

Chiral Lagrangians and the QCD String

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Abstract

We propose a method to derive the low-energy effective action of QCD assuming that the long-distance properties of strong interactions can be described by a string theory. We bypass the usual problems related to the existence of the tachyon and absence of the adequate Adler zero by using a sigma model approach where excitations above the correct (chirally non-invariant) QCD vacuum are included. Two-dimensional conformal invariance then implies the vanishing of the $\mathcal{O}(p^4)$ effective lagrangian coefficients. We interpret this result and discuss ways to go beyond this limit.

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1 Introduction

There are many theoretical and empirical reasons that make us believe that it should be possible to describe QCD in terms of a string theory[1], at least in some kinematical regime. The more commonly cited arguments are the dominance of planar diagrams in the large N limit[2] ‘filling in’ a surface (interpreted as the world-sheet of a string), the expansion in terms of surfaces built out of plaquettes in strong-coupling lattice QCD[3], and the success of Regge phenomenology[4], which can ultimately be understood in terms of string theory ideas (although, as we will discuss a little bit later, the actual Regge theory that corresponds to QCD cannot be derived, at present, from any known string theory).

To these we could add two more reasons. One is the appearance in string theory of the universal (at least at long distances) Lüscher term[5]. The static interquark potential provided by the string $V(r) = \sigma r + c$ gets modified by quantum fluctuations by a Coulomb-like piece $-\pi/12r$, a term which comes very handy when fitting the string interquark potential to heavy quark spectra. Finally, and in a completely different context, namely that of deep inelastic scattering, the evolution of the parton distribution down to low values of Q^2 (around $(2 \text{ GeV})^2$) leads[6] to a low x behaviour for the structure functions of the form $x^{-1.17}$, while Regge theory predicts x^{-1} , in striking good agreement.

Thus, that there is a string description of QCD is almost evident. Which is the appropriate string theory for QCD, however? To answer this question one should first ask oneself which is the kinematical regime where the string picture would be applicable. It is manifestly hard to reconcile the string picture and high energy processes, such as deep inelastic scattering, where the point-like structure of quarks and gluons is apparent. While it is quite conceivable that non-abelian gauge theories could one day be understood in terms of a given string theory (cf. the interesting recent developments about the AdS/CFT relationship[7]), it is also quite obvious that this will never be the natural language to understand high-energy processes. We should probably be less ambitious and satisfy ourselves with an effective description.

Not surprisingly, a lot of candidates have been put forward as possible candidates of the QCD string, ranging from the original Nambu-Goto string[8] to the supersymmetric string[9] and from the rigid string[10] to the five-dimensional string with manifest zig-zag symmetry[11]. Most, if not all, of the candidates are believed to be understood as effective theories, valid only up to some characteristic momentum transfer k_{max} , and at the (string) tree level, lest the inconsistencies of string theory away from the critical dimensions show up. We subscribe this point of view and think of strings as effective theories and not worry at all about their mathematical consistency as fundamental objects.

It is surprising that even with this modest and limited scope all known string theories are inconsistent and cannot provide a description of low energy QCD, even for $k \ll k_{\max}$. To see how this comes about let us remember the original Veneziano amplitude[12]. After decorating it with the appropriate Chan-Paton[13] factors¹ it is supposed to describe the scattering amplitude of four pions

$$\mathcal{A}(\pi^a \pi^b \rightarrow \pi^c \pi^d) \sim \text{Tr}(T^a T^b T^c T^d) A(s, t) + \text{non cyclic permutations}, \quad (1)$$

$$A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))}, \quad (2)$$

where $\alpha(s) = \alpha(0) + \alpha' s$ is the Regge trajectory. The inverse string tension α' is to be determined latter. In the Nambu-Goto string, from which this amplitude is deduced, $\alpha(0) = 1$.

In the above expression we immediately recognize that there are poles in the s -channel whenever $\alpha' s = n - \alpha(0)$. Thus a tachyon is present for $n = 0$.

