ICAE
Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE
FACULTAD DE ECONOMICAS
Campus de Somosaguas
28223 MADRID
Teléfono 394 26 11 - FAX 294 26 13

Documento de trabajo

The Welfare Cost of Fluctuations in Representative Agent Economies

Franck Portier
Luis A. Puch

No.9812 Junio 1998

ICAE
Instituto Complutense de Análisis Económico
UNIVERSIDAD COMPLUTENSE

THE WELFARE COST OF FLUCTUATIONS IN REPRESENTATIVE AGENT ECONOMIES

Franck Portier and **Luis A. Puch***

March 1997
Revised June 1998

ABSTRACT

In this paper we quantify the welfare cost of fluctuations in a representative agent dynamic equilibrium framework. In doing so, we argue that two key features of Intertemporal Stochastic General Equilibrium Models should not be forgotten: non-linearities and dynamics. We think that these features are often disregarded in the existing literature. We propose a structural measure of the welfare cost of fluctuations, and quantify the role played by dinamics and non-linearities in assessing this cost for some versions of the one sector stochastic growth model. We find that non-linearities do not magnify the cost of fluctuations for walrasian growth models, and our structural measure is close to what has been measured in the literature. That difference becomes sharply larger in non-walrasian cases, where fluctuations magnify equilibrium inefficiencies.

RESUMEN

En este artículo evaluamos el coste de bienestar de las fluctuaciones, en un marco de equilibrio general dinámico de agente representativo, prestando atención a dos elementos clave que a menudo se descuidan en la literatura: dinámica y no linealidades. Para ello, proponemos una medida estructural del coste de bienestar de las fluctuaciones y cuantificamos el papel que juegan la dinámica y las no linealidades para distintas versiones del modelo de crecimiento estocástico de un sector. Encontramos que en entornos walrasianos las no linealidades no magnifican el coste de las fluctuaciones. Sin embargo, en economías no walresianas nuestra medida modifica sustancialmente la evaluación del coste de las fluctuaciones al recoger cómo éstas magnifican las ineficiencias del equilibrio.

Keywords: Cost of Fluctuations - Non linearity - PEA- Business Cycles - Transitions.
JEL code: E21, E32, E37

*Portier, CREST-CEPREMAP and Puch, University Complutense of Madrid. We thank J. Adda, P. Beaudry. J.P. Bénassy, F. Collard, J. Durán, J.F. Fagnart, P. Fève, S. Grégoir, P.Y. Hénin, M. Juillard, T. Kehoe, F. Langot, O. Licandro, P. Pereira, J.V. Rios-Rull and seminar participants at Bilbao EHU, CEPREMAP, CREST, FEDEA, Federal Reserve Bank of Minneapolis, La Coruña WDM 97, London School of Economics, Louvain T2M 97, Madrid UCM, Oxford SED 97, Toulouse ESEM 97, Valencia UV, Vigo EAE 97 for helpful comments. We are indebted to M. Juillard for implementing the MATLAB version of DYNARE, and to G. Vandenbroucke for expert research assistance on that implementation. Address correspondence about this paper to: Luis A. Puch, Dpto. de Economía Cuantitativa. Universidad Complutense de Madrid. Campus de Somosaguas. 28223 Madrid (Spain). E-mail address. lpuch@ccee.ucm.es

1 Introduction

In an often quoted work, Lucas [1987] has shown that in a representative agent framework, the potential welfare gain from stabilizing consumption around its mean is small. Let us recall briefly Lucas' argument. If the process of aggregate consumption is given by a log linear process around a deterministic trend, $c_t = (1+\mu)^t e^{-\frac{1}{2}\sigma_z^2} z_t$, where $\{z_t\}$ is a stationary stochastic process with a stationary distribution given by $\ln z_t \rightsquigarrow N(0, \sigma_z^2)$, then the cost of instability can be computed as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption instability and a perfectly smooth consumption path. With a CRRA utility function with risk aversion coefficient ν, this cost is given by $\ell = \frac{1}{2}\nu\sigma_z^2$. In the following, we will refer to it as the linear measure of the welfare cost of fluctuations, or ℓ measure. With $\sigma_z = 0.013$ (Lucas' estimate), and $\nu = 5$, the linear welfare cost of fluctuations is only .042% of average consumption.

Such an evaluation is an argument supporting the idea that eliminating fluctuations is neither a feasible nor a desirable objective of policy. But it is only an informative "back of an envelope" computation, that needs to be confirmed within fully specified small dynamic general equilibrium models.

Two strands of the literature have challenged Lucas' view within general equilibrium models. The first relaxes the assumption of a representative agent and introduces incomplete insurance markets: without complete insurance markets, a recession does not reduce everyone's consumption by a small amount, but reduces the consumption of a small fraction of the population by a large amount. This argument has been developed by Imrohoroglu [1988] and Atkeson and Phelan [1994] among others. A second strand adopts more general utility functions, for which the intertemporal elasticity of substitution is not the inverse of the degree of relative risk aversion. Following the work of Epstein and Zin [1991], Obstfeld [1994] and Epaulard and Pommeret [1997] among others have shown that with non Von Neuman-Morgenstern preferences, a representative agent model can display higher welfare cost of fluctuations when computed as in Lucas [1987]. These departures suggest that simple stochastic growth models are seemingly unable to evaluate the cost of fluctuations while providing a somewhat accurate description of the business cycle.

In our opinion, economies with representative agents and Von Neuman - Morgenstern preferences have not been fully explored in the literature, and

deserve more attention before relaxing one of these two hypothesis. We argue that the "back of an envelope" computation done by Lucas has been wrongly implemented in fully specified dynamic general equilibrium models, forgetting the "discipline of applied general equilibrium" advocated by Kehoe and Prescott [1995]. More precisely, that evaluation (the ℓ evaluation) misses non-linearities and dynamics. On the one hand, we are aware that most widely used dynamic general equilibrium models we have consensus for exhibit relevant non-linearities on their fundamentals. On the other hand, when using those models for normative purposes we are often led to comparisons of economies with different steady state wealth. Consequently, in this paper we fully take into account non-linearites and we stress on dynamics.

Firstly, because the model is non-linear, precautionary motives are in action, and by Jensen inequality, means of economic variables are different from their non stochastic steady state levels. Assuming a log-linear process for consumption (most of the time by log-linearizing the model) misses this point which is an important feature of economies with ongoing uncertainty. In other words, linear approximations to non-linear policy rules abstract from potential first order effects if cycles are not symmetric. Given that we know those effects are meaningfully related with uncertainty we would like to see whether they are large or not when we fully keep track of non-linearities in our benchmark business cycle framework. Secondly, we should keep in mind that we are working with a dynamic model when comparing an economy with shocks and the same economy without shocks. What the ℓ evaluation suggests is to compare two steady states (one or both being stochastic steady states), and disregards the welfare effect of a transition between these two steady states (see for instance Greenwood and Huffman [1991] for the welfare cost of fluctuations, Cooley and Hansen [1989] for the welfare cost of inflation, Cho, Cooley and Phaneuf [1997] for the welfare cost of nominal wage contracting). The importance of taking into account transitions has been illustrated in a deterministic model by Cooley and Hansen [1992] and in a stochastic log-linearized one by Hairault, Langot and Portier [1998].

