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A risk-averse solution for the prescribed burning problem

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ABSTRACT

Hazard reduction is a complex task involving important efforts to prevent and mitigate the consequences of disasters. Many countries around the world have experienced devastating wildfires in recent decades and risk reduction strategies are now more important than ever. Reducing contiguous areas of high fuel load through prescribed burning is a fuel management strategy for reducing wildfire hazard. Unfortunately, this has an impact on the habitat of fauna and thus constrains a prescribed burning schedule which is also subject to uncertainty. To address this problem a mathematical programming model is proposed for scheduling prescribed burns on treatment units on a landscape over a planning horizon. The model takes into account the uncertainty related to the conditions for performing the scheduled prescribed burns as well as several criteria related to the safety and quality of the habitat. This multiobjective stochastic problem is modelled from a risk-averse perspective whose aim is to minimize the worst achievement of the criteria on the different scenarios considered. This model is applied to a real case study in Andalusia (Spain) comparing the solutions achieved with the risk-neutral solution provided by the simple weighted aggregated average. The results obtained show that our proposed approach outperforms the risk-neutral solution in worst cases without a significant loss of quality in the global set of scenarios.

1. Introduction

Wildfires are affecting communities all around the world with devastating consequences. Fuel management is employed to mitigate the effects of wildfires by modifying landscape vegetation. *Prescribed burning* is a fuel management strategy widely used in Australia, USA, and Canada (Fernandes, 2015) and is increasingly being implemented in the Mediterranean area (Fernandes et al., 2013). Prescribed burning involves reducing the fuel load in selected areas of the landscape. This reduces the spread and intensity of large wildfires (Boer et al., 2009; Coop et al., 2016; Lydersen et al., 2017), and thus the risk to human life and economic assets (Penman et al., 2011).

Selecting which treatment units to burn is a complex decision-making problem best solved with the aid of mathematical models. Fire spread tools, such as *FARSITE* (Finney, 1998), *FlamMap* (Finney, 2006) or *BehavePlus* (Andrews, 2014) allow the effectiveness of each treatment unit to be compared. Oliveira et al. (2016) and Salis et al. (2016). For instance, the minimum travel-time, fire-spread algorithm, which is implemented in *FlamMap*, generates data needed for evaluating different fuel management strategies.

The flammability of vegetation changes with time since it last burned and this also differs with species. Thus, the problem of determining a multiperiod schedule of treatments on a landscape is a complex spatio-temporal problem (Hof and Omi, 2003; Rönnqvist et al., 2015). This is especially true if the ecological effects of burning a particular area are also considered (León et al., 2019). Minas et al. (2014) formulated the fuel hazard problem on a regular grid, and Matsypura et al. (2018) proposed a mathematical programming model to allocate prescribed burns to treatment units over a planning horizon, in which fuel accumulation is accounted for using a type of Olson curve (Olson, 1963). Rachmawati et al. (2016) include multiple vegetation classes in a real landscape, and Driscoll et al. (2016) recognize the conflict between different criteria when performing prescribed burns, applying a multicriteria decision-making approach considering 8 objectives to evaluate 22 burn plans. Alcasena et al. (2018) also study the prescribed burning problem, with emphasis on the probability of fire occurrence, fire behaviour and assets at risk. Other recent works, Rachmawati et al. (2018) and León et al. (2019), consider a multiperiod model for fuel management that incorporate constraints relating to the sustainability of both flora and fauna. The latter work is the starting point for this research.

Emergencies and disaster management are fields in which many difficulties arise, such as high uncertainty and multiple conflicting objectives. As mentioned earlier, the prescribed burning problem includes

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multiple objectives (Driscoll et al., 2016; Rachmawati et al., 2018), but uncertainty around the completion rates of prescribed burn, for instance, should also be considered (Duff et al., 2019). Stochastic programming models are useful for decision making under conditions of probabilistic uncertainty. They tend to focus on optimizing the average of a stochastic objective function, but this can lead to poor performance of solutions in low probability scenarios. To overcome such difficulties, risk-averse decisions can be made, focusing on minimizing the worst consequences. Some measures to control the worst scenarios are *value at risk* (VaR) and *conditional value at risk* (CVaR), well known in financial applications (Mansini et al., 2015; Dixit and Tiwari, 2019). VaR is a percentile of the profit or loss distribution, and CVaR is the expected value from that percentile. CVaR has the advantage over VaR in that it can be modelled linearly.

A mathematical programming model considering uncertainty represented by a probability distribution and multiple objective functions to be optimized is called a Multiobjective Stochastic Programming (MSP) model. The probability distribution is usually is a discrete distribution, and its different values are called scenarios. It is especially used in multistage models.

The different techniques which can be used for solving the MSP problem can be categorized into those using a "stochastic transformation" (eliminating the multiple objectives and solving a single-objective stochastic problem) and "multiobjective transformation" (reducing the stochasticity and solving a multiobjective deterministic problem). These categories are considered in Caballero et al. (2004) and in Ben Abdelaziz (2012), where different solutions methods for the MSP problem are reviewed, categorizing them as stochastic approach or multiobjective approach. Stochastic goal programming was studied in Aouni et al. (2005), where the sum of the deviations of objective functions to some goals set beforehand to stochastic values was minimized. In Muñoz et al. (2010), an interactive reference point method is developed. The decision-maker (DM) gives targets and probabilities for each objective, and a chance constraint is included ensuring that the solution achieves the goal related to the target with the defined probability. If the model results to be infeasible, then the DM should either relax the targets or decrease the probabilities of achievement.

León et al. (2020) propose a solution concept for MSP problems searching risk-averse decisions, regarding both the uncertainty and the multiple criteria. Their proposed approach focuses on the worst scenarios and criteria performance, aggregating the former using the concept of CVaR (Rockafellar and Uryasev, 2002) and the latter with an ordered weighted averaging (OWA) operator (Yager, 1988; Yager and Alajlan, 2016) giving weights only to the worst achieved criteria.

The main objective of this research is the development of a mathematical programming model for the prescribed burn problem, taking into account safety and ecological criteria under an uncertain future. The model will lead to a risk-averse solution, robust when exposed to uncertainty with both scenarios and criteria controlled. A case study based on a real case in Andalusia (Spain) will be presented to assess the validity of the model, as well as interpreting its results and examining the sensitivity to its parameters.

The remainder of the paper is structured as follows. Section 2 summarizes the problem treated in León et al. (2019), which serves as a starting point for this research, and details the modifications for the problem tackled in this paper. Section 3 introduces the developed MSP model to obtain a risk-averse solution, and the methodology followed to solve it. The model is applied to a real case study located in the south of Spain. The description of the case study is given in Section 4 and its results are shown in Section 5. Finally, Section 6 draws some conclusions and limitations of the research.