The supersymmetric string[9] does not really fare any better. There are two sectors in supersymmetric strings. In the Neveu-Schwarz (bosonic) sector $\alpha(0) = \frac{1}{2}$ and there exist a (scalar/pseudoscalar) tachyon too. In Regge parlance the spectrum in this sector is described by the ‘pion’ trajectory $\alpha_\pi(s) = \alpha(0) + \alpha' s$, corresponding to negative G -parity, and by the ‘rho’ trajectory, corresponding to positive G -parity, $\alpha_\rho(s) = \alpha_\pi(s) + \frac{1}{2}$. Usually one performs the GSO projection[14], projecting out the tachyon. However one may choose not to do so and compute the four-tachyon amplitude, supposed to describe pion-pion scattering, which is mediated by the exchange of particles in the ρ -trajectory. The corresponding amplitude is

$$A(s, t) = \frac{\Gamma(1 - \alpha_\rho(s))\Gamma(1 - \alpha_\rho(t))}{\Gamma(1 - \alpha_\rho(s) - \alpha_\rho(t))}, \quad (3)$$

or

$$A(s, t) = \frac{\Gamma(-s\alpha')\Gamma(-t\alpha')}{\Gamma(-s\alpha' - t\alpha' - 1)}. \quad (4)$$

This is the Lovelace-Shapiro amplitude, which contains no tachyonic poles. Could it be a candidate to describe pion scattering? The answer is no. It does not have the appropriate Adler zero, i.e. the property that at $s = t = 0$ the amplitude vanishes.

A fix[15] to this problem is to replace *by hand* $\alpha_\rho(s)$ by $\alpha_\pi(s)$. The amplitude becomes

$$A(s, t) = \frac{\Gamma(\frac{1}{2} - s\alpha')\Gamma(\frac{1}{2} - t\alpha')}{\Gamma(-s\alpha' - t\alpha')}. \quad (5)$$

¹Due to difficulties with unitarity only orthogonal groups can be introduced in this way, but since there are other inconsistencies one should not worry too much at this point.

with poles in the s -channel when $\alpha's = n + \frac{1}{2}$. It has no tachyons and the first pole is identified with the ρ particle, thus fixing α' . Furthermore, the previous amplitude has the right Adler zero. Based on this amplitude Polyakov and Vereshagin[16] have derived the first coefficients of the effective chiral lagrangian and have found that

$$L_1 = \frac{1}{2}L_2, \quad L_2 = \frac{F_\pi^2}{8m_\rho^2} \ln 2, \quad L_3 = -2L_2. \quad (6)$$

Numerically they turn out to be quite acceptable values², but unfortunately there is no way to justify the apparently arbitrary change in the intercept.

An attempt to solve the difficulties associated to the presence of the tachyon is the formulation of the rigid string[10], where four derivative interactions contained in the second fundamental form modify the string behaviour at short distances. Classically at least, the Regge trajectories are modified[17], making it conceivable that the tachyon is avoided. Unfortunately, the classical trajectories are no more straight lines, something with ample phenomenological support and thus a highly desirable property to preserve. While the spectrum of the rigid string has not been determined at the quantum level (the theory is not exactly solvable), it seems unlikely at this point that it provides a satisfactory solution by itself, even though it may be part of the solution, as we will later see.

It has been thought for a long time that the ultimate reason for the presence of a tachyon in the spectrum lies in a wrong choice of the vacuum[18]. Since the choice of the vertex operator, $V(k) =: \exp ikx :$, is based on the Lorentz properties alone, it is the same both for scalar and pseudoscalars and, accordingly, both scalars and pseudoscalars have tachyonic poles in the s -channel on account of parity conservation. The situation is thus parallel to the one in multicomponent $\lambda\phi^4$ when perturbing around $\phi = 0$ gives negative m^2 values for all components. It is natural to assume that the amplitudes obtained through the use of the canonical vertex operators correspond to (unphysical) amplitudes for excitations perturbed around the wrong vacuum.

These ideas are certainly not new, but how could one obtain the amplitudes for excitations around the physically correct vacuum? We propose to use two ingredients to try and give a partial answer to this question.

The first one is to identify from the outset the proper physical degrees of freedom. In this case, the relevant degrees of freedom are the ones emerging after the spontaneous breaking of chiral symmetry. In the physical vacuum of QCD there is a clear distinction between scalars (sigma particle) and pseudoscalars (pions). The pseudoscalars can be collected in a unitary

²The first relation, $L_1 = L_2/2$, is a consequence of the large N (planar) limit.

matrix $U(x)$ which under chiral transformations belonging to $SU(3)_L \times SU(3)_R$ transforms as

$$U(x) \rightarrow U'(x) = LU(x)R^\dagger \quad (7)$$

$U(x)$ is nothing but a bunch of couplings involving the variables x , from the string point of view.

Nobody knows how to write a ‘vertex’ operator for string excitations above a non-trivial vacuum, such as the one existing in QCD. A possible way out is provided by our second ingredient, namely conformal invariance. We propose to use a sigma model technique and request the vanishing of the corresponding beta functional for the couplings $U(x)$. From these we shall eventually derive the appropriate long distance effective action of QCD.

