Y_t \mid i_t] = \mu_t$, being i_t the information accumulated by the agent.

So, today's effort determines future wages by improving reputation.

Agent's optimisation occurs by influencing positively it by means of effort (a_t):

- Max $\sum \delta_t (W_t - c(a_t))$, being δ the discount factor and $c(a_t)$ the convex cost of effort.

For Holmström, workers mainly care about how today's performance affects beliefs about them tomorrow. Effort today has no direct return, but it improves how the employer perceives the worker's ability tomorrow, being the essence of career concerns.

3.2. Our Model

The model we bring enriches the previous model considering a monopolistic market structure and asymmetric information.

We propose a discrete-time logistic model to represent referees' decisions to continue or exit (Figure 1). We reflect how effort adapt to expectations and own's opportunity cost (V_0). We introduce the level of transparency/feedback to show how it reduces premature exit.

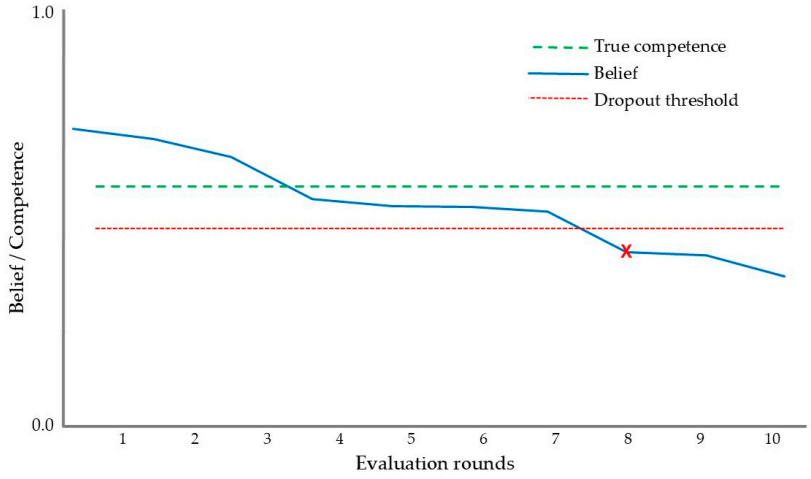

Figure 1. Independent variables influencing the intention of the referee to exit. **Source:** Authors.

Analytically we define:

- Time: t = 1, 2, 3, . . ., T
- Worker's decision at each period: at ϵ {1(=continue), 0(=exit)}
- Employer's contract Ct = (Wt, Pt), being W the wage and P the category based on performance.
- Signal about worker's reputation: St ~ f(St | et), being et the noise, where signal noise is stochastic and affects employer beliefs.
- Belief: ɲ t according to St

Where

- $U(W_t, at) = U(W_t) + U(a_t)$, being V_o if exiting.

Then the worker's decision will be:

- $V_t(ɲ_t) = Max\{U(W_t) - C(a_t) + \delta E[V_{t+1}(ɲ_{t+1})]$, for a_t = continue
- V_0, for a_t = exit

Being:

- δ (0, 1), the discount effect
- $E[V_t + 1]$, the expected value of future utility given current belief/signal

Then, the exit happens when:

- $U(W_t) - C - \delta E[V_{t+1}(ɲ_{t+1})] < V_0$

Where a noisy environment generates an increase of volatility (U_t) and premature exit. Monopoly makes decisions more sensitive to beliefs.

3.3. Comparison of the Two Models

Both models adopt a dynamic framework where past actions influence future outcomes. Our model retains similarities with Holmström's model:

- In the common dynamic framework past actions influence future outcome.
- Beliefs are updated and feedback is relevant.
- Effort is not directly observable, and evaluations are noisy, with feedback inferred from wage levels or category assignments.

In general, while Holmström's model estimates how much effort a worker chooses to exert, framed as the maximisation of future wages through continuous career effort, our model estimates the decision to continue or exit, formulated as a discrete-time optimal stopping problem:

- Sector labor market is monopsonistic.
- The worker seeks to maximise utility through the choice to continue or exit.
- The worker receives partial feedback through match appointments and informal evaluation.

We now test whether the theoretical conditions for dropout are consistent with observed referee behavior under asymmetric feedback conditions.

4. Analytical Results

This section derives and discusses the main analytical implications of the belief-based dynamic optimisation model under a monopsony employer and asymmetric information. We focus on the existence of an optimal decision rule, the characterisation of dropout thresholds, and the comparative statics regarding transparency, effort cost, and outside options. Finally, we reflect on the theoretical implications for institutional design and retention.

4.1. Existence of an Optimal Decision Policy

Worker's belief at time t is denoted by $ɲ_t$ that represents the subjective probability of being favorably evaluated.

Given the recursive function $V_t(ɲ_t)$, where $U(W_t)$ is continuous and bounded, C is a constant higher than 0, and δ ranges from 0 and 1, depending on the belief.

Then $V_t(ɲ_t)$ is defined by a threshold policy by a stopping rule $ɲ_t < \bar{o}$ that maps the utility level.

$\bar{o}$ satisfy that:

$V_0 = U(W(\bar{o})) - C + \delta\, E[V_{t+1}(ɲ_{t+1}) \mid ɲ_t = \bar{o}]$, being:

- $W(ɲ)$ the wage conditioned to belief ɲ
- $U_{t+1} \sim f(ɲ_t, S_t)$, governed by the signal distribution.

This defines $\bar{o}$ as the point where the agent is indifferent between continuing or exiting. This threshold is time-dependent in finite horizon cases.