2. Problem description

The problem addressed is to obtain a schedule for burning treatment units on a landscape over a planning horizon, as proposed in León et al. (2019) which serves as a starting point for this research. In that paper, a mixed integer programming model is proposed taking into account safety and ecological objectives subject to constraints on the annual burning capacity.

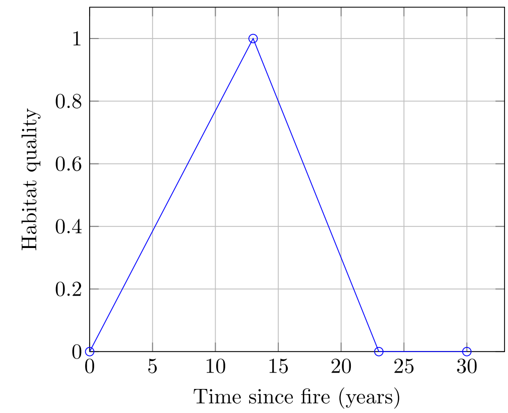


Fig. 1. Example of fire response curve (FRC), for a species preferring vegetation 10–15 years old and for which vegetation of above 23 years since fire is not suitable. Breakpoints of the piecewise linear function are located in 0, 13, 23 and 30, defining the pieces of the function jointly with the values of FRC in these points (0, 1, 0, 0, respectively).

The landscape is represented by polygons or cells, each of them representing a 'treatment unit' or 'burn unit'. Each cell is connected to its neighbours i.e. those polygons with which it has a common boundary. Burn units have an associated vegetation *age*, measuring the years that have passed since the last time the unit was burnt, evolving over time. Such 'age' works as a proxy for fuel load and is used for defining a fire risk measure. Units can be scheduled to burn over the planning horizon, as long as their ages are within some minimum and maximum Tolerable Fire Interval (TFI) thresholds. When a unit is burnt, its age is set to zero. Units with ages exceeding a given threshold are deemed risky both for ignition and for fire spread. They will be called *high-fuel load* units. Neighbours of these high-fuel load units form *high-risk connections*. High-fuel load units are a proxy of fire-ignition risk and high-risk connections of fire-spread risk. The age threshold to define a unit as high-fuel load depends on the type of vegetation.

Burning vegetation has implications for the habitat of fauna. The habitat requirements of fauna involve both the age and species of vegetation (Di Stefano et al., 2013). Thus it is sensible to search for solutions that reduce wildfire hazard in the landscape while maintaining a suitable mix of vegetation. This mix should provide for the requirements of existing fauna, both in terms of age and species.

León et al. (2019) define the habitat quality of a cell in terms of its age since fire, using a piecewise linear function called *fire response curve* (FRC) as the one shown in Fig. 1. The FRC represents that the fauna likes vegetation of a certain age and dislikes vegetation that is too young or too old. This FRC is specific for each species. The objective of the model is hence to determine the timing and location of prescribed burns that minimize fire risk while maintaining suitable habitat for the species living in the area.

León et al. (2019) did not develop a specific multiobjective approach for this problem, as it included the habitat quality via hard constraints or a lexicographical approach introducing a priority order between the objectives. A specific treatment will be given in this paper for the following four objectives considered (which extend those of the previous research):

1. Minimize high-fuel load connections, weighting each connection with the shared boundary. High-fuel load connections are deemed a proxy of fire-propagation risk.
2. Minimize high-fuel load units, weighting each unit with its size. High-fuel load units are deemed a proxy of fire-ignition risk.

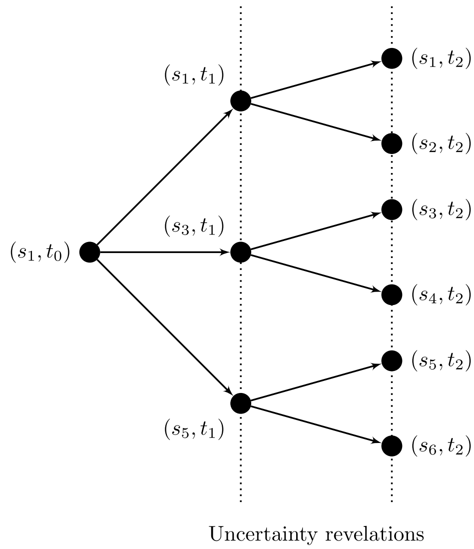


Fig. 2. Example of a compact representation of the scenarios, with uncertainty revealed after decision nodes. The first decisions made on this tree are at t_0, before it is revealed which of the three outcomes at t_1 will occur. Decisions at t_1 differ for each of the three outcomes that have occurred, and for each of them there is further uncertainty around which of two possible levels will occur. Note that in this problem there are no decisions in stage t_2, as these leaves are the final stages after the last uncertainty has been revealed. A scenario is a complete path from the origin to the leaves, making up six scenarios.

3. Maximize habitat quality for species 1.
4. Maximize habitat quality for species 2.

An essential issue of the problem is the yearly capacity for performing prescribed burns represented for each year t by a budget b_t. León et al. (2019) included a budget constraint measured in terms of area; it means, the total burnt area could not exceed a given quantity. While measuring the budget in terms of the area might seem the most logical choice, as an alternative this budget constraint could easily be replaced or complemented with limits on the number of units burnt or their perimeters (as long as they are independently burnt and not merged for burning).

However, very often not all units scheduled for treatment are burned at the appropriate time. Burns are not possible when conditions are too wet. It is also too dangerous to burn when conditions are too dry. Thus burning opportunities are limited by uncertain weather (O'Keefe, 1995; Duff et al., 2019).

If the probability distribution of this budget (area) can be estimated, then the problem can be addressed via stochastic programming. For this multistage problem, the probability distribution can be represented via scenarios. A *scenario* is a realization of the stochastic parameters from the beginning to the end of the planning horizon. The joint representation of scenarios is called a *scenario tree*.

The sequence of decisions and uncertainty leads to different states at each stage (year). Decisions are assumed to be made in each state, knowing the outcomes of the uncertainty and decisions of previous stages, before revealing the uncertainty in the current stage, and considering the uncertainty of the possible scenarios after this state. This is known as the principle of non-anticipation, which must be imposed in the decision model (Rockafellar and Wets, 1991). It can be done through an explicit inclusion of restrictions for those decisions common to different scenarios, or through a compact representation of the scenarios as a tree as can be seen in (Fig. 2). The second option is chosen, in which the decisions in each state (node) of one stage are based only on previous known nodes in the scenario tree and considering only the scenarios included in the branches following the current state.

