THE FORECASTING ABILITY OF FACTOR MODELS OF THE TERM STRUCTURE OF IRS MARKETS

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ABSTRACT

Using estimated principal components as factors, three-factor models are shown to produce forecasts comparable to those of autoregressive models for 2- to 10-year zero coupon interest rates IRS markets both, for short- and medium-term forecasting horizons. Evidence is provided for the Deutsche mark, Spanish peseta, Japanese yen and US dollar. Forecasts from factor models are also shown to preserve the correlation matrix of interest rates across a given term structure, an important property regarding risk management. The result is quite striking, because factor models are purely static, and forecasts for the factors must be obtained in advance of interest rate forecasts. The use of three-factor models greatly simplifies forecasting computations, since three univariate models, rather than nine, need to be used for prediction purposes. Besides, our results open the possibility that the type of simulations needed for VaR analysis could successfully be performed by just simulating the factors, again with a very substantial reduction in computational needs.

Keywords: Factor models, term structure of interest rates, Principal components, swap markets, IRS.

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1. Introduction

One of the more important issues in risk management deals with reducing the dimensionality of the set of assets in a given portfolio. This is particularly true of any analysis of a term structure of interest rates. When evaluating the level of risk associated to any given portfolio in the public debt market, we face a continuous zero coupon curve, and we will have to select a fairly large number of interest rates to follow in order not to miss any relevant information on fluctuations along the different maturities in the curve. In some other cases, a risk analyst deals with a term structure with specific maturities, as it is the case when characterizing money markets by combining interbank market rates for the shorter maturities with IRS (interest rates swap) rates for the longer ones. But even then, we still have a fairly large number of interest rate variables.

Dimensionality reduction is very important in practice. For instance, forecasting future bond prices requires forecasting interest rates at a large number of maturities. Computing Value at Risk (VaR) measures also requires estimating the variance-covariance matrix of the large number of interest rates usually involved in pricing the assets in the portfolio, as well as the sensitivity vector of the price of each bond to changes in interest rates at the relevant maturities. These two types of evaluations, crucial for risk management in fixed-income markets, very quickly become unmanageable for the standard sets of interest rates considered. Sometimes, rough simplifications are made by grouping the points in time at which cash-flow payments are to be made around a few interest rates, so as to reduce the dimensionality of the problem. Even though this is a somewhat standard practice, approximation errors can easily become higher than any sound risk manager would like to see.

It is therefore not surprising that a variety of attempts have been made at proposing methods directly designed to reduce the dimensionality of the vector of interest rates that need to be considered when evaluating risk. A particularly interesting approach emerges from Litterman and Sheinckman (1988), and was followed by Steeley (1990) and Knez et al. (1994). These authors use factor analysis to summarize the large-dimension interest rate vector in a few set of factors. Using data from different markets and time periods, they show that three factors are usually able to capture more than 95% of the fluctuation in the set of interest rates. Besides, in all these analysis, the three factors can always be naturally interpreted as a *level, steepness and curvature of the yield curve*.

More recently, Nifikeer et al. (2000) have used this approach on interest rate swap (IRS) markets data on different currencies to propose a specific method to compute VaR measures using the obtained factors. In their case, reducing the original set of fifteen interest rates to three factors drastically simplifies the computation of the variance-covariance matrix and sensitivity vector that are needed in VaR evaluation. These authors show that the reduction of dimensionality does not bias in any significant manner VaR results.

We continue in this paper with applications of factor analysis to the study of IRS markets. Besides providing additional information showing that the three-factor characterization seems quite robust to the choice of time period, currency and market, we are specifically interested in using factor models to forecast future interest rates across the term structure.

If they provide a good forecasting performance, forecasting interest rates through factor models could be of particular relevance for optimal portfolio design and risk management. First, a good forecasting performance of the factor model would be a good step towards simplifying the problem of forecasting future prices of fixed income assets, the crucial evaluation when designing a portfolio and hence, a central tool for fund management. Second, the forecasting ability of factor models could be exploited in Monte Carlo simulation exercises when computing VaR measures at any given future horizon. Again, in this type of exercises, in which a huge number of simulations is usually run, it would be computationally much simpler to just simulate a few number of factors and derive from their trajectories those for interest rates, than having to simulate each specific interest rate.

Section 2 contains a discussion of the methods to be used in the analysis, the data, and some preliminary statistical analysis. In section 3, the principal components are described and identified, discussing their ability to explain interest rate fluctuations. In section 4 we specify and estimate autoregressive and factor models for each interest rate and currency, and compare the forecasting ability of both models. The paper closes with some conclusions.

2. Factor analysis

2.1 *A description*

The object of *factor analysis* is to summarize fluctuations over time in a set of variables through those experienced by a small set of factors. The technique is specially interesting when reducing the dimensionality in a large set of interest rates from a given market, since correlations between all them are likely to be high. In factor analysis, observed variables are supposed to be linear combinations of the unobserved factors,

$$r_i = a_{i,1} z_1 + a_{i,2} z_2 + \ldots + a_{i,k} z_k + u_i \qquad (1)$$

where z_j denotes the j-th common factor, α_{ij} denote the loadings of the i-th interest rate on the j-th factor, and u_i denotes the interest rate-specific factor representing the component of r_i which is not explained by the common factors $z_1, z_2, \ldots z_k$. Factors are characterized up to scale and rotation transformations, and can be defined as linear combinations of observed variables,

$$z_j = \beta_{j,1} r_1 + \beta_{j,2} r_2 + \ldots + \beta_{j,q} r_q \tag{2}$$

where z_j represents the j-th factor and b_{ji} is the weight of the i-th variable in the j-th factor.

We use in this paper the *principal components* technique, a special case of factor analysis. This methodology constructs a specific linear transformation to the original variables, each transformed variable being a linear combination of the original, observed variables. These linear combinations are, by construction, uncorrelated with each other. The characteristic property of principal components is that they are obtained in a specific order: the first principal component is the linear combination that best explains fluctuations in the set of original variables, the second one is the linear combination of the original variables which best explains fluctuations among observed variables, orthogonal to those experienced by the first principal component, and so on. Principal components are defined by the eigenvectors associated to the larger eigenvalues of the variance-covariance matrix of interest rates, which are all non-negative.

2.2 *The data*

We apply in this paper the principal components technique to reduce the dimensionality of the vector interest rates from Interest Rate Swap (IRS) markets in different currencies. Daily data for interest rate swap rates at 2, 3, 4, 5, 7 and 10 year maturities, for the Deutsche mark, US dollar, Japanese yen and Spanish peseta were obtained from Datastream. Data for 6, 8 and 9 year maturities were obtained by linear interpolation. Zero coupon rates were then derived from swap rates by the *bootstrapping* method. To start the bootstrapping method, we used the 1-year rate from either the Eurodeposit or the domestic interbank market, depending on the currency. This rate was not used for any other purpose in our analysis.

We work with weekly data, obtained as the average of bid and ask rates at 18:00 hours GTM each Wednesday, from June 26, 1991 to December 31, 1998. As an illustration, Figure 1 shows 2-, 5- and 10-year rates for the Deutsche mark.

2.3 *First properties.*

Table 1 shows swap interest rates to be integrated variables for all maturities and currencies. A second unit root is clearly rejected in all cases. As shown in Table 2, correlations between interest rates in a same term structure are very large, although this result is contaminated by the nonstationary nature of interest rates. However, shows that correlations among weekly interest rate changes are also very high across the term structure in each currency. They decrease when maturities are farther apart, the lower correlation corresponding to the (2 year, 10 year) pair, which have correlation coefficients as low as .54 for the Spanish peseta. This already shows the existence of at least a significant common factor producing co-movements across the terms structure. It also suggests, however, that a single factor will not be enough to explain the observed discrepancies between fluctuations in 2- and 10-year rates.

3. Factor analysis results

3.1 *The number of factors*

We use principal components to summarize the extensive contemporaneous correlations among interest rates in a given currency. Using the ratios between partial sums of the eigenvalues in Table 3 to the sum of all them show that the first principal component explains at least 95% of the fluctuations in the vector of interest rates in all currencies. The first principal component, i.e., the eigenvector associated to the largest eigenvalue of the variance-covariance matrix of interest rates, is in each currency an approximate average of the set of nine interest rates, since it gives similar weights to all of them. It can therefore be interpreted as capturing the general *level* of interest rates. The second eigenvector gives weights of opposite sign to short- and long-term maturities, smoothly decreasing or increasing as we move from one end of the term structure to the other. Hence, it can be interpreted as a general measure of steepness along the term structure. This linear combination captures general changes in *the slope of the term structure*. The third eigenvector assigns positive weights to maturities at both the short- and the long-end of the term structure, and negative weights to intermediate maturities. Therefore, changes in this component will generally be associated to changes of opposite sign at short- and long- than at intermediate maturities, so that this linear combination can be interpreted as capturing changes in the *curvature* of the zero coupon curve. This interpretation, very similar to that obtained in previous work, is robust across the four currencies in our analysis, as shown in Table 3.

The right panel in Table 1 shows the three principal components to be integrated variables for all currencies, with the exception of the curvature factor in the case of the Spanish peseta and the US dollar. The hypothesis of a double unit root is overwhelmingly rejected in all cases.

3.2 *Identifying the principal components*

To further identify the forces behind each principal component, we estimated regressions having as dependent variable each one of the interest rates in our data set, and each principal component as the alternative single regressor. Estimated R-squared statistics are shown in Table 3 when each principal component is alternatively used as a single regressor, as well as when either the first two or three components are jointly used as regressors. As already mentioned, R-squared values from regressions of individual interest rates on the first component are all very large, being above .90 in most cases, the lower values corresponding to the two extreme maturities in the US dollar term structure.

The second component is able to explain a substantial amount of the fluctuations in all maturities for the Japanese yen. It also has significant explanatory power for changes in interest rates in the Spanish peseta and Deutsche mark term structures. In all currencies, R-squared values in regressions on the second principal component decrease with maturity. From its structure, as described in the previous section, this *slope factor*, should not be expected, by itself, to provide a good fit when explaining the level of interest rates. However, our results suggest that, at least during our sample period, changes in slope are mainly produced by changes in the shorter end of the term structure, probably produced as transmission of monetary policy interventions on very short-term rates.

As it should be expected, the *curvature factor* does not seem to have much explanatory power, by itself, to explain interest rates. Again, the US dollar is the exception, with changes in curvature being able to contain significant explanatory power for interest rates, specially at intermediate maturities, suggesting that these type of fluctuations have been more frequent in that country than in the other currencies considered.

