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THE SYMMETRY UNDERLYING REAL INTEREST RATE BEHAVIOUR AND THE LINK TO INVESTMENT FLOWS: AN EX ANTE FORMAL TREATRMENT

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ABSTRACT

From a purely speculative approach and under the usual assumptions, a well-known symmetrical structure appears, connecting neoclassical and Keynesian views of the markets. This framework admits graphical and formal explanation.

In previous work, we addressed this topic reaching some conclusions. Now that the credit bust spreads worldwide, we focus on formal analysis leading to more advanced results linked to our previous perspective that seems to hold. Using an ex ante formal treatment, we conclude that when applied to explain real interest rate behaviour, this symmetrical look shows a countercyclical pattern of response for this variable in neoclassical approach, while being procyclical from Keynesian view. This implies either a magnifying or a stabilizing role for the real rate in each case and could affect the financial to real investment flows ratio and, as a result, aggregate capital stock composition. The trend this ratio could follow, though difficult to explain, is of great interest to help explain the behaviour of financial markets. This appears as a key feature to approach the focal points of the financial markets reform.

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I. INTRODUCTION

Prior to consider the behaviour of the real interest rate from neoclassical or Keynesian views and under particular circumstances, we need to assume that financial markets tend to behave in a more or less competitive way, and that Central Bank's reference rate follows the path of a hypothetical 'notional rate' which would prevail if the perfect competition market-clearing conditions were met.

Under these circumstances, when the economy undergoes an expansive trend, caused by a rise in aggregate demand, the real rate shows a different pattern of behaviour depending on the approach applied; neoclassical or Keynesian. As a result, the ratio of financial to real investment flows, and the composition of the capital stock in the economy, could become affected.

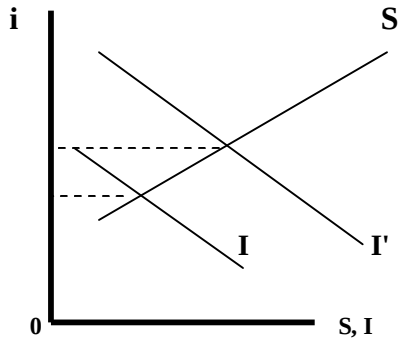
In previous work, we addressed this problem using mainly graphical tools and reaching some general conclusions in terms of a symmetrical structure connecting neoclassical and Keynesian approaches, which comes to light. Now we focus on a formal approach leading to a similar outcome that seems to hold. These ideas could show a narrow link to recent developments in international financial markets.

Our previous set of conclusions using graphical analysis

1. **In the figure below we depict the supply and demand schedules in neoclassical market for loanable funds and in Keynesian market for money. Market for goods and services and Phillips curve are also shown**

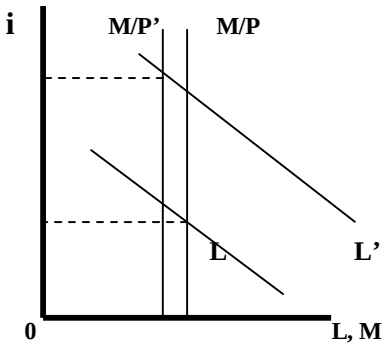
for each perspective.

NEOCLASSICAL MODEL

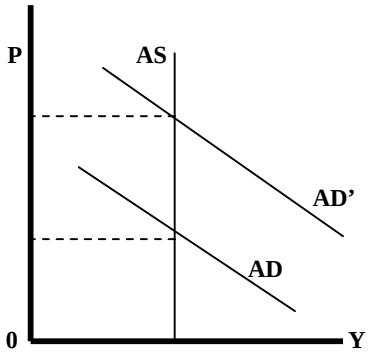


Market for Loanable Funds

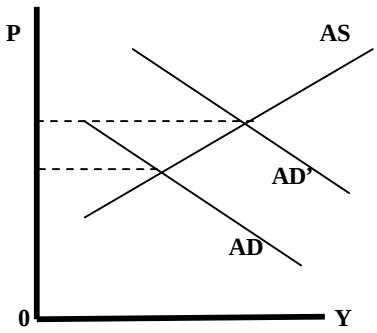
KEYNESIAN MODEL



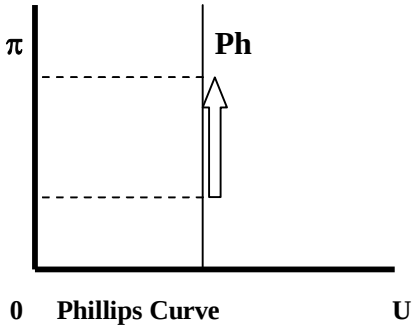
Money Market

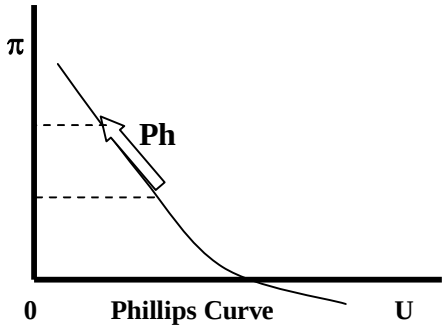


Market for Goods and Services



Market for Goods and Services





An exogenous expansive shock will produce a shift to the right in demand schedules, as well as a change in the corresponding equilibrium positions on the Phillips curve. Taking into account the different slopes