The tree starts with a node representing the current state where a common decision is made for all the considered scenarios. This decision is a partial order on the treatment units to be burnt. This partial order first establishes the units to be treated if the available budget is the smallest; next, the units to be treated are included to the previous ones if the available budget is the second smallest one; and so on, until the units to be burned with the highest budget are included. It is assumed that forest services will perform burns in this order while weather conditions permit. On each node, once the uncertainty is revealed, a number of units have been burnt. This will be the state (unit ages) on which decisions will be made during the next period. It is worth noting that stochastic programming does not intend to provide a long term schedule, its aim is to determine a decision for the current stage taking into account its future effects. Therefore, first-stage decisions are the only ones to be actually implemented, the rest of the tree captures the future effect of these decisions. For this reason, the uncertainty is usually represented with more accuracy in earlier periods. The model should be run again in the next period starting from the actual final state of the first stage.

Stochastic programming usually works on optimizing an average function. If the function is a simple average, then good achievements in some scenarios can compensate for poor performance in others. Dealing with criteria related to safety and fauna, which can never be recovered, this research focuses on finding robust solutions in which the worst outcomes are controlled. So, optimizing VaR or CVaR will be the objective from the stochastic point of view. CVaR, which is a conditional average, is chosen for its facility to be computed linearly, and because it takes into account all values over the β percentile defined. β will be the sum of the probabilities of the scenarios with the worst values for a given objective.

The problem is stochastic and multiobjective, with risk-aversion, using the CVaR as the value to be optimized for each objective. Each objective will have different importance (safety versus fauna), and a set of weights will be given for the different criteria. These weights represent a subjective scale, and hence an expert should be involved in fitting specific values for them. Obtaining these weights is a difficult task that involves much interaction with decision-makers. Probably the most extended method is the pairwise comparison proposed by Saaty in the Analytic Hierarchy Process (Saaty, 1977). Most multicriteria methods consider a trade-off between objectives based on these weights, but from a risk-averse perception, the value to be optimized would be focused on the criteria with the worst performance. Translating the CVaR idea to the multiobjective field, we propose an ordered weighted average (OWA) approach. It means, given a threshold r for weights, it should be optimized for the aggregated weighted values of the criteria with worst achievement whose added weights are lower than or equal to the threshold r.

Therefore, our problem is a multiobjective stochastic problem (MSP) with risk-aversion using CVaR and OWAs. The theoretical formulation of this type of problem was first introduced in León et al. (2020), where a general formulation and solving methodology can be seen. The present paper makes use of this methodology and applies to a real case study in which the need for a risk-averse solution is more than justified. The next section presents the model for the problem stated, paying special attention to the stochastic multiobjective with risk aversion treatment.

3. Multiobjective stochastic programming (MSP) model

The mathematical programming model for this prescribed burning problem is introduced in this section presenting the general variables and constraints first, followed by the special treatment of uncertainty and multiple criteria from a risk-averse point of view.

3.1. Indices, parameters and variables

In this subsection the parameters and variables used are first introduced.

Indices sets

I	set of all burn units in the landscape
$\mathcal{N}_i \subset I$	set of burn units connected to burn unit i (neighbours)
T	set of stages (years) in the planning horizon
M	set of animal species
N_m	set of breakpoints of the piecewise linear function of the fire response curve for the mth species $m \in M$
K	set of criteria considered
S	set of scenarios for budget over the planning horizon T
$\Psi_t \subset S$	set of active scenarios (nodes) at time t
$\Phi_{st} \subset S$	set of children (nodes) of scenario s at time t
Φ_{st}^{-1}	parent node (active) of scenario $s \in S$ at time $t \in T$
Λ_{st}	active scenario at time t of scenario s (ancestor)

Parameters

a_i	initial fuel age of burn unit $i \in I$
b_t^s	budget for fuel treatment at stage $t \in T$ in scenario $s \in S$
c_i	area of burn unit $i \in I$
h_i	high-fuel load threshold of burn unit $i \in I$
$\overline{TFI}_i$	maximum tolerable fire interval (TFI) of age for burning unit $i \in I$
$\underline{TFI}_i$	minimum tolerable fire interval (TFI) of age for burning unit $i \in I$
r_{nm}	nth breakpoint of the piecewise linear function of the FRC (see Fig. 1) for the mth species, $m \in M$, $n \in N_m$
v_{nm}	value of breakpoint r_{nm} according to the FRC for the mth species, $m \in M$, $n \in N_m$
l_{ij}	length of shared boundary of units i and j
U	upper bound coefficient
π_s	probability of scenario $s \in S$
w_k	importance of kth objective function, $k \in K$

Variables

X_{it}^s	decision variable of the model, which takes the value 1 if burn unit $i \in I$ is treated at stage $t \in T$ in scenario $s \in S$, 0 otherwise
H_{it}^s	1 if burn unit $i \in I$ is classified as high-fuel load at stage $t \in T$ in scenario $s \in S$, 0 otherwise
Q_{ijt}^s	1 if connected burn units $i \in I$ and $j \in I$ are both classified as high-fuel load at stage $t \in T$ in scenario $s \in S$, 0 otherwise
A_{it}^s	fuel age of burn unit $i \in I$ at stage $t \in T$ in scenario $s \in S$
FRC_{itm}^s	habitat quality of burn unit $i \in I$ at stage $t \in T$ for species $m \in M$ in scenario $s \in S$ (fire response curve)
Z_{itnm}^s	1 if the age of burn unit $i \in I$ at stage $t \in T$ in scenario $s \in S$ is between r_{nm} and $r_{(n+1)m}$, 0 otherwise
G_{itnm}^s	coefficient of the linear convex combination for the nth breakpoint of the piecewise linear function FRC of the mth species for burn unit i in time period t in scenario s

Uncertainty

S represents the set of scenarios, a scenario being a sequence of budget values over the planning horizon. The final states (unit ages) are the result of the sequence of budget values and decisions made. The option used for the non-anticipativity property is the compact representation of the scenarios tree, for which for any $t \in T$ only as many pairs (s,t) will be active as nodes in the tree. Thus, at each stage t, there will be only nodes corresponding to scenarios with different sequences of the stochastic parameters up to that moment. So, the pair (s,t) will belong to all scenarios sharing the same sequence until stage t. The following representation will be useful, where Φ_{st} denotes the set of possible states at time $t+1$ if at time t state s occurred (children), and Φ_{st}^{-1} which state happened at time $t-1$ given the state s of time t (parent):

$$\Phi_{st} = \{s' \in S | (s', t+1) \text{ is a child of } (s,t)\} = \{s' \in S | b_{t'}^s = b_{t'}^{s'} \quad \forall t' \le t\}$$

$$\Phi_{st}^{-1} = s' \Rightarrow s \in \Phi_{s'(t-1)}$$

The active scenario chosen for representing the parent node will be the one of lower ordinal, as usual. For instance, the tree of Fig. 2 has $\Phi_{s_1t_1} = \{s_1, s_3, s_5\}$ and $\Phi_{s_4t_2}^{-1} = s_3$.