It is also informative to examine the goodness of fit of interest rate regressions in Table 3. R-squared values from regressions on the first two components do not fall below .997 in any case. Besides, evidence against the presence of a unit root in the residuals in these regressions is overwhelming, so that the high R-squared values are not the reflection of spurious correlations. As a consequence, we can conclude that the curvature factor does not add much to the explanation of any interest rate when the level and slope factors are already taken into account. This is true even at the extreme maturities in the US dollar term structure, which as pointed out before, are the ones for which the level factor has the lowest explanatory power.

Estimated coefficients, which we review below, show in the four currencies that changes in the level factor tend to come together with changes of the same sign and similar size in all interest rates. Changes in the slope factor come together with changes of opposite sign at the shorter and the longer end of the term structure. Changes in the curvature factor tend to be associated with changes of the same sign at both ends of the term structure, and changes of opposite sign at intermediate maturities. All that agrees with the interpretation we just proposed for the three components.

Finally, Figure 2 shows the impact on interest rate at each maturity of a change in each principal component, in the case of the Deutsche mark. The continuous line shows the levels of interest rates which will be implied from the regressions described above, evaluated at the sample averages of the principal components. Dashed lines show the level of interest rates when each principal components deviates two standard deviations from above and below its sample average. The suggested interpretation is again the same as above.

3.3 *Factors and interest rate correlations*

Having risk management in mind as the final aim of factor analysis in a set of interest rates, we need to worry about the extent to which factor models are able to preserve cross-correlations over the term structure. This is important because, as clearly shown by VaR analysis, the correlation matrix of interest rates plays a central role in computing the chosen quartile of the distribution of market value portfolio.

The lower triangular matrices in Table 2 show the correlation matrix of the fitted part of the regressions on the first three factors, taken as proxies for the observed interest rates. We again present correlations among levels and first differences of the fitted interest rates. For each currency, the table shows that the three principal components, the level, slope and curvature factors, are able to fully capture the tight correlation among interest rates at near maturities, at the same time than the lower correlations for maturities farther apart from each other. In fact, differences are hard to see, particularly in the case of level interest rates.

4. Forecasting ability of term structure factors

As mentioned in the Introduction, the final goal of this paper is to analyze the extent to which term structure factors as characterized above: level-, slope- and curvature-factors, are able to provide good predictions of future interest rates. By good quality forecasts we understand forecasts at least as good as those that could be obtained with univariate models of each interest rate. This request should be regarded as being quite demanding, since we are comparing the forecasting performance of a purely contemporaneous projection on term structure factors, with that of a dynamic model. Besides, the fact that the factor model regression is contemporaneous, forces us to elaborate models to forecast the factors, previously to forecasting interest rates.

For each set of forecasts in the paper, we computed Mean Absolute Errors, Root Mean Square Errors, and Theil's U-statistic for each autoregressive models, for *static* as well as for *dynamic* forecasting, in each currency. Interest rates frequency distributions are asymmetric and platykurtic, so that the use of both, mean and median absolute forecast error is advised. In *dynamic* forecasting, models are estimated just once, using a sample that excludes data from the forecasting period. Forecasts are then obtained once-and-for all for the whole forecasting period. *Static* forecasts are a sequence of one-step ahead forecasts. Starting from the model estimated for dynamic forecasting, forecasts are obtained for the first week of the forecast period. Then, data for that first week is added to the sample, models are estimated again, and estimates are obtained for the second week, and so on. In static forecasting, actual past data are always used to compute forecasts, while in dynamic forecasting, previously obtained forecasts are progressively being incorporated, as we run out of actual data.

4.1 *Choosing an autoregressive model for forecasting*

As a base for comparison, we will use univariate models to forecast each interest rate as well as to forecast each factor. In the case of the factors, their orthogonality makes unnecessary the use of multivariate models. In general, among univariate structures, a third order autoregressive [AR(3)] model might be appropriate. That structure is able to capture possible cycles in interest rates through the presence of two complex roots, plus a possible unit root, thorough the third root in the characteristic equation of the third degree autoregressive polynomial.

This turns out to be the case when fitting autoregressive models to levels of interest rates for the Spanish peseta, for which AR(3) models produce for all maturities stationary residuals without much evidence of autocorrelation. For the Deutsche mark, Japanese yen and US dollar, an AR(3) model does not fully capture the persistence in interest rates levels, and an AR(4) model seems more appropriate, being the shorter dynamic structure that leaves no significant residual autocorrelation. The added lag is significant in most cases, and the model fits the data better. However, it is interesting that the AR(3) model would have already produced stationary residuals for all maturities in these three currencies as well.

A second line of analysis deals with the comparison between fitting models to level or differenced interest rates. In principle, an AR(3) model for level interest rates should be equivalent to an AR(2) model in first differences, while an AR(4) in levels would correspond to an AR(3) in first differences. However, while theoretically equivalent, significant differences arise between these two types of models in practice, as discussed below.

Something similar can be said about forecasting models for the principal components, with AR(3) models in levels producing stationary residuals in all cases and leaving no significant trace of residual autocorrelation for the Spanish peseta. For the Deutsche mark, Japanese yen and US dollar, an AR(4) structure is needed to account for residual autocorrelation. This should be expected, since as explained in Section 2, the first component is essentially the general level of interest rates, and will share the general characteristics of persistence across the term structure. For the slope and curvature components, the choice of order in the autoregressive model is irrelevant. For consistency, we estimated AR(3) models in levels to forecast the three factors for the Spanish peseta, and AR(4) models to forecast factors for the Deutsche mark, Japanese yen and US dollar.

This common choice of length in the autoregressive models for interest rates and principal components for each currency greatly simplifies the search for an adequate specification. Some informal experiments reveal that, from the point of view of forecasting performance, no much gain is achieved from an exhaustive search for the best dynamic specification each time models are estimated with additional data, as it is the case in *static* forecasting.

4.2 *Forecasting results*

Table 4 presents estimates of interest rates models. The left column shows in each case estimates of the autoregressive model, while the right column contains estimates of factor models. Adjusted R-squared values are very high in both cases, but regressions are not spurious, since residuals seem to be clearly stationary according to the Augmented Dickey-Fuller or Phillips-Perron tests. In addition to stationary tests for residuals, we provide for each estimated model, first- and fourth-order Lagrange Multiplier tests as well as third- and tenth-order Ljung-Box tests for residual autocorrelation. The comparison between standard errors of estimate (SEE) in both models and the values of the accompanying statistics suggests that, as expected, factor models exploit much better the information in the term structure at each point in time, while the autoregressive models are much better designed to capture the dynamic structure of interest rates.

Tests for first- and fourth-order ARCH structures in the residuals show significant evidence of this type of heteroskedasticity, specially when the three factors are used to explain the behavior of interest rates. Even though it leads to a loss of precision in forecasting, the presence of ARCH components should not be expected to introduce significant systematic biases in forecasting, so the comparison between forecasting strategies that we develop next should not be expected to be significantly contaminated by the presence of ARCH components in the residuals. Nevertheless, it would be interesting to estimate models capturing this feature and analyzing the effect, if any, on forecasting performance. It might also be the case that common ARCH factors can be found across the term structure, following the proposal by Engle and Kozicki (1993), which could then be explicitly incorporated into a forecasting strategy. This is left for further research.

We are interested in the quality of forecasts obtained with both types of models. For that analysis, we obtain *static* and *dynamic* forecasts over the last three months in our sample, October, November and December 1998. *Dynamic* forecasts are obtained with models estimated using data until the end of September 1998. These are once-and-for all predictions over the 13 weeks in those three months. We also computed *static* forecasts. For them, we estimate the previous models each Wednesday over the three months, each time computing a single one-step ahead forecast, for next Wednesday. That way, we have a sequence of 13 one-step ahead forecasts for each interest rate and currency.

Table 5 shows the mean value of each interest rate over the forecasting period, to be compared with the values of forecast error statistics. We have four forecasting error criteria for static and another four for dynamic forecasting for each maturity, 72 comparisons between autoregressive and factor models for each currency in total.

Among the 72 error criteria comparisons, the principal components model achieves a lower value of the error criterion in 27 cases (38% of the comparisons) for the Deutsche mark, 32 cases (44%) for the Spanish peseta, and 33 cases each (46%) for the Japanese yen and the US dollar. Among them, 16 correspond to dynamic forecasting for the Deutsche mark, 20 for the Spanish peseta, and 19 for the Japanese yen and US dollar. Hence, in a very significant number of comparisons, the principal component model performs better than the autoregressive model. Favorable comparisons are almost equally distributed between *static* and *dynamic* forecasting situations.

For each possible comparison, we computed the ratio between the value of each error criterion for the autoregressive model over the similar one for the factor model in Table 5. Median ratios over the 72 comparisons for the Deutsche mark, Spanish peseta, Japanese yen and US dollar are .985, .999, .992 and 1.000, suggesting that, *on average*, forecasts from factor models are comparable to those of autoregressive models, except for the Deutsche mark. Furthermore, skewness is -.0764, 1.0167, .7039 and 1.4886, showing that, again except for the Deutsche mark, distributions for these ratios are skewed to the right. This suggests that the forecast gain obtained when factor models perform better is superior to that obtained when autoregressive models produce better forecasts. Hence, except for the Deutsche mark, factor models seem to have a better forecasting performance than autoregressive models.

These results are quite striking because of the static nature of factor models and the need to use estimated autoregressive models for the factors to obtain forecasts for them, previously to computing interest rate forecasts. This way, we add to the sampling error in estimating factor regression models, that from estimating autoregressive models for the factors. Yet, in spite of this double estimation process, factor models often predict better than autoregressive models, which use the dynamics in interest rate processes for forecasting.

As mentioned in the Introduction, the practical relevance of our forecasting results stems from the fact that to forecast the set of nine interest rates we need to forecast only three principal components. For the set of our four currencies, this reduces the number of forecasting models needed from thirty-six to twelve.

Even bigger simplicity would be achieved by working with just two factors, level and slope. We explored that possibility by computing forecasts from projections of each interest rate on these two factors. Median ratios of criterion values for the three-components model over the two-components model are 0.674 for the Deutsche mark, 0.787 for the Spanish peseta, 0.941 for the Japanese yen, and 0.987 for the US dollar. Besides, the distributions of the ratios are skewed to the left in each currency, suggesting that the gain in forecasting performance from using a third factor is very significant.