and elasticities of aggregate supply and Phillips curves prevailing in each model, we can compare the reaction of both nominal interest rate and inflation rate, to reach some conclusions on the probable behaviour of the real interest rate, after the occurrence of an expansive exogenous shock the economy is supposedly undergoing. We can do this by virtue of Fisher's equation:

$$i_t = r_t + \pi_t^e$$

or

$$r_t = i_t - \pi_t^e$$

Assuming that agents make their conjectures following a process of expectations, we will approximate the expected rate by the ex post effective inflation rate, which allows us to arrive to the real interest rate by rewriting Fisher equation as:

$$r_t = i_t - \pi_t$$

Where: $r_t =$ **real interest rate for period** t

$i_t =$ **money interest rate for period** t

2. **When the economy undergoes an expansive shock -like the one depicted in the figure below-, the neoclassical Say's Law supply side approach, predicts such a comparative evolution of both nominal interest rate and inflation rate, that would probably cause the real interest rate to decrease. On the contrary, the Keynesian approach predicts a probable increase in the real rate of interest.**

3. **On the other hand, when a bust takes place, the neoclassical approach**

predicts a probable increase, in the real rate of interest, and the Keynesian demand side view, a probable decreasing trend for the real interest rate.

4. **As a result, and in spite of the fact that in economic literature real rate might be considered an acyclical variable, in a graphical ex ante analysis, its pattern of behaviour appears to be countercyclical in neoclassical Say's Law approach, while being procyclical from Keynesian view.**

5. **The different response of the real interest rate to exogenous demand side shocks, owes its explanation to the relative elasticity values of supply and demand functions considered in the analysis, and to the slope and elasticity of the Phillips curve in each case. The latter reflects the conditions in the markets for production factors, the degree of monetary illusion amongst workers, the particular assumptions on expectations, or the level of unemployment and its associated variance (Fernández Díaz, 1977). But the different elasticity values of the functions considered in the analysis correspond to the graphical translation of the initial assumptions and hypotheses of each model, owing them in no little part their explanation. Thus, the symmetry affecting the real rate pattern of behaviour and connecting both models is related to the particular set of assumptions of each, at least to a certain extent.**

6. **It remains to see whether or not is one of them today, and in the foreseeable future, more capable than the other to portray in its assumptions a more precise grasp of the economic reality.**

II. A SIMPLIFIED EX ANTE FORMAL ANALYSIS

Following a purely speculative perspective and initiative, we intend to show that our previous conclusions hold, when using a formal approach. In order to do so, we shall formalize both models, which will enable us to reach more obvious results, and express them in a more accurate form.

We focus first in the relationship between inflation rate and nominal interest rate. According to economic literature, both seem to follow a similar path throughout the time being procyclical and with some lags. An increase in the former, usually takes place when the latter as well as the output level are also increasing, and vice versa.

Nevertheless, the data doesn't seem to confirm the existence of a Fisher Effect. In other words, the inflation rate surge doesn't meet an identical nominal interest rate upturn. As a result, the real rate of interest doesn't remain steady, but follows a certain path depending on which of the former increments prevails. Thus, we shall compare both for each model.

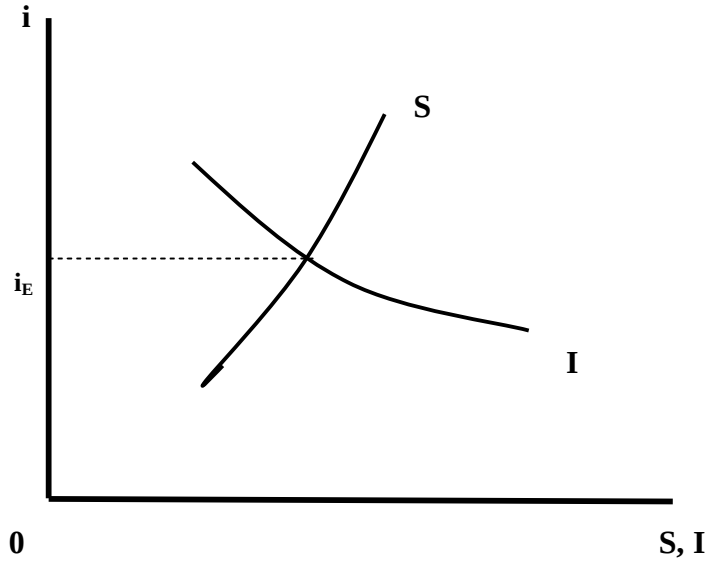
We will use formal analysis to observe changes in nominal interest rates and in effective inflation rates in response to an increase in aggregate demand, associated to an expansive trend the economy might be undergoing. In this way, comparing them we will observe the final effect on the real interest rate. However, in order to perform this task, each approach requires a particular set of analytic instruments and a particular set of assumptions on elasticity values.

From Keynesian demand side approach, changes occurring in real interest rates are studied by means of IS-LM analysis that underlies the Neoclassical-Keynesian Synthesis Model. Against this background, the elasticities express a positive price-sensitivity of the aggregate supply, a relatively high response of savings and investment

levels to changes in national income, and a relatively poor response of both to the nominal interest rate.

