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Weakly compact bilinear operators among real interpolation spaces

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ABSTRACT

We show a necessary and sufficient condition for weak compactness of bilinear operators interpolated by the real method. This characterization does not hold for interpolated operators by the complex method.

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1. Introduction

An important problem in interpolation theory is to study the behaviour under interpolation of properties that an operator may have. For example, the classical results of Riesz-Thorin and Marcinkiewicz refer to boundedness of operators between L_p spaces and the theorem of Krasnosel´skii to compactness (see the monographs by Bergh and Löfström [2] or Triebel [28]). Abstract versions of these results can be found in the papers by Lions and Peetre [20], Calderón [4], Cwikel [12] and Cobos, Kühn and Schonbek [9], among other papers. The behaviour under interpolation of weakly compact operators has also been study in depth. See, for example, the books by Beauzamy [1] and Brudnyĭ and Krugljak [3], and the papers by Heinrich [19], Maligranda and Quevedo [21], Mastyło [25] and Cobos and Martínez [10,11].

In the bilinear case, the behaviour of weakly compact operators have been studied in the recent papers by Manzano, Rueda and Sánchez-Pérez [22,23] when the couple in the target reduces to a single Banach space or both couples in the source reduce to single Banach spaces, and in the paper by Martínez and the present authors [8] for general couples. For the real method, the result of [8] shows that if the restrictions

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$T : A_j \times B_j \longrightarrow E_j$ are bounded for $j = 0, 1$ and any of them is weakly compact, then the interpolated operator $T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$ is also weakly compact provided that $0 < \theta < 1, 1 \leq p, q < \infty, 1 < r < \infty$ and $1/p + 1/q = 1 + 1/r$.

In this paper, we continue this research by showing that a necessary and sufficient condition for the weak compactness of the interpolated operator is that $T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1$ is weakly compact.

Our approach is based on the connection between the properties of a bilinear operator $T : A \times B \longrightarrow E$ and those of its linearization $\widetilde{T} : A \widetilde{\otimes}_\pi B \longrightarrow E$, the linear operator with source in the projective tensor product of the spaces A and B.

Furthermore, we show that a similar characterization does not hold for bilinear operators interpolated by the complex method.

We finish the paper with some comments on compactness of interpolated operators by the real method. By means of an example we establish that compactness of $T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1$ is not enough to imply that the interpolated operator $T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$ is compact.

2. Preliminaries

Let A, B be Banach spaces. A bounded linear operator $R \in \mathcal{L}(A, B)$ is said to be *weakly compact* if $R(U_A)$ is relatively weakly compact in B. Here U_A stands for the closed unit ball of A. We write $\mathcal{W}(A, B)$ for the space of weakly compact operators between the spaces A and B.

Let $\mathcal{W} = \bigcup_{A,B} \mathcal{W}(A, B)$. Then the class $\mathcal{W}$ of all weakly compact operators between Banach spaces is an *operator ideal* in the sense of [27] and [14].

The ideal $\mathcal{W}$ is *injective*. This means that for every isometric embedding $J \in \mathcal{L}(B, F)$ and every $R \in \mathcal{L}(A, B)$, it follows from $JR \in \mathcal{W}(A, F)$ that $R \in \mathcal{W}(A, B)$ (see [27, 4.6.12]). Here F is another Banach space.

Given any sequence of Banach spaces $(E_m)_{m\in\mathbb{Z}}$ and $1 < r < \infty$, we denote by $\ell_r(E_m)$ the vector valued ℓ_r-space formed by all sequences $x = (x_m)$ with $x_m \in E_m$ which have a finite norm

$$\|x\|_{\ell_r(E_m)} = \Big(\sum_{m=-\infty}^{\infty} \|x_m\|_{E_m}^r \Big)^{1/r}.$$

If R is a bounded linear operator acting between vector valued ℓ_r-spaces $R \in \mathcal{L}(\ell_r(E_m), \ell_r(F_m))$, then R may be imagined as a infinite matrix whose elements are $Q_k R P_n$, where $P_n : E_n \longrightarrow \ell_r(E_m)$ is the embedding $P_n x = (\delta_m^n x)$ with δ_m^n being the Kronecker's delta, and $Q_r : \ell_r(F_m) \longrightarrow F_k$ is the projection $Q_k(y_m) = y_k$. Clearly, $Q_k R P_n \in \mathcal{L}(E_n, F_k)$ for $n, k \in \mathbb{Z}$.

For $1 < r < \infty$ the ideal $\mathcal{W}$ satisfies the so-called Σ_r-*condition*: For any sequences of Banach spaces $(E_m), (F_m)$ and any operator $R \in \mathcal{L}(\ell_r(E_m), \ell_r(F_m))$, it follows from $Q_k R P_n \in \mathcal{W}(E_n F_k)$ for any $n, k \in \mathbb{Z}$ that $R \in \mathcal{W}(\ell_r(E_m), \ell_r(F_m))$ (see [19], and also [5]).