The set of active scenarios at each stage t will be denoted by Ψ_t, being for Fig. 2:

$$\Psi_1 = \{s_1\}$$

$$\Psi_2 = \{s_1, s_3, s_5\}$$

$$\Psi_3 = \{s_1, s_2, s_3, s_4, s_5, s_6\}$$

As uncertainty is revealed within the burn season, the following constraints represent the partial order in the units without anticipativity:

$$X_{it}^s \ge X_{it}^{s'} \qquad \forall i,t,s,s' \text{ such that } \begin{cases} s, s' \in \Psi_t \\ \Phi_{st}^{-1} = \Phi_{s't}^{-1} \\ b_t^s \ge b_t^{s'} \end{cases}$$

Condition $s, s' \in \Psi_t$ ensures that scenarios s and s' are active at time t, $\Phi_{st}^{-1} = \Phi_{s't}^{-1}$ checks that both nodes come from the same parent, and finally $b_t^s \ge b_t^{s'}$ makes certain that scenario s has at least as much budget as scenario s', and hence whatever is burnt in s' needs to be burnt in s.

The last piece of notation refers to all active nodes which conform a scenario s:

$$\Lambda_{st} = s' \iff \text{node } (s,T) \text{ is a descendant of the active node } (s',t)$$

In the example, $\Lambda_{s_4t_2} = s_3$, because the final scenario s_4 came from scenario s_3 at time t_2. Note that $\Lambda_{st_1} = s_1$ for all $s \in S$, corresponding to the first stage decisions made before any uncertainty is known.

3.2. Objectives

Four different objective functions will be simultaneously considered for all $s \in S$: high-fuel load connections, high-fuel load areas and habitat quality of two different species represented by their corresponding Fire Response Curves (FRC).

$$[minimize] \qquad f_1^s = \sum_{t=1}^{T} \sum_{i \in I} \sum_{\substack{j \in \mathcal{N}_i \\ j > i}} l_{ij} Q_{ijt}^{\Lambda_{st}} \tag{1a}$$

$$(minimize) \qquad f_2^s = \sum_{t=1}^{T} \sum_{i \in I} c_i H_{it}^{\Lambda_{st}} \tag{1b}$$

$$(maximize) \qquad f_3^s = \sum_{t=1}^{T} \sum_{i \in I} c_i FRC_{itm_1}^{\Lambda_{st}} \tag{1c}$$

$$(maximize) \qquad f_4^s = \sum_{t=1}^{T} \sum_{i \in I} c_i FRC_{itm_2}^{\Lambda_{st}} \tag{1d}$$

All the objective functions defined above depend on the scenario s, and by using the parameter Λ_{st} they take into account only the existing states for each t of the planning horizon. Eq. (1a) measures high-fuel load connections (Q_{ijt} equals one when at time t both i and j have a high-fuel load), weighting them by the length of the shared boundary l_{ij}. Eq. (1b) aggregates the total area of units with a high-fuel load (H_{it} equals one when i has a high-fuel load at time t). Finally, Eqs. (1c) and (1d) measure the habitat quality of two different species, weighting each unit with its area c_i.

In order to carefully manage trade-offs between objectives, functions f_k^s should be normalized. Lower and upper bounds will be obtained solving independently each scenario, minimizing and maximizing each of the objective functions to obtain such bounds $\underline{f}_k = \min_{x\in X} f_k(x)$ and $\overline{f}_k = \max_{x\in X} f_k(x)$, being X the set defined by the model constraints. The objective functions are redefined as:

$$\tilde{f}_1^s(x) = \frac{f_1^s(x) - \underline{f}_1}{\overline{f}_1 - \underline{f}_1} \tag{2a}$$

$$\tilde{f}_2^s(x) = \frac{f_2^s(x) - \underline{f}_2}{\overline{f}_2 - \underline{f}_2} \tag{2b}$$

$$\tilde{f}_3^s(x) = \frac{\overline{f}_3 - f_3^s(x)}{\overline{f}_3 - \underline{f}_3} \tag{2c}$$

$$\tilde{f}_4^s(x) = \frac{\overline{f}_4 - f_4^s(x)}{\overline{f}_4 - \underline{f}_4} \tag{2d}$$

Now, it is guaranteed that the values of $\tilde{f}_k^s(x)$ are in $[0,1]$ for every feasible solution and all of them need to be minimized. Weights w_k are assumed to be stated for the normalized objectives, representing the possible trade-off between the objectives.

3.3. Constraints

$$\sum_{i\in I} c_i X_{it}^s \le b_t^s \qquad \forall t\in T, \forall s\in \Psi_t \tag{3a}$$

$$A_{it_1}^{s_1} = a_i \qquad \forall i\in I \tag{3b}$$

$$A_{it}^s \ge A_{i(t-1)}^{s'} + 1 - U\cdot X_{i(t-1)}^{s'} \qquad \forall t\in T | t\ne t_1, \forall i\in I, \forall s\in \Psi_t, s' = \Phi_{st}^{-1} \tag{3c}$$

$$A_{it}^s \le A_{i(t-1)}^{s'} + 1 \qquad \forall t\in T | t\ne t_1, \forall i\in I, \forall s\in \Psi_t, s' = \Phi_{st}^{-1} \tag{3d}$$

$$A_{it}^s \le \overline{TFI}(1 - X_{i(t-1)}^{s'}) \qquad \forall t\in T | t\ne t_1, \forall i\in I, \forall s\in \Psi_t, s' = \Phi_{st}^{-1} \tag{3e}$$

$$\underline{TFI}\cdot X_{it}^s \le A_{it}^s \qquad \forall t\in T, \forall i\in I, \forall s\in \Psi_t \tag{3f}$$

$$A_{it}^s \le h_i - 1 + U\cdot H_{it}^s \qquad \forall t\in T, \forall i\in I, \forall s\in \Psi_t \tag{3g}$$

$$H_{it}^s + H_{jt}^s \le 1 + Q_{ijt}^s \qquad \forall t\in T, \forall i\in I, \forall j\in \mathcal{N}_i, j>i, \forall s\in \Psi_t \tag{3h}$$

$$\sum_{n\in N_m | n<card(N_m)} Z_{itnm}^s = 1 \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t \tag{3i}$$

$$Z_{itnm}^s \le G_{itnm}^s + G_{it(n+1)m}^s \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t, \forall n\in N_m | n<card(N_m) \tag{3j}$$

$$\sum_{n\in N_m} r_{nm} G_{itnm}^s = A_{it}^s \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t \tag{3k}$$

$$\sum_{n\in N_m} G_{itnm}^s = 1 \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t \tag{3l}$$

$$FRC_{itm}^s = \sum_{n\in N_m} v_{nm} G_{itnm}^s \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t \tag{3m}$$