We mentioned above the theoretical equivalence between an AR(4) model in levels and an AR(3) model in differenced interest rates. Ratios of the 72 forecasting error statistics for both models have median values below 1 and distributions which are skewed to the left, suggesting that the AR(4) model in levels produces better forecasts than the AR(3) model in differenced interest rates. For the Spanish peseta the comparison was made between an AR(3) model in levels and an AR(2) model in differenced interest rates.

Finally, stationary residuals in the right columns of Table 4 show interest rates to be cointegrated with the set of three factors for all maturities and currencies. Hence, it should be expected that an error correction model among the set of variables formed by each interest rate and the three factors would more adequately capture the long-run relationships between them, as well as their short term co-movements. However, searching for the right dynamic specification of the single-equation error equation model for each interest rate would imply an extensive amount of data mining. This would be contrary to our goal of setting relatively simple setups that allow for exploiting the contemporaneous correlations across the term structure, for interest rate forecasting. Testing the forecasting gain of an error correction model over our models in differences would constitute, however, an interesting issue for further research.

4.3 *The correlation structure of interest rate forecasts.*

As in Section 3.3, it is important to analyze the extent to which forecasting models can reproduce the observed pattern of correlations across the term structure, since the correlation matrix plays such an important role in most risk management exercises. A significant deviation from the correlation matrix of interest rates would lead to possibly important biases when using the set of forecasts to evaluate portfolio risk at a given future horizon.

Table 6 reproduces correlation matrices for autoregressive and factor model forecasts, both, for dynamic and static forecasts. In each case, the correlation coefficient between forecasts for any two maturities is compared to the sample correlation coefficient corresponding to those two maturities and currency over the forecasting period. The general result is that *static* forecasts in levels obtained from autoregressive models and from factor models retain the same cross-correlation structure that it is observed in the sample, although the fit for correlation coefficients between maturities farther apart deteriorates. *Dynamic* forecasts also reproduce sample cross-correlations, although deviations for correlation coefficients between faraway maturities are now rather large in the case of the Deutsche mark and US dollar. For the Spanish peseta and Japanese yen, the whole matrix of correlation coefficients is fairly well reproduced.

Not only forecasting but also simulating interest rates are useful tools for sound portfolio management. The good fit of interest rate models in Table 4 suggests that Monte Carlo simulations aimed at computing VaR values for fixed income portfolios at a given future horizon or at fixed income derivative pricing could successfully be conducted by just simulating future trajectories for three factors. Using the simulated paths in the estimated interest rate regressions would provide us with simulated trajectories for interest rates themselves. Again, the reduction in computational requirements would be extensive. Even though we have not explored this possibility, the relatively accurate preservation of correlations across the term structure in the forecasting exercises we have reported in this paper suggests that simulation exercises over the term structure could be conducted to a good approximation using univariate simulations for the factors together with estimated regressions on Table 4. That would avoid the need to draw realizations from the multivariate distribution of innovations for the vector of interest rates, which would be rather hard to estimate.

5. Conclusions

We have analyzed the extent to which factor models of the term structure of Interest Rate Swap (IRS) markets can be used to forecast future interest rates. We have used as factors principal components computed for a vector of zero coupon rates at maturities between 2- and 10-years. Data for 6-, 8- and 9-year maturities were obtained interpolating on market quotes at other maturities, and a zero coupon curve was then estimated by the bootstrapping method. We use weekly data from interest rate swaps denominated in Deutsche mark, Spanish peseta, Japanese yen and US dollar. Zero coupon rates at the mentioned maturities are all I(1) variables and display high correlations, not only in levels but also in first differences.

Most issues in risk management in fixed income markets require reducing the dimensionality of the vector of relevant interest rates. This is particularly true of forecasting changes in bond prices, the essential tool for fund management. It is also needed for VaR analysis, so that computation of variance-covariance matrices remains manageable.

Factor analysis, in the form of principal components has often been used to summarize a large vector of interest rate vector in a few set of factors. Usually, just three factors, naturally interpreted as the *level, steepness and curvature of the yield curve*, have been shown to account for a large amount of variation in a vector of interest rates representing a term structure. Factors of this kind have also been shown to drastically simplify the computation of the variance-covariance matrix and sensitivity vector that are needed in VaR evaluation, without introducing any noticeable bias. We complement that research by showing that the three-factor characterization of a term structure seems quite robust to the choice of time period, currency and market. The level factor captures, by itself, at least 95% of the fluctuation in each interest rate, while adding the slope factor allows for capturing at least 99,7% of the fluctuation at all maturities and currencies. Furthermore, we have shown the factors to be able to capture quite well sample correlations among interest rates.

We have specifically examined the ability of factors to forecast future interest rates. For this analysis, we have compared the forecasting performance of factor models to that of autoregressive models. A short autoregression in interest rate levels produces stationary residuals for all maturities and currencies, while leaving no significante trace of residual autocorrelation. A third autoregression in differenced interest rates should be an equivalent model, but it displays a slightly worse forecasting performance. As an alternative, we have used least-squares projections of each interest rate on the chosen factors to produce interest rate forecasts.

We use four criteria for forecasting performance: Mean and Median Absolute Errors, Root Mean Square Error, and Theil's U-statistic, and compare the forecasting ability of alternative models by examining the ratio between the error criteria obtained under each specification. Forecasts were obtained for the thirteen weeks in the last three months of our sample. Having nine interest rates for each currency, four forecasting criteria and static, as well as dynamic forecasts, we have 72 comparisons for each currency. Factor models perform better than autoregressive models in at least 44% of the comparisons for the Spanish peseta, Japanese yen and US dollar, and 38% of the cases for the Deutsche mark. Besides, an examination of the frequency distribution of forecast error criteria suggests that predictions from factor models may be preferable to those of autoregressive models.

This result is quite striking because of the static nature of factor models, and the need to obtain forecasts for the factors, previously to computing interest rate forecasts. That way, we add to the sampling error in estimating factor regression models, that from estimating autoregressive models for the factors. Yet, in spite of this double estimation process, factor models can sometimes predict better than autoregressive models, which fully incorporate the dynamics in interest rate processes for forecasting.

The good forecasting performance of factor models, as well as their ability to reproduce the sample correlation structure is very important for optimal portfolio design and risk management. First, forecasting future prices of fixed income assets can be greatly simplified, since we can proceed by using just forecasts for the factors to obtain forecasts for all interest rates involved. Also, in the type of Monte Carlo exercises needed to compute VaR measures at a given future horizon, a huge number of simulations is usually run. The described properties of term structure factors suggests a much simpler approach by just simulating the factors and deriving from their trajectories those for interest rates, as opposed to having to simulate each specific interest rate.

Finally, it is interesting that both forecasting models can approximately reproduce the observed pattern of correlations across the term structure, since the correlation matrix plays such an important role in most risk management exercises. This is specially relevant for VaR analysis, since it shows that drawing from univariate probability distributions for the innovations in either model is enough to roughly preserve during the forecast horizon the sample correlation matrix among interest rates. In this respect, *static* forecasts do specially well at retaining the sample cross-correlation structure, except between distant maturities, *dynamic* forecasts doing a little worse.

References

Engle, R.F. and S. Kozicki (1993), Testing for Common Features, Journal of Business and Economic Statistics 11, 369-380.

Knez, P.J., Litterman, R. and Scheinkman, J. (1994): “Explorations into factors explaining money market returns”, *Journal of Finance*, 49, 1861-1882.

Litterman, R. and Scheinkman, J. (1991): “Common factor affecting bond returns”, *Journal of Fixed Income*, 1, 54-61.

Niffikeer, C.I., Hewins, R.D. and Flavell, R.B. (2000): "A synthetic factor approach to the estimation of value-at-risk of a portfolio of interest rate swaps", *Journal of Banking and Finance*, 24, 1903-1932.

Steeley, J.M. (1990): “Modelling the dynamics of the term structure of interest rates”, *The Economic and Social Review*, 21, 337-661.

Table 1. Unit root tests

	Zero coupon rates									Principal Components		
	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year	First	Second	Third
a) DEM												
ADF-Level	-1.081	-1.038	-0.822	-0.729	-0.575	-0.369	-0.231	-0.072	0.105	-0.594	-1.427	-2.203
ADF-Differences	-7.480	-7.599	-7.717	-7.805	-8.083	-8.479	-8.783	-9.139	-9.545	-8.171	-7.784	-8.061
PP-Level	-1.323	-1.195	-0.981	-0.874	-0.717	-0.519	-0.429	-0.327	-0.217	-0.792	-1.434	-2.080
PP-Differences	-19.035	-18.358	-19.090	-19.639	-20.313	-21.259	-21.752	-22.348	-23.036	-20.288	-19.793	-21.017
b) ESP												
ADF-Level	-0.563	-0.550	-0.516	-0.454	-0.319	-0.180	-0.137	-0.093	-0.050	-0.315	-1.969	-3.133
ADF-Differences	-9.949	-9.233	-8.691	-8.205	-8.267	-8.454	-8.511	-8.633	-8.823	-8.581	-11.024	-12.840
PP-Level	-0.625	-0.534	-0.444	-0.437	-0.303	-0.185	-0.121	-0.062	-0.011	-0.268	-2.345	-4.512
PP-Differences	-23.687	-22.541	-20.974	-20.318	-20.064	-20.006	-19.853	-19.874	-20.094	-20.107	-28.911	-27.242
c) JPY												
ADF-Level	-2.187	-1.967	-1.712	-1.610	-1.498	-1.373	-1.377	-1.380	-1.384	-1.573	-2.952	-2.481
ADF-Differences	-8.042	-7.965	-7.775	-7.553	-7.323	-7.177	-7.109	-7.048	-6.993	-7.352	-9.140	-8.601
PP-Level	-2.321	-1.984	-1.748	-1.633	-1.480	-1.311	-1.300	-1.287	-1.277	-1.575	-2.789	-2.466
PP-Differences	-18.690	-18.919	-18.904	-18.973	-18.903	-18.924	-18.767	-18.679	-18.691	-18.745	-18.594	-24.207
d) USD												
ADF-Level	-2.248	-2.350	-2.300	-2.196	-2.088	-1.950	-1.860	-1.758	-1.654	-2.118	-1.267	-3.106
ADF-Differences	-7.377	-7.319	-7.368	-7.433	-7.627	-7.891	-8.106	-8.371	-8.681	-7.710	-7.826	-9.456
PP-Level	-2.130	-2.190	-2.169	-2.085	-2.031	-1.955	-1.886	-1.807	-1.726	-2.073	-1.153	-3.339
PP-Differences	-20.047	-20.287	-20.653	-21.098	-21.272	-21.583	-21.735	-21.962	-22.260	-21.223	-18.510	-22.755

Note: Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root statistics. Critical values for both statistics: -3.448 (1%), -2.869 (5%), -2.571 (10%) .