In the money market, the speculative component of money demand surpasses the transactional component, the supply of real money balances being exogenous and thus, inelastic to the nominal interest rate. The Phillips curve shows positive absolute values of inflation-elasticity, and a negative slope in a context of monetary illusion and adaptive expectations. **Although modern contributions by new Keynesians have been built on strong micro foundations incorporating the rational expectations hypothesis, it is not our intention to introduce here but the elementary versions. This will keep our attention focused on the underlying *from demand to supply* logical framework identifying Keynesian thought, and differing from the *from supply to demand* Say's law neoclassical approach.**

On the contrary, from neoclassical Say's law approach, IS-LM analysis cannot be used because of its incompatibility with the postulate of the neutrality of money and the loanable funds theory of interest. From this approach, according to neoclassical elasticities, income has little influence on savings supply and on investment demand, both being more clearly influenced by the money interest rate.



Loanable funds market. (Where i_E = equilibrium nominal interest rate)

Against this background the price elasticity of aggregate supply is null. The Phillips curve, being perfectly inelastic to inflation, shows a vertical shape over the natural unemployment rate (NUR), in a context of absence of monetary illusion.

Similarly, when considering the neoclassical view it won't be our intention, to introduce but an elementary version of the model. This will keep our attention focused on the underlying *from supply to demand* Say's law logical framework, underlying neoclassical thought.

1. Keynesian demand side approach. A simplified formal treatment

When examining this view, we need to admit some particular assumptions:

1. The economy reaches its equilibrium far from full-employment output level in a context of monetary illusion.

2. As a result, aggregate supply schedule has a positive slope showing positive price-elasticity value, while Phillips curve is downward sloping. Both reflect the market conditions associated to equilibrium with involuntary unemployment, in a context of monetary illusion. The slope of both curves is relatively reduced in absolute value.

3. The agents are supposed to form their conjectures on the evolution of price level following a process of adaptive expectations. As a result, we can approximate the expected inflation rate by **the ex post effective inflation rate**.

Thus,

$$\pi^{e}{}_{t} = \pi_{t}$$

where π stands for current or expected inflation rate.

4. It exists an underlying *from demand to supply* logical framework, opposite to Say's law. Not all savings are transformed into investment, because agents can hold idle money balances.

5. The money interest rate is no more determined in the market for loanable funds but in the money market, with an exogenous real money balances supply under the control of the Monetary Authority.

Taking into consideration the previous assumptions, the expressions reflecting macroeconomic equilibrium are;

1.1. Aggregate demand

The equilibrium in the market for goods and services, IS schedule, occurs when:

$$Y = \bar{\alpha}(A - bi)$$

where

Y: Aggregate level of output.

$\bar{\alpha} = \dfrac{1}{(1 - c(1-t) + m)} =$ Aggregate autonomous expenditure multiplier in open economy.

A = Aggregate autonomous expenditure in open economy

b = interest-elasticity of investment demand

The equilibrium in the Money market, LM Schedule, takes place when:

$$\frac{M}{p} = kY - hi$$

or: $$Y = \frac{1}{k}(\frac{M}{p} + hi)$$

where

k = Income-elasticity of the demand for real balances

$\frac{M}{p}$ = Supply of real balances

h = Interest-elasticity of the demand for real balances

The equilibrium in both markets may be expressed by means of the aggregate demand function.

$$Y = \beta A + \gamma \frac{M}{p} \quad \textbf{(1)}$$

where

$\beta = \frac{1}{(\frac{1}{\alpha} + b\frac{k}{h})}$ = Fiscal policy multiplier including monetary sector and Hicksian crowding-out

$\gamma = \frac{\frac{b}{h}}{(\frac{1}{\alpha} + b\frac{k}{h})}$ = Monetary policy multiplier

1.2. Aggregate supply

From this Keynesian approach, the supply curve is upward sloping and shows positive values for price-elasticity. From this demand-side view, the equilibrium level of output is first determined within the demand side of the economy by means of the IS-LM analysis. Then solving the production function, we can obtain the employment level needed to reach the equilibrium level of output. The former, in general not coincident with full employment level, will be remunerated at marginal labour productivity.

If an increase in production takes place, the additional employment needed to support the expansion, is more than proportional to the increase in output. This shows that the former can only occur under increasing unit labour costs conditions. And assuming that the mark-up pricing equation over the unit labour cost, is,

$$P = (1+\rho)\frac{w}{a} \qquad \textbf{(2)}$$

Where

$w =$ Nominal wage

$\rho =$ Mark up level ($\rho > 0$ if the firm has some kind of market power)

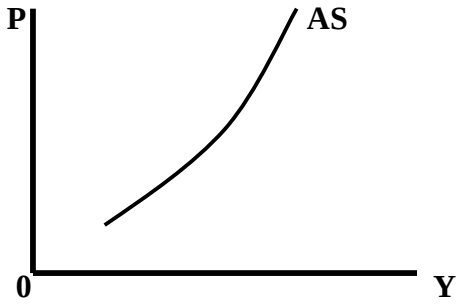
$a = \frac{Y}{L} =$ Average labour productivity

If ρ, remains steady; the rise in unit labour costs would be passed-through to final prices, which implies an upward-sloping aggregate supply curve,

$$Y = F\left(p\right) \qquad \textbf{(3)}$$

Where

$$\frac{dY}{dp} > 0$$



Until now we have:

$$Y = \beta A + \gamma \frac{M}{p} \qquad \text{(Aggregate demand)} \qquad (1)$$

$$Y = F(p) \qquad \text{(Aggregate Supply)} \qquad \textbf{(3)}$$

Now we have to bear in mind that our goal is no other than to determine the effect of a change in aggregate demand -associated to an either expansive or restrictive trend of the economy-, on the nominal interest rate and on the inflation rate. And afterwards, by calculating the difference between the both, we will observe the effect on the real interest rate.