We put $A \otimes B$ for the tensor product of the Banach spaces A, B. For $u \in A \otimes B$, let

$$\pi(u) = \inf\{\sum_{k=1}^{n} \|a_k\|_A \|b_k\|_B : u = \sum_{k=1}^{n} a_k \otimes b_k,\ a_k \in A,\ b_k \in B,\ 1 \leq k \leq n,\ n \in \mathbb{N}\}.$$

We write $A \widetilde{\otimes}_\pi B$ for the *projective tensor product* of A and B, that is to say, for the completion of $(A \otimes B, \pi)$ (see [13,15]).

Let E be another Banach space. We put $\mathcal{L}(A \times B, E)$ for the space of all *bounded bilinear operators* from $A \times B$ into E. We write $\widetilde{T} \in \mathcal{L}(A \widetilde{\otimes}_\pi B, E)$ for the linearization of T, that is, the unique bounded linear operator from $A \widetilde{\otimes}_\pi B$ into E such that $\widetilde{T}(a \otimes b) = T(a, b)$ for any $a \in A, b \in B$.

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According to [8, Lemma 2] if $T \in \mathcal{L}(A \times B, E)$ then

$$T(U_A \times U_B) \subseteq \widetilde{T}\big(U_{A\widetilde{\otimes}_\pi B}\big) \subseteq \overline{co\,\big(T\big(U_A \times U_B\big)\big)}. \tag{2.1}$$

Here $T(U_A \times U_B) = \{T(a,b) : a \in U_A, b \in U_B\}$ and we write $co(S)$ for the convex hull of the subset S.

A bounded bilinear operator $T \in \mathcal{L}(A \times B, E)$ is said to be *weakly compact* if $T(U_A \times U_B)$ is relatively weakly compact in E.

It is not hard to check that the class of all bilinear weakly compact operators is a *closed surjective 2-ideal* in the sense of [23].

By a *Banach couple* $\bar{A} = (A_0, A_1)$ we mean two Banach spaces A_0, A_1 which are continuously embedded in the same Hausdorff topological vector space. Then we can form their sum $A_0 + A_1$ and their intersection $A_0 \cap A_1$, which become Banach spaces when endowed with the norms

$$\|a\|_{A_0+A_1} = \inf\{\|a_0\|_{A_0} + \|a_1\|_{A_1} : a = a_0 + a_1\ ,\ a_j \in A_j\}$$

and $\|a\|_{A_0\cap A_1} = \max\{\|a\|_{A_0}\ ,\ \|a\|_{A_1}\}$, respectively.

Given any $t > 0$, we may equivalently renorm $A_0 + A_1$ by the *Peetre's K-functional*

$$K(t,a) = \inf\{\|a_0\|_{A_0} + t\|a_1\|_{A_1} : a = a_0 + a_1,\ a_j \in A_j\}.$$

For $1 \le q < \infty$ and $0 < \theta < 1$, the *real interpolation space* realized as a K-space in discrete form $\bar{A}_{\theta,q} = (A_0, A_1)_{\theta,q}$ is formed by all those $a \in A_0 + A_1$ which have a finite norm

$$\|a\|_{\theta,q} = \Big(\sum_{m=-\infty}^{\infty} \big(2^{-\theta m} K(2^m, a)\big)^q\Big)^{1/q}.$$

See the monographs [2,28,1,3] for properties of these spaces. We only recall that spaces $(A_0, A_1)_{\theta,q}$ are of class $\mathcal{C}_K(\theta; \bar{A})$ (see [2, Section 3.5] or [28, Section 1.10.1]). Real interpolation spaces make also sense for $q = \infty$ and they can also be equivalently defined using integrals instead of series, but this continuous description will not be used here.

If $\bar{A} = (A_0, A_1), \bar{B} = (B_0, B_1), \bar{E} = (E_0, E_1)$ are Banach couples and $T : (A_0+A_1)\times(B_0+B_1) \longrightarrow E_0+E_1$ is a bounded bilinear operator such that

$$T : A_j \times B_j \longrightarrow E_j \text{ boundedly for } j = 0, 1,$$

then the bilinear interpolation theorem establishes that the restriction

$$T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$$

is bounded provided that $0 < \theta < 1, 1 \le p, q, r \le \infty$ and $1/p + 1/q = 1 + 1/r$. See [20] and also [17].

3. Weak compactness of interpolated bilinear operators

Next we establish the central result of the paper.