Table 2. Contemporaneous correlation coefficients between observed and fitted zero coupon rates

	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
Level					**a) DEM**									**b) ESP**				
2-year	1.000	0.994	0.979	0.960	0.939	0.905	0.889	0.868	0.839	1.000	0.997	0.993	0.988	0.984	0.979	0.976	0.972	0.967
3-year	0.994	1.000	0.995	0.984	0.969	0.942	0.929	0.911	0.886	0.997	1.000	0.999	0.996	0.994	0.990	0.987	0.984	0.980
4-year	0.980	0.995	1.000	0.996	0.988	0.970	0.960	0.945	0.925	0.993	0.999	1.000	0.999	0.998	0.995	0.993	0.991	0.988
5-year	0.960	0.984	0.996	1.000	0.997	0.986	0.979	0.969	0.952	0.988	0.996	0.999	1.000	1.000	0.998	0.997	0.995	0.992
6-year	0.939	0.969	0.988	0.997	1.000	0.996	0.991	0.984	0.971	0.984	0.994	0.998	1.000	1.000	0.999	0.999	0.997	0.996
7-year	0.905	0.943	0.970	0.987	0.996	1.000	0.999	0.996	0.989	0.979	0.990	0.995	0.998	0.999	1.000	1.000	0.999	0.998
8-year	0.889	0.929	0.960	0.980	0.991	0.999	1.000	0.999	0.994	0.976	0.987	0.993	0.997	0.999	1.000	1.000	1.000	0.999
9-year	0.868	0.911	0.945	0.969	0.984	0.996	0.999	1.000	0.998	0.972	0.984	0.991	0.995	0.998	0.999	1.000	1.000	1.000
10-year	0.839	0.886	0.925	0.952	0.972	0.989	0.994	0.998	1.000	0.967	0.981	0.988	0.992	0.996	0.998	0.999	1.000	1.000
First difference																		
2-year	1.000	0.968	0.907	0.875	0.836	0.766	0.738	0.700	0.650	1.000	0.957	0.860	0.703	0.645	0.567	0.568	0.558	0.537
3-year	0.970	1.000	0.961	0.934	0.903	0.840	0.814	0.777	0.728	0.937	1.000	0.955	0.846	0.800	0.729	0.729	0.718	0.692
4-year	0.922	0.986	1.000	0.986	0.970	0.922	0.902	0.871	0.828	0.821	0.968	1.000	0.948	0.919	0.863	0.868	0.859	0.836
5-year	0.874	0.958	0.991	1.000	0.987	0.943	0.928	0.902	0.865	0.709	0.908	0.984	1.000	0.988	0.950	0.945	0.927	0.894
6-year	0.831	0.922	0.970	0.993	1.000	0.984	0.974	0.953	0.921	0.659	0.870	0.963	0.993	1.000	0.987	0.982	0.963	0.929
7-year	0.767	0.865	0.929	0.968	0.991	1.000	0.996	0.982	0.957	0.590	0.813	0.923	0.971	0.992	1.000	0.994	0.976	0.942
8-year	0.737	0.832	0.900	0.946	0.978	0.997	1.000	0.995	0.979	0.560	0.785	0.900	0.954	0.982	0.998	1.000	0.993	0.972
9-year	0.698	0.789	0.863	0.916	0.957	0.987	0.996	1.000	0.994	0.523	0.748	0.869	0.930	0.966	0.991	0.998	1.000	0.993
10-year	0.649	0.736	0.814	0.875	0.926	0.967	0.984	0.996	1.000	0.478	0.704	0.831	0.900	0.944	0.978	0.989	0.997	1.000
Level					**c) JPY**									**d) USD**				
2-year	1.000	0.997	0.991	0.984	0.977	0.967	0.965	0.963	0.961	1.000	0.977	0.923	0.857	0.809	0.748	0.714	0.675	0.630
3-year	0.998	1.000	0.998	0.994	0.989	0.981	0.980	0.978	0.976	0.977	1.000	0.984	0.947	0.915	0.872	0.846	0.816	0.780
4-year	0.991	0.998	1.000	0.999	0.996	0.991	0.990	0.989	0.987	0.924	0.984	1.000	0.989	0.972	0.945	0.927	0.905	0.878
5-year	0.984	0.994	0.999	1.000	0.999	0.996	0.995	0.994	0.992	0.857	0.947	0.989	1.000	0.996	0.982	0.972	0.957	0.937
6-year	0.977	0.989	0.997	0.999	1.000	0.999	0.998	0.997	0.996	0.809	0.916	0.972	0.996	1.000	0.995	0.989	0.979	0.964
7-year	0.967	0.982	0.992	0.996	0.999	1.000	1.000	0.999	0.999	0.748	0.872	0.945	0.983	0.995	1.000	0.999	0.994	0.985
8-year	0.965	0.980	0.990	0.995	0.998	1.000	1.000	1.000	0.999	0.714	0.847	0.927	0.972	0.989	0.999	1.000	0.998	0.993
9-year	0.963	0.978	0.989	0.994	0.997	1.000	1.000	1.000	1.000	0.675	0.816	0.905	0.957	0.979	0.994	0.998	1.000	0.998
10-year	0.961	0.976	0.987	0.992	0.996	0.999	0.999	1.000	1.000	0.630	0.780	0.878	0.937	0.964	0.985	0.993	0.998	1.000
First difference																		
2-year	1.000	0.965	0.920	0.883	0.852	0.781	0.769	0.747	0.713	1.000	0.988	0.969	0.943	0.925	0.894	0.883	0.864	0.836
3-year	0.975	1.000	0.965	0.937	0.909	0.839	0.826	0.803	0.767	0.990	1.000	0.991	0.973	0.961	0.935	0.926	0.908	0.882
4-year	0.927	0.986	1.000	0.978	0.960	0.901	0.889	0.866	0.830	0.968	0.994	1.000	0.987	0.980	0.959	0.952	0.937	0.913
5-year	0.889	0.963	0.993	1.000	0.988	0.937	0.930	0.912	0.880	0.942	0.980	0.996	1.000	0.994	0.975	0.971	0.959	0.938
6-year	0.850	0.932	0.974	0.993	1.000	0.980	0.974	0.957	0.927	0.925	0.968	0.989	0.998	1.000	0.993	0.989	0.978	0.956
7-year	0.788	0.876	0.931	0.966	0.989	1.000	0.996	0.982	0.955	0.900	0.949	0.976	0.990	0.997	1.000	0.996	0.985	0.964
8-year	0.767	0.850	0.906	0.946	0.977	0.997	1.000	0.995	0.978	0.883	0.933	0.963	0.979	0.990	0.998	1.000	0.996	0.983
9-year	0.739	0.815	0.872	0.917	0.956	0.987	0.996	1.000	0.994	0.860	0.911	0.943	0.961	0.977	0.990	0.997	1.000	0.996
10-year	0.701	0.768	0.825	0.877	0.924	0.967	0.983	0.995	1.000	0.831	0.882	0.916	0.937	0.958	0.975	0.988	0.997	1.000

Note: The upper triangular matrix contains correlation coefficients between sample interest rates. The lower triangular matrix contains correlation coefficients between the fitted components in regressions of interest rates on principal components.

Table 3. Principal Components in the term structure

	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
a) DEM									
Variance-covariance matrix of interest rates									
Eigenvalues	155089.7	707.02	8.31	0.34	0.25	0.10	7.66E-04	8.91E-05	7.18E-08
First eigenvector	-0.3078	-0.3153	-0.3230	-0.3298	-0.3352	-0.3410	-0.3445	-0.3483	-0.3523
Second eigenvector	0.6451	0.4166	0.2184	0.0637	-0.0598	-0.1928	-0.2535	-0.3191	-0.3895
Third eigenvector	0.5790	-0.1229	-0.3703	-0.4057	-0.2736	-0.1410	0.0448	0.2381	0.4411
R-squared coefficients on individual principal components									
First component	0.9152	0.9617	0.9889	0.9984	0.9966	0.9790	0.9672	0.9487	0.9205
Second component	0.8612	0.7888	0.7059	0.6268	0.5581	0.4673	0.4301	0.3854	0.3324
Third component	0.0320	0.0723	0.1109	0.1422	0.1615	0.1857	0.1876	0.1895	0.1909
R-squared coefficients on subset principal components									
On first two components	0.9987	0.9998	0.9991	0.9987	0.9994	0.9997	0.9999	0.9994	0.9977
On first three components	1.0000	0.9999	0.9999	0.9998	1.0000	0.9998	1.0000	1.0000	0.9999
b) ESP									
Variance-covariance matrix of interest rates									
Eigenvalues	321531.4	367.02	13.75	1.14	0.50	0.30	2.45E-04	1.78E-04	5.45E-07
First eigenvector	-0.3287	-0.3296	-0.3312	-0.3323	-0.3336	-0.3350	-0.3357	-0.3365	-0.3373
Second eigenvector	0.6050	0.4035	0.2278	0.0735	-0.0539	-0.1889	-0.2643	-0.3452	-0.4315
Third eigenvector	0.6222	-0.0972	-0.3760	-0.4433	-0.2657	-0.0887	0.0616	0.2141	0.3707
R-squared coefficients on individual principal components									
First component	0.9766	0.9919	0.9976	0.9992	0.9993	0.9970	0.9952	0.9923	0.9880
Second component	0.6583	0.5998	0.5541	0.5172	0.4895	0.4577	0.4425	0.4250	0.4050
Third component	0.0873	0.1255	0.1504	0.1669	0.1696	0.1728	0.1727	0.1728	0.1730
R-squared coefficients on subset principal components									
On first two components	0.9990	0.9999	0.9995	0.9992	0.9997	0.9998	1.0000	0.9998	0.9994
On first three components	1.0000	0.9999	0.9999	0.9999	1.0000	0.9999	1.0000	1.0000	0.9999
c) JPY									
Variance-covariance matrix of interest rates									
Eigenvalues	60006.8	535.22	7.70	0.67	0.26	0.14	1.42E-03	7.31E-04	3.74E-05
First eigenvector	-0.2744	-0.2967	-0.3163	-0.3307	-0.3403	-0.3510	-0.3552	-0.3597	-0.3644
Second eigenvector	0.6303	0.4414	0.2512	0.0931	-0.0371	-0.1829	-0.2443	-0.3102	-0.3813
Third eigenvector	0.5372	-0.0004	-0.3536	-0.3753	-0.3119	-0.2112	0.0050	0.2403	0.4959
R-squared coefficients on individual principal components									
First component	0.9664	0.9857	0.9963	0.9992	0.9989	0.9945	0.9934	0.9917	0.9891
Second component	0.8987	0.8589	0.8118	0.7755	0.7429	0.7021	0.6946	0.6863	0.6768
Third component	0.0188	0.0404	0.0651	0.0804	0.0913	0.1038	0.1014	0.0987	0.0954
R-squared coefficients on subset principal components									
On first two components	0.9990	0.9999	0.9994	0.9993	0.9997	0.9998	1.0000	0.9998	0.9990
On first three components	0.9999	0.9999	0.9999	0.9999	1.0000	0.9998	1.0000	1.0000	0.9999
d) USD									
Variance-covariance matrix of interest rates									
Eigenvalues	162570.9	252.63	3.07	0.28	0.09	0.02	4.95E-04	2.16E-04	1.18E-04
First eigenvector	-0.2939	-0.3095	-0.3213	-0.3310	-0.3373	-0.3441	-0.3481	-0.3524	-0.357
Second eigenvector	0.6767	0.4167	0.2065	0.0383	-0.0653	-0.1797	-0.2389	-0.3031	-0.3727
Third eigenvector	0.4759	-0.0573	-0.2774	-0.4216	-0.3253	-0.2247	0.0210	0.2715	0.5339
R-squared coefficients on individual principal components									
First component	0.7124	0.8811	0.9697	0.9984	0.9958	0.9743	0.9567	0.9316	0.8976
Second component	0.4742	0.2699	0.1295	0.0481	0.0176	1.23E-03	2.22E-04	4.86E-03	0.0167
Third component	0.3138	0.4606	0.5627	0.6253	0.6398	0.6444	0.6317	0.6142	0.5913
R-squared coefficients on subset principal components									
On first two components	0.9993	0.9999	0.9996	0.9992	0.9996	0.9995	0.9999	0.9997	0.9988
On first three components	0.9999	0.9999	0.9998	0.9998	1.0000	0.9997	0.9999	1.0000	0.9999