In order to do so, let's begin by considering an increase in aggregate autonomous expenditure (A), linked to an expansive trend the economy is supposedly undergoing. According to the aggregate demand schedule, (1) the income and production level would increase by

$$dY = \beta dA$$

And according to the expression of the aggregate supply (3), that can be rewritten as follows:

$$p = f(Y)$$

The following change in prices would occur,

. $$dp = f'(Y)dY$$

or,

$$dp = f'(Y)\beta dA \quad \textbf{(4)}$$

Where

$dY = \beta dA$

1.3. Change in inflation rate

The change in the price level accelerates inflation rate. This change in inflation rate has to be compared to the one affecting nominal interest rate, in order to observe the evolution of real interest rate which constitutes our main concern.

But the change in inflation rate has to be observed through Phillips curve that in Keynesian view shows a regular downward sloping profile because the effective inflation rate differs from its expected level (i.e., $\pi_t^e \neq \pi_t$)

According to expectations-augmented Phillips Curve:

$$U_t = U_{NUR} + \phi\left(\pi_t - \pi_t^e\right) \quad \textbf{(5)}$$

where

U_t = Current Unemployment level

U_{NUR} = Natural Unemployment Level

$-1 > \phi$ = Adjustment parameter, that depends on the proximity of the economy's productive potential to the full employment level. When $|\phi| > 1$ the economy is relatively far from full employment as is seems in line with Keynesian demand side approach opposite to Say's law. The downward sloping Phillips curve would in this case show a relatively flat profile.

It is possible to put (5) as an expectations-augmented aggregate supply curve

$$Y_t = \overline{Y_t} - \phi(\pi_t - \pi_t^{\,e})$$

where

$\overline{Y_t} =$ NUR related income level.

The latter is the 'surprise' aggregate supply schedule, representing the output firms are willing to produce at different inflation rates. This curve considers together the production function and the employment decisions in the labour market, and can be rewritten as

$$\pi_t = \pi^{e}{}_t - \varepsilon\left(Y_t - \overline{Y_t}\right) \qquad \textbf{(6)}$$

where

$$-1 < \varepsilon = \frac{1}{\phi} < 0$$

This relationship will allow us to obtain the first derivative of the inflation rate, with respect to the output level.

$$d\pi_t = -\varepsilon dY_t$$

where

$$1 > -\varepsilon > 0$$

In other terms,

$$d\pi_t = -\varepsilon\beta dA_t \qquad \textbf{(7)}$$

where

$$dY_t = \beta dA_t$$

The expression (7) is a measure of the change in inflation, caused by an upsurge in output level in response to an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

When $|\varepsilon\beta|$ alternatively surpasses, equals or falls behind the unit, the changes in inflation rate will be respectively more than proportional, proportional or less than proportional to the changes in aggregate autonomous private expenditure (the changes in aggregate demand). From an *ex ante* approach it is only possible to affirm that in the Keynesian demand side view, opposite to Say's law, $-\varepsilon\beta$ would be greater than zero. Nonetheless, the Keynesian approach opposite to Say's law, implies that the economy is relatively far from full employment and from the economy full productive potential level, which suggests $|\varepsilon|<1$ in line with a relatively flat downward sloping profile for the Phillips curve.

Moreover, $-\varepsilon\beta$ would normally be greater than one ($-\varepsilon\beta>1$), because β (autonomous expenditure multiplier) is always greater than one.

As we could expect, inflation rate seems procyclical.

1.4. Change in nominal interest rate

Now we focus on the increase in nominal interest rate taking place in the money market, because of the excess demand caused by the elevation of the transactional component of money demand in response to the expansion in output level in response to an upsurge in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

In the equilibrium equation for the money market,

$$\frac{M}{p} = kY - hi$$

we solve for the nominal interest rate:

$$i = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$$

In this expression, output –through the money demand-, and prices -through the money supply-, are the two variables affecting the nominal interest rate. Therefore, we shall derive the expression with respect to production and prices.

$$di = \frac{k}{h}dY - \frac{1}{h}d\left(\frac{M}{p}\right)$$

Substituting:

$$di = \frac{k}{h}\beta dA - \frac{1}{h}d\left(\frac{M}{p}\right)$$

Where

$$\beta dA = dY$$

or,

$$di = \frac{k}{h}\beta dA + \left(\frac{M}{p^2 h}\right)dp$$

where

$$d\left(\frac{M}{p}\right) = -\left(\frac{M}{p^2}\right)dp$$

or,

$$di = \frac{k}{h}\beta dA + \left(\frac{M}{p^2 h}\right)f'(Y)\beta dA$$

Where, according to (4)

$$dp = f'(Y)\beta dA$$

Simplifying,

$$di = \left(\frac{k}{h} + f'(Y)\frac{M}{p^2 h}\right)\beta dA \qquad \textbf{(8)}$$

As we may see, this relationship shows the effect in nominal interest rate, of changes taking place in money demand and money supply, in response to the expansion in the production and price level caused by an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

Nominal interest rates seem procyclical as well.

