From Nearly Tilted Waves to Cavity Phase Solitons in Broad Area Lasers with Squeezed Vacuum

Oscar G. Calderón,* Eduardo Cabrera, M. A. Antón, I. Gonzalo, F. Carreño, and J. M. Guerra

Departamento de Optica, Universidad Complutense de Madrid, Ciudad Universitaria s/n, 28040 Madrid, Spain

(Received 30 September 2003; published 23 April 2004)

Phase domains and phase solitons in two-level amplifying media damped by a squeezed vacuum are predicted for the first time. Two different types of pattern formation are found depending on the relative value of the cavity detuning to the squeezed parameter: the usual one in lasers via a supercritical Hopf bifurcation and a new one via pitchfork bifurcation.

DOI: 10.1103/PhysRevLett.92.163901 PACS numbers: 42.65.Sf, 42.50.Dv, 42.65.Tg, 89.75.Kd

The formation and dynamics of transverse light patterns in broad area lasers and other nonlinear optical resonators have been a field of intense research in recent years [1–8]. In lasers, the phase of the radiation can have an arbitrary value and the transition from the nonlasing to the lasing regime is described by a supercritical Hopf bifurcation. For positive cavity detunings, transverse traveling waves are favored above threshold. In this situation, the laser emission is off axis (tilted waves), which helps the laser to emit on resonance. This phenomenon has been experimentally observed by Staliunas *et al.* [8] and by Hegarty *et al.* [9]. Laser systems can be modified in order to show amplitude optical bistability (OB) response which corresponds to a change of the nature of the Hopf bifurcation being in this case a subcritical Hopf bifurcation. This phenomenon leads to the formation of bright localized structures or spatial cavity solitons (CSs). These localized structures are of great interest due to their potential applicability to information processing [10,11]. Laser CSs have been only shown to exist provided a passive element (a saturable absorber) is placed inside the resonator [12,13], or in the presence of two-photon amplification [14]. Cavity solitons have also been obtained recently in purely two-level amplifying medium including local-field effects as the mechanism responsible for the amplitude OB [15]. However, the threshold value of the local-field parameter to achieve OB, and thus CSs, involves a high density medium. In this regime, the effect of radiation trapping should not be neglected since the excited state population is considerable and could prevent the appearance of OB [16]. On the other hand, it has been pointed out that a bad cavity condition is required in order to allow local-field effects to become appreciable [17].

In this Letter we propose an alternative scheme of generation of CSs in purely two-level lasers which overcomes the above mentioned difficulties. Furthermore, in our model, phase OB is obtained instead of the usual amplitude OB, i.e., the localized structures that appear in our system are cavity phase solitons. Besides, this system does not require any intracavity optical element such as saturable absorbers [12,13] or spatial filters [15]. In the new schema that we propose, the atoms are damped by a squeezed vacuum field. Squeezed light sources have become available in laboratories, and in recent years attention has turned to their interaction with optical systems [18,19]. The underlying physics relies on the fact that the squeezed vacuum introduces a phase-dependent relaxation process in the polarization quadratures which decay at different rates. This phenomenon breaks the phase invariance of the laser field and leads to a phase-locked steady-state laser field [20,21].

The starting point for our analysis is with the Maxwell-Bloch equations for a broad area homogeneously broadened two-level laser with plane and parallel mirrors in the rotating wave, slowly varying amplitude, and single-longitudinal-mode approximations, and by considering the two-level atomic medium damped by the squeezed vacuum field. The Maxwell-Bloch equations read as follows:

$$\frac{\partial E}{\partial \tau} = ia\Delta_{\perp}E + \sigma(P - E), \tag{1}$$

$$\frac{\partial P}{\partial \tau} = -(1 + i\Delta)P - MP^{*} + ED, \tag{2}$$

$$\frac{\partial D}{\partial \tau} = -\gamma\left[D - r + \frac{1}{2}(E^{*}P + EP^{*})\right]. \tag{3}$$