In this paper we show that the scope of models for which is meaningful abstracting from non-linearities and dynamics is extremely restrictive. Once proved that the ℓ evaluation is not correct, the question remains of the quantitative importance of the error. Using simple extensions of the one sector growth model, we are able to show with examples that the correction we propose does not matter for walrasian economies, but does in non walrasian ones.

The paper is organized as follows. In Section 2, we formally define our measure of the welfare cost of fluctuations, and we characterize sufficient conditions under which it coincides with the ℓ evaluation. Section 3 presents simple extensions of the neoclassical growth model for which the equilibrium path is optimal. Section 4 adds real wage rigidity to the model, and shows how welfare results are affected. Section 5 gathered some concluding comments.

2 Definition and Measure of the Welfare Cost of Fluctuations

In this section, we define a stochastic dynamic general equilibrium and the welfare cost of fluctuations. We then show that the ℓ evaluation coincides with this definition in only very restrictive cases, in which the type of computation done by Lucas [1987] can be directly implemented.

2.1 Definitions

Let us state what we call a dynamic general equilibrium of a model economy.

Definition 1 *A dynamic general equilibrium of a model economy is recursively given by*
(i) a distribution for the exogenous shocks with cumulative H_ε on a support $\Omega\varepsilon$, with respect to which ε_t is iid; we denote $\bar{\varepsilon}$ the mean of the shock distribution ($\bar{\varepsilon} = \int \varepsilon dH_\varepsilon$),
(ii) a decision rule for state variables S: $S_t = \Phi(S_{t-1}, \varepsilon_t; H_\varepsilon)$ $\forall t \geq 0$,
(iii) a decision rule for controls C: $C_t = \phi(S_{t-1}, \varepsilon_t; H_\varepsilon)$ $\forall t \geq 0$,
(iv) a separable intertemporal utility function $W_t = \sum_{j=0}^{\infty} \beta^j u(C_{t+j})$ $\forall t \geq 0$, where $\beta \in]0,1[$ and u is an instantaneous utility function,
(v) an initial condition S_{-1}.

Note that the presence of non linearities implies that decision rules depend on the whole distribution of the shock, and not only its mean, as in linear models. Let us assume that there exist in the economy under consideration a unique deterministic steady state, defined as follows.

Definition 2 *A deterministic steady state of the economy is a pair (S_{SS}, C_{SS}) that satisfies*

$$S_{SS} = \Phi\left(S_{SS}, \bar{\varepsilon}; H_\varepsilon^0\right) \quad (1)$$

$$C_{SS} = \phi\left(S_{SS}, \bar{\varepsilon}; H_\varepsilon^0\right) \quad (2)$$

with $H_\varepsilon^0(\varepsilon) = 0$ if $\varepsilon < \bar{\varepsilon}$ and $H_\varepsilon^0(\varepsilon) = 1$ if $\varepsilon \geq \bar{\varepsilon}$

$W_{SS} = \frac{1}{1-\beta} u(C_{SS})$ will denote the level of welfare at the deterministic steady state, — *i.e.* the utility that one gets from living in an economy that started with a state variable at level S_{SS}, and in which shocks are always at their mean $\bar{\varepsilon}$. We will refer to this economy as the economy without shocks. Let us also assume that the economy has a unique stochastic steady state, defined as follows.

Definition 3 *A stochastic steady state of the economy is a pair (H_S, Ω_S) of invariant distribution and support of the state variable S such that, for all $t > 0$, if $S_{t-1} \in \Omega_S$ is distributed according to H_S and if ε_t is distributed according H_ε, then $S_t = \Phi(S_{t-1}, \varepsilon_t; H_\varepsilon)$ is an element of Ω_S and is distributed according to the same distribution H_S.*

How should we evaluate the welfare cost of fluctuations in such an economy? The evaluation we propose can be understood as the outcome of the following experiment of structural change: let us assume that we have been in an economy with shocks from $-\infty$ to $T-1$, and that from T to eternity, fluctuations will be eliminated by setting $\varepsilon_t = \bar{\varepsilon}$ $\forall t \geq T$. We evaluate the welfare gain of this structural change by comparing the discounted flows of utility of paths $\left\{S_t^A, C_t^A, \varepsilon_t\right\}_{t\geq T}$ and $\left\{S_t^B, C_t^B, \bar{\varepsilon}\right\}_{t\geq T}$. Path A corresponds to an economy that started with initial condition S_{T-1} and with shocks, path B to an economy that started with initial condition S_{T-1} and without shocks. These paths are computed as

$$\begin{aligned} S_{T-1}^A &= S_{T-1} \text{ given} \\ S_t^A &= \Phi\left(S_{t-1}^A, \varepsilon_t; H_\varepsilon\right) \quad \forall\, t \geq T \\ C_t^A &= \phi\left(S_{t-1}^A, \varepsilon_t; H_\varepsilon\right) \quad \forall\, t \geq T \end{aligned}$$

and

$$S_{T-1}^B = S_{T-1} \text{ given}$$

$$S_t^B = \Phi\left(S_{t-1}^B, \overline{\varepsilon}; H_\varepsilon^0\right) \quad \forall\, t \geq T$$
$$C_t^B = \phi\left(S_{t-1}^B, \overline{\varepsilon}; H_\varepsilon^0\right) \quad \forall\, t \geq T$$

and we denote W^A and W^B the intertemporal utility of these two paths.

In economy B (the economy without shocks), the only dynamics is the one related to the transition from an initial condition S_{T-1} to the non stochastic steady state S_{SS}. Conditionally on a sequence of shocks $\{\varepsilon_t\}_T^\infty$ and on a starting point for the structural change $T-1$ (or equivalently to a initial condition S_{T-1}), the welfare cost of fluctuations is given by $W^A - W^B$. By repeating this experiment for many different sequences of ε, so that the measure will be unconditional with respect to the shocks, and for many different starting point $T-1$, so that S_{T-1} will be drawn in its ergodic distribution, one will get an unconditional measure of the welfare cost of fluctuations $E\left[W^A\right] - E\left[W^B\right]$. We therefore have the following definition.

Definition 4 *The welfare cost of fluctuations is the difference between the unconditional welfare of being in an economy with shocks, — i.e. in an economy where shocks are drawn from their distribution H_ε at each period and where the initial condition S_{-1} is drawn from the ergodic distribution H_S, and the welfare of being in an economy without shocks, — i.e. in an economy where shocks are always equal to $\overline{\varepsilon}$ and where the initial condition S_{-1} is drawn from the ergodic distribution H_S.*

We now compare our measure with the ℓ evaluation that is usually implemented in the literature.

2.2 ℓ Evaluation of the Welfare Cost of Fluctuations

We want to compare our measure $E\left[W^A\right] - E\left[W^B\right]$ with the ℓ one that corresponds to $E\left[W^A\right] - W_{SS}$, and which is defined as follows:

Definition 5 *The linear (or ℓ) welfare cost of fluctuations is the difference between the unconditional welfare of being in an economy with shocks and the welfare of being in an economy without shocks and with an initial condition equal to the deterministic steady state level of the state variables S_{SS}.*

Such an evaluation compares steady states (the deterministic and the stochastic ones), and does not take into account the transition from one

steady state to the other. This is perfectly admissible in Lucas original text, where no formal model is proposed, but such a measure should not be used when a model is available. In such a case, such an evaluation does not fully takes into account the dynamic aspect of the economy, by disregarding the transitional path from a stochastic steady state to a deterministic one.

