Cosmological tensor perturbations in theories beyond ΛCDM

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Abstract. We study for the first time a complete analysis of the imprint of tensor anisotropies on the Cosmic Microwave Background for a class of $f(R)$ gravity theories within the CAMB-PPF framework. Herein we present the most relevant equations, both for the cosmological background and gravitational wave perturbations, taking care to include all effects which arise from $f(R)$ modifications. We find that that the dominant contribution to deviations from General Relativity in the temperature and polarization spectra can be attributed to modifications in the background. This demonstrates the importance of using the correct background in perturbative studies of $f(R)$ gravity.

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INTRODUCTION

Modified gravity [1] has been shown to be able to mimic both the dark energy and the inflationary eras [2]. However the sole use of large scale observations (Ia type supernova, baryon acoustic oscillations, or the cosmic microwave background), which only depend upon the expansion history of the Universe is not enough to determine uniquely the nature and the origin of dark energy. In other words, identical evolutions for the cosmological background can be explained by a diverse number of theories. This is the so-called *degeneracy problem*, whose breaking requires the use of measurements that are not only sensitive to the expansion history but, among others, the evolution of scalar perturbations [3], the stability of cosmological solutions against small perturbations [4] and the existence of GR-predicted astrophysical objects such as black holes [5].

In this sense, the simplest and in fact the most studied modification of the Hilbert-Einstein action is generalized to a general function of the Ricci scalar R, the so-called $f(R)$ gravities [6]-[7]. In addition to reproducing the entire cosmological history [8] and despite some shortcomings [7], these theories may behave quite well on local scales, where the GR limit must be recovered [9].

The study of the CMB tensor perturbations in alternative gravity theories has not received much interest in comparison with the study of scalar perturbations. For the latter case, the required tensor perturbed equations are usually of higher order and the analysis often relies on the use of simulations performed by several codes available such as CAMB [10] or CMB-Easy, both based on CMBFast [11].

Several attempts have been made for a number of modified gravity scenarios, namely cosmic strings [12] and brane-world theories [13, 14]. In the case of $f(R)$ theories, the only investigation was presented in [15], where authors considered a flat thick domain wall branes supplemented with $f(R)$ gravity. Nonetheless the full calculation of tensor perturbations for $f(R)$ theories in metric formalism has never been presented before. In the present investigation we sketch the main features and steps to study this issue. We refer the reader to [16] for further and more detailed explanations.

GENERALITIES AND DYNAMICS

The most general action for fourth order gravity [7] can be written as an analytic function of the Ricci scalar only :

$$\mathscr{A} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[f(R)+\mathscr{L}_m\right], \tag{1}$$

where $\mathscr{L}_m$ represents the matter contribution. Varying the action with respect to the metric provides the generalized Einstein equations in the metric formalism:

$$f'G_{ab} = T^m_{ab} + \frac{1}{2}\left(f - Rf'\right)g_{ab} + \nabla_b\nabla_a f' - g_{ab}\nabla_c\nabla^c f', \tag{2}$$

where $f \equiv f(R)$, the prime holds for derivative with respect to R and T^m_{ab} represents the stress-energy tensor of standard matter. Expression (2) can be recast as,

$$G_{ab} \equiv R_{ab} - \frac{1}{2}R\,g_{ab} = T^{tot}_{ab} = \tilde{T}^m_{ab} + T^R_{ab}, \tag{3}$$

where explicit expressions for $\tilde{T}^m_{ab}$ and T^R_{ab} can be easily found from (2). These quantities respectively represent two effective *fluids*: the *effective matter fluid* (associated with $\tilde{T}^m_{ab}$) and the *curvature fluid* (associated with T^R_{ab}).