E, P, and D are the dimensionless envelopes of the electric field, the electric polarization, and the population inversion, respectively. $\gamma \equiv \gamma_{\parallel}/\gamma_{\perp}$ and $\sigma = \kappa/\gamma_{\perp}$ are the population inversion decay rate and the cavity losses, respectively, in units of the polarization decay rate ($\gamma_{\perp}$). $\Delta = (\omega_{21} - \omega)/\gamma_{\perp}$ is the rescaled detuning between the atomic line center and the cavity frequency. r represents the pumping parameter. Light diffraction is taken into account by means of the transverse Laplacian term in the field equation, and is measured by the diffraction coefficient $a = c^2/(2\omega\gamma_{\perp}d^2)$, where d is the spatial transverse size of the laser. $\Delta_{\perp} = \partial_x^2 + \partial_y^2$ is the transverse Laplacian where x and y are normalized with the spatial scale d. The time τ is normalized versus the polarization decay rate ($\tau = \gamma_{\perp}t$). Finally, M is the parameter that characterizes the squeezing of the vacuum field. The term proportional to P^* in Eq. (2) is responsible for the change in the behavior of the pattern formation. For the sake of simplicity, we will assume that M has a real and positive value as in Refs. [20,21]. Equations (1)–(3) reduce to those derived by Marte *et al.* [20,21] by setting $a = 0$ and $\Delta = 0$. We have numerically integrated Eqs. (1)–(3) in a square bidimensional lattice of 126×126 cells with periodic boundary conditions by means of a finite-difference algorithm.

In the following we study the spatiotemporal dynamics for pump values close to the threshold. We consider the following parameters: $\sigma = 0.5$, $\gamma = 0.2$, and $a = 0.0001$. We use a squeezing parameter $M = 0.5$ in agreement with Marte *et al.* [20,21]. We start by analyzing the case of near-zero or small cavity detuning ($\Delta = 0.3$) for the pump value $r = 1$. After an initial transient the system reaches a steady state where a homogeneous solution is present. This state corresponds to the spatially homogeneous stationary solution of the system of Eqs. (1)–(3) which can be written as $E = |E| \exp(i\phi)$, with $|E| = r - 1 + \sqrt{M^2 - \Delta^2}$, and $\sin(2\phi) = \Delta/M$. This expression represents two physically equivalent solutions with the same amplitude but two phases differing by π. At small cavity detunings the solution asymptotically leads to one of these two homogeneous distributions. At large and negative values of the detuning, we find stationary patterns showing separated domains, characterized by one of the two values of the phase inside each domain. These phase domains present a labyrinth structure as displayed in Figs. 1(a)–1(c). We can see that the intensity field vanishes along the lines separating the two phases, which are called domain boundaries. The power spectrum shows a ringlike structure with a wave vector $k \simeq 47$. This

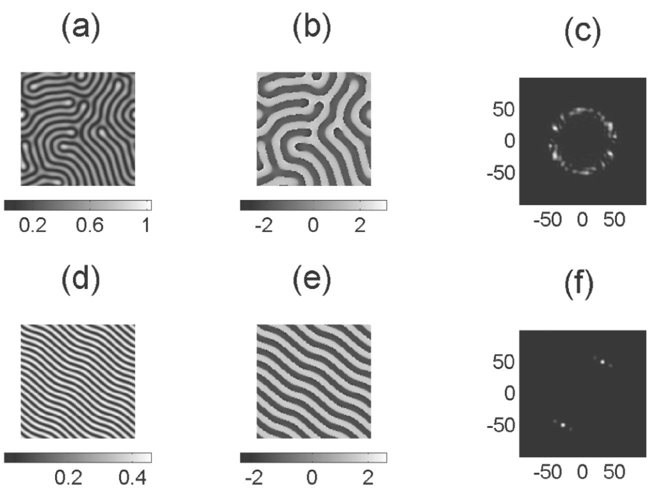

FIG. 1. (a),(d) The intensity field; (b),(e) the phase field; and (c),(f) the power spectrum, for (a),(b),(c) $\Delta = -0.6$, $r = 1.2$, and (d),(e),(f) $\Delta = -1$, $r = 1.5$. The rest of parameters are $\sigma = 0.5$, $\gamma = 0.2$, $a = 0.0001$, and $M = 0.5$. All the magnitudes presented in this figure are dimensionless.