$$X_{it}^s \ge X_{it}^{s'} \qquad \forall t\in T, \forall i\in I, \forall s, s'\in \Psi_t | \Phi_{st}^{-1} = \Phi_{s't}^{-1}, b_t^s \ge b_t^{s'} \tag{3n}$$

$$X_{it}^s \in \{0,1\} \qquad \forall t\in T, \forall i\in I, \forall s\in \Psi_t \tag{3o}$$

$$H_{it}^s \in \{0,1\} \qquad \forall t\in T, \forall i\in I, \forall s\in \Psi_t \tag{3p}$$

$$Z_{itnm}^s \in \{0,1\} \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t, \forall n\in N_m | n<card(N_m) \tag{3q}$$

$$Q_{ijt}^s \ge 0 \qquad \forall t\in T, \forall i\in I, \forall j\in \mathcal{N}_i, j>i, \forall s\in \Psi_t \tag{3r}$$

$$A_{it}^s \ge 0 \qquad \forall t\in T, \forall i\in I, \forall s\in \Psi_t \tag{3s}$$

$$FRC_{itm}^s \ge 0 \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall s\in \Psi_t \tag{3t}$$

$$G_{itnm}^s \ge 0 \qquad \forall t\in T, \forall i\in I, \forall m\in M, \forall n\in N_m, \forall s\in \Psi_t \tag{3u}$$

The feasible region of the model, X, is given by Eqs. (3a) to (3u). Eq. (3a) limits the area that can be burnt each year by the budget in that scenario and stage. Eq. (3b) initialize the age variables. Eqs. (3c) to (3f) are constraints concerning the age of the cells: Eqs. (3c) and (3d) state that ages are increased by 1 if they are not burnt, Eq. (3e) decreases ages to 0 when they are burnt, and Eqs. (3e) and (3f) maintain the vegetation age for burning within some minimum and maximum tolerable fire interval. Eqs. (3g) and (3h) activate the variables representing high fuel load cells and high-risk connections respectively.

The habitat quality of a cell FRC is a function of its age and is determined using a piecewise linear function as represented by Eqs. (3i) to (3m). The points of the piecewise linear function for species m are denoted by (r_{nm}, v_{nm}). For each scenario, unit and species, the value A_{it}^s will be expressed by Eq. (3k) and Eq. (3l) as a convex combination of the breakpoints of the piecewise linear function, being G_{itnm}^s the coefficients of the combination. As for only one n (pieces of the piecewise linear function) the variable Z is one (Eq. (3i), the non-zero values of G_n are consecutive (Eq. (3j)), ensuring a correct representation. A simplified example of this modelling scheme is shown in Fig. 3.

Eq. (3n) enforces that uncertainty is revealed within the burn season. Finally, Eqs. (3o) to (3u) set the domains of the variables.

Note that this is a compact model in which the non-anticipativity constraints are implicitly expressed using Ψ_t and Φ_{st}^{-1}. Ψ_t limiting the decisions set to those whose uncertainty has already been revealed in previous stages, and Φ_{st}^{-1} in such a way that each child of a given node always refers to the same decision in the previous stage.

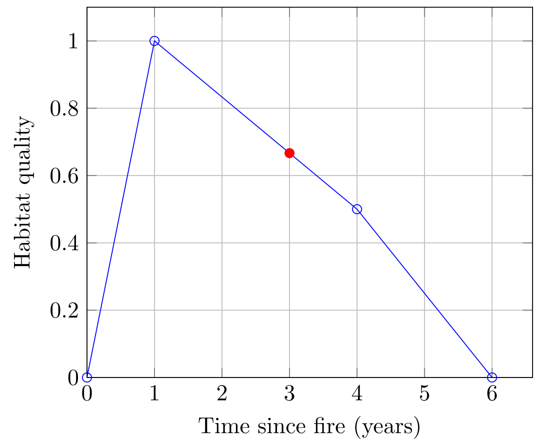
3.4. Formulating the MSP model with risk-aversion

The resulting problem is a MSP problem. The proposed solution is the risk-averse solution developed in León et al. (2020), whose concept has been already introduced in the Problem Description section and is formalized below.

Let K be the set of criteria to be minimized, with importance w_k; S the set of uncertainty scenarios, with probabilities π_s, and $\tilde{f}_k^s(x)$ the normalized objective functions. For each criterion k and a given parameter $\beta \in (0,1]$, the function representing the CVaR $g_k^\beta(x)$ is constructed, measuring the worst values of $\tilde{f}_k^s(x)$ with probability adding up to β. Then, given a parameter $r\in(0,1]$ the function $h_r^\beta(x)$ is defined, aggregating the worst values of $g_k^\beta(x)$, with importance adding up to r. With this definition function $h_r^\beta(x)$ measures the worst possible outcomes of x, in terms of both scenarios and criteria, limited by probability β and importance r, respectively. The feasible x minimizing the value of $h_r^\beta(x)$ will be the solution for this problem. How to obtain it is described below concisely, more details can be seen in León et al. (2020).

For a given k and $x\in X$ the value $g_k^\beta(x)$ is the maximum value that a combination of $\tilde{f}_k^s(x)$ can achieve limiting each scenario by its probability. This can be obtained solving model (4), or equivalently, model (5):

$$\begin{aligned} \max_{\tilde{u}_s} \quad & \frac{1}{\beta}\sum_{s=1}^{S} \tilde{u}_s \cdot \tilde{f}_k^s(x) \\ \text{s.t.} \quad & \sum_{s=1}^{S} \tilde{u}_s = \beta \\ & 0 \le \tilde{u}_s \le \pi_s \quad \forall s \end{aligned} \tag{4}$$



n	1	2	3	4
r_n	0	1	4	6
v_n	0	1	0.5	6
Z_n	0	1	0	-
G_n	0	$\frac{1}{3}$	$\frac{2}{3}$	0

Fig. 3. Example of FRC modelling with Eqs. (3i) to (3m), for fixed (and omitted) values of ι, s, m and i. When the age of the unit is set at $A = 3$, it can be seen that $r_2G_2 + r_3G_3 = 1\frac{1}{3} + 4\frac{2}{3} = 3 = A$, and such unit receives the habitat quality $FRC = v_2G_2 + v_3G_3 = 1\frac{1}{3} + 0.5\frac{2}{3} = \frac{2}{3}$.

$$
\begin{aligned}
g_k^{\beta}(x) = \max_{u_s} \quad & \sum_{s=1}^{S} u_s \cdot \tilde{f}_k^s(x) \\
\text{s.t.} \quad & \sum_{s=1}^{S} u_s = 1 \\
& 0 \le u_s \le \frac{\pi_s}{\beta} \quad \forall s
\end{aligned} \tag{5}
$$

Model (4) will give to each scenario s a value $\tilde{u}_s$, not exceeding its probability π_s and assigning a total probability of β. As the value of x is fixed, the values of $\tilde{u}_s$ will be filled starting with those of the scenarios with the largest (worst) values of $\tilde{f}_k^s(x)$, hence effectively computing the CVaR, as the weighted worst scenarios with probability adding up to β.