It is only in very specific economies that $E\left[W^B\right] = W_{SS}$, — *i.e.* that the two evaluations coincides. This is stated in Proposition 1 .

Proposition 1 *Let a dynamic general equilibrium be given by definition 1. The ℓ evaluation coincides with the welfare cost of fluctuations in the two following economies:*

1. *instantaneous utility is logarithmic, decision rules are log-linear, independent of the distribution of the shocks and shocks are log-normally distributed*
2. *instantaneous utility is linear, decision rules are linear, independent of the distribution of the shocks and shocks are normally distributed*

Proof of Proposition 1: see appendix A.

The intuition of Proposition 1 is quite simple. The ℓ evaluation coincides with the welfare cost of fluctuations when some certainty equivalence property hold, which implies restrictions on the shape of the decision rules, on the distribution of the shocks and on the utility function.

Corollary 1 *In the standard one sector optimal growth model, the ℓ evaluation coincides with the welfare cost of fluctuations if shocks are log-normally distributed, if instantaneous utility is logarithmic and if there is full depreciation.*

Proof of Corollary 1 To prove this corollary, it is sufficient to notice that in that model, decision rules are log-linear and independent of H_ε if and only if utility is logarithmic and if there is full depreciation. If it is the case, Proposition 1 applies. *Q.E.D.*

We shall notice that conditions under which the ℓ measure coincides with the welfare cost of fluctuations are quite restrictive. For instance, in a model with log-linear decision rules, log-normally distributed shocks but with non logarithmic utility (let say CRRA with $\nu \neq 1$), the linear evaluation does not coincide with the welfare cost of fluctuations, because there is no certainty equivalence with respect to the uncertainty on the initial condition for the state variable.

In the two following sections, we propose a quantitative implementation of our measure.

3 Illustration in the Optimal Growth Model and in Two Simple Extensions

3.1 Models

We consider a standard version of the stochastic growth model with inelastic labor supply. The social planner orders individuals' stochastic sequences of consumption to maximize the expected utility function of the representative individual:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{(1-\nu)} (c_t^{1-\nu} - 1) \tag{3}$$

where $\beta \in (0,1)$ is the time discount factor; c_t is consumption and $\nu > 0$ is the relative risk aversion coefficient.

The aggregate resource constraint is given by

$$c_t + k_t - (1-\delta)\, k_{t-1} \leq \theta_t\, k_{t-1}^{\alpha} \tag{4}$$

where k_{t-1} is the beginning-of-period capital stock; δ is the depreciation rate; α is the capital elasticity and θ_t is a productivity shock that evolves according to:

$$\ln(\theta_t) = \rho \ln(\theta_{t-1}) + \epsilon_t + \ln(\Theta(\sigma_\epsilon)) \tag{5}$$

Here, ϵ_t is the innovation to $\ln(\theta_t)$, which is assumed to follow an *i.i.d.* process with zero mean and standard deviation σ_ϵ and $|\rho| < 1$. $\Theta(\sigma_\epsilon)$ is a correction parameter, increasing in σ_ϵ, that guarantees that the mean of θ is always equal to one, for any level of σ_ϵ. Therefore, variations in the level of σ_ϵ will be *mean preserving spread* variations of uncertainty.

The social planning problem of this economy is to maximize (3) subject to (4) and (5) with K_{-1} and θ_0 given, by choice of contingency plans for $\{c_t, k_t : t \geq 0\}$.

After that, we explore two departures from the standard optimal growth model. First, we consider a constrained version with irreversible investment

$$k_{t+1} - (1-\delta)\, k_t \geq 0 \tag{6}$$

Second, we consider a constrained version with an upper bound on consumption. We require that consumption will not exceed deterministic steady state consumption more that some fraction λ, that is,

$$c_t \leq c_{ss}\,(1+\lambda) \equiv \tilde{c} \tag{7}$$

where c_{ss} denotes deterministic steady state consumption which is the same regardless the version of the model under consideration.

We take both of these alternative formulations as good abstractions for the question we want to address. Our aim is to capture the non-linearities associated to some types of occasionally binding constraints. These specifications can be somewhat far from being empirically relevant but we take them for expository purposes as abstractions of more elaborated model economies with capital markets imperfections[1]. The choice of these two specifications can be justified as follows: models with trivial welfare costs of fluctuations are models in which good times and bad times can be smoothed by agents through savings, so that consumptions flows stay relatively smooth. Bad times and good times are more or less similar, so that they almost compensate each others on average: in terms of welfare, the economy with shocks is quantitatively not that different from an economy without shocks. The two constraints we consider are two alternative ways of limiting such a consumption smoothing in one of the two phases of the cycle: booms and recessions are qualitatively different, and do not compensate on average. When investment is constrained to be positive, the agent cannot eat capital in periods of low productivity, and has to reduce its consumption while he is allowed to save in expansions. When consumption is constrained, it cannot go upward in good times as far as it goes downward in bad times. Again, recessions and expansions will not anymore compensate each others on average.

As stated before, we could have considered alternative specifications, for instance through asymmetric adjustment costs for investment, which incorporates some differentiability in the constraint, rather than an inequality

[1]From a methodological point of view, this strategy is in line with Christiano and Fisher [1997].

constraint which is binding in some periods. We think of our model specifications as a proxy for this class of models in which non linearities matter for the welfare cost of fluctuations.

In what follows, we will refer to our version of the standard optimal growth model as *MU*, to the model augmented with irreversible investment as *MI*, and to the model augmented with upper bound on consumption as *MC*. We will relate across experiments those implementations for which the constraint is binding roughly the same fraction of the time at the solution.

3.2 Models Resolution

Table 1 reports the calibrated economy's parameters values. The top rows of the table report those parameter values that are common across experiments. All these values are standard in the literature. The bottom rows of the table report those parameter values that guarantee that the constraints are binding the same fraction of time at the solution. For *MC*, we play with the constant λ. We find that $\lambda = 0.10$ makes the constraint binding 11% of the time at the solution. We consider this a relatively mild constraint on consumption since a 10% deviation is never observed in actual data when consumption cycle is measured relatively to a smooth trend (for example a Hodrick-Prescott trend). For *MI* we retain the non-negativity constraint and we increase the variance of the shock until we find the targeted x% level. We solve for the *MU* economy with all the corresponding sizes of the shock.

Since we are interested in non-linearities, we do not want to solve the model by quadratic approximation. Furthermore, the constraint on consumption is occasionally binding, so that the model is non-differentiable. We then need to adopt a non-linear approximation of the model's solution.