Note: Each panel contains eigenvalues and eigenvectors for the variance-covariance matrix of zero coupon rates. R-squared statistics are presented for regressions of each interest rate on each individual principal component as well as on the first two and three principal components.

Table 4. Estimated Models: a) DEM

	2-year		3-year		4-year		5-year		6-year		7-year		8-year		9-year		10-year	
Own lag																		
i=1	1.0665		1.1117		1.0713		1.0416		1.0066		0.9580		0.9319		0.9018		0.8688	
	(0.0491)		(0.0489)		(0.0486)		(0.0486)		(0.0485)		(0.0486)		(0.0487)		(0.0488)		(0.0489)	
i=2	-0.0598		-0.1450		-0.0959		-0.0641		-0.0273		0.0180		0.0423		0.0678		0.0926	
	(0.0719)		(0.0730)		(0.0712)		(0.0701)		(0.0688)		(0.0671)		(0.0664)		(0.0656)		(0.0647)	
i=3	0.1048		0.1818		0.2029		0.1992		0.1984		0.1900		0.1751		0.1600		0.1457	
	(0.0715)		(0.0728)		(0.0710)		(0.0700)		(0.0686)		(0.0670)		(0.0663)		(0.0655)		(0.0646)	
i=4	-0.1150		-0.1521		-0.1817		-0.1799		-0.1803		-0.1677		-0.1506		-0.1303		-0.1072	
	(0.0488)		(0.0486)		(0.0484)		(0.0485)		(0.0485)		(0.0485)		(0.0487)		(0.0489)		(0.0491)	
First component		-0.3101		-0.3121		-0.3211		-0.3303		-0.3366		-0.3444		-0.3457		-0.3474		-0.3496
		(0.0003)		(0.0005)		(0.0004)		(0.0005)		(0.0002)		(0.0004)		(0.0002)		(0.0001)		(0.0003)
Second component		0.6397		0.4245		0.2229		0.0626		-0.0633		-0.2009		-0.2563		-0.3169		-0.3832
		(0.0007)		(0.0014)		(0.0011)		(0.0014)		(0.0006)		(0.0012)		(0.0005)		(0.0002)		(0.0009)
Third component		0.6020		-0.1570		-0.3880		-0.4013		-0.2588		-0.1071		0.0563		0.2292		0.4142
		(0.0043)		(0.0079)		(0.0066)		(0.0080)		(0.0033)		(0.0069)		(0.0032)		(0.0013)		(0.0052)
R-squared	0.9964	1.0000	0.9958	0.9999	0.9951	0.9999	0.9945	0.9998	0.9941	1.0000	0.9932	0.9998	0.9928	1.0000	0.9922	1.0000	0.9914	0.9998
SEE	0.1128	0.0099	0.1087	0.0184	0.1045	0.0152	0.1009	0.0186	0.0966	0.0077	0.0951	0.0159	0.0934	0.0073	0.0926	0.0029	0.0926	0.0121
ARCH(1)	0.16	185.17	2.06	114.28	4.07	178.60	2.13	176.44	3.69	196.91	6.76	166.25	6.47	174.51	6.06	173.69	5.63	149.89
	[0.69]	[0.00]	[0.15]	[0.00]	[0.04]	[0.00]	[0.14]	[0.00]	[0.06]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.02]	[0.00]
ARCH(4)	1.27	183.67	3.96	114.60	5.41	169.33	3.43	177.32	6.57	196.19	12.51	173.70	13.36	179.99	13.29	175.67	13.07	161.44
	[0.87]	[0.00]	[0.41]	[0.00]	[0.25]	[0.00]	[0.49]	[0.00]	[0.16]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]
Q(3)	0.10	601.03	0.07	460.16	0.10	575.60	0.15	709.81	0.21	663.18	0.23	619.42	0.21	625.74	0.18	672.28	0.16	644.79
	[0.99]	[0.00]	[1.00]	[0.00]	[0.99]	[0.00]	[0.99]	[0.00]	[0.98]	[0.00]	[0.97]	[0.00]	[0.98]	[0.00]	[0.98]	[0.00]	[0.98]	[0.00]
Q(10)	4.23	1370.90	3.31	1028.00	3.47	1084.40	6.07	1769.50	7.94	1502.40	10.12	1035.30	10.31	1088.60	10.10	1399.10	9.53	1106.50
	[0.94]	[0.00]	[0.97]	[0.00]	[0.97]	[0.00]	[0.81]	[0.00]	[0.64]	[0.00]	[0.43]	[0.00]	[0.41]	[0.00]	[0.43]	[0.00]	[0.48]	[0.00]
LM(1)	1.57	266.03	1.74	223.50	2.55	248.80	3.92	292.90	4.97	282.33	5.52	262.99	5.28	265.03	4.92	281.01	4.48	271.71
	[0.21]	[0.00]	[0.19]	[0.00]	[0.11]	[0.00]	[0.05]	[0.00]	[0.03]	[0.00]	[0.02]	[0.00]	[0.02]	[0.00]	[0.03]	[0.00]	[0.03]	[0.00]
LM(4)	4.00	272.32	3.20	230.50	3.70	254.74	4.41	296.55	5.72	287.05	7.11	268.15	7.86	270.22	8.83	286.52	10.04	276.35
	[0.41]	[0.00]	[0.53]	[0.00]	[0.45]	[0.00]	[0.35]	[0.00]	[0.22]	[0.00]	[0.13]	[0.00]	[0.10]	[0.00]	[0.07]	[0.00]	[0.04]	[0.00]
ADF	-8.14	-3.89	-8.40	-4.72	-8.55	-5.23	-8.53	-3.39	-8.76	-3.70	-9.01	-4.47	-9.16	-4.36	-9.33	-3.79	-9.53	-4.33
PP	-20.45	-6.31	-20.51	-7.66	-20.67	-6.96	-20.74	-5.48	-20.79	-5.82	-20.78	-6.53	-20.76	-6.46	-20.72	-5.87	-20.68	-6.26

Note: Estimates of autoregressive and factor interest rate models. Both models were estimated using levels of interest rates and principal components. Fourth order autoregressions were used in all cases except Spain, where a third order autoregression was used. A constant (generally non-significant) was included in all models. Standard deviations are shown in parentheses. Statistics for each regression include: adjusted R-squared, standard error of estimate (SEE), Lagrange multiplier statistics to test for ARCH(1) and ARCH(4) structure in residuals, Ljung-Box autocorrelation statistics of orders 3 and 10 (Q(3), Q(10)), Breusch-Godfrey autocorrelation statistics of orders 1 and 4 (LM(1), LM(4)), and Augmented Dickey-Fuller and Phillips-Perron statistics to test for a unit root in the residuals. *p*-values are included in square brackets.