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In the matter frame u_a the total energy momentum tensor can be decomposed as,

$$T^{tot}_{ab} = \mu\, u_a\, u_b + p\, h_{ab} + q_a\, u_b + q_b\, u_a + \pi_{ab}\,, \tag{4}$$

where the symbols in the previous expression keep their usual meanings. In terms of the aforementioned effective fluids, the thermodynamical quantities can be written as,

$$\begin{aligned} \mu &= \frac{\mu^m}{f'} + \mu^R\,, p = \frac{p^m}{f'} + p^R\,, q_a = \frac{q^m_a}{f'} + q^R_a\,, \\ \pi_{ab} &= \frac{\pi^m_{ab}}{f'} + \pi^R_{ab}\,, \qquad (5) \\ \pi^R_{ab} &\equiv \frac{1}{f'}\left(f''\widetilde{\nabla}_{\langle a}\widetilde{\nabla}_{b\rangle}R + f'''\widetilde{\nabla}_{\langle a}R\widetilde{\nabla}_{b\rangle}R - \sigma_{ab}f''\dot{R}\right) (6) \end{aligned}$$

The effective thermodynamical quantities for the curvature fluid are presented in [16].

The background and tensor perturbations dynamics

For homogeneous and isotropic, i.e. Robertson-Walker, space-time with vanishing 3-curvature and barotropic perfect fluid – with equation of state $p = \omega\rho$ – as the standard matter source, the independent field equations for general $f(R)$ gravity can be written as

$$\begin{aligned} &\dot{\Theta} + \frac{1}{3}\Theta^2 + \frac{1}{2\,f'}(\mu^m + 3p^m) + \frac{1}{2\,f'}(\mu^R + 3\,p^R) = 0\,, \\ &\Theta^2 = 3\,\frac{\mu^m}{f'} + 3\,\mu^R\,,\ \dot{\mu^m} + \Theta\,(\mu^m + p^m) = 0. \qquad (7) \end{aligned}$$

The linearization of the exact propagation and constraint equations around this background for pure tensor perturbations then leads to the system [17]:

$$\dot{\sigma}_{ab} + \frac{2}{3}\,\Theta\,\sigma_{ab} + E_{ab} - \frac{1}{2}\pi_{ab} = 0\,, \tag{8}$$

$$\dot{H}_{ab} + H_{ab}\,\Theta + (curl\,E)_{ab} - \frac{1}{2}(curl\,\pi)_{ab} = 0\,, \tag{9}$$

$$\begin{aligned} &\dot{E}_{ab} + E_{ab}\,\Theta - (curl\,H)_{ab} + \frac{1}{2}\,(\mu + p)\,\sigma_{ab} \\ &+\frac{1}{6}\Theta\,\pi_{ab} + \frac{1}{2}\dot{\pi}_{ab} = 0\,, \qquad (10) \end{aligned}$$

together with the linearized conservation equations for the effective matter and curvature fluids (see [16] for detailed expressions). Taking the time derivative of equations (8)-(10) and performing a harmonic decomposition, the system reduces to

$$\begin{aligned} &\ddot{\sigma}_k + \Theta\,\dot{\sigma}_k + \left(\frac{k^2}{a^2} - \frac{\mu + 3\,p}{3}\right)\sigma_k \\ &= \frac{a}{k}\left(\mu\,\dot{\pi}_k - \frac{\mu + 3\,p}{3}\,\Theta\,\pi_k\right). \qquad (11) \end{aligned}$$

On the other hand, from equation (6) and for pure tensor modes, the anisotropic pressure takes the form

$$\pi^R_k = -\frac{k}{a\,\mu}\,\frac{f''\,\dot{R}}{f'}\,\sigma_k\,. \tag{12}$$

Unlike the scalar counterparts for $f(R)$ theories [3], where the involved equations are usually fourth order, the last equation (12) guarantees that the tensor perturbations equations are governed by a second order differential equation.