The solution for *MU* is the solution of the following equations

$$c_t^{-\nu} = \beta E_t\left[c_{t+1}^{-\nu}(\alpha\theta_{t+1}k_t^{\alpha-1}+1-\delta)\right] \quad (8)$$
$$c_t + k_t = \theta_t k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \quad (9)$$
$$\theta_t = \theta_{t-1}^{\rho}e^{\varepsilon_t}\Theta(\sigma_\varepsilon), \quad (10)$$
$$(11)$$

the solution for *MI* is the solution of the following equations

$$c_t^{-\nu} - \eta_t^i = \beta E_t\left[c_{t+1}^{-\nu}(\alpha\theta_{t+1}k_t^{\alpha-1}+1-\delta) - \eta_{t+1}^i(1-\delta)\right] \quad (12)$$
$$\eta_t^i(k_{t+1} - (1-\delta)\,k_t) = 0 \quad (13)$$

Table 1: Calibrated economy's parameters

		Common
Preferences		
Subjective discount factor	β	0.99
Relative risk aversion	ν	3
Technology		
Capital elasticity	α	0.33
Depreciation rate	δ	0.025
Shock process		
Serial Correlation of innovation	ρ	0.95
		Changing
Shock process		
Std. dev. of Tech. shock	σ_ϵ	0.02
		0.05
		0.08
		0.10
Policy parameters		
Constraint on consumption	λ	0.10
		0.13
		0.15

$$c_t + k_t = \theta_t k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \quad (14)$$
$$\theta_t = \theta_{t-1}^{\rho}e^{\varepsilon_t}\Theta(\sigma_\varepsilon) \quad (15)$$
$$\eta_t^i \geq 0 \quad (16)$$
$$(17)$$

and the solution for *MC* is the solution of

$$c_t^{-\nu} - \eta_t^c = \beta E_t\left[(c_{t+1}^{-\nu} - \eta_{t+1}^c)(\alpha\theta_{t+1}k_t^{\alpha-1}+1-\delta)\right] \quad (18)$$
$$\eta_t^c(\tilde{c} - c_t) = 0 \quad (19)$$
$$c_t + k_t = \theta_t k_{t-1}^{\alpha} + (1-\delta)k_{t-1} \quad (20)$$
$$\theta_t = \theta_{t-1}^{\rho}e^{\varepsilon_t}\Theta(\sigma_\varepsilon) \quad (21)$$
$$\eta_t^c \geq 0 \quad (22)$$

where η_t^i and η_t^c are the Lagrange multipliers associated to the investment (6) and consumption constraint (7), respectively.

We solve for by using the method of parameterizing expectations developed by Marcet [1988] and Den Haan and Marcet [1990][2]. This method substitutes the conditional expectation in the model Euler equation ((12) or (18)) by a particular function $\psi(a^\star, s_t)$ where s_t are the state variable of the model. $s_t = \{\theta_t, k_{t-1}\}$.

Let us take for instance the *MC* case and denote

$$\phi(s_{t+1}) = \beta(c_{t+1}^{-\nu} - \eta_{t+1}^{c})(\alpha\theta_{t+1}k_t^{\alpha-1} + 1 - \delta)$$

ψ and $a^\star$ are chosen so that the function $\psi(a^\star, \cdot)$ is close to the conditional expectation in (12) or (18). In what follows, $a^\star$ is chosen according to the following metric:

$$a^\star = \text{ArgMin}_a \quad \frac{1}{T}\sum_T (\phi(\tilde{s}_{t+1}) - \psi(a, \tilde{s}_t))^2$$

where $\tilde{s}_t$ is simulated by substituting the conditional expectation in the Euler equation with the parameterized expectation $\psi(a^\star, \tilde{s}_t)$. The approximation has been checked to be accurate, according to Den Haan and Marcet [1990]'s test (see appendix B)

To compute our measure for the welfare cost of fluctuations we implement an algorithm in light of the issues discussed in section 2 (see appendix C for computational details). This numerical procedure allows us to compute the unconditional welfare associated to economies with and without shocks. Alternative measures of the welfare benefits associated with switching from one economy to another are then computed based on comparison between permanent consumption in the economy with shocks according to either a linear, c^ℓ, or non linear, c^A, evaluation and permanent consumption in the economy either in the transition, c^B, or at its deterministic steady state, c_{SS}. Thus, the (correct) welfare cost of fluctuations, in percentage points of steady state consumption, will therefore be given by $(c^B - c^A)/c_{SS} - 1$, the welfare cost of fluctuations disregarding the transition by $(c_{SS} - c^A)/c_{SS} - 1$ and the ℓ one by $(c_{SS} - c^\ell)/c_{SS} - 1$. We now turn to the quantitative evaluation of these costs in our three models.

3.3 Findings

In this subsection we discuss the welfare effects associated with switching from a world with aggregate fluctuations to a world without fluctuations.

[2]See Marcet and Marshall [1994] for convergence properties of such an approximation method.

By using the measures outlined in the previous subsection across experiments we will address three issues: *i)* the contribution to the welfare cost of fluctuations of incorporating non-differentiabilities to the standard stochastic growth model for a given size of the shock, *ii)* how does the fact of taking into account the transition modify the evaluation of the welfare costs, and *iii)* the size of the correction for non-linearity to Lucas' measure.

First, it is important to notice that shocks always lead to an accumulation of extra capital so much so that the mean of consumption at the stochastic steady state is always above the deterministic steady state value of consumption[3]. This result has been already pointed out in the literature. For instance, Den Haan and Marcet [1994], when comparing linear and nonlinear methods, find that in the standard growth model there are two reasons for which the mean of the capital stock (and therefore the mean of consumption) becomes higher with an increase in uncertainty. The first is that the mean of θ goes up with σ_ε. We do not have this effect here since in each simulation, we keep the mean of θ constant with the term $\Theta(\sigma_\varepsilon)$, so that the simulated model is a mean preserving spread transformation of the model without shocks. But the second reason still applies: the representative agent uses capital as an asset for insuring against periods of low productivity. An increase in uncertainty leads to more insurance and therefore higher capital (and consumption) on average.

Second, if fluctuations are costly, which is always the case as far as preferences are concave, then permanent consumption in the economy with shocks is lower than permanent consumption in the economy without shocks. Thus, we always find a welfare gain from shutting down fluctuations.

Third, and related with the two previous arguments, the welfare cost of fluctuations is roughly the negative of the welfare gain from having extra capital to eat during the transition to the deterministic steady state plus the welfare gain of eliminating the variability in the long-run equilibrium, the latter being proportional to the size of fluctuations and corresponding to the measure.

Table 2 report the welfare results. As discussed above, all of the figures are expressed in percentage terms of steady state consumption.

At first sight, it is clear that a precise evaluation of the welfare effects,

[3]Such a result is conditional on the fact that no other asset or good are available in this economy. If we introduce leisure of a storage technology, it might be the case that the mean of consumption at the stochastic steady state is lower than its deterministic steady state level.

Table 2: Welfare Cost of fluctuations

	Unconstrained Economy				Constrained Economy		
	Correct	No transition	ℓ evaluation		Correct	No transition	ℓ evaluation
						Consumption	
				$\lambda = 0.10$	0.476%	0.393%	0.609%
$\sigma_\varepsilon = 0.02$	0.489%	0.442%	0.651%	$\lambda = 0.13$	0.464%	0.434%	0.634%
				$\lambda = 0.15$	0.461%	0.448%	0.644%
						Investment	
$\sigma_\varepsilon = 0.05$	2.764%	2.722%	3.666%		2.750%	2.841%	3.699%
$\sigma_\varepsilon = 0.08$	6.726%	7.057%	8.443%		7.153%	7.804%	9.093%
$\sigma_\varepsilon = 0.10$	11.233%	12.044%	13.398%		12.883%	13.705%	14.946%

taking into account transitions and non-linearities, does not result in quantitatively relevant changes. Let us first consider the unconstrained model. For empirically relevant values for the volatility of the shock ($\sigma = 2\%$) the ℓ evaluation is fairly small, around 0.651% of steady state consumption on average, while our measure is 0.489%. Increasing the size of the shock correspondingly increases the welfare cost of fluctuations, up to 11.233% when $\sigma_\varepsilon = 0.10$. The ℓ evaluation *overestimates* the cost of fluctuations by working as if the model were linear. If we turn to both constrained versions of the model, this conclusion is preserved throughout all experiments.