Model (4) is then substituted with model (5), by letting $u_s = \frac{\tilde{u}_s}{\beta}$. This way, u_s represents the proportion of the β tail covered by scenario s. Next, replacing model (5) by its dual formulation and letting variable x be feasible, model (6) efficiently finds a feasible x minimizing $g_k^{\beta}(x)$ for a given criterion k:

$$
\begin{aligned}
\min_{x \in X} g_k^{\beta}(x) = \min_{z, y_s, x} \quad & z + \sum_{s=1}^{S} \frac{\pi_s}{\beta} y_s \\
\text{s.t.} \quad & z + y_s \ge \tilde{f}_k^s(x) \quad s = 1, \dots, S \\
& z \text{ free}, y_s \ge 0 \quad s = 1, \dots, S \\
& x \in X
\end{aligned} \tag{6}
$$

The above model is able to find, for a single criterion k, a feasible x with lowest $g_k^{\beta}(x)$. In addition to that, if the value of x is fixed, then the value of $g_k^{\beta}(x)$ is computed. A similar reasoning is followed to aggregate the worst values of $g_k^{\beta}(x)$ for a given $x \in X$:

$$
\begin{aligned}
\max_{t_k} \quad & \sum_{k=1}^{K} t_k \cdot g_k^{\beta}(x) \\
\text{s.t.} \quad & \sum_{k=1}^{K} t_k = 1 \\
& 0 \le t_k \le \frac{w_k}{r} \quad \forall k
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\min_{z, v_k} \quad & z + \sum_{k=1}^{K} \frac{w_k}{r} v_k \\
\text{s.t.} \quad & z + v_k \ge g_k^{\beta}(x) \quad k = 1, \dots, K \\
& z \text{ free}, v_k \ge 0 \quad k = 1, \dots, K
\end{aligned} \tag{8}
$$

Model (7) searches for the worst possible combination of criteria, with importances limited by the importances of each criterion. Model (8) is obtained by taking the dual formulation. Replacing the value of $g_k^{\beta}(x)$ by its computation from model (6) leads to model (9).

$$
\begin{aligned}
h_r^{\beta}(x) = \min_{z, v_k} \quad & z + \sum_{k} \frac{w_k}{r} v_k \\
\text{s.t.} \quad & z + v_k \ge \begin{pmatrix} \min_{z_k, y_{ks}} & z_k + \sum_{s=1}^{S} \frac{\pi_s}{\beta} y_{ks} & \\ \text{s.t.} & z_k + y_{ks} \ge \tilde{f}_k^s(x) & \forall s \\ & z_k \text{ free}, y_{ks} \ge 0 & \end{pmatrix} \forall k \\
& z \text{ free}, v_k \ge 0 \quad \forall k
\end{aligned} \tag{9}
$$

Finally, model (10) is proposed to find a feasible x minimizing $h_r^{\beta}(x)$ (a proof of this equivalence can be seen in León et al. (2020)):

$$
\begin{aligned}
\min_{x \in X} h_r^{\beta}(x) = \min_{z, v_k, z_k, y_{ks}, x} \quad & z + \sum_{k} \frac{w_k}{r} v_k \\
\text{s.t.} \quad & z + v_k \ge z_k + \sum_{s=1}^{S} \frac{\pi_s}{\beta} y_{ks} \quad \forall k \\
& z_k + y_{ks} \ge \tilde{f}_k^s(x) \quad \forall k, s \\
& y_{ks} \ge 0 \quad \forall k, s \\
& z_k \text{ free}, v_k \ge 0 \quad \forall k \\
& z \text{ free} \\
& x \in X
\end{aligned} \tag{10}
$$

With the functions $\tilde{f}_k^s$ determined by Eqs. (2a) to (2d) and X given by Eqs. (3a) to (3u), this mixed-integer linear programming model is able to compute a risk-averse solution for the prescribed burning problem proposed.

4. Case study

A prescribed burning problem with real data will be solved with the previously laid out approach. Data is provided in the context of the GEO-SAFE project by INFOCA, the wildfire prevention and suppression plan in Andalusia, in the south of Spain. The case study consists of a landscape of 1820 km^2 divided into 193 units. This landscape representation has been carried out by INFOCA based on basins, vegetation type and fuel load, and land uses and owners. Fig. 4 shows the area of study, as well as the limitations on where prescribed burning can be conducted, basically related to private properties and protected fauna. These limitations result in 113 units where prescribed burning may be conducted, representing approximately half of the total area. Table 1 shows median values of some characteristics of the burn units, grouped by their restrictions for burning.

Some characteristics of this case study are described below.

Fig. 4. 193 burn units of the case study in Andalucía. Blue reflects public property where prescribed burns can be performed, green indicates private property and yellow areas where prescribed burning is not allowed due to conservation constraints.

Table 1
Median characteristics of the landscape, grouped by prescribed burning limitations.

Limitation	Area (km^2)	Current age	Minimum TFI	Hazard age	Maximum TFI
No limitation	5.79	2.0	5.0	10.0	15.0
Private property	7.98	2.0	–	5.0	–
Endangered species	14.01	6.5	–	5.0	–

- As mentioned before, some land is not eligible for prescribed burning. The parameter $public_i$ is defined, which equals 1 if prescribed burning can be realized in unit i, 0 otherwise. Treatment variables and constraints will be defined only for those units with $public_i = 1$. The rest of the units will be taken into account in the different objective functions, even though it is not possible to intervene there.
- Different types of vegetation are present on the landscape, each unit having a single type, determining the values of $\underline{TFI}_i, \overline{TFI}_i$ and h_i for each unit i. Figs. A.8 to A.11 show the initial conditions for this case study. Fig. A.8 shows at which age the vegetation of each unit is deemed dangerous, that is, the high fuel load threshold. Figs. A.9 and A.10 respectively show for each burn unit its minimum and maximum tolerable fire interval (the earliest and the latest a unit can be burnt after their last prescribed burn). A thorough discussion on tolerable fire interval for different species is found in Cheal (2010). Finally Fig. A.11 displays the initial age of each of the units.
- For this case study, yearly time periods (one burning season per year) and a 5 year horizon have been chosen.
- Yearly prescribed burning budget is set around 20 km^2. However, fire services in the area do not perform the burns in the entire units, they are instead limited to the boundary of the unit. This is done with the goal of decreasing fire connectivity at a lower expense, and as a means for creating fire attack opportunity areas. This is included in the model considering that budget limitation affects only a percentage of the unit area. After observing the areas of the units and their maximum TFIs, such percentage is set to 15%.
- The scenarios were created around the average 20 km^2 budget value: some more optimistic, some more pessimistic. In the beginning more uncertainty is considered, decreasing in the last years where the scenario tree is simpler as the decisions planned on those nodes are never actually implemented. The scenario tree is outlined in Fig. 6, in which 12 scenarios have been considered.
- The yearly budgets scenarios for prescribed burning vary from 15 to 25 km^2, all of them being equally probable.
- Two undetermined species are included in the model, whose habitat qualities are determined by the curves shown in Fig. 5.