labyrinth structure evolves to a roll pattern by increasing the cavity detuning to $\Delta = -1$ [see Figs. 1(d)–1(f)]. Note that a zigzag behavior remains in the roll pattern. As we can see in the power spectrum [see Fig. 1(f)], two opposite waves govern the dynamics. Then, from Eqs. (1)–(3) and assuming a stationary laser field formed by stripes $E(\vec{x}) \propto \cos(\vec{k} \cdot \vec{x})$, we obtain the resonant wave vector $k = \sqrt{-\Delta\sigma/[a(1 + M)]}$, which agrees with the numerical simulation results. It means that the characteristic spatial scale of the pattern can be controlled by means of the squeezing parameter M. For intermediate and negative values of the detuning ($\Delta = -0.3$) we show in Fig. 2 a stationary pattern formed by four minimum phase domains. The interest of these localized structures or cavity phase solitons is increasing [22–24] since they are easier to achieve experimentally than the usual spatial solitons which appear in subcritical systems. Let us analyze the case of a cavity detuning larger than the squeezing parameter. Figure 3 shows the intensity field, the phase field, and the power spectrum for $\Delta = 1$ and $r = 1.2$. After an initial transient the system reaches a steady state where strong and weak counterpropagating traveling waves are present (wave vector $k \simeq 87$). This leads to a small modulation of the intensity field [see Fig. 3(a)]. We call this pattern a nearly tilted wave. We would like to point out that following the recent work by Valcárcel and Staliunas in a passive system (self-oscillatory system) [25], these nearly tilted waves could be interpreted as Bloch-like stripe patterns.

In summary, we have found two main regimes in the pattern formation in lasers with squeezed vacuum which can be controlled by means of the ratio between the cavity detuning and the squeezing parameter. A first regime takes place for cavity detunings lower than the squeezing parameter. The pattern dynamics is based on the selection of the phase field showing phase domains and phase CSs. These types of spatial structures have been previously found in passive optical media such as degenerate optical parametric oscillators (DOPOs) and degenerate four-wave mixers (DFWMs) and are associated with a pitchfork bifurcation [26–28]. This pattern formation is completely different from the usual case in standard lasers. A second regime appears for cavity detunings

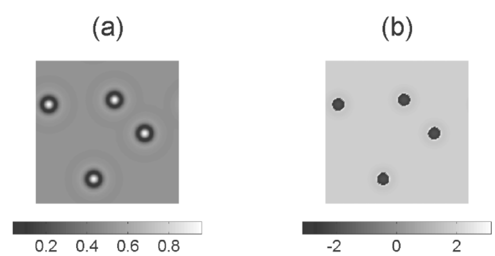

FIG. 2. (a) The intensity field and (b) the phase field for $\Delta = -0.3$ and $r = 1.1$. The rest of the parameters are the same as in Fig. 1.

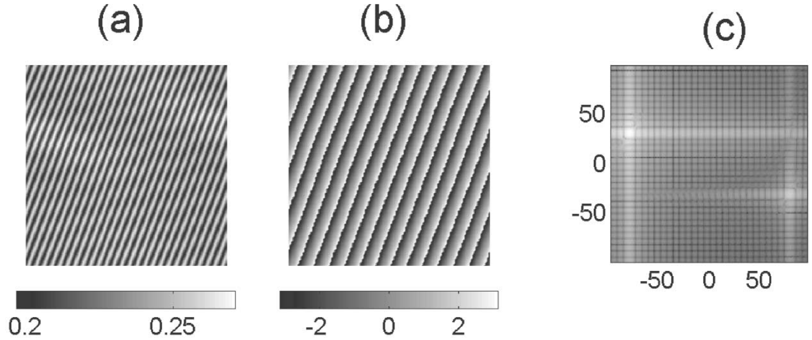

FIG. 3. (a) The intensity field, (b) the phase field, and (c) the power spectrum in logarithmic scale, for $\Delta = 1$ and $r = 1.2$. The rest of the parameters are the same as in Fig. 1.