Conformal invariance amounts to demanding that the theory is independent of the specific conformal factor chosen to describe the two-dimensional world sheet. While this is a desirable property of fundamental strings, it need not be necessarily so (if we look at the QCD string with a magnifying glass we shall eventually see quarks and gluons, not the string itself!), so we should rather demand ‘conformal covariance’. Let us assume for the time being that conformal invariance is approximately true, however, and we shall later briefly mention how to move away from this ‘zeroth order’ approximation.

2 The model

In order to obtain the long distance QCD effective action from the QCD string we follow the strategy used[19] to derive Einstein equations from string theory, namely, the non-linear σ model approach.

We couple, in a chiral invariant manner, the matrix in flavour space $U(x)$, containing the meson fields, to the string degrees of freedom while preserving general covariance in the two dimensional coordinates and conformal invariance under local scale transformations of the two-dimensional metric tensor.

The equations of motion for the U field will be obtained from the condition that the quantum theory must be conformally invariant, i.e. the β functional for the $U(x)$ couplings must vanish.

Since the string variable x does not contain any flavor dependence, we have to invent a way to couple it to the background U variable. We introduce two fermion families living on the boundary of the string sheet. They carry flavor indices. The action for the fermions is

$$\frac{1}{2} \int_{\partial\Sigma} d\tau (\bar{\psi}_L U \frac{\partial \psi_R}{\partial \tau} - \frac{\partial \bar{\psi}_L}{\partial \tau} U \psi_R) + h.c., \quad (8)$$

where τ is the coordinate along the (open) string boundary. Physically the labels R and L remind us that these one dimensional Grassmann variables represent massless quarks of a given chirality moving along the ends of the string, and coupled to the external source $U(x)$. Under $SU(3)_L \times SU(3)_R$ they transform as left- and right-handed fields do. The above lagrangian is not unique, but it appears to be the simplest one with the desired properties.

The above coupling may appear surprising at first and somewhat ad-hoc. To see that this is not so, let us expand the non-linear field $U(x)$, i.e. $U(x) \simeq 1 + i\pi(x)/v + \dots$ and retain the first two terms. The first term just gives rise to a θ -function propagator which eventually leads to the familiar ordering in the usual string amplitudes $t_1 < t_2 < \dots$. The second term just provides (after integrating the fermions out) the usual (tachyonic!) vertex. In short, if we ignore the non-linearities in the theory we are back to the usual difficulties.

In order to simplify the calculations, we treat the couplings U and U^+ as independent. The constraint

$$UU^+ = 1 \tag{9}$$

will be imposed after finding the equations of motion for an arbitrary matrix U . The reason is simply that we do not know of an easy way to find the beta function for constrained coupling constants.

It is easy to see that the previous action is invariant under general coordinate transformations of the two dimensional world sheet by writing it as follows

$$\frac{1}{2} \int_{\partial\Sigma} d\tau \frac{dx^\mu}{d\tau} (\bar{\psi}_L U \frac{\partial\psi_R}{\partial x^\mu} + \dots = \frac{1}{2} \int_{\partial\Sigma} dx^\mu (\bar{\psi}_L U \frac{\partial\psi_R}{\partial x^\mu} + \dots, \tag{10}$$

where $\frac{dx^\mu}{d\tau}$ is the tangent vector to the boundary of the two dimensional surface of the string and the fermions are treated as scalars under general coordinate transformations. The fermion action is automatically conformally invariant, because it does not contain the two dimensional world sheet metric tensor since it can be written as a line integral.

Notice that $U(x)$ has support only on the boundary of the string. The above boundary action has to be supplemented with the usual bulk action for the string in the conformal gauge. Namely

$$\frac{1}{\alpha'} \int d\sigma d\tau \partial_a x^\mu \partial_a x_\mu. \tag{11}$$

Unless otherwise indicated we take $\alpha' = 1$.

3 One loop

Now we expand $U(x(\tau))$ around a constant background x_0 and look for the potentially divergent One Particle Irreducible diagrams (OPI). We classify them according to the number of loops.