Theorem 3.1. *Let $\bar{A} = (A_0, A_1)$, $\bar{B} = (B_0, B_1)$, $\bar{E} = (E_0, E_1)$ be Banach couples. Assume that $T : (A_0 + A_1) \times (B_0 + B_1) \longrightarrow E_0 + E_1$ is a bounded bilinear operator such that the restrictions $T : A_j \times B_j \longrightarrow E_j$ are bounded for $j = 0, 1$. Let $0 < \theta < 1$, $1 \le p, q < \infty$ and $1 < r < \infty$ with $1/p + 1/q = 1 + 1/r$. Then a necessary and sufficient condition for*

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$$T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r} \text{ to be weakly compact}$$

is that

$$T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1 \text{ is weakly compact.}$$

Proof. Since the following continuous embeddings hold $A_0 \cap A_1 \hookrightarrow (A_0, A_1)_{\theta,p}$, $B_0 \cap B_1 \hookrightarrow (B_0, B_1)_{\theta,q}$ and $(E_0, E_1)_{\theta,r} \hookrightarrow E_0 + E_1$, it is clear that if $T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$ is weakly compact, then

$$T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1 \text{ is also weakly compact.}$$

Next we show that the condition is sufficient. By the bilinear interpolation theorem, we know that

$$T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r} \text{ boundedly.}$$

Hence, in view of (2.1), it suffices to show that

$$\widetilde{T} : (A_0, A_1)_{\theta,p} \widetilde{\otimes}_\pi (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r} \text{ is weakly compact.} \tag{3.1}$$

With this aim, for $m \in \mathbb{Z}$ we put $F_m = (E_0 + E_1, K(2^m, .))$ and we write $j : (E_0, E_1)_{\theta,r} \longrightarrow \ell_r(2^{-\theta m} F_m)$ for the isometric embedding defined by $j(x) = (\cdots, x, x, x, \cdots)$. Here $2^{-\theta m} F_m$ is the space F_m with the norm $2^{-\theta m} \| \cdot \|_{F_m}$. The injectivity of the ideal $\mathcal{W}$ of weakly compact linear operators yields that (3.1) holds if and only if

$$\widetilde{T} : (A_0, A_1)_{\theta,p} \widetilde{\otimes}_\pi (B_0, B_1)_{\theta,q} \longrightarrow \ell_r(2^{-\theta m} F_m) \text{ is weakly compact.} \tag{3.2}$$

Using that $\mathcal{W}$ satisfies the $\sum_r$-condition, we have that (3.2) is equivalent to

$$\widetilde{T} : (A_0, A_1)_{\theta,p} \widetilde{\otimes}_\pi (B_0, B_1)_{\theta,q} \longrightarrow E_0 + E_1 \text{ is weakly compact.} \tag{3.3}$$

By (2.1), this in turn is equivalent to

$$\text{weak compactness of } T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow E_0 + E_1. \tag{3.4}$$

To establish (3.4) we observe that by the assumption we have that

$$T : (A_0 + A_1) \times (B_0 + B_1) \longrightarrow E_0 + E_1 \text{ is bounded}$$

and that

$$T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1 \text{ is weakly compact.}$$

Since $(A_0, A_1)_{\theta,q}$ is of class $\mathcal{C}_K(\theta; \bar{A})$ and $(B_0, B_1)_{\theta,q}$ is of class $\mathcal{C}_K(\theta; \bar{B})$, using [23, Corollary 4.7], we derive that

$$T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow E_0 + E_1 \text{ is weakly compact.}$$

This completes the proof. □

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Note that in the special case when the couple in the target (E_0, E_1) reduces to a single Banach space (i.e. $E_0 = E_1$), then Theorem 3.1 follows from [23, Corollary 4.7] applied to the 2-ideal of weakly compact operators.

Martínez and the present authors have shown in [8] that weakly compact bilinear operators can be also interpolated by the complex method (we refer to [4,2,28] for the definition and properties of this interpolation method). According to [8, Theorem 4.2] if $T : (A_0 + A_1) \times (B_0 + B_1) \longrightarrow E_0 + E_1$ is bilinear and bounded, with the restrictions $T : A_j \times B_j \longrightarrow E_j$ being bounded for $j = 0, 1$ and one of these two restrictions being weakly compact, then $T : [A_0, A_1]_\theta \times [B_0, B_1]_\theta \longrightarrow [E_0, E_1]_\theta$ is weakly compact. However, a similar result to Theorem 3.1 does not hold for the complex method as we show next by means of an example.