The initial conditions

In the radiation dominated era, the anisotropic stress π is dominated by the radiation fluid contribution. Therefore, in this scenario and after having assumed homogeneity of the early universe, i.e. that the radiation anisotropic stress vanishes, equation (11) can be further simplified and reduces to

$$\frac{\mathrm{d}^2\sigma_k}{\mathrm{d}\tau^2} + (aA - aH)\,\frac{\mathrm{d}\sigma_k}{\mathrm{d}\tau} + a^2B\sigma_k = 0\,, \tag{13}$$

where the coefficients A and B are defined in [18] and τ holds for conformal time. By performing the variable change $u_k = a^m\sigma_k$, equation (13) reads

$$\frac{\mathrm{d}^2u_k}{\mathrm{d}\tau^2} + \left(-\frac{1}{2}maH\frac{f''}{f'} - \frac{m}{a}\frac{\mathrm{d}^2a}{\mathrm{d}\tau^2} + a^2B\right)u_k = 0\,. \tag{14}$$

where we chose $m = \frac{A-H}{2H}$. Note that in the derivation of the previous equation, the exponent m has been assumed to be constant as is the case for R^n models.

BACKGROUND AND TENSOR PERTURBATIONS FOR R^n MODELS

In order to illustrate the formalism described in the previous section, we considered the one-parameter class of gravity theories, given by $f(R) = R^n$.

Background and evolution equations

The background setup of these models can be studied following the dynamical system method [17, 19]. Let us define the following set of dynamical variables :

$$x = \frac{\dot{R}(n-1)}{HR}\,,\ y = \frac{R(1-n)}{6nH^2}\,,\ \Omega_{d,r} = \frac{\mu_{d,r}}{3H^2nR^{n-1}}\,, \tag{15}$$

where $\mu_{d,r}$ are dust and radiation densities respectively. In terms of these variables, the Friedmann equation in (7) takes the simple form,

$$1 + x + y - \Omega_d - \Omega_r = 0\,. \tag{16}$$

Thus, an autonomous system, which is equivalent to the cosmological equations (7) can be derived by differentiating the dynamical variables defined in (15) [16] and the constraint equation (16) can be used to reduce the dimensionality of the system. By imposing initial conditions deep in the radiation-dominated era as the ones given by ΛCDM model one can get values of the exponent n that evolve in a similar way to the ΛCDM. In order to illustrate this procedure, we chose the value $n = 1.29$.

Perturbations setup

For $f(R) = R^n$ ($n \neq 2$), the equation (14) reduces to [18],

$$\frac{\mathrm{d}^2 u_k}{\mathrm{d}\tau^2} + \left(k^2 - 2\,\tau^{-2}\right) u_k = 0\,, \tag{17}$$

where $m = \frac{2-n}{n}$, due to the fact that for R^n models, the scale factor in the radiation dominated era satisfies $a(\tau) = \tau^{\frac{n}{2+n}}$ [20] and therefore the parameter m is constant as it was assumed in order to obtain (14). The result in (17) is exactly the same as the one for tensor perturbations in GR [10]. Thus, equation (12) becomes

$$\pi_k^R = -\frac{k}{a^2}\frac{(n-1)}{\mu R}\frac{\mathrm{d}R}{\mathrm{d}\tau}\sigma_k. \tag{18}$$

CMB SPECTRA FOR R^n MODELS

We used the latest version of CAMB, known as CAMB-PPF [21]. The equation of state for the dark energy contribution is provided to the code through a data file. In our case, the curvature fluid is expected to play the role of dark energy in the $f(R)$ theories, thus equation (7) can be used to generate a data file for the equation of state. This procedure was usually missing in previous investigations that for the sake of simplicity assumed GR background when studying the tensor perturbations of modified gravity theories. Once the correct background and perturbation evolution is implemented, there exist notable differences in both the c_l^{TT} and c_l^{EE} coefficients which are produced by modifications in the background and tensor perturbations, when compared to the usual GR calculations. Let us consider the value $n = 1.29$ and summarise the results as follows:

c_l^{TT} features

According to Figure 1 (left panel), we see that the amplitude for the c_l^{TT} coefficients is suppressed for large l's with respect to the usual GR simulations by approximately one order of magnitude, when passing from $2\cdot 10^{-3}$ (GR) to $5\cdot 10^{-4}$ ($f(R)$) at $l \approx 3000$. For small l's, the amplitude remains approximately at the same value found in GR. We also observe a horizontal shift to smaller $l's$ for the modified c_l^{TT} with respect to the GR simulations at intermediate scales ($l \approx 100-200$). Concerning the number of relative maxima and minima, we find they remain invariant although their location is shifted to the left. Finally, by studying separately the simulations involving only modifications in the background or in the perturbations, we notice that the reduction can be attributed mainly to the modification of the background evolution. The modification introduced just by the $f(R)$-perturbations is negligible.

c_l^{EE} features

Unlike the c_l^{TT} patterns, there is a slight amplitude suppression at the lowest $l's$ with respect to the GR case. The number of relative maxima-minima remains invariant, although their location is shifted to the left. We note that the amplitude of thec_l^{EE} coefficients is suppressed for large l's with respect to GR by about one order of magnitude when passing from 10^{-4} (GR) to 10^{-5} ($f(R)$) at $l \approx 3000$. By studying separately the simulations involving only modifications in the perturbations and keeping the background as GR, we note that the reduction can be attributed mainly to the modification of the background evolution introduced by the $f(R)$ models. All these feature can be seen by straightforward comparison with the right panel in Figure 1.

CONCLUSIONS

In this work we have presented a detailed analysis of the CMB features for a simple class of $f(R)$ modified gravity theories using the CAMB-PPF implementation. These simulations used the correct cosmological background evolution as provided by these fourth order gravities as well as the required tensor perturbations equations. Our results demonstrate the importance of considering the correct background when alternative theories of gravity are subjected to this kind of analysis.

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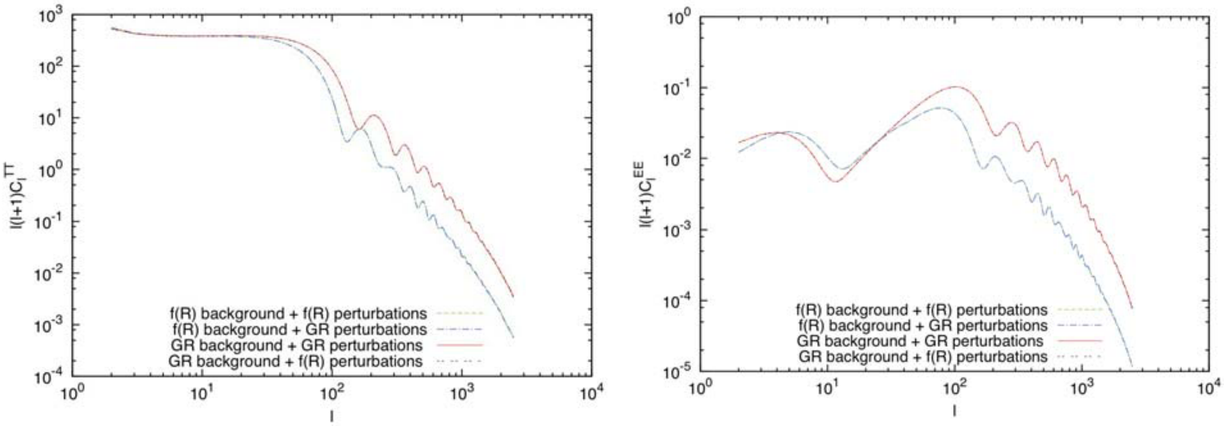

FIGURE 1. The temperature (left panel) and electrical (right panel) power spectra for tensor perturbations in all the possible background and perturbations scenarios. R^n model for $n = 1.29$: It is observed how the most relevant suppression comes from the $f(R)$ background consideration.

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