Table 4. Estimated Models: b) ESP

	2-year		3-year		4-year		5-year		6-year		7-year		8-year		9-year		10-year	
Own lag																		
i=1	0.8590		0.9040		0.9705		1.0003		1.0128		1.0161		1.0236		1.0223		1.0115	
	(0.0490)		(0.0490)		(0.0492)		(0.0494)		(0.0494)		(0.0494)		(0.0494)		(0.0494)		(0.0494)	
i=2	0.2659		0.2150		0.1101		0.0075		-0.0257		-0.0526		-0.0450		-0.0305		-0.0089	
	(0.0635)		(0.0654)		(0.0685)		(0.0698)		(0.0703)		(0.0703)		(0.0706)		(0.0706)		(0.0702)	
i=3	-0.1278		-0.1213		-0.0826		-0.0094		0.0118		0.0358		0.0208		0.0076		-0.0030	
	(0.0490)		(0.0491)		(0.0493)		(0.0494)		(0.0495)		(0.0495)		(0.0495)		(0.0495)		(0.0495)	
First component		-0.3287		-0.3299		-0.3311		-0.3310		-0.3338		-0.3367		-0.3364		-0.3362		-0.3362
		(0.0002)		(0.0004)		(0.0003)		(0.0003)		(0.0002)		(0.0003)		(0.0001)		(0.0001)		(0.0003)
Second component		0.6051		0.4020		0.2288		0.0817		-0.0550		-0.1996		-0.2686		-0.3437		-0.4247
		(0.0015)		(0.0028)		(0.0025)		(0.0021)		(0.0015)		(0.0026)		(0.0010)		(0.0006)		(0.0021)
Third component		0.6225		-0.0936		-0.3790		-0.4666		-0.2623		-0.0578		0.0741		0.2097		0.3507
		(0.0057)		(0.0107)		(0.0095)		(0.0082)		(0.0056)		(0.0099)		(0.0040)		(0.0023)		(0.0082)
R-squared	0.9926	1.0000	0.9936	0.9999	0.9943	0.9999	0.9940	0.9999	0.9939	1.0000	0.9935	0.9998	0.9937	1.0000	0.9938	1.0000	0.9936	0.9999
SEE	0.2499	0.0170	0.2244	0.0320	0.2053	0.0283	0.2026	0.0244	0.1983	0.0168	0.1990	0.0295	0.1913	0.0120	0.1860	0.0067	0.1835	0.0245
ARCH(1)	10.30	0.39	10.92	1.62	7.21	0.59	31.86	79.44	60.95	0.04	82.93	3.13	55.91	6.95	29.26	0.08	12.11	1.33
	[0.00]	[0.53]	[0.00]	[0.20]	[0.01]	[0.44]	[0.00]	[0.00]	[0.00]	[0.85]	[0.00]	[0.08]	[0.00]	[0.01]	[0.00]	[0.77]	[0.00]	[0.25]
ARCH(4)	19.26	2.22	33.50	2.86	30.33	121.36	32.99	48.66	60.81	41.29	83.60	58.57	60.53	37.30	44.11	112.50	36.26	118.89
	[0.00]	[0.70]	[0.00]	[0.58]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Q(3)	0.15	213.98	0.30	265.41	0.14	181.50	0.00	309.01	0.17	147.60	0.61	147.62	0.70	165.75	0.67	129.94	0.49	142.54
	[0.99]	[0.00]	[0.96]	[0.00]	[0.99]	[0.00]	[1.00]	[0.00]	[0.98]	[0.00]	[0.90]	[0.00]	[0.87]	[0.00]	[0.88]	[0.00]	[0.92]	[0.00]
Q(10)	8.63	289.98	6.48	338.72	7.66	259.06	6.52	570.32	7.99	210.79	9.33	203.66	9.10	232.72	8.75	173.77	8.39	190.50
	[0.57]	[0.00]	[0.77]	[0.00]	[0.66]	[0.00]	[0.77]	[0.00]	[0.63]	[0.00]	[0.50]	[0.00]	[0.52]	[0.00]	[0.56]	[0.00]	[0.59]	[0.00]
LM(1)	0.12	120.56	0.21	150.62	0.08	90.98	0.01	169.38	0.01	80.33	0.55	90.42	0.74	102.77	0.62	73.58	0.01	84.72
	[0.73]	[0.00]	[0.65]	[0.00]	[0.78]	[0.00]	[0.92]	[0.00]	[0.93]	[0.00]	[0.46]	[0.00]	[0.39]	[0.00]	[0.43]	[0.00]	[0.94]	[0.00]
LM(4)	4.43	135.55	1.49	156.81	0.56	128.38	1.47	178.44	1.21	114.51	0.99	108.28	2.61	115.60	1.01	105.83	0.61	107.85
	[0.35]	[0.00]	[0.83]	[0.00]	[0.97]	[0.00]	[0.83]	[0.00]	[0.88]	[0.00]	[0.91]	[0.00]	[0.63]	[0.00]	[0.91]	[0.00]	[0.96]	[0.00]
ADF	-10.09	-6.25	-9.47	-6.39	-8.94	-5.52	-8.25	-6.00	-8.25	-5.99	-8.33	-6.41	-8.45	-6.45	-8.61	-6.16	-8.81	-6.27
PP	-20.32	-11.49	-20.33	-10.32	-20.30	-12.78	-20.27	-9.75	-20.28	-13.20	-20.31	-12.44	-20.30	-11.88	-20.28	-13.41	-20.27	-12.77

Table 4. Estimated Models: c) JPY

	2-year		3-year		4-year		5-year		6-year		7-year		8-year		9-year		10-year	
Own lag																		
i=1	1.0844		1.0724		1.0619		1.0465		1.0379		1.0248		1.0300		1.0322		1.0300	
	(0.0494)		(0.0492)		(0.0494)		(0.0493)		(0.0495)		(0.0497)		(0.0496)		(0.0495)		(0.0495)	
i=2	-0.1354		-0.1382		-0.0918		-0.0485		-0.0049		0.0407		0.0208		0.0012		-0.0157	
	(0.0727)		(0.0717)		(0.0719)		(0.0714)		(0.0715)		(0.0714)		(0.0714)		(0.0714)		(0.0711)	
i=3	0.1950		0.2463		0.1923		0.1719		0.1178		0.0510		0.0786		0.1078		0.1361	
	(0.0725)		(0.0717)		(0.0718)		(0.0714)		(0.0715)		(0.0714)		(0.0715)		(0.0714)		(0.0711)	
i=4	-0.1500		-0.1858		-0.1668		-0.1736		-0.1537		-0.1186		-0.1312		-0.1429		-0.1517	
	(0.0491)		(0.0490)		(0.0492)		(0.0493)		(0.0495)		(0.0498)		(0.0498)		(0.0497)		(0.0497)	
First component		-0.2785		-0.2933		-0.3128		-0.3279		-0.3419		-0.3575		-0.3584		-0.3591		-0.3594
		(0.0004)		(0.0006)		(0.0006)		(0.0005)		(0.0002)		(0.0006)		(0.0003)		(0.0001)		(0.0004)
Second component		0.6123		0.4564		0.2667		0.1053		-0.0440		-0.2112		-0.2587		-0.3079		-0.3589
		(0.0019)		(0.0026)		(0.0025)		(0.0024)		(0.0010)		(0.0027)		(0.0012)		(0.0003)		(0.0019)
Third component		0.5884		-0.0430		-0.3988		-0.4081		-0.2920		-0.1317		0.0457		0.2340		0.4324
		(0.0071)		(0.0098)		(0.0095)		(0.0091)		(0.0037)		(0.0101)		(0.0046)		(0.0013)		(0.0072)
R-squared	0.9971	0.9999	0.9964	0.9999	0.9961	0.9999	0.9960	0.9999	0.9962	1.0000	0.9962	0.9998	0.9963	1.0000	0.9963	1.0000	0.9962	0.9999
SEE	0.1039	0.0146	0.1116	0.0201	0.1126	0.0197	0.1091	0.0187	0.1019	0.0077	0.0970	0.0208	0.0935	0.0094	0.0906	0.0027	0.0887	0.0148
ARCH(1)	31.20	173.35	39.48	117.58	22.64	120.63	16.61	208.03	20.29	166.45	19.16	103.72	16.20	99.25	13.06	168.35	10.57	119.96
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
ARCH(4)	32.42	180.26	41.75	127.85	23.35	128.67	16.94	206.40	21.11	171.10	21.87	118.99	19.39	112.97	17.05	173.06	15.55	135.22
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Q(3)	0.01	909.58	0.031	648.93	0.02	892.08	0.03	767.76	0.02	866.17	0.02	966.20	0.01	908.96	0.02	921.06	0.03	1028.70
	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
Q(10)	8.20	1404.60	11.29	820.34	7.90	1354.40	7.01	1015.60	6.06	1268.90	4.47	1617.20	3.57	1476.10	3.00	1484.60	2.95	1755.50
	[0.61]	[0.00]	[0.34]	[0.00]	[0.64]	[0.00]	[0.72]	[0.00]	[0.81]	[0.00]	[0.92]	[0.00]	[0.97]	[0.00]	[0.98]	[0.00]	[0.98]	[0.00]
LM(1)	0.04	272.92	0.00	231.61	0.07	255.71	0.38	258.89	0.44	262.40	0.45	276.86	0.52	268.84	0.62	272.34	0.79	284.17
	[0.84]	[0.00]	[0.96]	[0.00]	[0.79]	[0.00]	[0.54]	[0.00]	[0.51]	[0.00]	[0.50]	[0.00]	[0.47]	[0.00]	[0.43]	[0.00]	[0.38]	[0.00]
LM(4)	0.24	278.44	1.47	236.02	0.98	265.59	0.67	262.13	0.70	269.38	0.56	283.97	0.57	275.66	0.78	278.32	1.23	291.83
	[0.99]	[0.00]	[0.83]	[0.00]	[0.91]	[0.00]	[0.96]	[0.00]	[0.95]	[0.00]	[0.97]	[0.00]	[0.97]	[0.00]	[0.94]	[0.00]	[0.87]	[0.00]
ADF	-9.23	-4.15	-9.34	-4.96	-9.05	-3.92	-8.90	-4.98	-8.71	-4.30	-8.50	-3.94	-8.49	-4.10	-8.47	-4.07	-8.40	-3.69
PP	-20.26	-6.14	-20.26	-7.55	-20.28	-6.57	-20.33	-6.89	-20.30	-6.49	-20.25	-5.98	-20.25	-6.26	-20.26	-6.14	-20.27	-5.68