The appropriate Feynman rules for the bosonic and fermionic propagators are

$$\langle x^a(\tau)x^b(\tau') \rangle = \delta^{ab} \Delta_F(\tau - \tau') \quad (12)$$

$$\langle \psi_L(\tau)\bar{\psi}_R(\tau') \rangle = U(x_0)^{-1}\theta(\tau - \tau') \equiv D_0(\tau - \tau') \quad (13)$$

Here Δ_F is the Feynman propagator for the string coordinate x . The vertices obtained after the expansion of $U(x)$ around x_0 lead to the following Feynman rule for a vertex with n external x fields

$$V_n = -\frac{1}{n!}\partial_{\mu_1, \mu_2, \dots, \mu_n} U(x_0)\partial_\tau \quad (14)$$

To renormalize the propagator we have, at this order, the diagrams shown in figure 1. These are the only one loop graphs with two fermion legs and zero string legs. The calculation is straightforward and we immediately get for the divergent part of the propagator

$$-\frac{1}{2\epsilon}U^{-1}\left(\frac{1}{2}\square U - \partial_\mu U U^{-1}\partial^\mu U\right)U^{-1}\theta(\tau - \tau') \quad (15)$$

where dimensional regularization has been used. The ϵ pole comes from the singular part of $\Delta_F(0)$ which also contains the factor $e^{\phi\epsilon}$. Thus conformal invariance will be broken at the one loop level unless

$$\frac{1}{2}\square U - \partial_\mu U U^{-1}\partial^\mu U = 0 \quad (16)$$

These are the equations of motion of the U field. Later we will supplement them with the unitarity constraint. The fermion propagator can be made finite by using minimal subtraction and redefining accordingly

$$U(x_0)^{-1} \rightarrow U(x_0)^{-1} + \delta^{(2)}U(x_0)^{-1}. \quad (17)$$

with $\delta^{(2)}U^{-1}$ given by

$$\delta^{(2)}U^{-1} = \frac{1}{2\epsilon}U^{-1}\left(\frac{1}{2}\square U - \partial_\mu U U^{-1}\partial^\mu U\right)U^{-1}. \quad (18)$$

Next we turn to the vertices with one x - and two x -fields. The relevant diagrams are shown in figures 2 and 3, respectively. A direct calculation, and the use of

$$\delta U = -U\delta U^{-1}U, \quad (19)$$

shows that the counterterm needed to cancel the divergent part for the former is just

$$\delta^{(2)}V_1 = -\partial_\mu\delta U\partial_\tau, \quad (20)$$

while the counterterm of the latter is

$$\delta^{(2)}V_2 = -\frac{1}{2}\partial_\mu\partial_\nu\delta U\partial_\tau. \quad (21)$$

These expressions will be needed for the two loop calculation.

The vanishing of the beta functional for $U(x)$ can be obtained as the Euler-Lagrange variation of a given action \tilde{S} . The true action will however be

$$S = \tilde{S} + \int d^n x \operatorname{tr}(\lambda(x)(U(x)U^+(x) - 1)) \quad (22)$$

It is easy to see that the variation of S produces the equations of motion

$$U\Box U^+ - \Box U U^+ = 0, \quad (23)$$

which are the ones derived from the chiral lagrangian at lowest order (see appendix).

Thus we have succeeded in deriving a long-distance effective action for QCD with all the required properties, at least at this order.

4 Two loops

In order to compute the two-loop corrections to the fermion propagator in eq.13 one has to consider first the diagrams in figure 4. The total result amounts to

$$-\frac{1}{2}D_0(\tau - \tau')\Delta_F^2(0)T \quad (24)$$

where T is given by

$$T = \frac{1}{4}O_1 - O_2 - O_3 - O'_3 + 2O_4 + O_5 - 2O_6 - 2O_7 + 2O_8 + 2O'_8 \quad (25)$$

and the O_i operators are defined as

$$O_1 = \Box^2 U U^{-1}$$

$$\begin{aligned}
O_2 &= \partial_\mu \partial_\nu U U^{-1} \partial^\mu \partial^\nu U U^{-1} \\
O_3 &= \partial_\mu U U^{-1} \square \partial^\mu U U^{-1} \\
O'_3 &= \square \partial_\mu U U^{-1} \partial^\mu U U^{-1} \\
O_4 &= \partial_\mu U U^{-1} \partial^\mu \partial^\nu U U^{-1} \partial_\nu U U^{-1} \\
O_5 &= \partial_\mu U U^{-1} \square U U^{-1} \partial^\mu U U^{-1} \\
O_6 &= \partial_\mu U U^{-1} \partial_\nu U U^{-1} \partial^\mu U U^{-1} \partial^\nu U U^{-1} \\
O_7 &= \partial_\mu U U^{-1} \partial_\nu U U^{-1} \partial^\nu U U^{-1} \partial^\mu U U^{-1} \\
O_8 &= \partial_\mu \partial_\nu U U^{-1} \partial^\mu U U^{-1} \partial^\nu U U^{-1} \\
O'_8 &= \partial_\mu U U^{-1} \partial_\nu U U^{-1} \partial^\mu \partial^\nu U U^{-1}
\end{aligned} \tag{26}$$

Notice the appearance of four derivatives in the above expressions. The two-loop calculation is the relevant one for the $\mathcal{O}(p^4)$ terms of the chiral lagrangian.