Counterexample 3.2. Consider the couple of Lorentz spaces $(\Lambda_\varphi, \Lambda_\psi)$ constructed by Mastyło in [24, page 161]. These spaces satisfy that the embedding $\Lambda_\varphi \cap \Lambda_\psi \hookrightarrow \Lambda_\varphi + \Lambda_\psi$ is weakly compact and that $[\Lambda_\varphi, \Lambda_\psi]_\theta$ contains a subspace isomorphic to ℓ_1. Choose $(A_0, A_1) = (E_0, E_1) = (\Lambda_\varphi, \Lambda_\psi), (B_0, B_1) = (\mathbb{C}, \mathbb{C})$, where $\mathbb{C}$ is the field of complex numbers, and let T be the bilinear operator defined by $T(f, \lambda) = \lambda f$. Clearly, T is bounded acting among the following spaces

$$T : (A_0 + A_1) \times (B_0 + B_1) \longrightarrow E_0 + E_1,$$
$$T : A_j \times B_j \longrightarrow E_j\,,\ j = 0, 1.$$

Moreover, since $A_0 \cap A_1 = \Lambda_\varphi \cap \Lambda_\psi, B_0 \cap B_1 = \mathbb{C}$ and $E_0 + E_1 = \Lambda_\varphi + \Lambda_\psi$, weak compactness of the embedding $\Lambda_\varphi \cap \Lambda_\psi \hookrightarrow \Lambda_\varphi + \Lambda_\psi$ yields that

$$T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1 \text{ is weakly compact}$$

but

$$T : [A_0, A_1]_\theta \times [B_0, B_1]_\theta \longrightarrow [E_0, E_1]_\theta$$

fails to be weakly compact because $[B_0, B_1]_\theta = \mathbb{C}, [A_0, A_1]_\theta = [E_0, E_1]_\theta = [\Lambda_\varphi, \Lambda_\psi]_\theta$ and the space $[\Lambda_\varphi, \Lambda_\psi]_\theta$ is not reflexive.

4. Remarks on compactness of interpolated bilinear operators

We close the paper with some comments on compact bilinear operators, that is, bilinear operators which send the product of the unit balls onto a relatively compact set. Their interpolation properties have been extensively studied recently. See, for example, the papers [16–18,6,26,7]. In particular, it follows from [6, Theorem 4.9] that if $T : (A_0 + A_1) \times (B_0 + B_1) \longrightarrow E_0 + E_1$ is bilinear and bounded, with the restrictions $T : A_j \times B_j \longrightarrow E_j$ being bounded for $j = 0, 1$ and one of these two restrictions being compact, then $T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$ is compact provided that $0 < \theta < 1, 1 \le p, q \le \infty, 1 \le r \le \infty$ and $1/p + 1/q = 1 + 1/r$.

In the proof of Theorem 3.1 we have used the $\sum_r$-condition, but compact linear operators do not satisfy that condition. Hence, the approach of Theorem 3.1 does not allow to replace weak compactness by compactness. Clearly, if $T : (A_0, A_1)_{\theta,p} \times (B_0, B_1)_{\theta,q} \longrightarrow (E_0, E_1)_{\theta,r}$ is compact then $T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1$ is also compact. However, the converse does not hold as we show next by means of an example.

Counterexample 4.1. Subsequently, we put $\ell_2(2^n)$ for the space ℓ_2 with the weight (2^n), formed by all sequences of scalars $\xi = (\xi_n)$ such that $(2^n \xi_n) \in \ell_2$. Choose $A_0 = E_0 = \ell_2$, $A_1 = E_1 = \ell_2(2^n)$ and $B_0 = B_1 = \mathbb{K}$, where $\mathbb{K}$ is the scalar field, $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Let $T(x, \lambda) = \lambda x$. Since the embedding $A_0 \cap A_1 = \ell_2(2^n) \hookrightarrow \ell_2 = E_0 + E_1$ is compact, then

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$$T : (A_0 \cap A_1) \times (B_0 \cap B_1) \longrightarrow E_0 + E_1 \text{ is compact.}$$

Choose $p = 2, q = 1$ and $r = 2$, so $1/p + 1/q = 1 + 1/r$, and let $0 < \theta < 1$. We have $(A_0, A_1)_{\theta,2} = \ell_2(2^{\theta n}) = (E_0, E_1)_{\theta,2}$ (see [28, Theorem 1.18.2] and $(B_0, B_1)_{\theta,1} = \mathbb{K}$. Therefore, the interpolated operator is

$$T : \ell_2(2^{\theta n}) \times \mathbb{K} = (A_0, A_1)_{\theta,2} \times (B_0, B_1)_{\theta,1} \longrightarrow (E_0, E_1)_{\theta,2} = \ell_2(2^{\theta n})$$

which is not compact because $\ell_2(2^{\theta n})$ is not finite dimensional.

Therefore, the characterization of Theorem 3.1 fails if we replace weak compactness by compactness.

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