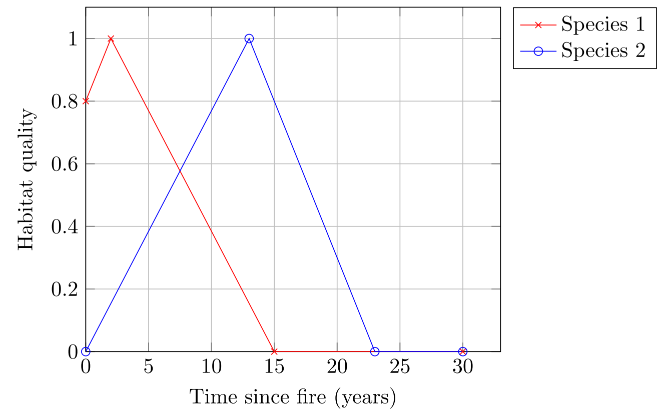


Fig. 5. Habitat quality for two species, one preferring young vegetation and another preferring old vegetation.

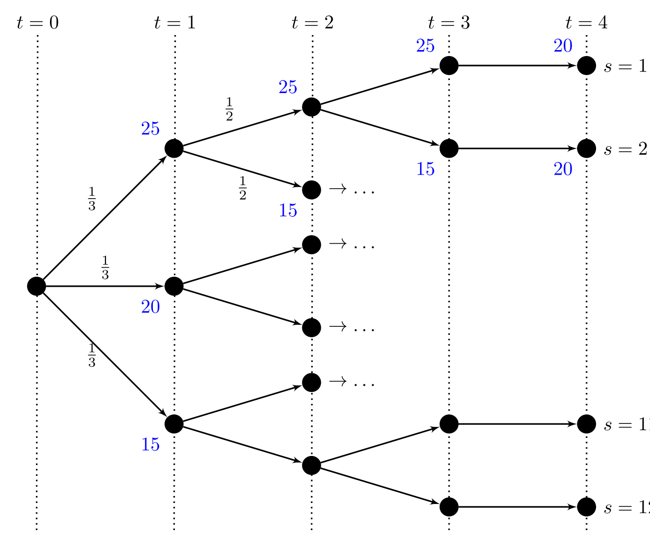


Fig. 6. Scenario tree in case study. The numbers around the vertices represent the budget (maximum burnt area) in each node, in km^2.

5. Experiments and results

In order to illustrate the model, the properties of the solutions and their sensitivity to different parameters, the model was solved with different parameters of β and r, and the solutions compared with the risk-neutral solution provided by the weighted average. The following five experiments were conducted:

1. Risk-neutral, minimizing the weighted average $f_{\text{AVG}}(x) = \sum_{k\in K}\sum_{s\in S} w_k \pi_s \tilde{f}_k^s(x)$
2. Risk-averse, minimizing $h_r^\beta(x)$, with $r = \beta = 0.5$
3. Risk-averse, minimizing $h_r^\beta(x)$, with $r = 0.5, \beta = 0.25$
4. Risk-averse, minimizing $h_r^\beta(x)$, with $r = 0.25, \beta = 0.5$
5. Risk-averse, minimizing $h_r^\beta(x)$, with $r = \beta = 0.25$

Table 2 shows the results for the five experiments conducted, in which each row corresponds to one of the experiments. For each of the five solutions obtained, their performance in the other metrics has been measured, as well as the solution time, the final integrality gap, and the time until a 1% integrality gap is achieved. The experiments were conducted using an HP computer with 8 threads and 16Gb of RAM. The optimizer employed was Gurobi 8.1.1 with the JuMP package of

(a) Scenario after applying treatments with smallest budget (15km^2). Units burnt are displayed with a dot on top

(b) Scenario with median budget (20km^2). Additional units burnt in this scenario are displayed with a dot on top

(c) Scenario with largest budget (25km^2). Additional units burnt in this scenario respect to the median budget are displayed with a dot on top

Fig. 7. Three scenarios at time $t = 1$, for the experiment with $r = \beta = 0.25$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the Julia 1.2.0 programming language. The models solved have 89,249 constraints and 78,288 variables, with 40,789 binary variables and the remaining 37,499 continuous. Note that only the variables related to the active scenarios in each stage (corresponding to the uncertainty revealed up to that moment) are included in the models to ensure the non-anticipativity of the solution, without including any specific constraints.

None of the executions was able to reduce the gap to 0 within the time limit (1 h). However, all of them finished with a gap of less than 0.5% and except for one case reached a 1% gap in less than 6 min. It can be observed that lower values of r and β lead to slower executions. As β and r increase, the risk-aversion is reduced and the problem is closer to the weighted average, and the time to solve the model is also reduced. A reasonable explanation for this is that the smaller r and β are, the fewer scenarios and criteria are considered for obtaining the value of $h_r^\beta(x)$, leading to situations with multiple optima. This slows down the integer programming solving algorithm. However, solution times in this problem are not as crucial as in fire response problems, as it is for tactical purposes.

Comparing the solutions obtained, $h_r^\beta(x)$ provides good outputs in seeking risk-averse solutions. Take into account when reading the table that all functions are bounded by 1. Table 2 clearly shows that

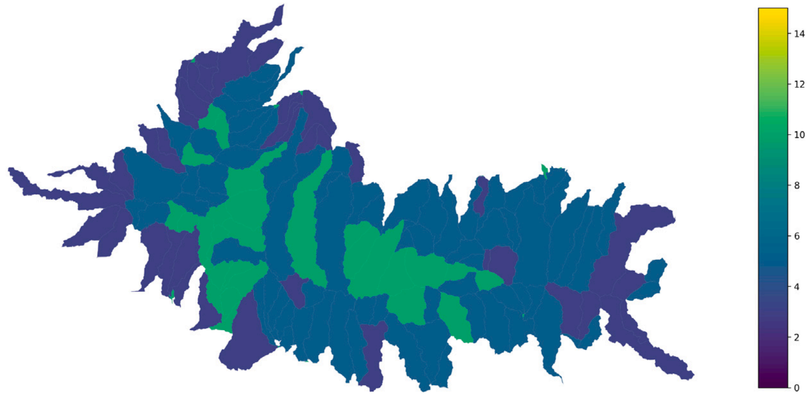


Fig. A.8. High fuel load threshold by unit in case study.