In addition we have also the contribution coming from the counterterm diagrams *I*, *II* and *III* appearing in figure 5

$$D_0(\tau - \tau') \frac{1}{2\epsilon} \Delta_F(0) (D_I + D_{II} + D_{III}) \tag{27}$$

where

$$\begin{aligned}
D_I &= \frac{1}{2} O_5 - O_7 \\
D_{II} &= -\frac{1}{2} (O_3 + O'_3) + 2O_4 - 2O_6 + O_8 + O'_8 \\
D_{III} &= \frac{1}{4} O_1 - \frac{1}{2} (O_3 + O'_3) - O_2 + O_8 + O'_8 - O_7 + \frac{1}{2} O_5
\end{aligned} \tag{28}$$

Thus the counterterm contribution is also proportional to the *T* operator.

Finally, the complete two loop divergent part of the fermion propagator is

$$\frac{1}{8\epsilon^2} D_0(\tau - \tau') T \tag{29}$$

so that no simple ϵ pole appears. The two-loop fermion propagator is made finite with the help of the counterterm

$$\delta^{(4)} U^{-1} = -\frac{1}{8\epsilon^2} U^{-1} T \tag{30}$$

The absence of simple poles at the two loop level implies in minimal subtraction that the two loop contribution to the beta functional is zero. Thus there is no net contribution to the equation of motion at order $(\frac{1}{\alpha'})^2$. (Notice that Δ_F actually contains a $\frac{1}{\alpha'}$ factor.) Therefore the requirement of conformal invariance implies $L_1 = L_2 = L_3 = 0$.

5 Discussion

Is it possible to understand the vanishing of the $\mathcal{O}(p^4)$ coefficients? Here we provide a tentative argument.

One must first realize that the matrix $U(x)$ is dimensionless and thus cannot solely depend on x^μ , some dimensional quantity is required. Let us call this quantity v (of course, $v = F_\pi$, but we do not need to know this at his point). Then the full action we have written (eqs. (8) plus (11)) is trivially invariant under the following set of transformations

$$x \rightarrow e^t x, \quad v \rightarrow e^{-t} v, \quad \alpha' \rightarrow e^{2t} \alpha', \quad (31)$$

One may say that this is not really an invariance since we change both fields (which are integrated over) and couplings (which are not). In fact if this were the full story this would imply nothing for L_1 , L_2 and L_3 since, on dimensional grounds and counting powers of α' , they must be of the form $L_i \propto \alpha' v^2$.

However, it turns out that conformal invariance would imply that the invariance is stronger, since the change in α' can be absorbed by the shift $\phi \rightarrow \phi - 2t/\epsilon$ once the theory is regulated by continuing it away from 2 dimensions. Conformal invariance would guarantee independence of the conformal factor and thus the real invariance of the theory would be

$$x \rightarrow e^t x, \quad v \rightarrow e^{-t} v. \quad (32)$$

This would imply $L_1 = L_2 = L_3 = 0$, in fact all higher order coefficients appear to vanish for exactly the same reasons if this argument holds. However, the argument is only tentative, since the theory is after all not conformally invariant in all sectors since we are away from the critical dimension. The calculation we have just presented is what really settles the issue.

In real QCD the $\mathcal{O}(p^4)$ coefficients are known to be of order 10^{-3} . It appears thus that assuming conformal invariance of the string propagating in a chirally-non invariant vacuum is not such a bad approximation. (Of course the smallness of the L_i can be understood on other grounds, but these have nothing to do with the string.)

In large N QCD (the theory the string is supposedly reproducing) F_π is known to be of order \sqrt{N} . Hence quantum loop effects are absent in the chiral lagrangian. On the other hand, all resonances are narrow. Consequently $L_i \sim \sum_n f_n^2/m_n^2$. If no higher spins are included, then in the large N limit $L_i = 0$. This is precisely what we get. The consistency with large N QCD is striking.

To obtain more physical values for the L_i we should extend our program to include higher spin external fields (such as vectors and axial vectors), in a way similar to what was done in

[21] for the open bosonic string (perturbed around the usual vacuum). Demanding conformal invariance of the effective action via the vanishing of the beta functionals would lead to a system of coupled differential equations for all these degrees of freedom, from where an effective lagrangian containing pions, vector mesons, etc could be inferred. The subsequent integration of the higher spin states would give non-zero values for the L_i .