This apparently unexpected result deserves justification. Let us restrict ourselves to the unconstrained version of the model with $\sigma_\varepsilon = 8\%$. In this case, the stochastic steady state mean of consumption is 2.3% greater than deterministic steady state consumption. As stated above, the total welfare cost of fluctuations will be the negative of the gain from having the opportunity to eat some extra capital during the transition plus the gain from shutting down fluctuations in the long-run. The latter is well approximated by the ℓ evaluation and it turns to be around an 8.5% of deterministic steady state consumption. Let us compute the welfare of a transition path that starts from the stochastic steady state means of k and θ, i.e. the means of the stationary distributions of the economy with shocks. We can therefore compute a welfare cost of fluctuations as if there were no uncertainty on the levels of the state variables at the time of the shocks shutdown. In this case we find that permanent consumption after the policy change is 3.23% above deterministic steady state consumption and total welfare cost is 9.93%,

consequently greater than the ℓ evaluation (8.5%).

This result says that taking into account the transition increases the welfare cost of fluctuations compared to ℓ when we compute only the "average" transition. However, when we compute our measure of the correct cost of fluctuations we find a substantial reduction compared to ℓ. This means that uncertainty about the initial conditions for the transition dominates the gain on average, so much so that the transition is globally costly, therefore reducing the gain from shutting down fluctuations.

This general result when compared with the traditional ℓ evaluation can be stated more precisely by decomposing the contribution of the transition to the correct welfare cost of fluctuations. If we look across experiments it is clear that for low values of σ the transition generates a welfare gain so that the measure of total welfare costs is greater than the measure of welfare costs that abstracts from the transition. However, for high values of σ we obtain the opposite result. Thus, the less dispersed are the initial conditions the higher will be the expected utility of the transition so the more will be the welfare gain of shutting down fluctuations.

Summarizing, the bigger the size of the shock the more the economy accumulates extra capital on average so the larger should be the welfare gain of shutting down fluctuations. However, at the same time, the dispersion of the initial conditions may more than compensate the gain on average so much so that the total welfare cost of fluctuations can be smaller. All in all, our correction is quantitatively small in those transformed optimal growth models.

4 A Non Walrasian Case

In our former examples, the welfare cost of fluctuations was not magnified by the introduction of occasionally binding constraints. In this section, we show that a non walrasian model does so, because allocation inefficiencies are magnified by fluctuations. The model we use as an example is a model with one period real wage contracts. The model is the analog of Bénassy [1995], but with real rather than nominal wage rigidities.

4.1 The Model

Let us now consider a simple non walrasian extension of the Neoclassical growth model. To simplify the analysis, we restrict ourselves to a fully analytically computable case, i.e. a case with logarithmic utility and full depreciation. Output is produced with capital k and labor n according to a Cobb Douglas technology:

$$y_t = \theta_t k_{t-1}^{\alpha} n_t^{1-\alpha} \tag{23}$$

with

$$\theta_t = \theta_{t-1}^{\rho} e^{\varepsilon_t} \Theta(\sigma_\varepsilon). \tag{24}$$

Preferences are given by the following intertemporal utility function U:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\log c_t + \gamma \log(\overline{n} - n_t)\right)$$

It is easy to show (see appendix D) that in the walrasian regime (no real wage rigidity), the dynamics of the model is given by equations (23), (24) and:

$$\begin{aligned} n_t &= \tilde{n} & (25)\\ c_t &= (1-\alpha\beta) y_t & (26)\\ k_t &= \alpha\beta y_t & (27)\\ w_t^w &= (1-\alpha)\theta_t k_{t-1}^{\alpha} n_t^{-\alpha} & (28) \end{aligned}$$

where w^w will be referred as the walrasian wage.

In the non walrasian case, we assume that the real wage that prevails in period t, w_t, has been set in period $t-1$ at the level that clears the labor market one period ahead:

$$w_t = E_{t-1}\left[w_t^w\right] = (1-\alpha) k_{t-1}^{\alpha} \tilde{n}^{-\alpha} E_{t-1}\left[\theta_t\right] \tag{29}$$

In period t, the labor market does not clear, and we assume that the quantity of labor effectively transacted is the minimum of labor demand and labor supply. It can be shown that in the real wage contract economy, the non walrasian equilibrium is given by equations (23), (24), (29) and:

$$n_t = \min\left(\tilde{n}, w_t^{-1/\alpha}\left((1-\alpha)\theta_t k_{t-1}^{\alpha}\right)^{1/\alpha}\right) \tag{30}$$

$$\begin{aligned} c_t &= (1-\alpha\beta) y_t & (31)\\ k_t &= \alpha\beta y_t & (32)\\ E_t\left[\theta_{t+1}\right] &= \theta_t^{\rho} & (33)\\ & & (34) \end{aligned}$$

We now turn to the quantitative evaluation of the welfare cost of fluctuations.

4.2 Quantitative Evaluation

As it can be seen from equations (23), (24), (29) and (30) to (33), the model solution is recursive, and is fully computable once a k_0 and a sequence of θ_t are given. The model is log-linear except for the "min" operator in (30). The calibration used in the simulation of the model is given in table 3.

Table 3: Calibrated economy's parameters

Preferences		
Subjective discount factor	β	0.99
Time endowment	$\tilde{n}$	1
Disutility of labor	γ	3.9712
Technology		
Capital elasticity	α	0.42
Depreciation rate	δ	1
Shock process		
Serial Correlation of Tech. shock	ρ	0.95
Std. dev. of innovation	σ	0.01

Let us recall that in the walrasian case (no real wage contract), the model fulfills the requirements under which, according to Proposition 1, the ℓ evaluation correctly measures the welfare cost of fluctuations. It is shown in table 4 that the introduction of real rigidities sharply modifies the welfare cost of fluctuations.

Two points deserve some comments. First, our corrected measure is about one order of magnitude higher that the ℓ evaluation. With low risk aversion ($\nu = 1$) and small shocks ($\sigma_\varepsilon = 0.01$), we measure the welfare cost of fluctuations to be about 1% of deterministic steady state consumption, compared to 0.16% with an (incorrect) ℓ evaluation. Second, it should be notice that the

Table 4: Welfare Cost of fluctuations

	Correct	No transition	ℓ evaluation
Walrasian Economy	0.14%	0.14 %	0.14%
Non Walrasian Economy	0.99%	1.00%	0.16 %

correction matters only because of the non-linearities and inefficiencies in the model, given that the cost is trivial in the walrasian version of the economy (0.14%). This shows that our correction, even in a representative agent with CRRA preferences framework, substantially increases the evaluation of the welfare cost of fluctuations, in non walrasian models.