4.2. Characterisation of the Belief Threshold

The referee's decision to remain in the profession can be represented by a cutoff rule defined through the belief threshold $\tilde{ɳ}$. At this point, the referee is indifferent between dropping out for the outside option V_0 or continuing as an active referee.

Proposition: There exists a unique belief threshold η such that the referee continues if and only if $ɲ \geq \tilde{ɳ}$. This threshold is defined by the indifference condition:

$$E[V(ɲ) \mid ɲ = \tilde{ɳ}) = V_0$$

which can be expressed as:

$$V_0 = U(W(\tilde{ɳ})) - C + \delta P^{+}(\tilde{ɳ}; \tau^2)\, E[J(\eta' \geq \tilde{ɳ}, \eta = \tilde{ɳ}]/1 - \delta P^{-}(\tilde{ɳ}; \tau^2)$$

Moreover, the threshold increases monotonically with both the outside option and the variance of evaluation noise:

$$\delta\tilde{ɳ}/\delta V_0 > 0 \text{ and } \delta\tilde{ɳ}/\delta\sigma 2 > 0$$

By definition, the referee continues if the expected value of staying, conditional on belief η exceeds the outside option V_0. Because the continuation value is strictly increasing in η, there exists a unique cutoff η at which indifference holds.

This representation decomposes into three components. The numerator includes the immediate payoff $U(W(\tilde{ɳ}))$, net of refereeing costs C, and the discounted continuation value conditional on surviving evaluation, weighted by the success probability $P^{+}(\tilde{ɳ}; \tau 2)$.

Intuitively, the belief threshold is the point at which the expected value of staying equals the outside option. When outside opportunities improve, referees must be more confident in their ability to continue. Similarly, noisier evaluation environments force referees to hold stronger beliefs to offset uncertainty. This structural definition directly links dropout dynamics to observable parameters, wages, costs, outside opportunities, and evaluation accuracy, providing a rigorous foundation for understanding career continuation in refereeing and, more broadly, in other monopsonistic labor markets with noisy performance feedback.

Behavioral biases can be understood within the model as distortions of these parameters. For example, overconfidence can be represented as an underestimation of τ^2, which makes signals appear less noisy and increases the subjective probability of achieving favorable evaluations. This bias shifts the threshold $\tilde{ɳ}$ downward, making continuation more likely than the rational model predicts. Conversely, loss aversion can be captured

as an increase in the perceived evaluation cost C, raising the required threshold $\tilde{\eta}$ and leading to earlier dropout. Likewise, present bias modifies the effective discount factor δ, assigning less weight to future payoffs and thus raising the continuation threshold. By linking these behavioral mechanisms to the structural parameters of the model, the framework highlights how deviations from rational updating systematically translate into biased thresholds, offering a unified interpretation of both normative and descriptive decision rules.

4.3. Comparative Statistics

When considering how the core parameters affect the dropout threshold:

- A higher signal precision (or transparency) makes beliefs more reliable. It implies that T^2 (variance of the signal) decreases and:

  $\delta\bar{o}/\delta T^2 > 0$

  Then, greater noise raises dropout threshold, leading to an earlier exit.
- Effort cost (C), lowers the reward of continuing:

  $\delta\bar{o}/\delta C > 0$

  Then, higher cost leads to an earlier exit.
- Opportunity cost (V_0) presents the value of outside options:

  $\delta\bar{o}/\delta V_0 < 0$

4.4. Policy Levers and Theoretical Implications

The model demonstrates that retention is not purely a function of true ability, but rather of:

- How agents believe they are doing,
- How noisy the evaluation system is, and
- What alternatives they perceive to exist.

Thus, institutions that wish to reduce premature exits in monopolistic settings (e.g., referee federations, academic programs, civil services) should:

- Increase transparency: clearer feedback reduces noise and stabilizes beliefs.
- Provide cost support: reduce the burden of effort, especially in early stages.
- Offer structured progression paths: minimize uncertainty in promotion or selection processes.
- Improve or diversify outside pathways: paradoxically, offering lateral exit options may increase initial retention if it reduces perceived trap-like dynamics.

To operationalise the theoretical framework, we translate the dynamic decision into a discrete-time logistic regression model. In this specification, the probability of continuing in a given period depends on observable characteristics and time-varying beliefs, through variables such as feedback, category progression, and institutional support.

5. Numerical Simulation and Empirical Context

In this section, we present a data-informed illustration to exemplify the model.

We choose the case of football referees in Europe employing the recent extensive survey in Aliende and Webb promoted by UEFA [31]. Football refereeing career complies with the characteristics described for our model:

- Comprises an essentially amateur career
- It is regulated by a sole employer in each country, usually the national federation/association

- Referee's decision to continue or exit is season-discrete and subject to noisy evaluation where the referees committees of the federations/associations do not count on a detailed list of indicators to provide transparency to the evaluation process.

5.1. Data

To empirically test the applicability of our belief-based model under conditions of monopolistic employment and asymmetric feedback, the extensive dataset generated by the UEFA-sponsored study conducted by Aliende and Webb offers a rare, large-scale dataset that meets the requirements of our framework by capturing both objective characteristics and subjective perceptions of referees operating within federated systems [31].

The dataset was drawn from a census-type survey of referees in England, Spain, and Italy [31], in which all registered referees were invited to participate. No ex-ante sample size calculation was needed since the study targeted the entire eligible population.

The dataset includes anonymised responses from 8545 football referees affiliated with the English, Spanish, and Italian national federations or associations, achieving substantial statistical power for logistic regression analysis.