1.5. Change in real interest rate

Knowing that:

$$r_t = i_t - \pi_t$$

It's enough to compare (7) to (8) to reach some conclusions on the evolution of the real interest rate after an expansive trend the economy is supposedly undergoing.

When (7) > (8), or, in other words $-\varepsilon > \left(\frac{k}{h} + f'(Y)\frac{M}{p^2 h}\right)$ then, an increase in aggregate autonomous expenditure linked to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase more than the nominal interest rate does**. As a result the real interest rate tends to decrease, despite a Fisher Effect may appear in the long run.

When (7) < (8) or, in other words $-\varepsilon < \left(\frac{k}{h} + f'(Y)\frac{M}{p^2 h}\right)$ then, an increase in aggregate autonomous expenditure, connected to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase less than the nominal interest rate does**. As a result the real interest rate tends to increase, despite a Fisher Effect may appear in the long run.

When (7) = (8), or, in other words $-\varepsilon = \left(\frac{k}{h} + f'(Y)\frac{M}{p^2 h}\right)$ then, an increase in aggregate autonomous expenditure, connected to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase in the same amount than the nominal interest rate does**. As a result, the real interest rate tends to stay unchanged, and the Fisher Effect holds even in the short run.

In the Keynesian demand-side view, opposite to Say's law, in order to compare (7) to (8), we need to estimate the relationships' elasticities and parameters. Notwithstanding the fact that Keynesianism gives a central role to the speculative demand for money, depending on interest rates, there is empirical evidence supporting a bigger role for wealth and output in money demand function that depends on income level and distribution. Furthermore, there are economies of scale in the possession of money.

Goldfeld, Koyck and Almon data for the USA show some 0.19 value for short-run income elasticity of money demand (0.68 for the long-run) (Goldfeld, 1973). It has always to be lesser than one, because technological and institutional changes encourage agents to reduce their money demand which becomes compensated by an increase in money velocity, associated to financial innovation processes.

The demand for money is negatively correlated to interest rates, Goldfeld's data

showing some -0,045 value for short-run interest elasticity of money demand (-0,16 for the long run).

According to these data, in the relationship (8), $\left|\frac{k}{h}\right|$, would be between $\left|\frac{k}{h}\right| = 4,2$ (in the short run) and $\left|\frac{k}{h}\right| = 6,25$ (in the long run)

Nevertheless, money demand has lost stability because of the continuous changes in money velocity. What's more, financial innovation and uncertainty seem to play a significant role in the heterogeneous behaviour of agents (Cuthbertson, K., 1997). In the Spanish financial system for example the hoarding of great amounts of cash, mainly in 500 euros bills led to a recent fall in money velocity. This phenomenon is related to the nonofficial origin of money or coming from certain sectors of activity. The housing bubble caused by real estate speculation, may be an explanation.

According to a Keynesian view, and as stated before $0 \leq |-\varepsilon| \leq 1$, which reflects a soft downward sloping Phillips curve, in narrow relationship with factor markets far from the full productive capacity utilisation, and thus, far from full employment output level.

According to the previous values, we may conclude that (7) < (8).

Or:

$$-\varepsilon < \left(\frac{k}{h} + f'(Y)\frac{M}{p^2 h}\right)$$

If and only if $M > p^2 h$, which may be considered as normal.

The previous results allows us to conclude that an increase in aggregate autonomous expenditure linked to the expansive trend the economy is supposedly

undergoing, causes **the inflation rate to increase less than the nominal interest rate does**. As a result the real interest rate tends to increase.

This seems to provide a formal treatment for our previous set of conclusions, -reached by geometrical analysis-, about real rate dynamics from Keynesian opposite to Say's Law, demand side view. In other words, that the real interest rate seems to behave in a procyclical way. This surely has an impact on investment decisions, introducing a shift in financial to real capital ratio in the aggregate structure of capital stock. The trend this ratio will undergo isn't easy to forecast, but is of great interest to help explain the behaviour of financial markets.

We may conclude that real interest rate probably plays the role of a procyclical variable from Keynesian demand side approach, what tends to affect in a certain way investment decisions and, as a result, aggregate capital stock composition.

2. Neoclassical say's law supply side approach. A simplified formal analysis

When assuming this view, we admit some particular assumptions:

1. No involuntary unemployment exists within the labour market. The economy reaches its equilibrium over full-employment output level in a context where no monetary illusion exists.

2. The economic individual agents are supposed to form their conjectures on the evolution of the price level following a process of rational expectations, without predicting error. Thus:

$$\pi_t^{\,e} = \pi_t$$

This allows us to arrive to the real interest rate using current instead of expected inflation rate. In this way, we can write Fisher equation as:

$$r_t = i_t - \pi_t$$

where: $r_t =$ real interest rate for period t

$i_t =$ money interest rate for period t

3. It exists an underlying *from supply to demand* logical framework, linked to Say's law which states that supply creates its own demand. In other words; aggregate output is produced to satisfy the aggregate demand which has in fact been determined by aggregate supply. All savings are transformed into investment in the period, because the economic agents can't maintain idle money cash balances.

4. As a result, aggregate supply schedule is vertical over full employment output level, showing no price-elasticity, while Phillips curve is vertical over the natural unemployment rate (NUR). Both reflect the market conditions associated to equilibrium without involuntary unemployment, in a context without monetary illusion.

5. The money interest rate is no longer determined in the money market but in the market for loanable funds, and is no more under the complete control of the Monetary Authority, though remaining exogenous to a certain extent.