5 Concluding remarks

In this paper we quantify the welfare cost of fluctuations in a representative agent dynamic equilibrium framework. In doing so we formalize a measure of welfare effects and we formally characterize the settings for which our measure coincides with, at least, part of existing measures discussed in the literature.

To illustrate the empirical performance of our argument we implement our measure of welfare effects in a standard version of the general equilibrium standard growth model. We stress the potential role that non-linearities generated by occasionally binding constraints aimed at inducing asymmetries can have in welfare evaluations in terms of first-order (mean) consequences. To this end, we solve for three departures of our benchmark model: a model with irreversible investment, a model with an upper-bound in consumption and a non walrasian model with real wage contracts.

In the walrasians models, our correction for transitions and non-linearities is not quantitatively dramatic. This is so because in our model economy equilibrium is pareto optimal. On the one hand, as Lucas [1987] pointed out risk by itself is not that important in standard versions of the stochastic growth model. On the other hand, intratemporal inefficiency generated by occasionally binding constraints does not really matter for the welfare evaluation. As far as the economy is producing efficiently then the economy is allocating intertemporally in an efficient way whatever will be the non-linear constraint. To illustrate this point, we have shown that our correction sharply increases

the measure of the welfare cost of fluctuations in a non walrasian model, namely a model with one period real wage contracts.

Therefore, we consider the contribution of the paper is methodological. If we think the welfare cost of fluctuations is large then we will need for its evaluation a model incorporating inefficiencies in the way agents allocate intertemporally. This can be a natural result in models with countercyclical mark-ups (Rotemberg and Woodford [1992] or Gali [1994]) which give rise to more market power in recessions and therefore more inefficiencies in recessions. Alternatively, models explaining the equity premium puzzle such those incorporating habit persistence (Constantinides [1990] or Lettau and Uhlig [1995]) or small probabilities of deep recessions (Danthine and Donaldson [1997]) are good candidates for evaluating the role of stabilization policies. Exploring the implications of our measure of welfare effects in these frameworks is left for future research.

References

A. Atkeson and C. Phelan. *Reconsidering the costs of business cycles with incomplete markets.* Working paper 4719, National Bureau of Economic Research, 1994.

J.P. Bénassy. Money and wages contracts in an optimizing model of the business cycle. *Journal of Monetary Economics*, 35(2):303–15, 1995.

R. Boucekkine. An alternative methodology for solving nonlinear forward-looking models. *Journal of Economics Dynamics and Control*, 19(4):711–34, 1995.

J.O Cho, T. Cooley and L. Phaneuf. The welfare cost of nominal wage contracting. *Review of Economics Studies*, 64:565–84, 1997.

L. Christiano and J. Fisher. *Algorithms for solving dynamic models with occasionally binding constraints.* Technical working paper 218, NBER, january 1997.

G. Constantinides. Habit formation: a resolution of the equity premium puzzle. *Journal of Political Economy*, 98:519–43, 1990.

T. Cooley and G. Hansen. The inflation tax in a real business cycle model. *American Economic Review*, 79(4):733–748, September 1989.

T. Cooley and G. Hansen. Tax distortions in a neoclassical monetary economy. *Journal of Economic Theory*, 58:2090–316, 1992.

T. Cooley and L. Ohanian. Postwar British economy growth and the legacy of Keynes. *Journal of Political Economy*, 105(3):439–72, 1997.

J.P. Danthine and J. Donaldson. *Productivity growth, consumer confidence and the business cycle.* Working paper 9711, University of Lausanne, 1997.

W. Den Haan and A. Marcet. Solving the stochastic growth model by parameterizing expectations. *Journal of Business and Economic Statistics*, 8:31–34, 1990.

W. Den Haan and A. Marcet. Accuracy in simulations. *Review of Economic Studies*, 61:3–17, 1994.

A. Epaulard and A. Pommeret. *Evaluating risky consumption paths: a Lucas' critique.* Working paper, ERASME, Ecole Centrale de Paris, 1997.

L. Epstein and S. Zin. Substitution, risk aversion and the temporal behavior of consumption and asset returns: an empirical analysis. *Journal of Political Economy*, 99(2):261–86, 1991.

J. Gali. Monopolistic competition, business cycles, and the composition demand. *Journal of Economic Theory*, 63(1):73–96, 1994.

J. Greenwood and G. Huffman. Tax analysis in a real business cycle model: on measuring Harberger triangles and Okun gaps. *Journal of Monetary Economics*, 27:195–232, 1991.

J.O. Hairault, F. Langot and F. Portier. *Efficiency and stabilization: reducing Harberger triangles and Okun gaps.* Working paper, CEPREMAP, Paris, 1998.

A. Imrohoroglu. Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy*, 97:1664–83, 1988.

M. Juillard. *DYNARE: a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm.* Working paper, CEPREMAP, Paris, 1996.

T. Kehoe and E. Prescott. Introduction to the symposium: the discipline of applied general equilibrium. *Economic Theory*, 6:1–11, 1995.

J.P. Laffargue. Résolution d'un modèle macroéconomique avec anticipations rationnelles. *Annales d'Economie et de Statistiques*, 17:97–119, 1990.

M. Lettau and H. Uhlig. *Can habit formation be reconciled with business cycle facts?* Working paper, CentER for Economic Research, Tilburg University, 1995.

R. Lucas. *Models of Business Cycles.* Basil Blackwell, Oxford, 1987.

A. Marcet. *Solution of Nonlinear Models by Parameterizing Expectations.* manuscript, GSIA, Carnegie Mellon University, 1988.

A. Marcet and D. Marshall. *Solving nonlinear rational expectations models by parameterized expectations: convergence to stationary solutions.* Discussion Paper 91, Research Department, Federal Reserve Bank of Minneapolis, Minneapolis, 1994.

M. Obstfeld. Evaluating risky consumption paths: the role of intertemporal substitutability. *European Economic Review*, 38:1471–86, 1994.

J. Rotemberg and M. Woodford. Oligopolistic competition and the effect of aggregate demand on economic activity. *Journal of Political Economy*, 100(6):1153–1207, 1992.

Appendix

A Proof of Proposition 1

In what follows, we restrict ourselves to the case where C, S and ε are uni-dimensional. The extension to a multi-dimensional case is straightforward.