These three systems share the structural features described in our theoretical model: each functions as a single-employer labor market, governs referee appointments through discretionary and often opaque processes, and offers no parallel private labor market for professional refereeing, characteristics broadly consistent with monopsonistic labor markets [26].

Data were collected retrospectively through an online questionnaire administered in early 2024. The survey instrument was explicitly designed to capture the biographical background, motivational profile, institutional experience, and career trajectories of active and former referees. Variables include both objective dimensions (e.g., age, gender, category achieved, years of experience, academic level, number of dependents) and subjective assessments (e.g., perceptions of institutional support, belief in promotion opportunities, motivation to continue, and perceived fairness of treatment).

This rich combination of structural and perceptual data allows us to empirically approximate the belief-updating process central to our model. In line with the behavioral literature on expectations and decision-making under uncertainty [12,32], this approach recognizes that retention decisions are shaped not only by observed outcomes but also by the agent's evolving interpretation of signals regarding institutional support, reward predictability, and opportunity costs.

Although we estimate logistic regression in Section 5.2, this empirical model is derived from the theoretical optimal stopping framework defined in Section 3. The logistic probability reflects the likelihood that an agent's belief-based expected utility from continuation falls below the utility of exiting: $V_t(\eta_t) < V_0$. Therefore, the regression captures a reduced-form approximation of the utility-maximizing decision rule under noisy evaluation and monopsonistic employment.

5.2. Empirical Approximation of the Optimal Stopping Rule: Logistic Regression

To evaluate the empirical relevance of the proposed theoretical model, we estimate a logistic regression in which the dependent variable is a binary indicator reflecting whether the referee believed their career exit was avoidable by the organisation. Specifically, the item asked respondents to indicate whether they considered their departure preventable through changes in support or evaluation processes. A response affirming this view was coded as 1 ("avoidable dropout"), while other responses were coded as 0. This dependent variable serves as a proxy for belief-based dropout, capturing the agent's perception that the system failed to respond adequately to their efforts or signals.

This strategy aligns with theoretical work suggesting that exit decisions often stem from expectation violations and unmet institutional promises, particularly under conditions of asymmetric information and constrained mobility [11,27]. In our case, the dependent variable reflects the referee's belief that, had the institutional environment been more responsive, transparent, or developmental, the decision to exit could have been averted.

Logistic regression is an appropriate method to model the referee's belief-based dropout decision as a binary outcome, given its interpretability and robustness in estimating the influence of multiple factors on discrete behavior. In addition to its widespread use in labor economics and behavioral modelling [33], logistic regression has also been applied in optimal stopping contexts, where binary decision rules (to continue or exit) must be evaluated against complex, belief-dependent thresholds [13].

Our model benefits from recent mathematical insights into the structure of optimal exit under noise. For example, Entwistle et al. emphasize how asymptotic properties of stopping times vary with the precision of feedback, a concept that supports our operationalisation of "noisy evaluation" [13].

Although a simulated method of moments (SMM) approach could, in principle, provide a closer structural test of the theoretical model by directly calibrating parameters such as signal variance, discount factor, or belief thresholds, we deliberately rely on a reduced-form logistic regression in this article. Logistic regression offers two advantages in our context: first, it provides a transparent way to map survey-based variables onto the conceptual dimensions of the model (effort, reward, opportunity cost, and noise), ensuring clear interpretability; and second, it avoids the heavy parameterisation and data demands of structural estimation, which would require stronger assumptions than our empirical setting can support. Our methodological choice therefore emphasizes transparency and empirical alignment with the survey data, rather than pursuing full structural calibration, which would exceed the scope and aims of this article. In this sense, SMM should be viewed as a promising complementary method for future research once richer longitudinal data are available, while our present approach is better suited to establishing robust empirical alignment between the belief-based optimal stopping framework and observed dropout behavior.

The regression incorporates a range of explanatory variables grounded in the theoretical framework described in Section 3.2. (Table 1). Besides the report that provides the empirical base for this study [31] that is fed by the most relevant studies on the topic of football referees careers, independent variables are substantiated by other extensive studies [34,35].

- Effort-related factors (e.g., exposure to physical threats, perceived burden of fitness demands),
- Reward-related indicators (e.g., seniority-adjusted category, highest level reached, presence or absence of promotion expectations),
- Opportunity cost measures (e.g., academic attainment, number of dependents),
- Indicators of noise and unmet developmental support (e.g., quality of relationship with the federation, participation in or desire to join development programs).

The selected variables enable us to empirically assess how career exit is influenced by perceived misalignments between effort and institutional reward, or by noisy and unmet feedback mechanisms. This empirical design is consistent with research on belief-driven labor decisions [36,37], as well as with expectations-based models of dropout and turnover. Additionally, logistic regression is well suited to analysing dichotomous outcomes in behavioral contexts [33], especially when combined with diagnostics such as AUC, pseudo R-squared, and RESET tests to evaluate model fit and robustness.

Table 1. Independent variables influencing the intention of the referee to exit.

Factor	Variable	Type	Coded as
Effort	They felt physically threatened from 0 to 3 (lowest to highest)	Ordinal	threaten
Effort	Physical requirements and tests are perceived as onerous	Binary	RtL_requir
Reward	Category divided by the number of years of experience (synthetic indicator)	Numeric	Cat_by_Seniority
Reward	Highest category reached from 1 to 10 (down to top)	Ordinal	Top_category_level
Reward	They keep promotion expectations	Binary	RtC_promo
Opportunity cost	Academic level from 0 (lowest) to 6 (highest)	Ordinal	academic_level
Opportunity cost	Number of dependents	Numeric	dependents
Noise	Relationship with the federation as a variable to stay	Binary	RtC_fedtreat
Noise	Participant in development programs	Binary	dev_prog
Noise	Non-participant in development programs but they would have liked	Binary	dev_prog_idlike

Source: The Author.