larger than the squeezing parameter and consists of strong and weak counterpropagating waves (nearly tilted waves) which seem to be closer to the usual pattern dynamics in lasers. So this result represents conceptually a new scenario in the pattern formation, where two different mechanisms lead to the spatial structures. Now, a natural question arises: what is the behavior of this type of laser near the transition between both regimes ($\Delta \simeq M$)? In order to show this behavior we analyze the stationary state of the laser field at $\Delta = 0.5$ and $r = 1.1$(see Fig. 4). We can see in Fig. 4 that both the intensity field and the phase field have the same shape. The power spectrum [see Fig. 4(c)] reveals that the pattern is formed by a combination of a homogeneous solution ($k = 0$) and two weak counterpropagating waves ($k \simeq 70$). That is, the two characteristic spatial structures that appear for cavity detunings just below and above the squeezing parameter value, combine to form a new spatial structure.

In order to obtain a further physical insight on the numerical simulations, we have developed a linear stability analysis of the nonlasing solution ($E = P = 0$, $D = r$). Following the same procedure as in Ref. [5], we linearize about this trivial solution and expand the variables as Fourier series of transversal modes of wave vectors k. The eigenvalue problem has been approached numerically. Let us first analyze the case with a cavity detuning larger than the squeezing parameter. The stationary pattern was shown in Fig. 3 which is formed by strong and weak counterpropagating waves. We find that the bifurcation takes place through a complex eigenvalue, that is, by means of a Hopf bifurcation. This is the usual case in standard lasers and this is the reason why we call this pattern as a nearly tilted wave. We have studied the dependence of the wave vector selected above threshold on the cavity detuning and we found the same usual relation, i.e., $k = \sqrt{\Delta/a}$. Let us continue with the case of a cavity detuning lower than the squeezing parameter. In this regime we have found several patterns [see Figs. 1 and 2] which are characteristic of passive optical media such as DOPOs and DFWMs. We find that the bifurcation is governed by a real eigenvalue which indicates that a pitchfork bifurcation is responsible for the dynamics. We have checked that the most unstable spatial size agrees with the numerical simulation results. Finally, we have analyzed the bifurcation for the case $\Delta = M$, and we have found that the two previous instabilities occur simultaneously. Therefore, a combination of both regimes is expected to form a new type of pattern (see Fig. 4).

In conclusion, we have shown that a very rich laser pattern formation takes place when the two-level atomic medium is damped by a squeezed vacuum field. We have found two main regimes depending on the ratio between the cavity detuning (Δ) and the parameter that measures the squeezing (M) of the vacuum field. When the cavity detuning is lower than the squeezing parameter the phase field is fixed at threshold and the pattern formation is completely different in comparison to the usual case in lasers. Here, we have obtained phase domains and phase solitons which are associated to a pitchfork bifurcation. This is the first prediction of these types of patterns in amplifying cavities (i.e., in lasers). On the other hand, when the cavity detuning is larger than the squeezing

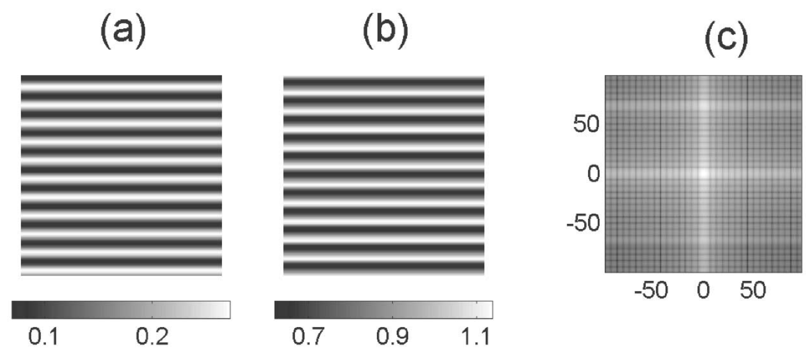

FIG. 4. (a) The intensity field, (b) the phase field, and (c) the power spectrum in logarithmic scale, for $\Delta = 0.5$ and $r = 1.1$. The rest of the parameters are the same as in Fig. 1.