Of course another way to obtain non-zero values for L_1 , L_2 and L_3 is to give up conformal invariance. For that one must use a string action which manifestly breaks conformal invariance. The simplest possibility is to include the extrinsic curvature term. The L_i would then be get a term proportional to the rigidity coefficient. Perhaps this would be the appropriate way to include the $1/N$ corrections. A hint in this direction come from the well-known fact that integration of fermions (suppressed by $1/N$) in the supersymmetric string leads, amongst other things, to the appearance of extrinsic curvature.

In conclusion, we have seen that while attempting to build string operators describing excitations above the ‘right’ physical vacuum is probably hopeless, the sigma model approach bypasses this difficulty by determining which are the ‘classical’ backgrounds where propagation of the bosonic string is consistent. The ‘perturbative’ vacuum built of tachyons, massless vectors, etc. is a consistent one (from the string point of view, not of QCD, of course!). But a chirally non-invariant vacuum with massless scalars (interacting with a non-linear lagrangian) is consistent too and certainly a lot more physical. The tachyon is gone. We are perfectly well aware that the string action we have used is a sick one and cannot be used beyond the string tree level, but, as said, this is not really a fundamental difficulty for an effective theory. We have found that conformally symmetric string actions are a good starting point, contrary to a common belief. Perhaps the old ideas of Cremmer and Scherk could be finally implemented following the present lines.

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Appendix

At long distances QCD is described by the chiral lagrangian. This is an effective lagrangian organized in powers of momenta (see e.g. [20] for a general discussion)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots \quad (33)$$

where

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4} \text{tr} \partial_\mu U \partial^\mu U^\dagger, \quad (34)$$

$$\mathcal{L}^{(4)} = L_1 (\text{tr}(\partial_\mu U \partial^\mu U^\dagger))^2 + L_2 \text{tr}(\partial_\mu U \partial_\nu U^\dagger) \text{tr}(\partial^\mu U \partial^\nu U^\dagger) + L_3 \text{tr}(\partial^\mu U \partial^\nu U^\dagger \partial_\mu U \partial_\nu U^\dagger). \quad (35)$$

The matrix $U = \exp i\tau^a \pi^a / 2F_\pi$ collects the Goldstone bosons associated to the $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ breaking.

The experimental values for these low-energy constants are (at the m_η scale): $L_1 = (0.65 \pm 0.28) \times 10^{-3}$, $L_2 = (1.90 \pm 0.26) \times 10^{-3}$ and $L_3 = (-3.06 \pm 0.92) \times 10^{-3}$. They are, generally speaking, well accounted for by either the chiral quark model or vector meson saturation (the latter, incidentally, explains the good agreement with the Lovelace-Shapiro amplitude predictions).

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Figure 1: One-loop diagrams for the propagator.

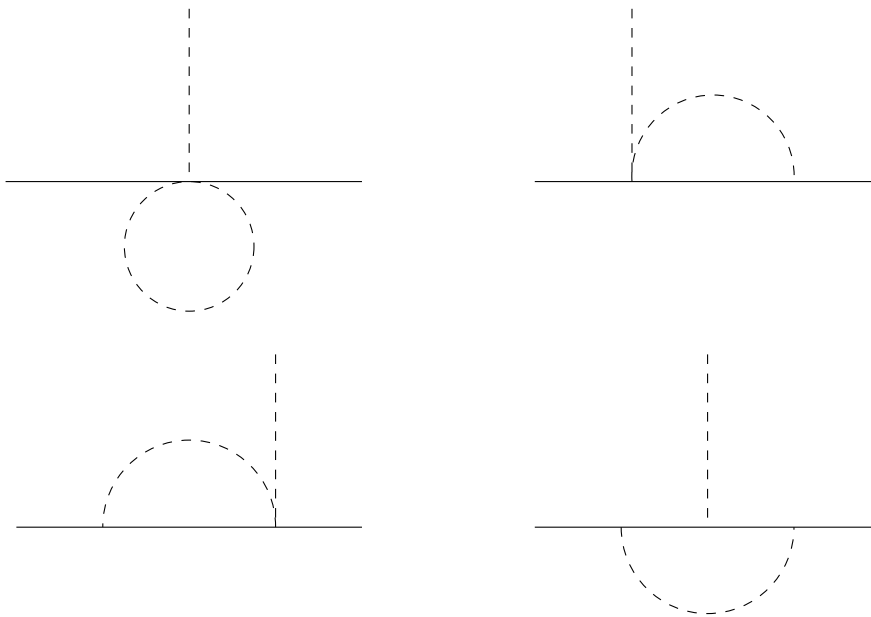


Figure 2: One-loop diagrams for the vertex with one x -field.