By linking these perceptual and structural indicators to the belief-based dropout proxy, the analysis provides a direct test of the central premise of this study: that subjective belief trajectories (shaped by opaque signals, limited alternatives, and institutional design) are critical in explaining voluntary career exits in monopolistic employment environments.

5.3. Results and Model Validation

By means of the software R-Studio (version 2025.05.1) we estimate the logistic regression, employing the described variables as independent variables. The regression results were as follows:

Logistic regression (GLM)—Model 1

Response variable. 'avoidable_dropout'
Level: 1 in 'avoidable_dropout'
Explanatory variables (x): 'threaten', 'RtL_requir', 'academic_level', 'dependents_', 'Cat_by_Seniority', 'Top_category_level', 'RtC_promo', 'RtC_fedtreat', 'dev_prog', 'dev_prog_idlike'
Null hyp.: there is no effect of x on 'avoidable_dropout'
Alt. hyp.: there is an effect of x on 'avoidable_dropout'

	OR	OR%	Coeff.	std.Error	ME * 100	*p*.Value	
(Intercept)			−5.274	0.432	−2.67	<0.001	***
threaten	1.493	49.30%	0.401	0.086	+0.20	<0.001	***
RtL_requir	0.625	−37.50%	−0.47	0.305	−0.23	0.124	
academic_level	1.237	23.70%	0.212	0.075	+0.10	0.005	**
dependents_	0.98	−2.00%	−0.02	0.069	−0.01	0.77	
Cat_by_Seniority	0.474	−52.60%	−0.746	0.24	−0.38	0.002	**
Top_category_level	1.255	25.50%	0.227	0.034	+0.12	<0.001	***
RtC_promo	1.466	46.60%	0.382	0.177	+0.19	0.031	*
RtC_fedtreat	4.534	353.40%	1.512	0.295	+0.77	<0.001	***
dev_prog	1.457	45.70%	0.376	0.209	+0.19	0.073	.
dev_prog_idlike	2.473	147.30%	0.906	0.197	+0.46	<0.001	***

Pseudo R-squared: 0.115, Adjusted Pseudo R-squared: 0.099
AUC: 0.763, Log-likelihood: −574.909, AIC: 1171.818, BIC: 1236.494
Chi-squared: 149.101 df(10), *p*.value < 0.001
Nr obs.: 2643
***** <0,001; ** <0.01; * <0.05; . <0.10**

In logistic regression, coefficients (Coeff.) are expressed in log-odds units, and odds ratios (OR) indicate the multiplicative change in the odds of the outcome for a one-unit

change in the predictor. However, both log-odds and odds ratios can be difficult to interpret in terms of the actual probability of the event of interest. Marginal effects (ME) translate these results into more terms by estimating how much the predicted probability of dropout changes when an independent variable increases by one unit (for continuous variables) or changes category (for binary/dummy variables), holding all other variables at their observed values.

For example, the odds ratio of 1.493 for the threaten variable indicates that referees who report being threatened are 49.3% more likely to drop out. The marginal effect expresses how much the probability of dropout itself increases (in percentage points) when referees report being threatened. In this way, marginal effects provide a direct and interpretable measure of the substantive impact of each variable on dropout probability.

The pseudo-*R*2 values obtained are relatively low, which is expected in behavioral logistic regression models where individual career decisions are influenced by multiple unobserved and idiosyncratic factors. In such contexts, low pseudo-*R*2 does not imply poor model fit but rather reflects the inherent unpredictability of individual decisions. What is more relevant are the sign, magnitude, and consistency of the estimated coefficients across specifications, which indicate robust relationships between institutional factors and the probability of career continuation. Similar magnitudes of pseudo-*R*2 are commonly reported in empirical labor economics studies using cross-sectional survey data. For instance, ref. [38] demonstrate through simulations that models with substantively meaningful effects can yield pseudo-R2 values well below 0.10, while McFadden (via summary interpretations) considers values between 0.2 and 0.4 as excellent fit.

Robustness checks were also estimated to ensure the results. For multicollinearity, VIF score ranked from 1.012 to 1.237 for all the regressors. According to Ramsey (RESET) test, it is not evident that new explanatory variables add a better adjustment (RESET = 0.92, p-value = 0.56). With an AUC of 0.76, the model demonstrates a good ability to distinguish between the two groups: referees who find their exit avoidable (coded as 1) and those who do not (coded as 0).

To address potential concerns regarding endogeneity in our model, we argue that the variables RtC_promo (promotion expectations), RtC_fedtreat (relationship with the federation), and dev_prog_idlike (desire to participate in development programs) can reasonably be treated as exogenous within our identification strategy. Although promotion expectations might be shaped by past evaluations or optimism bias, in this context they are formed in relation to the federation's formal promotion communication. Similarly, referees' perceptions of their relationship with the federation reflect accumulated institutional treatment over time, rather than post hoc rationalisations of dropout beliefs, and thus can be interpreted as structurally grounded. Finally, the desire to have participated in development programs is considered an expression of unmet opportunity, more indicative of structural program availability than of subjective dissatisfaction tied to the intention to leave. In all three cases, the variables are measured before the explicit belief-based dropout outcome, supporting their temporal and conceptual exogeneity.