Table 4. Estimated Models: d) USD

	2-year		3-year		4-year		5-year		6-year		7-year		8-year		9-year		10-year	
Own lag																		
I=1	1.0087		0.9933		0.9789		0.9567		0.9454		0.9302		0.9253		0.9193		0.9124	
	(0.0497)		(0.0496)		(0.0496)		(0.0497)		(0.0497)		(0.0498)		(0.0498)		(0.0499)		(0.0499)	
I=2	0.0830		0.0755		0.0719		0.1063		0.0982		0.0924		0.0933		0.0929		0.0918	
	(0.0709)		(0.0702)		(0.0697)		(0.0690)		(0.0686)		(0.0681)		(0.0680)		(0.0677)		(0.0675)	
I=3	0.0061		0.0456		0.0646		0.0460		0.0614		0.0723		0.0651		0.0592		0.0546	
	(0.0704)		(0.0701)		(0.0696)		(0.0690)		(0.0690)		(0.0687)		(0.0684)		(0.0681)		(0.0675)	
I=4	-0.1124		-0.1305		-0.1317		-0.1241		-0.1197		-0.1091		-0.0975		-0.0850		-0.0722	
	(0.0493)		(0.0494)		(0.0494)		(0.0496)		(0.0499)		(0.0501)		(0.0502)		(0.0501)		(0.0501)	
First component		-0.2927		-0.3115		-0.3220		-0.3301		-0.3368		-0.3439		-0.3478		-0.3525		-0.3575
		(0.0003)		(0.0003)		(0.0004)		(0.0004)		(0.0002)		(0.0005)		(0.0002)		(0.0000)		(0.0003)
Second component		0.6775		0.4153		0.2061		0.0390		-0.0649		-0.1796		-0.2387		-0.3031		-0.3730
		(0.0007)		(0.0007)		(0.0009)		(0.0008)		(0.0003)		(0.0011)		(0.0005)		(0.0001)		(0.0007)
Third component		0.4467		-0.0121		-0.2615		-0.4423		-0.3382		-0.2302		0.0145		0.2733		0.5458
		(0.0094)		(0.0100)		(0.0124)		(0.0108)		(0.0049)		(0.0151)		(0.0071)		(0.0013)		(0.0097)
R-squared	0.9798	0.9999	0.9769	0.9999	0.9756	0.9998	0.9761	0.9998	0.9762	1.0000	0.9762	0.9997	0.9761	0.9999	0.9760	1.0000	0.9756	0.9999
SEE	0.1379	0.0099	0.1375	0.0105	0.1354	0.0131	0.1325	0.0114	0.1313	0.0052	0.1313	0.0159	0.1303	0.0075	0.1300	0.0013	0.1308	0.0102
ARCH(1)	0.02	204.35	0.02	174.67	0.14	97.37	0.43	49.87	0.14	152.90	0.01	171.38	0.08	156.81	0.20	188.12	0.31	189.10
	[0.89]	[0.00]	[0.89]	[0.00]	[0.71]	[0.00]	[0.51]	[0.00]	[0.71]	[0.00]	[0.92]	[0.00]	[0.78]	[0.00]	[0.65]	[0.00]	[0.58]	[0.00]
ARCH(4)	3.07	218.54	4.03	189.03	5.83	135.54	8.10	64.63	8.612	178.20	9.06	200.53	9.65	190.55	10.26	215.96	10.89	212.53
	[0.55]	[0.00]	[0.40]	[0.00]	[0.21]	[0.00]	[0.09]	[0.00]	[0.07]	[0.00]	[0.06]	[0.00]	[0.05]	[0.00]	[0.04]	[0.00]	[0.03]	[0.00]
Q(3)	0.26	620.13	0.22	515.30	0.21	354.98	0.20	227.77	0.11	477.79	0.05	497.87	0.03	467.47	0.02	587.75	0.01	533.91
	[0.97]	[0.00]	[0.97]	[0.00]	[0.98]	[0.00]	[0.98]	[0.00]	[0.99]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]	[1.00]	[0.00]
Q(10)	8.54	1042.00	9.25	840.56	11.47	637.96	12.02	283.11	10.35	829.17	8.12	753.63	6.98	696.56	5.74	949.55	4.53	831.63
	[0.58]	[0.00]	[0.51]	[0.00]	[0.32]	[0.00]	[0.28]	[0.00]	[0.41]	[0.00]	[0.62]	[0.00]	[0.73]	[0.00]	[0.84]	[0.00]	[0.92]	[0.00]
LM(1)	6.27	264.53	6.34	225.96	6.30	160.08	6.43	111.25	4.81	219.80	3.03	205.97	1.92	196.07	0.87	250.96	0.16	220.70
	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]	[0.03]	[0.00]	[0.08]	[0.00]	[0.17]	[0.00]	[0.35]	[0.00]	[0.69]	[0.00]
LM(4)	8.85	270.03	7.50	235.11	8.07	179.15	7.51	126.74	5.90	225.78	4.75	225.19	3.77	215.62	3.25	257.74	3.20	237.70
	[0.07]	[0.00]	[0.11]	[0.00]	[0.09]	[0.00]	[0.11]	[0.00]	[0.21]	[0.00]	[0.31]	[0.00]	[0.44]	[0.00]	[0.52]	[0.00]	[0.53]	[0.00]
ADF	-8.19	-4.70	-8.16	-5.02	-8.12	-4.93	-8.16	-6.12	-8.24	-4.94	-8.34	-4.91	-8.42	-4.99	-8.52	-4.91	-8.66	-4.81
PP	-20.62	-6.89	-20.65	-7.87	-20.65	-9.98	-20.61	-11.82	-20.55	-8.08	-20.47	-8.63	-20.42	-8.94	-20.37	-7.40	-20.32	-8.17

Table 5. Forecasting performance indicators (Forecasting period: 10/7/98-12/30/98)

Forecasting Model		2-year		3-year		4-year		5-year		6-year		7-year		8-year		9-year		10-year	
		AR	Factor	AR	Factor	AR	Factor	AR	Factor	AR	Factor	AR	Factor	AR	Factor	AR	Factor	AR	Factor
a) DEM																			
Mean		0.0538	0.0605	0.0467	0.0713	0.0579	0.0629	0.0611	**0.0595**	0.0653	**0.0640**	0.0691	0.0822	0.0691	0.0720	0.0709	0.0737	0.0729	0.0842
Median	**Static**	0.0510	0.0526	0.0444	0.0648	0.0438	0.0681	0.0473	**0.0436**	0.0477	0.0566	0.0504	0.0998	0.0569	0.0593	0.0558	0.0693	0.0527	0.0797
RMSE	**Forecasts**	0.0604	0.0749	0.0576	0.0846	0.0667	0.0737	0.0715	0.0735	0.0774	0.0817	0.0841	0.1026	0.0853	0.0866	0.0873	0.0904	0.0904	0.1078
U-Theil		0.9975	**0.9959**	0.9977	**0.9950**	0.9970	**0.9962**	0.9966	**0.9963**	0.9960	**0.9956**	0.9953	**0.9937**	0.9953	0.9955	0.9952	**0.9950**	0.9950	**0.9929**
Mean		0.0917	0.0927	0.0933	0.0996	0.1123	0.1144	0.1230	**0.1191**	0.1304	**0.1189**	0.1377	**0.1163**	0.1436	**0.1284**	0.1552	0.1591	0.1686	0.2035
Median	**Dynamic**	0.0684	0.0778	0.0916	0.0992	0.1258	0.1258	0.1333	0.1359	0.1396	**0.1157**	0.1704	**0.1156**	0.1721	**0.1500**	0.1739	0.1965	0.1763	0.2395
RMSE	**Forecasts**	0.1169	**0.1131**	0.1088	0.1104	0.1215	0.1245	0.1332	**0.1288**	0.1460	**0.1310**	0.1682	**0.1291**	0.1723	**0.1520**	0.1792	0.1878	0.1897	0.2285
U-Theil		0.9898	0.9907	0.9921	**0.9920**	0.9913	**0.9907**	0.9899	0.9906	0.9876	0.9905	0.9837	0.9911	0.9834	0.9871	0.9828	**0.9810**	0.9817	**0.9737**
Sample mean		3.3821		3.4923		3.6329		3.7864		3.9335		4.0892		4.2028		4.3224		4.4485	
b) ESP																			
Mean		0.0581	**0.0521**	0.0517	0.0518	0.0514	0.0524	0.0512	0.0569	0.0549	**0.0537**	0.0586	0.0712	0.0621	0.0640	0.0659	0.0689	0.0699	0.0824
Median	**Static**	0.0498	**0.0458**	0.0463	0.0501	0.0367	0.0415	0.0376	0.0509	0.0338	0.0342	0.0441	0.0747	0.0537	**0.0438**	0.0533	0.0589	0.0528	0.0743
RMSE	**Forecasts**	0.0684	**0.0628**	0.0629	**0.0606**	0.0639	0.0640	0.0675	0.0710	0.0713	0.0718	0.0765	0.0907	0.0800	0.0822	0.0845	0.0896	0.0899	0.1086
U-Theil		0.9966	0.9972	0.9973	0.9976	0.9971	0.9971	0.9967	**0.9961**	0.9964	**0.9963**	0.9960	**0.9950**	0.9957	**0.9955**	0.9954	**0.9945**	0.9949	**0.9922**
Mean		0.1614	**0.1039**	0.0973	**0.0823**	0.0918	**0.0895**	0.0971	0.1076	0.1050	**0.1018**	0.1288	**0.0991**	0.1442	**0.1277**	0.1611	0.1793	0.1794	0.2363
Median	**Dynamic**	0.1329	**0.0968**	0.0445	0.0634	0.0751	0.0827	0.1175	**0.1097**	0.1119	**0.1045**	0.1418	**0.0913**	0.1516	**0.1375**	0.1608	0.1828	0.1695	0.2276
RMSE	**Forecasts**	0.1931	**0.1267**	0.1309	**0.1058**	0.1054	**0.1013**	0.1080	0.1195	0.1245	**0.1168**	0.1543	**0.1161**	0.1669	**0.1519**	0.1812	0.1984	0.1969	0.2502
U-Theil		0.9741	0.9885	0.9876	0.9918	0.9928	0.9935	0.9932	**0.9915**	0.9907	0.9920	0.9863	0.9922	0.9847	0.9871	0.9829	**0.9797**	0.9809	**0.9701**
Sample mean		3.4128		3.5152		3.6506		3.8007		3.9462		4.1001		4.2121		4.3299		4.4540	
c) JPY																			
Mean		0.0480	0.0565	0.0673	0.0871	0.0788	0.1043	0.0884	0.1024	0.0958	**0.0902**	0.1081	**0.0909**	0.1105	**0.0933**	0.1128	0.1149	0.1151	0.1653
Median	**Static**	0.0360	0.0437	0.0528	0.0961	0.0728	0.1177	0.0786	0.0923	0.1015	**0.0875**	0.1298	**0.0766**	0.1274	**0.0947**	0.1243	0.1256	0.1208	0.1218
RMSE	**Forecasts**	0.0625	0.0657	0.0834	0.1114	0.0986	0.1322	0.1083	0.1296	0.1200	**0.1083**	0.1353	**0.1061**	0.1386	**0.1138**	0.1425	0.1436	0.1475	0.1957
U-Theil		0.9770	0.9809	0.9688	**0.9482**	0.9635	**0.9417**	0.9656	**0.9515**	0.9632	0.9703	0.9593	0.9779	0.9609	0.9745	0.9621	**0.9620**	0.9629	**0.9395**
Mean		0.0843	0.0982	0.1154	0.1532	0.1464	0.1780	0.1784	0.1947	0.2129	**0.2018**	0.2600	**0.2309**	0.2938	**0.2421**	0.3336	**0.2897**	0.3773	**0.3665**
Median	**Dynamic**	0.0690	**0.0568**	0.0821	**0.0664**	0.0661	0.0697	0.0786	**0.0498**	0.0602	0.0790	0.1182	**0.1112**	0.1274	**0.0913**	0.1243	0.1256	0.1392	0.1415
RMSE	**Forecasts**	0.1100	0.1458	0.1691	0.2374	0.2389	0.2863	0.2990	0.3195	0.3527	**0.3339**	0.4156	**0.3535**	0.4493	**0.3923**	0.4873	**0.4433**	0.5295	**0.5090**
U-Theil		0.9355	**0.8795**	0.8713	**0.7681**	0.7891	**0.7238**	0.7340	**0.7079**	0.6899	0.7127	0.6430	0.7150	0.6307	0.6904	0.6174	0.6605	0.6036	0.6240
Sample mean		0.5399		0.6874		0.8248		0.9583		1.0743		1.2051		1.3149		1.4362		1.5710	
d) USD																			
Mean		0.0992	0.1020	0.0929	0.1006	0.0906	0.0938	0.0900	**0.0867**	0.0883	**0.0873**	0.0870	0.0880	0.0893	0.0895	0.0917	0.0932	0.0942	0.0975
Median	**Static**	0.0696	0.0886	0.0699	0.0736	0.0538	0.0590	0.0444	0.0458	0.0441	**0.0394**	0.0443	**0.0437**	0.0536	**0.0481**	0.0639	**0.0582**	0.0713	0.0818
RMSE	**Forecasts**	0.1338	0.1339	0.1259	**0.1243**	0.1211	**0.1186**	0.1208	0.1228	0.1211	0.1233	0.1226	0.1250	0.1250	0.1277	0.1284	0.1309	0.1328	0.1354
U-Theil		0.9895	**0.9894**	0.9914	0.9919	0.9926	0.9930	0.9925	**0.9920**	0.9922	**0.9917**	0.9917	**0.9912**	0.9910	**0.9905**	0.9902	**0.9900**	0.9892	0.9893
Mean		0.1946	**0.1864**	0.1736	**0.1350**	0.1422	**0.1235**	0.1246	0.1268	0.1281	0.1319	0.1318	0.1362	0.1461	0.1488	0.1605	**0.1560**	0.1749	**0.1631**
Median	**Dynamic**	0.1757	**0.1687**	0.1364	0.1453	0.1579	**0.1182**	0.1454	0.1493	0.1266	**0.1240**	0.1091	**0.0961**	0.1048	0.1259	0.1364	0.1503	0.1691	**0.1586**
RMSE	**Forecasts**	0.2163	**0.2088**	0.1962	**0.1519**	0.1666	**0.1398**	0.1490	0.1516	0.1501	0.1548	0.1525	0.1594	0.1661	0.1722	0.1806	0.1807	0.1960	**0.1900**
U-Theil		0.9781	0.9795	0.9817	0.9892	0.9870	0.9908	0.9898	**0.9894**	0.9897	**0.9890**	0.9895	**0.9884**	0.9879	**0.9868**	0.9860	**0.9857**	0.9838	0.9844
Sample mean		4.8815		4.9680		5.0499		5.1276		5.1981		5.2689		5.3323		5.3983		5.4686	