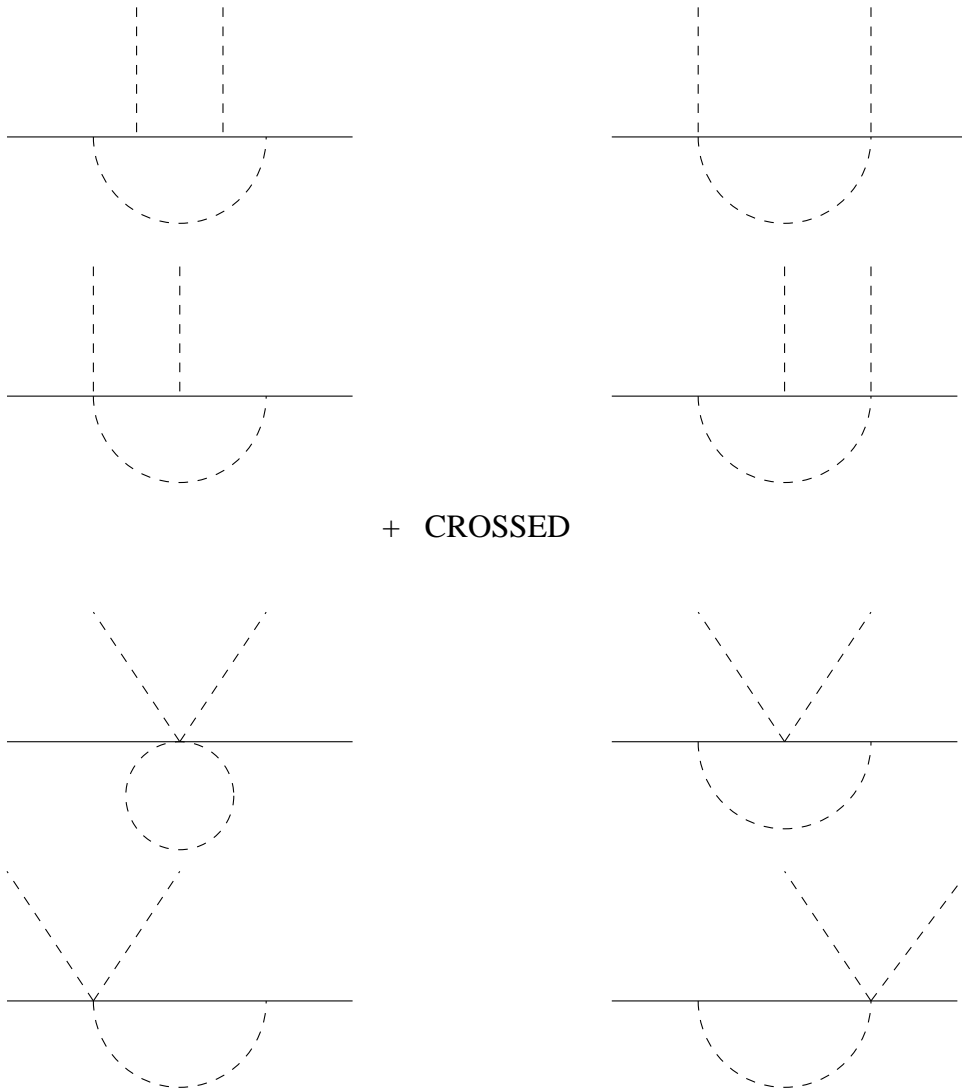


Figure 3: One-loop diagrams for the vertex with two x -fields.