Fig. A.9. Minimum tolerable fire interval by unit in case study.

Table 2
Experiment results. Each row corresponds to the solution for each of the objective functions considered, and columns show the objective values of each of the optimal solutions in the other objective functions. Columns at the end show the integrality gap, and the time taken in seconds to find a solution with a 1% gap.

Min.	Values of:					Gap (%)	Time to 1% (s)
	f_{AVG}	$h_{0.5}^{0.5}$	$h_{0.5}^{0.25}$	$h_{0.25}^{0.5}$	$h_{0.25}^{0.25}$		
$f_{\text{AVG}}(x)$	0.203	0.418	0.456	0.606	0.651	0.21	191
$h_{0.5}^{0.5}(x)$	0.269	0.331	0.338	0.444	0.449	0.04	205
$h_{0.5}^{0.25}(x)$	0.266	0.332	0.333	0.451	0.452	0.06	305
$h_{0.25}^{0.5}(x)$	0.292	0.341	0.344	0.426	0.429	0.45	341
$h_{0.25}^{0.25}(x)$	0.296	0.343	0.344	0.426	0.427	0.45	1638

function $f_{\text{AVG}}(x)$ should not be used to reduce the stochasticity and the multiple criteria in risky contexts. Regarding the values obtained for this solution in the worst scenarios and criteria, for $r = \beta = 0.25$, a value of 0.651 is obtained, when it could be limited to 0.427. On the other hand, if the solution of $h_{0.5}^{0.5}(x)$ is taken, its evaluation under $r = \beta = 0.25$ yields a value of 0.449. This value is much closer to the optimal of 0.427, even if the function $h_{0.5}^{0.5}(x)$ is not aiming mainly to reduce the case of $r = \beta = 0.25$.

All solutions obtained with risk-aversion produce very similar values, both in the average and tails, with a maximum difference of 0.022 for tails (in the extreme tail $r = \beta = 0.25$) and a maximum of 0.027 for the simple average. Also, Table 2 shows that values for the simple average when optimizing the worst cases do not get significantly worse, being the maximum difference 0.093 when focused on the smaller tail, and 0.066 when optimizing $h_{0.5}^{0.5}(x)$.

Consequently, it can be seen that the solutions found using the h_r^{β} functions are appropriate in risk-averse contexts without significant loss of quality in the global set of scenarios and criteria, and the specific values chosen for r and β do not need to be too small to control risk in the worst cases. Therefore, the solution is not very sensitive to the specific values given to the parameters under a certain threshold so in order to reduce run times it is not necessary to choose very exigent values.

Finally, to illustrate behaviour of the solutions, Fig. 7 shows the three possible landscapes after time $t = t_1$ with $r = \beta = 0.25$, where red dots display units burnt in each scenario but not in the previous scenario with a lower budget. The first illustration can be compared with Fig. A.11 seeing how a significant reduction of ages is obtained for the units allowed to be treated, focusing the intervention on the older units (note that no yellow or light green areas remain). Also, it can be seen that when the budget increases new areas in green are chosen.

6. Conclusions and future work

Prescribed burning is an important tool in fuel management for reducing the hazard of large wildfires. Interventions on landscapes must be developed taking into account safety and ecological criteria, as well as future planning and forest evolution. Decisions are also subject to uncertainty on the actual conditions when performing the burns, and must be robust under this uncertainty for all criteria considered. A multiobjective stochastic programming model including all these elements from a risk-aversion perspective has been developed using the methodology introduced in León et al. (2020). The model aims to minimize an ordered weighted average (OWA) considering only the criteria with the weighted worst achievements adding up to a limited

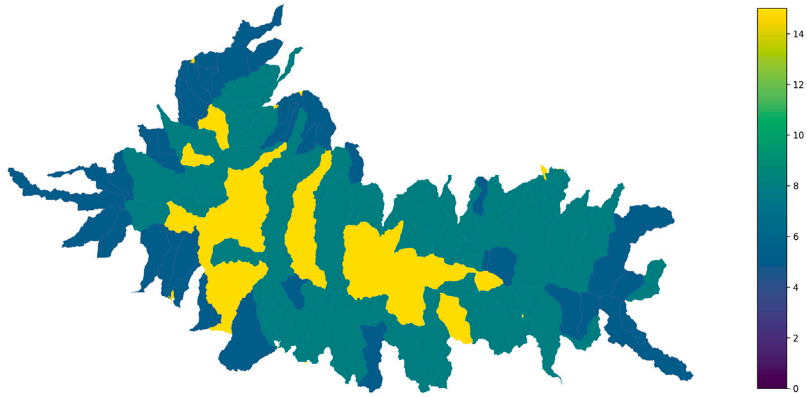


Fig. A.10. Maximum tolerable fire interval by unit in case study.

Fig. A.11. Initial age by unit in case study.

value. The achievement of each criterion is measured as the expected value in the worst scenarios under a fixed probability (the conditional value-at-risk for a prefixed percentile). The model has been applied to a case study considering four different objectives, two related to safety and two to habitat quality for animal species. The case study, located in Andalusia (Spain), includes almost 200 burn units and an extent of 1820 km^2. A limitation of the case study is that the prescribed burning problem developed from the one in León et al. (2019) might not accurately reflect how prescribed burning is performed in Andalusia. The model considers burning whole units, while fire services in the area do not perform the burns in the entire units as they are instead limited to the boundary of the unit. A proxy has been done considering that burning the perimeter accounts for burning around 15% of the total area. Future work includes considering the boundaries explicitly and not necessarily for all the perimeters. Partitioning the units into smaller units might also resolve this issue.

Results show that using this risk-averse solution instead of an average-average greatly improves the expected outcome of unfavourable conditions, without significantly compromising average outcomes. Moreover, they also show that solutions are not very sensitive to the threshold given to the probability of the tails or the importance limit, if they are lower than some general values such as 0.5 for both parameters. So, the specific values chosen are not crucial and can be chosen with some freedom. Future work will consider other sensitivity analyses for some parameters included in the model, like those representing the FRC of species or thresholds, which are more related with the representation of the system itself than with the risk-averse model.

Computational times for the different experiments are provided, showing higher but manageable times for the risk-aversion approach, increasing as the parameters decrease. However, for real problems, the dimensions of the model may increase and therefore its execution time, which could lead to having to implement strategies to reduce it, such as including integrality gap or advanced decomposition methods.

CRediT authorship contribution statement

Javier León: Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Begoña Vitoriano:** Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing. **John Hearne:** Project administration, Resources, Supervision, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Supplementary figures

See Figs. A.8–A.11.

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