However, changes in promotion rules across cohorts or regions, which shape referees' expectations without being influenced by individual dropout choices, could serve as useful instrumental variables. Likewise, differences in access to development programs, often limited by administrative capacity or selective invitations, provide an external source of identification for referees' implicit desire to participate. At the same time, although our specification emphasizes the main belief-based dropout mechanism, we recognize that heterogeneous effects may arise across subgroups of referees, such as by experience level, league tier, or age. Including interaction terms or conducting subgroup regressions

would allow us to examine whether the belief threshold operates differently across these characteristics.

Outliers could represent a problem for Model 1, since 277 observations of 2643 non-empty values from the dependent variable had omitted at least one of the values of the regressors. Then, we proceeded eliminating the non-influential variables of the regression. In Model 1, the number of observations was N = 2643 from the original 8545 answers, due to listwise deletion across a larger set of explanatory variables, as some respondents omitted items such as development-program participation. In Model 2, focusing on a smaller set of regressors increased the available sample (N = 6545). This step minimized loss of information while retaining key theoretical factors, and robustness checks confirmed that the results remained consistent across specifications.

Logistic regression (GLM)—Model 2
Response variable: avoidable_dropout
Level: 1 in avoidable_dropout
Explanatory variables (x): threaten, academic_level, Cat_by_Seniority, Top_category_level, RtC_promo, RtC_fedtreat, dev_prog_idlike
Null hyp.: there is no effect of x on avoidable_dropout
Alt. hyp.: there is an effect of x on avoidable_dropout

	OR	OR%	Coeff.	std.Error	ME * 100	*p*.Value	
(Intercept)			−5.622	0.289	−2.02	<0.001	***
threaten	1.498	49.80%	0.404	0.056	+0.15	<0.001	***
academic_level	1.456	45.60%	0.376	0.053	+0.13	<0.001	***
Cat_by_Seniority	0.365	−63.50%	−1.008	0.169	−0.36	<0.001	***
Top_category_level	1.206	20.60%	0.188	0.024	+0.07	<0.001	***
RtC_promo	1.278	27.80%	0.245	0.113	+0.09	0.029	*
RtC_fedtreat	3.32	232.00%	1.2	0.193	+0.43	<0.001	***
dev_prog_idlike	4.258	325.80%	1.449	0.128	+0.52	<0.001	***

Pseudo R-squared:0.151, Adjusted Pseudo R-squared:0.146
AUC: 0.79, Log-likelihood: −1320.18, AIC: 2656.36, BIC: 2710.652
Chi-squared: 468.172 df(7), *p*.value < 0.001
Nr obs: 6545
***** <0,001; ** <0.01; * <0.05; . <0.10**

This second model reduces the proportion of observations with high Cook's distance (466 of 6545).

5.4. Mapping of the Variables Used in the Logistic Regression

To directly address how our theoretical parameters map onto the empirical specification, Table 2 organizes the logistic regression variables under the corresponding factors from our adaptation of Holmström's model to a monopsonistic context with asymmetric information. This mapping ensures conceptual alignment between the optimal stopping framework and the observed dropout determinants.

Overall, the regression results strongly support the adapted Holmström framework we have introduced, in a monopsonistic labor market with asymmetric information, where uncompensated effort (e.g., threats) leads to exit, perceived reward stagnation weakens motivation, opportunity costs increase dropout among higher-skilled individuals, and unmet developmental expectations exacerbate the sense of disengagement.

In our adaptation, we contextualize the model to the monopsony of labor demand (referees can only work through the federation) and the presence of asymmetric information (referees receive noisy or incomplete feedback on their performance and future prospects). Our logistic regression results provide strong empirical support for the relevance of this framework.

Table 2. Results by variable and factor.

Factor	Variable	Regression Result
Effort	They felt physically threatened (threaten)	Yes | Being threatened significantly increases odds of dropout.
Effort	Physical requirements and tests are perceived as onerous (RtL_requir)	No significant effect, although with a negative coefficient, implying that those concerned with tests show a higher propensity to stay
Reward	Category divided by the number of years of experience (synthetic indicator) (Cat_by_Seniority)	Yes | Being in a higher category protects against avoidable dropout.
Reward	Highest category reached from 1 to 10 (down to top) (Top_category_level)	Refereeing in higher competition categories slightly increases dropout odds, possibly because most referees exit when they had reached their top category.
Reward	They keep promotion expectations (RtC_promo)	Finding promotion criteria is positively related to dropout, may reflect dissatisfaction considering the actual limited possibilities of promotion.
Opportunity cost	Academic level from 0 (lowest) to 6 (highest) (academic_level)	Yes | Higher academic level raises dropout odds, reinforcing the influence of opportunity cost that generates tension between academic life and refereeing.
Opportunity cost	Number of dependents (dependents)	No effect
Noise	Relationship with the federation as a variable to stay (RtC_fedtreat)	Expecting good treatment by the federation greatly increases dropout odds.
Noise	Participant in development programs (dev_prog)	No effect
Noise	Non-participant in development programs but they would have liked (dev_prog_idlike)	Yes | Very strong predictor: referees who wanted but didn't receive development support are over 4x more likely to drop out. Highlights unmet expectations.

Effort Dimension

Being physically threatened (threaten variable) shows a strong and significant positive association with dropout odds. This aligns perfectly with Holmström's premise: when high effort leads to non-monetary costs such as risk or stress without clear compensation or protection, agents are more likely to opt out.

