Quantum phase transitions and spontaneous symmetry-breaking in Dicke Model

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Abstract. A method to find the Excited-States Quantum Phase Transitions (ESQPT's) from parity-symmetry in the Dicke model is studied and presented. This method allows us to stablish a critical energy where ESQPT's take places, and divides the whole energy spectrum in two regions with different properties.

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The Dicke model describes the interaction between an ensemble of two-level atoms and a single electromagnetic field mode, as a function of the radiation-matter coupling [1]. Its main property is a second order Quantum Phase Transition (QPT) which produces a macroscopic population of the upper atomic level. The Hamiltonian can be written as follows:

$$H = \omega_0 J_z + \omega a^\dagger a + \frac{2\lambda}{\sqrt{N}}\left(a^\dagger + a\right) J_x, \tag{1}$$

where the N atoms are represented by J, the angular momentum operator with a pseudo-spin lenght $J = N/2$. Photons are represented by the usual annihilation and creation operators, and ω and ω_0 represent the frecuency of the cavity mode and the transition frecuency respectively. The system is governed by λ, the intensity of radiation-matter coupling, and its critical value for QPT is $\lambda_c = \sqrt{\omega\omega_0}/2$. We consider that $\hbar = 1$ and $\omega = \omega_0 = 1$. The parity of the system is $\Pi = e^{i\pi\left(J+J_z+a^\dagger a\right)}$, which is a conserved quantity since $[H,\Pi] = 0$. The symmetry-parity is given by the invariance of H under $J_x \to -J_x$ and $a \to -a$ [2]. This symmetry is spontaneously broken when the critical point of QPT is crossed [3].

Since the parity Π is a conserved quantity, we can label the eigenstates of H with a certain value of the parity, positive or negative. Hence, we obtain two sets of eigenstates, for a i-th eigenstate we have another with opposite parity. Fixing he number of atoms N, we analyze the relative difference of the energy between two i-th eigenstates with opposite parity. We have shown that for $\lambda > \lambda_c$ every couple of eigenstates with opposite parity are degenerated below a certain critical energy [4]. This critical energy $E_c^N(\lambda)$ separates two regions in the spectrum and is a function of λ and the number of atoms N. Here we show how to find the E_c in the thermodinamical limit by fitting the finite precursor $E_c^N(\lambda)$ to a linear function $E_c^N(\lambda)/J = A_N + B_N\lambda$, and studying the finite-size scaling of coefficients A_N and B_N.

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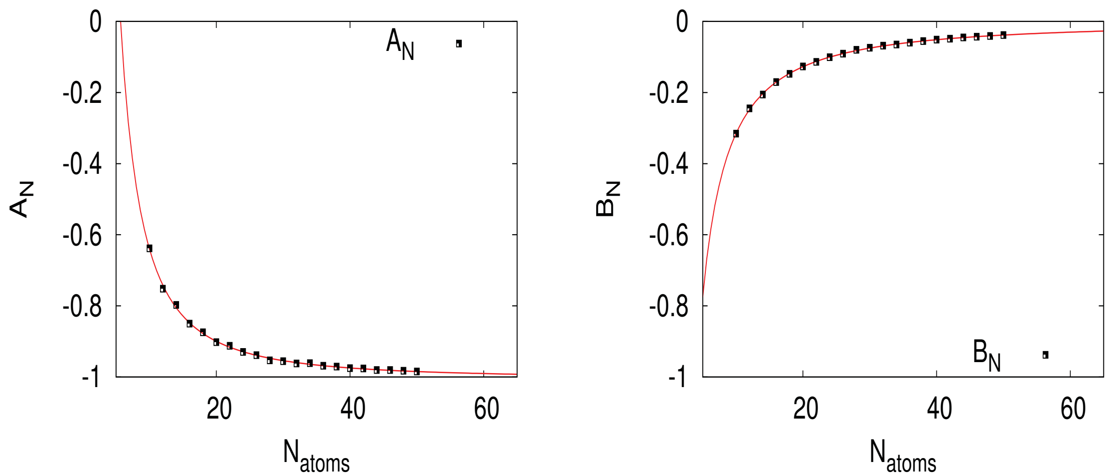

FIGURE 1. Finite-size scaling of the linear coefficients of the critical energy $E_c^N(\lambda)/J = A_N + B_N\lambda$. Left figure corresponds to the coefficient A_N while B_N is represented at the right one. Squares represent the numerical values of the coefficients and solid lines correspond to the fit to a power-law $N^{-\alpha}$.

Numerical results are represented in the figure 1, where it is clear that $A_N \to -1$ and $B_N \to 0$. In this way, we obtain the critical energy in the thermodinamical limit $E_c/J = -1$, which is independent of λ. This critical energy is valid for all $\lambda > \lambda_c$ and represent the border between two phases with different properties, where the ESQPT's take place, supporting previous results [5]. Properties of this two phases can be explored analyzing the dynamical behavior of certain observables [4].

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REFERENCES

1. R. H. Dicke, Phys. Rev. Lett. **93**, 99 (1954).
2. C. Emary, and T. Brandes, Phys. Rev. Lett. **90**, 044101 (2003); Phys. Rev. E **67**, 066203 (2007).
3. K. Baumann, R. Mottl, F. Brennecke, and T. Esslinger, Phys. Rev. Lett. **107**, 140402 (2011).
4. R. Puebla, A. Relaño and J. Retamosa, arXiv:1209.5320 (2012).
5. P. Pérez-Fernández, P. Cejnar, J. M. Arias, J. Dukelsky, J. E. García-Ramos, and A. Relaño, Phys. Rev. A **83**, 033802 (2011); P. Pérez-Fernández, A. Relaño, P. Cejnar, J. M. Arias, J. Dukelsky, and J. E. García-Ramos, Phys. Rev. E **83**, 046208 (2011).