Note: Forecasts obtained from estimated models in Table 4. Mean and Median denote the mean and median absolute values of the forecasting errors. RMSE denotes the Root Mean Square Error, while U-Theil denotes Theil´s statistic. Boldface figures denote cases when factors models forecast better than autoregresive models.

Table 6. Contemporaneous correlation coefficients between level zero coupon rate forecasts

	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
Static forecasts					**a) DEM**									**b) ESP**				
2-year	1.000	0.973	0.947	0.937	0.922	0.901	0.885	0.862	0.830	1.000	0.982	0.948	0.912	0.881	0.840	0.808	0.766	0.710
3-year	0.983	1.000	0.993	0.986	0.976	0.958	0.947	0.931	0.906	0.987	1.000	0.989	0.966	0.944	0.911	0.888	0.855	0.808
4-year	0.957	0.993	1.000	0.997	0.990	0.976	0.968	0.954	0.932	0.965	0.994	1.000	0.993	0.980	0.958	0.941	0.915	0.876
5-year	0.931	0.979	0.996	1.000	0.997	0.989	0.982	0.970	0.951	0.938	0.980	0.996	1.000	0.996	0.984	0.971	0.950	0.917
6-year	0.906	0.961	0.986	0.997	1.000	0.997	0.993	0.984	0.968	0.908	0.961	0.985	0.996	1.000	0.995	0.988	0.971	0.943
7-year	0.870	0.933	0.967	0.985	0.996	1.000	0.998	0.993	0.980	0.866	0.929	0.962	0.982	0.995	1.000	0.997	0.986	0.965
8-year	0.850	0.915	0.953	0.976	0.990	0.999	1.000	0.998	0.990	0.840	0.908	0.947	0.971	0.988	0.999	1.000	0.996	0.982
9-year	0.825	0.892	0.934	0.961	0.980	0.994	0.998	1.000	0.997	0.808	0.882	0.926	0.955	0.977	0.994	0.998	1.000	0.995
10-year	0.792	0.862	0.909	0.941	0.965	0.984	0.992	0.998	1.000	0.770	0.850	0.899	0.933	0.961	0.984	0.992	0.998	1.000
Dynamic forecasts																		
2-year	1.000	0.998	0.998	0.996	0.996	0.996	0.996	0.997	0.997	1.000	0.999	0.995	0.987	0.985	0.983	0.984	0.985	0.986
3-year	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	0.999	0.994	0.992	0.991	0.992	0.993	0.993
4-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.998	0.997	0.996	0.997	0.997	0.997
5-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
8-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10-year	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Static forecasts					**c) JPY**									**d) USD**				
2-year	1.000	0.997	0.990	0.976	0.962	0.941	0.927	0.909	0.886	1.000	0.984	0.943	0.881	0.828	0.753	0.691	0.620	0.544
3-year	0.997	1.000	0.996	0.985	0.971	0.951	0.939	0.923	0.902	0.988	1.000	0.985	0.945	0.909	0.853	0.804	0.745	0.679
4-year	0.994	0.997	1.000	0.995	0.986	0.971	0.961	0.947	0.929	0.947	0.985	1.000	0.985	0.964	0.924	0.885	0.836	0.779
5-year	0.987	0.987	0.996	1.000	0.997	0.988	0.981	0.971	0.957	0.874	0.938	0.983	1.000	0.993	0.968	0.938	0.897	0.847
6-year	0.974	0.971	0.985	0.996	1.000	0.997	0.993	0.986	0.975	0.800	0.882	0.949	0.990	1.000	0.991	0.972	0.942	0.902
7-year	0.955	0.949	0.968	0.987	0.997	1.000	0.999	0.995	0.988	0.693	0.794	0.886	0.954	0.986	1.000	0.994	0.977	0.949
8-year	0.945	0.937	0.958	0.980	0.993	0.999	1.000	0.999	0.994	0.630	0.738	0.841	0.923	0.967	0.996	1.000	0.994	0.977
9-year	0.933	0.923	0.946	0.971	0.988	0.997	0.999	1.000	0.998	0.557	0.672	0.786	0.881	0.938	0.981	0.995	1.000	0.994
10-year	0.920	0.909	0.933	0.962	0.982	0.994	0.997	0.999	1.000	0.476	0.598	0.721	0.829	0.898	0.957	0.980	0.995	1.000
Dynamic forecasts																		
2-year	1.000	0.998	1.000	0.998	0.996	0.995	0.997	0.998	0.999	1.000	0.989	0.982	0.985	0.980	0.974	0.975	0.974	0.973
3-year	1.000	1.000	0.997	0.992	0.988	0.987	0.989	0.991	0.994	0.999	1.000	0.999	1.000	0.998	0.997	0.997	0.997	0.996
4-year	1.000	1.000	1.000	0.999	0.997	0.996	0.997	0.999	0.999	0.998	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.999
5-year	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	0.999	0.999	0.999	0.998
6-year	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999
7-year	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
8-year	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9-year	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10-year	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note: The upper triangular matrix contains correlation coefficients between forecast interest rates from autoregressive model. The lower triangular matrix contains correlation coefficients between forecast interest rates from factor models. Forecasting period: 10/7/98-12/30/98.

Figure 1. Zero coupon rates: DEM

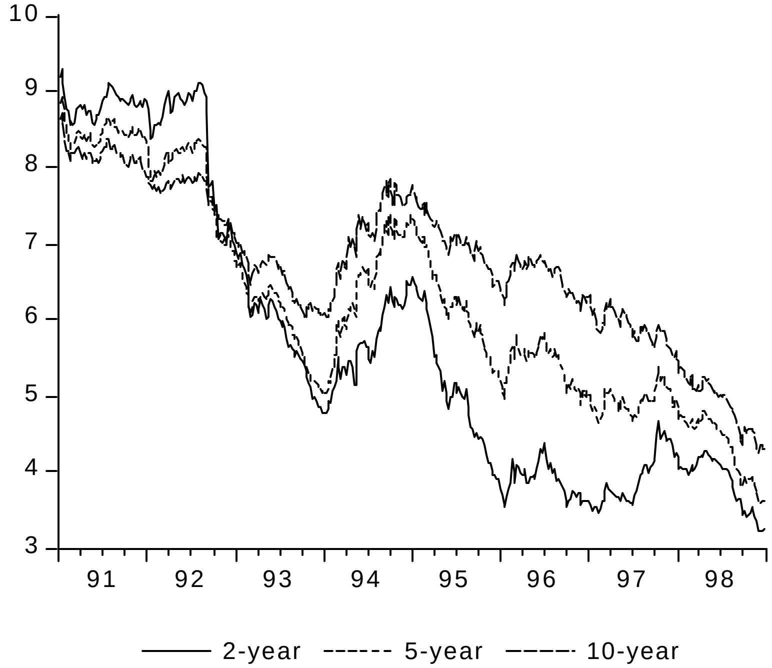

Figure 2. The impact on term structure of a change in each principal component