Perceived burden of physical requirements or tests ('RtL_requir' variable) showed no effect. This suggests that general physical demands, while accounted by referees, are not sufficient on their own to trigger resilience or dropout unless accompanied by other factors.

Therefore, the overall presence of unaddressed threats acts as a negative externality on effort. Since referees cannot negotiate safer conditions (monopsony employer), dropout becomes a rational response under limited agency.

Reward Dimension

Seniority-adjusted category ('Cat_by_Seniority' variable) significantly reduces dropout odds. This aligns well with incentive theory: referees who perceive a trajectory of advancement relative to tenure are more likely to stay, as they view the system as rewarding.

Top category reached ('Top_category_level' variable), however, increases dropout risk slightly. This may reflect a non-decisive effect: once a referee reaches their peak, the marginal value of further effort diminishes. In Holmström's terms, the incentive contract becomes "flat" at the top.

Promotion as a motivational trigger ('RtC_promo' variable) was associated with increased dropout. While this could be counterintuitive, it may reflect the disillusionment due to bottlenecks or noisy promotion criteria.

Therefore, overall rewards are effective when advancement is perceived as real and attainable. When promotions are perceived as inaccessible, the system signals low returns to effort, reducing retention, especially when the employer has full control over mobility.

Opportunity Cost Dimension

Academic level ('academic_level' variable) is a strong predictor of dropout. Higher academic achievers are more likely to leave, likely because their alternative career options offer higher returns or more predictable reward structures. This matches Holmström's insight: when effort is not matched by clear returns, and agents have outside options, they leave.

Opportunity cost plays a decisive role, especially when refereeing competes with other professional pathways offering more control and fewer barriers to advancement.

Number of dependents (dependents variable) had no effect, indicating that family responsibility alone does not alter dropout decisions, perhaps because the time costs are already internalized or uniform across groups.

Noise and Feedback Dimension

Expectations of good treatment by the federation ('RtC_fedtreat' variable) strongly increases dropout odds. We interpret this as dissonant signal: referees visualise a scenario that hardly is happening, remaining frustrated by lack of support, feedback and transparency.

Participation in development programs ('dev_prog' variable) showed no effect, suggesting mere participation is insufficient to retain referees. However, having wished to participate in development programs but not having had the chance ('dev_prog_idlike' variable) is one of the strongest predictors of dropout. This underscores a key insight from Holmström: uncertainty in feedback or access to skill-building intensifies dropout risk, especially in systems with poor communication and lack of perceived fairness.

The mismatch between expectations and reality in feedback and development creates noise in the reward structure. In monopolistic systems, where referees cannot "shop around" for better support, dropout becomes a rational reaction to unmet informational needs.

As a conclusion, we have demonstrated that when we apply empirically reasonable parameter values based on referee survey data, our model predicts that retention improves when feedback is transparent, effort/reward is less costly, or outside options are weaker. These results mirror real-world observations and demonstrate the practical value of our theoretical model.

6. Discussion

This study provides a structured and analytically tractable model of career continuation decisions under monopsonistic employment and informational asymmetries. By extending Holmström's model of career concerns [1], the agent's decision to exit is modelled as a belief-based optimal stopping problem. The two key enrichments, (1) monopsony employer and (2) noisy, agent-held evaluations, lead to endogenous exit thresholds shaped by belief dynamics, feedback precision, and external opportunity values.

The central research question guiding this study is whether belief-driven dropout among referees can be explained within an optimal stopping framework adapted to a

monopsonistic labor market with asymmetric information. The evidence from our logistic regression analysis provides a partial but robust answer. Factors such as threats, promotion expectations, and unmet developmental opportunities closely align with the theoretical model's parameters, underscoring the role of beliefs shaped by noisy signals and institutional constraints. At the same time, other predictors, such as dependents or physical requirements, showed weaker effects, suggesting that the explanatory power of the framework is uneven across dimensions. Taken together, the results reinforce the model's relevance for understanding referee dropout, while highlighting areas where extensions are needed to fully capture the heterogeneity of career decisions.

The model demonstrates that dropout does not necessarily arise from underperformance, but rather emerges as a rational response to deteriorating subjective beliefs about future prospects. This complements earlier work by Dewatripont et al., Gibbons and Murphy, and Meyer and Vickers, who concentrated on incentive provision but did not explicitly model the discontinuation decision [7–9]. Our contribution lies in shifting the focus from incentive alignment to belief-dependent exit behavior, offering a dynamic threshold logic that can be used to test retention under varying institutional conditions. This perspective aligns with insights from Milgrom and Roberts, and Bénabou and Tirole, who show how individuals respond to incomplete or ambiguous signals about their own performance and ability [11,12].

Although our empirical application has focused on football referees, the mechanisms highlighted by the model, limited outside options, noisy performance evaluations, and asymmetric information, are not unique to sports. Similar dynamics have been documented in other labor markets characterized by monopsonistic structures.

For instance, Azar et al. provide large-scale evidence that labor market concentration depresses wages across diverse U.S. industries, including academia and services [18]. Schubert et al. further demonstrate that monopsonistic power plays a significant role in shaping employment outcomes in professional occupations in health and education where mobility is limited [20]. Likewise, Staiger et al. examine the healthcare sector, showing that nurses' wages are heavily influenced by employer concentration and the scarcity of alternative opportunities [4]. These studies reveal that the core features of our model, dependence on a dominant employer, uncertainty in performance feedback, and the importance of outside options, are broadly applicable across professional contexts.