parameter, the system behaves very close to the usual lasers, that is, the pattern formation is governed by a supercritical Hopf bifurcation. However, instead of the usual transverse traveling wave (tilted wave) we obtain a new pattern formed by strong and very weak counter-propagating traveling waves. We denominated to this pattern as a nearly tilted wave. Finally, a new type of pattern arises at the transition between both regimes, which presents features of both behaviors. A linear stability analysis of the nonlasing solution has been carried out confirming the above results.

This work was supported by Project No. BFM2000-0796 (Spain). We are very grateful to M. Ritsch-Marte and J. Lega for their helpful advice.

*Email address: oscargc@opt.ucm.es
Electronic address: http://www.ucm.es/info/laserlab

[1] P. Coullet, L. Gil, and F. Rocca, Opt. Commun. **73**, 403 (1989).
[2] P. K. Jakobsen *et al.*, Phys. Rev. A **45**, 8129 (1992).
[3] F. T. Arecchi *et al.*, Phys. Rev. Lett. **70**, 2277 (1993).
[4] F. Prati, M. Brambilla, and L. A. Lugiato, Riv. Nuovo Cimento **17**, 1 (1994).
[5] P. K. Jakobsen *et al.*, Phys. Rev. A **49**, 4189 (1994).
[6] G. Huyet *et al.*, Phys. Rev. Lett. **75**, 4027 (1995).
[7] F. Encinas-Sanz, I. Leyva, and J. M. Guerra, Phys. Rev. Lett. **84**, 883 (2000).
[8] K. Staliunas, G. Slekys, and C. O. Weiss, Phys. Rev. Lett. **79**, 2658 (1997).
[9] S. P. Hegarty, G. Huyet, and J. G. McInerney, Phys. Rev. Lett. **82**, 1434 (1999).
[10] V. B. Taranenko and C. O. Weiss, IEEE J. Sel. Top. Quantum Electron. **8**, 488 (2002).
[11] S. Barland *et al.*, Nature (London) **419**, 699 (2002).
[12] N. N. Rosanov, J. Opt. Soc. Am. B **7**, 1057 (1990).
[13] V. B. Taranenko, K. Staliunas, and C. O. Weiss, Phys. Rev. A **56**, 1582 (1997).
[14] R. Vilaseca *et al.*, Phys. Rev. Lett. **87**, 083902 (2001).
[15] V. Ahufinger *et al.*, Phs. Rev. Lett. **91**, 083901 (2003).
[16] M. Fleischhauer and S. F. Yelin, Phys. Rev. A **59**, 2427 (1999).
[17] O. G. Calderón *et al.*, Phys. Rev. A **67**, 043812 (2003).
[18] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).
[19] D. F. Walls, Nature (London) **306**, 141 (1983).
[20] M. A. M. Marte and D. F. Walls, Phys. Rev. A **37**, 1235 (1988).
[21] M. A. Marte, H. Ritsch, and D. F Walls, Phys. Rev. Lett. **61**, 1093 (1988).
[22] M. Le Berre *et al.*, J. Opt. B **1**, 153 (1999).
[23] G. L. Oppo, A. J. Scroggie, and W. J. Firth, J. Opt. B **1**, 133 (1999).
[24] M. Tlidi *et al.*, Phys. Rev. A **61**, 043806 (2000).
[25] G. J. Valcárcel and K. Staliunas, Phys. Rev. E **67**, 026604 (2003).
[26] K. Staliunas and V. J. Sánchez-Morcillo, Phys. Rev. A **57**, 1454 (1998).
[27] S. Trillo, M. Haelterman, and A. Sheppard, Opt. Lett. **22**, 970 (1997).
[28] V. B. Taranenko, K. Staliunas, and C. O. Weiss, Phys. Rev. Lett. **81**, 2236 (1998).