From this neoclassical view, the IS-LM analysis is not of use, because no long-term interaction between real and monetary sectors is accepted.

Cambridge approach to the quantity theory, explains money demand as a function of the transactions component being proportionate to the general income level.

Thus:

$$M^d = kpY$$

Or: $$M^d V = pY$$

where

M^d = Demand for money.

k = 1/V: Portion of the money income that agents wish to maintain in cash. The inverse to money velocity. In neoclassical view, it is assumed to remain steady in the short run as it depends on institutional and technological factors.

V = Velocity of money.

Y = Real general income level.

The **equilibrium in the market for money** can be expressed as:

$$M^s = kpY$$

$$M^s V = pY$$

where

M^s = Money Supply

Assume that despite a financial innovation processes is taking place, V remains steady.

And, solving for Y

$$Y = \frac{MV}{p} \qquad \textbf{(9)}$$

On the other hand, the following equation expresses the **equilibrium in the market for loanable funds:**

$$S(Y,i) = I(Y,i) + (G - T) + (X - M)$$

or,

$$sY + ai = IY - bi + (G - T) + (X - M)$$

where

$S(Y,i) = sY + ai$

$s =$ Marginal propensity to save.

$a =$ Interest-elasticity of loanable funds supply.

$i =$ Money interest rate.

$I(Y,i) = IY - bi$; $I =$ Income-elasticity of investment demand. It gathers or includes, non observable arguments among which we find "animal spirits" with strong influence on firms expectations.

$b =$ Interest-elasticity of investment demand.

s and I, the more *"Keynesian"* elasticities, are of scarce influence here, because the function is supposed to be more neoclassical. We need to bear in mind the fact that it is the nominal interest rate the one that is determined here in the market for loanable funds. It is a *loanable market* theory of interest. See, for example, Fisher (1977 (1930), p.43), or Sargent (1987, chap.1).

Nevertheless, in the neoclassical view, the **aggregate equilibrium in the market for goods and services** can be expressed as the **equilibrium in the market for loanable funds**, by means of the following equation:

$$Y = C + I + (G - T) + (X - M)$$

Where

$C = (1 - s)Y - (1 - a)i$

$G = \overline{G}; T = \overline{T}; X = \overline{X}; M = \overline{M} + mY$

Operating,

$$Y = \frac{1}{1-(1-s)(1-t)-I+m}\left[A - \left[b + (1-a)\right]i\right]$$

Or, $$Y = \mu\left[A - \left[b + (1-a)\right]i\right] \qquad \textbf{(10)}$$

Where

$\mu = \dfrac{1}{1-(1-s)(1-t)-I+m}$ = multiplier of autonomous expenditure without monetary sector. This fact owes its explanation to the dichotomy between real and monetary sectors that characterizes the neoclassical model, based upon Say's Law.

$A = (\overline{G} - \overline{T}) + (\overline{X} - \overline{M}) = \text{Autonomous expenditure}$

Finally, equating (9) and (10) we arrive to the **equilibrium in both markets**

$$\mu\left[A - \left[b + (1-a)\right]i\right] = \frac{MV}{p} \qquad (11)$$

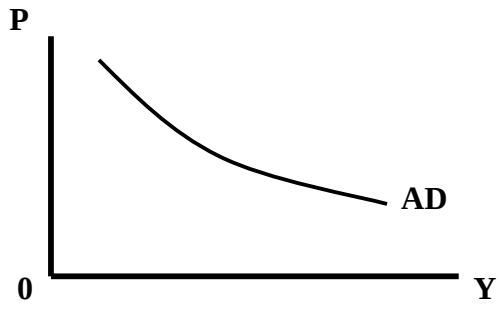
or:

$$kY = \frac{M}{p}$$

equivalent to the prior equation

$$Y = \frac{MV}{p} \qquad \textbf{(9)}$$

This expression shows an inverse relationship between price and production levels that can be graphed as the **downward sloping aggregate demand curve of the neoclassical model**. The values verifying it are equilibrium values for money and goods and services markets.



Let us begin again by considering an increase in aggregate autonomous expenditure (A), linked to an expansive trend the economy is supposedly undergoing.

As a consequence, the income and output level would increase in:

$$dY = \mu dA \qquad (12)$$

as no link can be found between real and monetary sectors, through the speculative demand for money.

In the same measure would have to increase the right side of (11), as money in the neoclassical view is considered only a means of payment.

2.1. Change in price level

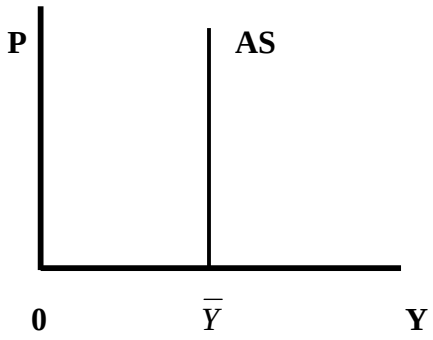
From this neoclassical approach, the supply curve shows a perfectly rigid vertical profile -zero price-elasticity-. From this supply-side view, the equilibrium level of output is determined by labour market equilibrium and production technology. The economy is at full-employment level of output, because no involuntary unemployment exists.

Without monetary illusion a price surge will lead to an equivalent rise in money wages to preserve workers' purchasing power. This will discourage employers to increase their labour demand because real wages remain unchanged. Therefore, the output level remains unchanged while price rise hold.