If decision rules are independent of H_ε, they can be written, when shocks are always equal to their mean for $t>0$, as:

$$S_t = \Phi(S_{t-1}, \overline{\varepsilon}; H_\varepsilon) = \Phi(S_{t-1}, \overline{\varepsilon}; H_\varepsilon^0) = \Psi(S_{t-1})$$

$$C_t = \phi(S_{t-1}, \overline{\varepsilon}; H_\varepsilon) = \phi(S_{t-1}, \overline{\varepsilon}; H_\varepsilon^0) = \psi(S_{t-1})$$

With these notations,

$$\begin{aligned} C_t &= \psi(S_{t-1}) \\ &= \psi(\Psi(S_{t-2})) \\ &\dots \\ &= \psi(\Psi^t(S_{-1})) \end{aligned}$$

and the welfare associated to this path is given by

$$\widetilde{W}_0(S_{-1}) = \sum_{t=0}^{\infty} \beta^t u\left(\psi\left(\Psi^t(S_{-1})\right)\right)$$

Let us notice that given S_{-1}, $\widetilde{W}_0(S_{-1})$ is known with certainty. By taking expectation over the possible values of S_{-1} (drawn into the distribution H_S), one gets the unconditional welfare of living in an economy without shocks from $t=0$ to ∞, denoted $E_{S_{-1}}\left[\widetilde{W}_0\right]$ and defined formally by

$$E_{S_{-1}}\left[\widetilde{W}_0\right] = \int_{S_{-1}} \widetilde{W}(S_{-1}) dH_S$$

This welfare has to be compared with the welfare of staying in an economy with shocks, that will be denoted by $E[W_0]$, which is defined formally by

$$E[W_0] = \int_{S_{-1}} \int_{\varepsilon} \dots \int_{\varepsilon} \left\{ \sum_{t=0}^{\infty} u\left(\phi\left(\Phi\left(\Phi\left(\dots\Phi\left(S_{-1}, \varepsilon_0\right), \dots, \varepsilon_{t-1}\right), \varepsilon_t\right)\right)\right) \right\} dH_\varepsilon \dots dH_\varepsilon dH_S$$

With these notations, Lucas [1987]' evaluation of the welfare cost of fluctuations is equal to $E[W_0] - W_{SS}$, while it should be defined as $E[W_0] - E_{S_{-1}}\left[\widetilde{W}\right]$. To prove Proposition 1, it is sufficient to show that $E_{S_{-1}}\left[\widetilde{W}\right] = W_{SS}$ in the two cases (log-linear– log-normal and linear-normal). We shall give the proof for the log-linear case, the same argument applying directly for the second case.

In the log-linear case with independence to H_ε, decisions rules can be written as follows:

$$\begin{aligned} \Phi(S, \varepsilon; H_\varepsilon) &= \Phi_0 S^{\Phi_1} \exp\left(\varepsilon^{\Phi_2}\right) \\ \phi(S, \varepsilon; H_\varepsilon) &= \phi_0 S^{\phi_1} \exp\left(\varepsilon^{\phi_2}\right) \end{aligned}$$

and one has

$$\log(S_t) = \log(\Phi_0) + \Phi_1 \log(S_{t-1}) + \Phi_2 \varepsilon_t \qquad (A.1)$$

From $(A.1)$, given that normality is preserved with the sum, we have the result that if ε is normally distributed, H_S will be a log-normal distribution. We shall then show that in such a case, $\log(S_{SS})$ and $E_S[\log(S)]$ coincide, which will be a useful result for the rest of the proof.

Taking expectations,

$$E_{S_{t-1}, \varepsilon_t}\left[\log(S_t)\right] = \log(\Phi_0) + \Phi_1 E_{S_{t-1}}\left[\log(S_{t-1})\right] + \Phi_2 E_{\varepsilon_t}\left[\varepsilon_t\right] \qquad (A.2)$$

Given the definition of the stochastic steady-state distribution of S, H_S, and the fact that it is log-normal, one has $E_{S_{t-1}, \varepsilon_t}\left[\log(S_t)\right] = E_{S_{t-1}}\left[\log(S_{t-1})\right] = E\left[\log(S)\right]$ where the last expectation is an unconditional one. Therefore, $(A.2)$ implies

$$E\left[\log(S)\right] = \Phi(E\left[\log(S)\right], \overline{\varepsilon})$$

which is the definition of S_{SS}, that has been assumed to be unique. Therefore, one has

$$E\left[\log(S)\right] = \log(S_{SS}) \qquad (A.3)$$

We shall know compute the welfare $E_{S_{-1}}\left[\widetilde{W}_0\right]$. The function Ψ is given by:

$$\Psi(S) = \Phi_0 S^{\Phi_1} \exp(\overline{\varepsilon})^{\Phi_2}$$

and Ψ^t by

$$S_t = \Psi^t(S_{-1}) = \Phi_0^{\alpha_t} S_{-1}^{\Phi_1^t} \exp(\overline{\varepsilon})^{\Phi_2 \alpha_t} \qquad (A.4)$$

with

$$\alpha_t = \sum_{j=0}^{t-1} \Phi_1^j$$

Taking $(A.4)$ in logarithm and with expectations, and using $(A.3)$

$$\begin{aligned}
E_{S_{-1}}\left[\log(S_t)\right] &= E_{S_{-1}}\left[\alpha_t \log(\Phi_0) + \Phi_1^t \log(S_{-1}) + \Phi_2 \alpha_t \overline{\varepsilon}\right] \\
&= \alpha_t \log(\Phi_0) + \Phi_1^t E_{S_{-1}}\left[\log(S_{-1})\right] + \Phi_2 \alpha_t \overline{\varepsilon} \\
&= \alpha_t \log(\Phi_0) + \Phi_1^t E\left[\log(S_{-1})\right] + \Phi_2 \alpha_t \overline{\varepsilon} \\
&= \alpha_t \log(\Phi_0) + \Phi_1^t \log(S_{SS}) + \Phi_2 \alpha_t \overline{\varepsilon} \\
&= \Psi^t(S_{SS})
\end{aligned}$$

Given that S_{SS} is uniquely defined by

$$S_{SS} = \Psi^t(S_{SS}),$$

one gets

$$E_{S_{-1}}\left[\log(S_t)\right] = \log(S_{SS}) \tag{A.5}$$

Using $(A.5)$, the expected instantaneous utility of C_t can be written

$$\begin{aligned}
E_{S_{-1}}\left[u(C_t)\right] &= E_{S_{-1}}\left[u(\psi(S_t))\right] \\
&= \log(\phi_0) + \phi_1 E_{S_{-1}}\left[\log(S_t)\right] + \phi_2 \overline{\varepsilon} \\
&= \log(\phi_0) + \phi_1 \log(S_{SS}) + \phi_2 \overline{\varepsilon} \\
&= u(\psi(S_{SS})) \\
&= u(C_{SS})
\end{aligned}$$

Therefore, $E_{S_{-1}}\left[\widetilde{W}_0\right]$ is given by

$$\begin{aligned}
E_{S_{-1}}\left[\widetilde{W}\right] &= E_{S_{-1}}\left[\sum_{t=0}^{\infty} \beta^t u\left(C_t\right)\right] \\
&= \sum_{t=0}^{\infty} \beta^t u\left(C_{SS}\right) \\
&= W_{SS}
\end{aligned}$$

which proves Proposition 1 in the log-linear–log-normal case. It is straightforward to apply the same argument in the linear–normal case. *Q.E.D.*

B Accuracy of the Approximated Solutions

To approximate the model's solution, one has to decide of a particular shape of the ψ function. We use polynomials in $\log k$ and $\log \theta$ whose order is chosen on the basis of the accuracy test proposed in Den Haan and Marcet [1990].

The fixed point a^* is computed for a very lengthy simulation (T periods), and once the approximated solution is obtained, the model is simulated s times with horizon t to check for accuracy. In doing so, we test for the orthogonality of the Euler equation residuals with respect to $(1, k_{t-1}, k_{t-2}, k_{t-3}, \theta_t, \theta_{t-1}, \theta_{t-2})$. Table 5 reports the algorithm's parameters values.