Meta-analytic evidence confirms that employer wage-setting power is pervasive, while occupation-level studies across the U.S. show that higher employer concentration systematically reduces wages and hiring [17–19]. These results suggest that the mechanisms captured in the belief-threshold model are not limited to the institutional setting of refereeing but may extend to other professional environments where outside options are scarce and employers hold disproportionate power.

The propositions of this enriched model are consistent with findings across a range of empirical literatures. For example, Hustinx and Willems et al. highlight the importance of evolving contextual expectations and social belonging in volunteer retention [23,39]. Similarly, Mazerolle et al. and England et al. point to misalignment between role expectations and organisational feedback in care work and athletic training, respectively, cases in which opaque evaluation and high effort costs drive disengagement. Our model formalises these dropout mechanisms as belief trajectories shaped by imprecise signals [25,31].

From a theoretical standpoint, this approach is aligned with Sannikov's continuous-time principal–agent models but operates at the intersection of decision theory and organisational sociology [5]. Unlike models that assume transparent output and contract renegotiation, our framework accounts for unobservable performance and fixed employer structures, as seen in refereeing [22], training [16], and mission-driven careers [40].

With respect to optimal stopping theory, our work parallels that of Entwistle et al., Liu and Mu, and Hunt and Caliendo, who have shown that uncertainty over timing and payoff resolution can lead to strategic delays or early exits [13–15]. Our model applies these insights to labor dynamics and demonstrates that institutional design, particularly the transparency of feedback and the structure of continuation costs and benefits, significantly alters retention curves. The simulation results reinforce these findings, showing that greater transparency reduces dropout, and that small changes in signal variance or outside utility produce large changes in exit probability.

This approach also connects with the structural labor economics literature. In particular, Keane and Wolpin developed a dynamic discrete choice model of career decision-making, in which young individuals choose among education, employment, and military service while accumulating experience and responding to labor market signals [3]. Their model, though rooted in general labor markets, shares with ours the forward-looking nature of decision-making, the role of uncertain returns, and the endogenous nature of career transitions. What differentiates our framework is its focus on agent-held belief dynamics, rather than externally modelled productivity shocks, and the specific context of monopsonistic employment, in which exit implies withdrawal rather than switching. Nonetheless, both approaches capture how structural uncertainty and evolving expectations shape real-world career pathways.

The role of monopsony power, as described by Manning, emerges as a structural force that intensifies the psychological impact of internal evaluations [26]. In such contexts, feedback transparency becomes a form of retention infrastructure, not merely a procedural detail. The organisational implications resonate with Frey and Osterloh, who advocate for institutional designs that buffer motivation from volatility through trust, fairness, and clarity [27].

The model also engages with findings from Xu, and Willems et al., who emphasise that subjective assessments and context-dependent factors critically influence continuation across both voluntary and professional trajectories [24,39]. Dropout due to contextual misfit, volunteer persistence, and divergence between individual motives and institutional reality are all encompassed within our belief-based dropout mechanism [23,31,39]. Moreover, Giarlotta et al. highlight that context-sensitive rationality and salience effects play a crucial role in shaping decisions under uncertainty. Their insights support the idea that our belief-threshold mechanism could extend to academic and civil service careers, where performance assessments are opaque and exit opportunities restricted [30].

While the foundational insights of Kahneman & Tversky, and Camerer & Lovallo underscore the role of biases, reference points, and overconfidence in shaping economic decisions [28,29], the present article contributes by embedding these behavioral dynamics in a structured institutional context characterized by monopoly power and asymmetric information. In our framework, individuals do not exit solely due to underperformance or poor outcomes, but because their subjective beliefs—formed through noisy and often opaque evaluative signals—evolve below a personal threshold of expected success or fairness. Unlike the environments studied in Camerer and Lovallo's experimental setups, where market entry is voluntary and competition is symmetric, the context examined here features constrained mobility, institutional path dependence, and information asymmetries that interact with cognitive biases. Thus, while overconfidence may delay exit and loss aversion may accelerate it, the key innovation in our analysis lies in modelling belief trajectories under constrained feedback systems, offering a dynamic account of how persistence and attrition unfold over time in monopolistic career settings.

Finally, by framing the persistence and exit decision as a dynamic choice under uncertainty, this study also complements the formal modelling strategies outlined in

Aguirregabiria and Mira [10], offering a behavioral microfoundation to dropout decisions that are often modelled using static or frictionless assumptions.

The proposed framework offers a foundation for future research on institutional feedback reform, dynamic career contracts, and the prediction of dropout risk. By explicitly incorporating feedback opacity and constrained employment structures, it bridges optimal stopping theory with behavioral labor economics and vocational dropout research.

7. Policy Implications

The results of this study carry several actionable implications for institutions operating under monopsonistic employment conditions and facing high dropout rates due to opaque performance systems. Our belief-based framework demonstrates that agent retention is shaped not only by objective performance or effort but significantly by the transparency, frequency, and interpretability of evaluative feedback.

First, institutional feedback practices must be understood as key levers influencing career continuity. When evaluations are ambiguous or inconsistently communicated, agents are likely to experience volatile or declining beliefs about their career prospects, increasing the risk of premature dropout. Implementing systematic feedback mechanisms, such as appointment rationales, evaluation rubrics, or structured debriefings, can help agents recalibrate expectations and maintain more stable beliefs. According to the theoretical model, greater signal precision should reduce dropout thresholds. Our empirical findings are consistent with this prediction, although the cross-sectional nature of the data indicates that causal claims should be interpreted cautiously.