We would have a perfectly rigid vertical profile supply curve, over the NUR-associated income level.

$$Y_t = \overline{Y_t}$$

(13)



Until now we have

$$Y = \frac{MV}{p} \qquad \text{(Aggregate demand)} \qquad \textbf{(9)}$$

$$Y_t = \overline{Y_t} \qquad \text{(Aggregate supply)} \qquad \textbf{(13)}$$

Under these circumstances, to observe the change in prices caused by a certain change in the income level due to an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing, we need only use the demand schedule previously considered

$$Y = \frac{MV}{p} \qquad \textbf{(9)}$$

Solving for the price level

$$p = \frac{MV}{Y}$$

The derivative of p with respect to Y is

$$\frac{dp}{dY} = -\frac{MV}{Y^2}$$

or

$$dp = -\left(\frac{MV}{Y^2}\right)dY \qquad \textbf{(14)}$$

or

$$dp = -\left(\frac{MV}{Y^2}\right)\mu dA \qquad \textbf{(14)}$$

where

$dY = \mu dA$

Which gives the response of the price level to a certain change in income due to an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

But we have to bear in mind that our goal is to determine the effect of such a change in a component of aggregate demand, -associated to an either expansive or restrictive trend of the economy-, on the nominal interest rate and in the inflation rate. And afterwards, by calculating the difference between the both, we would observe the effect on the real interest rate.

2.2. Change in inflation rate

The change in the price level accelerates inflation rate. This change in inflation rate has to be compared to the change in nominal interest rate, in order to observe the evolution of real interest rate which constitutes our main concern.

But the change in inflation rate has to be observed through Phillips curve (5),

which in neoclassical view will show a perfectly rigid vertical profile because the effective inflation rate equals its expected level (i.e., $\pi^e_t = \pi_t$), and production stays at NUR-associated output level.

And again, according to the expectations-augmented Phillips curve

$$U_t = U_{NUR} + \phi\left(\pi_t - \pi_t^{\,e}\right) \qquad \textbf{(5)}$$

Being now

$-1 < \phi \leq 0 =$ adjustment parameter which depends on the neighbouring of the economy to full productive capacity, The neighbouring of the productive potential to full employment level.

When $|\phi| \approx 0$, the economy is relatively close to full employment as it seems in line with Say's law neoclassical supply side approach. The Phillips curve would show a vertical profile because unemployment stays at NUR level.

We can put (5) as

$$\pi_t = \pi_t^{\,e} - \alpha\left(U_{NUR} - U_t\right)$$

Where

$$-1 >> \alpha = \frac{1}{\phi}$$

Or as an expectations-augmented aggregate supply schedule, which can be rewritten as

$$\pi_t = \pi^{e}{}_t - \varepsilon\left(Y_t - \overline{Y}_t\right) \qquad (6)$$

Where

$$-1 >> \varepsilon = \frac{1}{\phi}$$

Again, (6) will allow us to obtain the first derivative of the inflation rate with respect to the output level.

$$d\pi_t = -\varepsilon dY_t$$

Where

$1 << -\varepsilon > 0$

In other terms,

$$d\pi_t = -\varepsilon\mu dA_t \qquad (15)$$

where

$dY_t = \mu dA_t$

The expression (15) is a measure of the change in inflation, caused by an increase in output level in response to an upsurge in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

When $|-\varepsilon\mu|$, alternatively surpasses, equals or falls behind the unit, the changes in inflation rate will be respectively more than proportional, proportional or less than proportional to the changes in aggregate autonomous private expenditure (the changes in aggregate demand). From an *ex ante* approach it is only possible to affirm that in the neoclassical Say's law, supply side view $-\varepsilon\mu$ would be positive and greater than one. Nonetheless, the neoclassical approach implies that the economy has reached full employment and full productive potential level, which suggests $|\varepsilon| >> 1$ in line with a perfectly rigid vertical profile Phillips curve.

Moreover $-\varepsilon\mu$ would be even much bigger, because μ, (autonomous expenditure multiplier without monetary sector and under neoclassical elasticities conditions), is always greater than one.

In line with the former statements, and with the rationality of economic agents, we would have as previously stated a vertical profile Phillips curve, over the NUR level, and a perfectly rigid vertical profile supply curve, with the equilibrium staying at NUR-associated income level, with: $|\phi| = 0$, $|\varepsilon| = \infty$ and $|\alpha| = \infty$.

Under these circumstances, in a context of rationality and without monetary illusion, an expansive aggregate demand shock would produce an increase in expected inflation, which will cause the effective rate to soar. A new equilibrium point will be reached both on the Phillips curve and on the aggregate supply schedule. The new equilibria will be met for higher price and inflation rate levels. In both cases, over NUR or NUR-associated income level respectively.

As it could be expected, inflation rate seems to be procyclical.