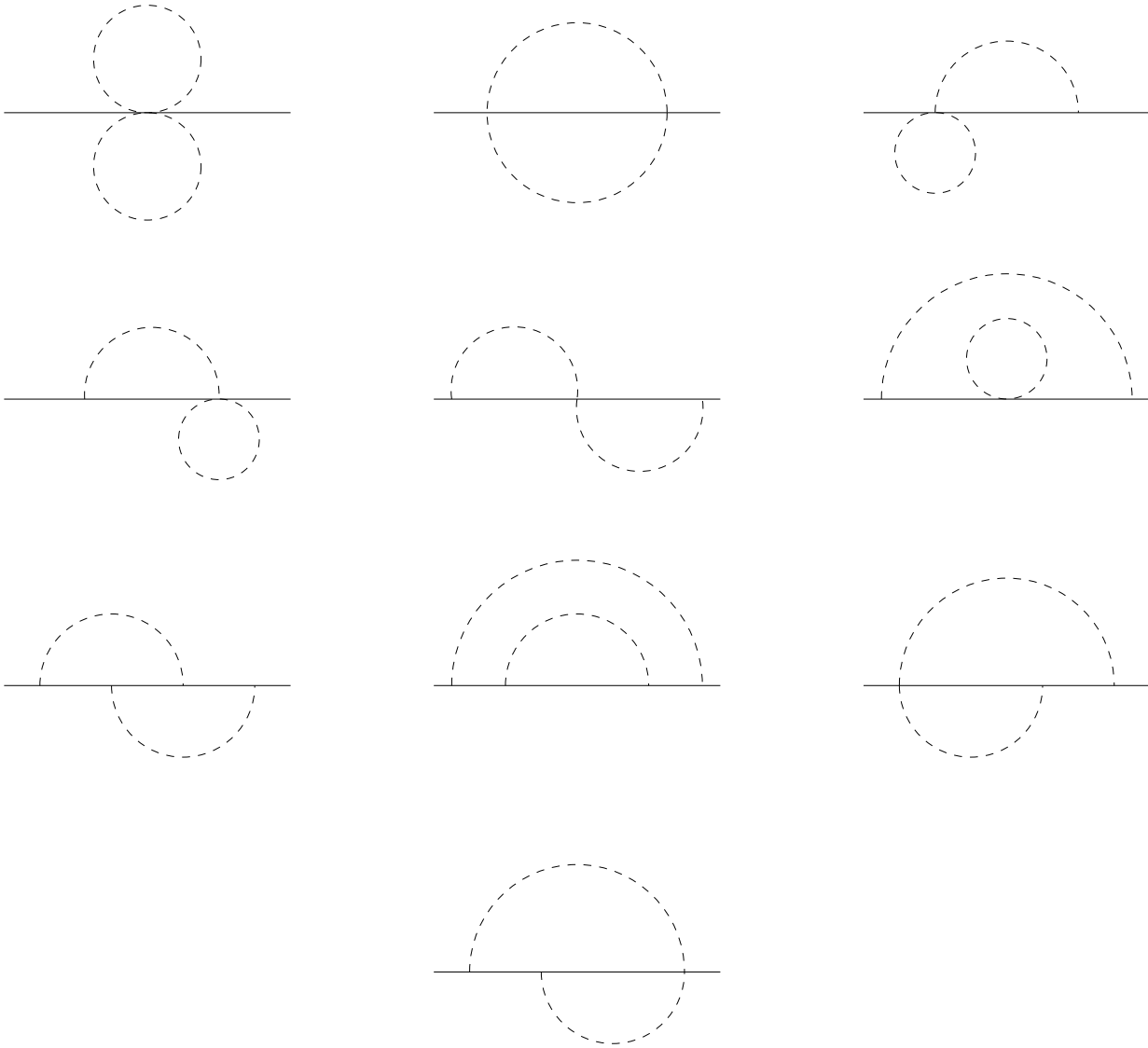


Figure 4: Two-loop diagrams for the propagator.

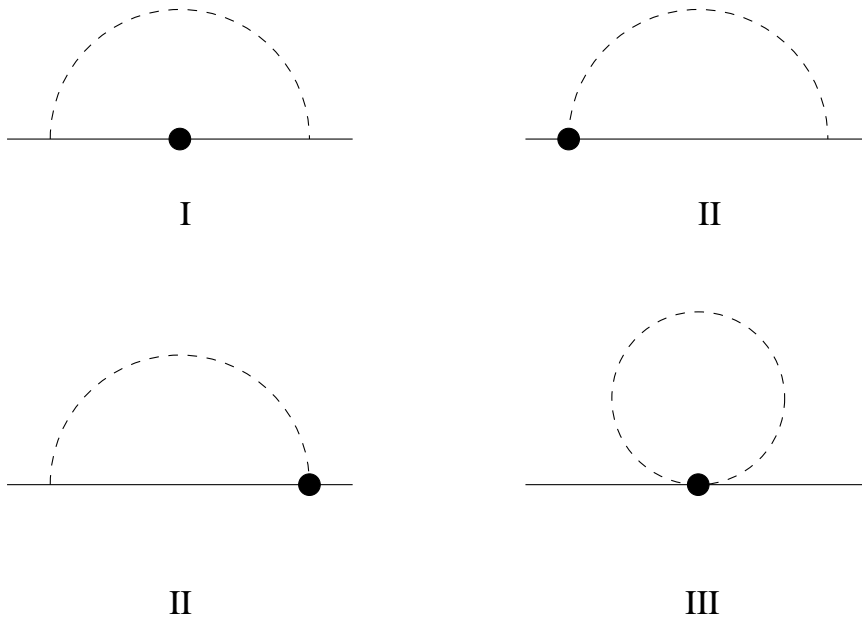


Figure 5: One-loop counterterms.