This has direct implications for refereeing systems, where performance is often judged informally and appointments are perceived as opaque. Federations should consider instituting feedback transparency initiatives, including standardized observer assessments, performance dashboards, and mentoring structures that make career trajectories clearer. These interventions are also transferable to academia, where early-career researchers face uncertain progression and ambiguous review outcomes, and to public service roles with delayed promotion and performance feedback cycles.

A further implication relates to institutional design. Organisations may consider redesigning their evaluation procedures to balance perceived fairness with performance sensitivity. This involves not only clarifying criteria but ensuring timely, interpretable, and developmental feedback. As shown in our model, better signal quality not only informs agents but encourages them to stay engaged longer.

In addition to improving the objective quality of feedback, institutions must consider how individuals process and interpret signals. Behavioral insights show that people respond strongly to feedback that violates their expectations, particularly when it is perceived as a setback. Ambiguous or inconsistent messages can erode confidence and lead to premature disengagement, not necessarily because of actual underperformance but due to belief instability. Conversely, overly optimistic feedback can create inflated expectations that later collapse when reality contradicts them. This highlights the need for feedback practices that are not only precise but also well-calibrated, contextualized, and framed in a way that promotes constructive belief updating. By reducing perceived losses and uncertainty, institutions can support more resilient commitment over time.

These implications emphasise that individuals are not passive recipients of feedback, but active interpreters whose persistence is shaped by evolving expectations and interpretations of their own trajectory. Interventions that reduce ambiguity, manage expectations, and promote realistic self-assessment are therefore central to retaining talent under conditions of constrained mobility.

Also, institutions should acknowledge the potential unintended effects of inflexible retention strategies. When individuals feel trapped or unable to safely revise their career path, they are more likely to exit entirely—even in the absence of poor performance. Providing visible off-ramps, lateral moves, or re-entry opportunities reduces the perceived risks of staying and allows individuals to interpret persistence as a flexible and agentic choice rather than a structural imposition. Paradoxically, the availability of exit options can increase long-term engagement by creating a sense of control and psychological safety.

The integration of behavioral insights into models of career persistence under monopolistic employment structures suggests several key implications. First, institutions should recognize that how feedback is framed, not just its content, can significantly influence individuals' perceptions of progress and fairness. Even objectively neutral evaluations may be interpreted as setbacks if they are misaligned with prior expectations or delivered in ambiguous terms. Second, overconfidence and self-selection highlight the need for more transparent and calibrated feedback mechanisms that help individuals update beliefs realistically over time. Without clear signals and credible benchmarks, individuals may persist too long based on inflated expectations or exit prematurely due to misperceived stagnation. These findings underscore that career exit decisions are not purely rational responses to performance or mobility constraints, but are also shaped by subjective belief dynamics. Institutions seeking to improve retention, especially in systems with limited external mobility, must therefore attend not only to structural conditions but also to the psychological and informational environment in which individuals interpret their trajectories.

8. Conclusions

This article has presented a novel adaptation of Holmström's dynamic incentive framework to explain dropout behavior in career paths governed by monopsonistic employment and asymmetric information. Through the development of a belief-based optimal stopping model, we show that individuals decide whether to continue or exit their careers not merely based on objective performance or outcomes, but on evolving subjective beliefs shaped by noisy signals and constrained mobility.

Empirical results based on logistic regression analyses using European football referee data provide strong support for the theoretical model. Specifically, we find that dropout is more likely when agents experience unreciprocated effort (e.g., feeling physically threatened), perceive rewards as stagnant or unfairly distributed (e.g., lacking meaningful promotion opportunities), face high opportunity costs (e.g., higher academic credentials), or experience unmet expectations in developmental feedback. Conversely, perceived progression (e.g., category advancement relative to seniority) protects against exit. Notably, the model successfully reproduces key behavioral patterns found in monopsonistic and opaque institutional settings, providing practical insights into retention dynamics.

The relevance of these findings spans three domains. Theoretically, our contribution lies in integrating optimal stopping logic with belief-updating under asymmetric feedback, extending Holmström's framework to discontinuous career decisions. This allows for analytical characterisation of dropout thresholds and their dependence on transparency, effort cost, and outside option valuation. Methodologically, our simulation and empirical validation bridge abstract theory with concrete vocational behaviors. Institutionally, the results underscore how dropout in monopolistic systems is not a sign of individual failure but often a rational response to misaligned beliefs and perceived stagnation, emphasizing the importance of feedback quality, progression clarity, and developmental equity.

Limitations of the model include its assumption of rational updating and a stationary environment. In practice, individuals may experience emotion-driven belief shifts, overreact to salient events, or be influenced by social dynamics [41,42]. Furthermore, em-

ployer feedback policies are treated here as exogenous, though in reality they may evolve strategically in response to observed attrition patterns.

Future research should explore several promising directions. First, incorporating multi-agent simulation can reveal how belief contagion, social comparisons, or peer norms influence career continuation in closed systems. Second, endogenising employer behavior, e.g., allowing federations or institutions to adapt their feedback or promotion strategies over time, would deepen understanding of retention dynamics. Third, contract theory extensions could consider long-term incentive-compatible structures under belief-driven dropout risks, offering normative models for career design in academia, voluntary service, and federated professions. Finally, further empirical validation using other institutional settings noted in this study (e.g., civil service, health, teaching, non-profit management) would test the model's generalizability and refine its policy recommendations. Also, the dependent variable, modeled through stated intentions rather than observed dropout behavior in this study could incorporate actual dropout events over time, which would provide stronger validation of the model's predictive capacity.

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