2.3. Change in nominal interest rate

Now we focus on the increase in nominal interest rate. This is caused by the elevation of the demand schedule for loanable funds in response to an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

The previous relationship expressing the equilibrium in the market for loanable funds:

$$S = I + (G - T) + (X - M)$$

rewritten as:

$$sY + ai = IY - bi + (\overline{G} - \overline{T}) + (\overline{X} - \overline{M} - mY)$$

allows us to solve for i:

$$i = \left(\frac{I - s}{a + b} - m \right) Y + \overline{G} - \overline{T} + \overline{X} - \overline{M}$$

A change in output level in response to a variation in aggregate autonomous expenditure (A) transmits its effect to the money interest rate. Thus, we compute the derivative of i with respect to Y, which gives:

$$di = \left(\frac{I - s}{a + b} - m \right) dY$$

and substituting we have:

$$di = \left(\frac{I - s}{a + b} - m \right) \mu dA \qquad \textbf{(16)}$$

where: $dY = \mu dA$

As we may see, this relationship shows the effect in the financial markets equilibrium money interest rate, of changes taking place in demand schedule for loanable funds, in response to the expansion in the output level caused by an increase in aggregate autonomous expenditure (A), linked to the expansive trend the economy is supposedly undergoing.

As expected, it seems that nominal interest rate is a procyclical variable.

2.4. Change in real interest rate

Knowing that:

$$r_t = i_t - \pi_t$$

It's enough to compare (15) to (16) **in absolute value** to reach some conclusions on the evolution of the real interest rate after an expansive trend the economy is supposedly undergoing.

When (15) > (16), or, in other words $-\varepsilon > \left(\frac{I-s}{a+b} - m\right)$, then, an increase in aggregate autonomous expenditure linked to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase more than the nominal interest rate does**. As a result the real interest rate tends to decrease, despite a Fisher Effect may appear in the long run.

When (15) < (16), or, in other words $-\varepsilon < \left(\frac{I-s}{a+b} - m\right)$, then, an increase in aggregate autonomous expenditure, connected to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase less than the nominal interest rate does**. As a result the real interest rate tends to increase, despite a Fisher Effect may appear in the long run.

When (15) = (16), or, in other words $-\varepsilon = \left(\frac{I-s}{a+b} - m\right)$ then, an increase in aggregate autonomous expenditure, connected to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase in the same amount than the nominal interest rate does**. As a result, the real interest rate tends to stay unchanged, and the Fisher Effect holds even in the short run.

In the neoclassical Say's law supply-side view, in order to compare (15) to (16), we need to estimate the elasticities. **The left side of the equation takes a value similar to infinite:** $-\varepsilon \approx -\infty$. And to consider the right side, we need again to estimate a, b, I,

and s.

Recent research (Mulligan & Sala-i-Martin, 2000), shows a serious overestimation of the predictive power of the Baumol-Tobin approach to money demand, to estimate interest rate elasticities at low nominal rates (under 5 or even 6%). Precisely those have been the prevailing rates over the last years both in Europe and the US.

Furthermore, from the neoclassical Say's law supply side perspective, $|a|$ and $|b|$ would always be greater than Keynesian elasticities $|I|$ and $|s|$, because savings and investment are, in this view, more sensitive to relative prices –interest rates-, than to quantities –wealth, income and output level-.

As a result, $|s|$ would probably be greater than $|I|$, because the latter includes a number of non-observable conjectures and arguments related to entrepreneurial expectations and their sensitivity to output growth. In the same way, and according to Say's law which states that all savings are transformed into investment, we could imagine that a change in interest rates, would have a similar or equivalent effect on both investment and savings. This implies that $a \leq b$. Or else, slightly lesser than **b** for it seems reasonable to think that a rise in rates, would have the effect of restraining investment more than it would encourage savings: $\left|\frac{\partial I}{\partial i}\right| > \left|\frac{\partial S}{\partial i}\right|$

Thus, the right side would take a relatively reduced absolute value and always lesser than the left side. As a result, an increase in aggregate autonomous expenditure linked to the expansive trend the economy is supposedly undergoing, causes **the inflation rate to increase more than the nominal interest rate does**. As a result the real interest rate will tend to decrease, despite a Fisher Effect may appear in

the long run.

Thus, we have:

$$-\varepsilon > \left(\frac{I-s}{a+b} - m \right)$$

We may conclude that real interest rate probably plays the role of a countercyclical variable from Say's Law neoclassical supply side approach, what tends to affect in a certain way investment decisions and, as a result, aggregate capital stock composition in terms of financial to real capital ratio. The trend this ratio could follow, besides difficult to explain is of great interest to help explain the behaviour of financial markets.

III. SUMMARY OF CONCLUSIONS

According to the previous simplified formal ex ante analysis, we conclude that:

1. Inflation rate is procyclical. Nominal interest rates are procyclical. Both are usually considered lagging variables.

2. In our analysis we didn't study whether the real rate is a leading a coincident or a lagging variable, but in spite of the fact that real rate might be considered in economic literature an acyclical variable, in a formal ex ante analysis, its pattern of behaviour appears to be countercyclical in neoclassical Say's Law approach, while being procyclical from Keynesian view.

3. This implies a fluctuations enhancement role for the real interest rate in neoclassical Say's Law approach and a kind of automatic stabilizing role for the real rate in Keynesian theory.

4. The latter could affect the ratio of financial to real investment flows, and as a result the composition of the capital stock in the economy. The outcome might have something to do with recent developments in financial markets.

It remains to see whether the empirical evidence on correlations between real GDP rate of growth and real interest rate, supports better neoclassical or Keynesian results. In the extent that these results reflect the particular assumptions of each model, the realism of the assumptions and specially the prevalence of this realism over time, might be crucial. Nevertheless, the case for regulating international financial markets is becoming widely accepted. In the light of recent developments that shattered world markets, our previous considerations could be of some interest.

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