Table 5: Algorithm's parameters

Fixed point		
Horizon	T	45000
Accuracy		
Horizon	t	3000
Number of simulations	s	500
Welfare		
Deterministic transition	t_d	1000
Number of simulations	s_d	2500

The test-statistic follows a $\chi^2(7)$ under the null (orthogonality). In Table 6 we report the percentage of draws in the lower and upper 5% tails for the empirical *cdf* for our candidate solution to each of the model economies.

Table 6: Accuracy test

	Unconstrained Economy			Constrained Economy	
	lower 5%	upper 5%		lower 5%	upper 5%
				Consumption	
			$\lambda = 0.10$	7.6%	3.6%
$\sigma = 0.02$	5.4%	5.8%	$\lambda = 0.13$	8.8%	5.2%
			$\lambda = 0.15$	8.6%	4.3%
				Investment	
$\sigma = 0.05$	4.8%	5.6%		5.0%	6.4%
$\sigma = 0.08$	5.0%	5.8%		5.2%	6.4%
$\sigma = 0.10$	6.6%	6.4%		3.0%	14.0%

In Table 7 we report the order of ψ and the percentage of time the constraint is binding at the solution. The n order for the polynomial size indicates that some, but not necesarily all, terms up to order n are included.

Table 7: Shape of the solution

		Polynomial size	% of time the constraint binds
			Consumption
	$\lambda = 0.10$	Fifth order	11%
$\sigma = 0.02$	$\lambda = 0.13$	Fifth order	4.1%
	$\lambda = 0.15$	Fifth order	2.1%
			Investment
$\sigma = 0.05$		Fourth order	0.15%
$\sigma = 0.08$		Fourth order	2.9%
$\sigma = 0.10$		Fifth order	5.7%

C Computation of the Welfare Cost of Fluctuations

Once we have obtained a solution for a model, we simulate it over 45.000 periods and build upon an empirical estimate of the invariant distribution $f(k,\theta)$ of capital stock and productivity to obtain an evenly spaced grid of 50×50 points in the $k \times \theta$ space. We also keep the series of consumption flows, denoted c^A, according to the notations of section 2.1. Then we draw initial conditions (k_{-1}, θ_{-1}) in that probability distribution of the economy with shocks[4] to compute a 500 periods deterministic transition for consumption to the non-stochastic steady state consumption of the economy without shocks. In doing so, we use a version of Boucekkine [1995] application of the the algorithm proposed by Laffargue [1990] as implemented and extended in *DYNARE* by Juillard [1996]. This algorithm solves non-linear deterministic models with lags and leads using a Gauss-Raphson method. Given initial conditions (on capital and productivity) and terminal ones (reaching the deterministic steady state after 500 periods), the algorithm stacks up the

[4]We extensively compute transition paths for all the cells of the 50 by 50 (k,θ) matrix, and then weight the utility of each of these path with the density of its initial conditions.

equations of the model for all periods in the simulation and solve in block the resulting system[5]. It has been shown by Laffargue [1990] and Juillard [1996] that the problem can be reduced to manageable proportions by exploiting the special structure of the jacobian matrix. Once these transition paths computed, we add to each path an extra 500 periods sequence with $c_t = c_{ss}$. These paths are denoted $\left\{c_{k_{-1},\theta_{-1}}(t)\right\}_{t=0}^{999}$.

The unconditional flow of utility of living in an economy with shocks can be approximated by a second order expansion around its deterministic steady state c_{SS} (as in Lucas [1987]), but taking into account first order terms (the fact that the empirical mean of c, $\bar{c}$, is not equal to the deterministic steady state level). It is given by:

$$\begin{aligned} W^A &= \tfrac{1}{1-\beta}\left(\frac{1}{1-\nu}c_{SS}^{1-\nu} + \left(\frac{\bar{c}}{c_{SS}} - 1\right)c_{SS}^{1-\nu} - \frac{1}{2}\nu\left(\frac{\bar{c}}{c_{SS}} - 1\right)^2 c_{SS}^{1-\nu} \right. \\ &\left. -\frac{1}{2}\nu c_{SS}^{1-\nu}\left(\bar{c}/c_{SS}\right)^2\sigma_c^2\right) \end{aligned}$$

where $\sigma_c^2 = E\left(\frac{c_t^A - \bar{c}}{\bar{c}}\right)^2$ is the empirical variance of consumption. Then, the permanent consumption flow associated to W^A is given by

$$c^A = \left((1-\beta)(1-\nu)W^A\right)^{1/(1-\nu)}$$

c^A will be referred as the permanent consumption in the economy with shocks. This permanent consumption has to be contrasted with the one that would have been computed according to the linear ℓ evaluation, i.e. a second order expansion around c_{SS} without taking into account the effect on the mean:

$$W^\ell = \frac{1}{1-\beta}\left(\frac{1}{1-\nu}c_{SS}^{1-\nu} - \frac{1}{2}\nu c_{SS}^{1-\nu}\sigma_c^2\right)$$

and the corresponding permanent consumption in the economy with shocks will be equal to

$$c^\ell = \left((1-\beta)(1-\nu)W^\ell\right)^{1/(1-\nu)}$$

Finally, the unconditional welfare associated to the transition from an economy with shocks to an economy without shocks will be given by

$$W^B = \sum_{k_{-1},\theta_{-1}} f(k_{-1},\theta_{-1})W^B(k_{-1},\theta_{-1})$$

[5]This is basically what is done for example in Cooley and Ohanian [1997]

with

$$W^B(k_{-1},\theta_{-1}) = \sum_{t=0}^{999} \beta^t \frac{c^{1-\nu}_{k_{-1},\theta_{-1}}(t)}{1-\nu}$$

and c^B will denote the permanent consumption in the economy in the transition:

$$c^B = \left((1-\beta)(1-\nu)W^B\right)^{1/(1-\nu)}$$

D The model with real wage rigidities

The model is a slightly modified version of Bénassy [1995], with real rather that nominal wage contracts and no money.

The walrasian case : The representative firm maximizes its profit, which gives the two following first order conditions:

$$w^w_t = (1-\alpha)\theta_t k^\alpha_{t-1} n_t^{-\alpha} \quad (B.1)$$

$$z_t = \alpha\theta_t k^{\alpha-1}_{t-1} n_t^{1-\alpha} \quad (B.2)$$

The household first order conditions, combined with (B.2), give at the competitive equilibrium

$$c_t = (1-\alpha\beta)y_t \quad (B.3)$$

$$k_t = \alpha\beta_t \quad (B.4)$$

while combined with (B.1) give

$$n_t = \tilde{n} = \frac{(1-\alpha)\overline{n}}{1-\alpha+\gamma(1-\alpha\beta)} \quad (B.5)$$

(B.3) and (B.4) are obtained independently of the labor market institutional organization, and will therefore still hold in the non walrasian case.

The non walrasian case : The expected clearing market level of the real wage is given by

$$w_t = E_{t-1}[w^w_t] = (1-\alpha)k^\alpha_t \tilde{n}^{-\alpha} E_{t-1}[\theta_t]$$

where we use the fact that k_t is known at period $t-1$. Labor is therefore given by the minimum of labor demand and labor supply:

$$n_t = \min\left(\tilde{n}, w_t^{-1/\alpha}((1-\alpha)\theta_t k^\alpha_{t-1})^{1/\alpha}\right) \quad (B.5)$$

In the non walrasian regime, equation (B.5) replaces equation (B.5) of the walrasian regime.