Evaluating the performance of the skewed distributions to forecast Value at Risk in the Global Financial Crisis*

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Executive summary:

This paper evaluates the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and forecasting Value at Risk (VaR). First, we used some goodness of fit tests to analyze which distribution best fits the data. The comparisons in terms of VaR have been carried out examining the accuracy of the VaR estimate and minimizing the loss function from the point of view of the regulator and the firm. The results show that the skewed distributions outperform the normal and Student-t (ST) distribution in fitting portfolio returns. Following a two-stage selection process, whereby we initially ensure that the distributions provide accurate VaR estimates and then, focusing on the firm´s loss function, we can conclude that skewed distributions outperform the normal and ST distribution in forecasting VaR. From the point of view of the regulator, the superiority of the skewed distributions related to ST is not so evident. As the firms are free to choose the VaR model they use to forecast VaR, in practice, skewed distributions will be more frequently used.

Keywords: Value at Risk, Parametric model, Skewness t-Generalised Distribution, GARCH Model, Risk Management, Loss function.

1. Introduction

A primary tool for financial risk assessment is Value at Risk (VaR). It is defined as the maximum loss expected of a portfolio of assets over a certain holding period at a given confidence level (probability). Since the Basel Committee on Bank Supervision at the Bank for International Settlements requires the financial institution to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions.

Despite VaR´s conceptual simplicity, its calculation could be rather complex. Many approaches have been developed to forecast VaR: non parametric approaches, e.g. Historical Simulation; semi-parametrics approaches, e.g. Extreme Value Theory and the Dynamic quantile regression CaViar model (Engle and Manganelli (2004)); and parametric approaches e.g. Riskmetrics (J.P. Morgan (1996)).

The parametric approach is one of the most used by financial institutions. This approach usually assumes that the asset returns follow a normal distribution. This assumption simplifies the computation of VaR considerably. However, it is inconsistent with the empirical evidence of asset returns, which finds that the distribution of asset returns is skewed, fat-tailed, and peaked around the mean (see Bollerslev (1987)). This implies that extreme events are much more likely to occur in practice than would be predicted by the symmetric thinner-tailed normal distribution. Furthermore, the normality assumption can produce VaR estimates that are inappropriate measures of the true risk faced by financial institutions.

Since the ST distribution has fatter tails than the normal one, this distribution has been commonly used in finance and risk management, particularly to model conditional asset returns (Bollerslev (1987)). The empirical evidence of this distribution performance in estimating VaR is ambiguous. Some papers show that the ST distribution performs better than the normal distribution (see Abad and Benito (2013), Orhan and Köksal (2012) and Polanski and Stoja (2010)) while other papers report that the ST distribution overestimates the proportion of exceptions (see Angelidis et al. (2007) and Guermat and Harris (2002)).

The ST distribution can often account well for the excess kurtosis found in common asset returns, but this distribution does not capture the skewness of the returns. Taking this into account, one direction for research in risk management involves searching for other distribution functions that capture this characteristic. The skewness Student-t distribution (SSD) of Hansen (1994), the exponential generalized beta of the second kind (EGB2) of McDonald and Xu (1995), the generalised error distribution (GED) of Nelson (1991), the skewness generalised-t distribution (SGT) of Theodossiou (1998), the skewness error generalised distribution (SGED) of Theodossiou (2001) and the inverse hyperbolic sign (IHS) of Johnson (1949) are the most used in VaR literature. Some applications of skewness distributions to forecast the VaR can be found in Chen et al. (2012), Polanski and Stoja (2010), Bali and Theodossiou (2008), Bali et al. (2008), Haas et al.

(2004), Zhang and Cheng (2005), Haas (2009), Ausín and Galeano (2007), Xu and Wirjanto (2010) and Kuester et al. (2006). Chen et al. (2012) compared the ability to forecast the VaR of a normal, ST, SSD and GED. In this comparison the SSD and GED distributions provide the best results. Polanski and Stoja (2010) compared the normal, ST, SGT and EGB2 distributions and found that just the latter two distributions provide accurate VaR estimates. Bali and Theodossiou (2008) compared a normal distribution with the SGT distribution and showed that the SGT provided a more accurate VaR estimate.

In this paper we carry out a comprehensive comparison of the skewed distributions aforementioned: SSD, SGT, SGED and IHS. Besides, in this comparison we include both the normal and the ST distribution. The comparative is performed following two directions. First, we compare the distributions in statistical terms to determine which is the best for fitting financial returns. Then, we compare the distributions in terms of VaR, in order to select which is best for forecasting VaR.

The main differences with the previous literature are as follows: (1) we consider a larger number of skewed distributions; (2) the comparison in statistical terms is made using a large battery of tests: Likelihood ratio, Chi-square (Chi2) of Pearson (1900) and Kolmogorov-Smirnov (KS) test (Kolmogorov (1933), Smirnov (1939) and Massey (1951)); the papers aforementioned only used the likelihood ratio test; 3) to carry out the comparison in terms of VaR we evaluate the results on the basis of two criteria: (i) the accuracy of VaR and (ii) the minimization of two loss functions which reflect the concerns of the financial regulator and the firm (Sarma et al. (2003)).

In the next section, we present the methodology used to estimate the VaR and summarize the statistical tests and the loss functions that we have used to evaluate the VaR estimates. In section 3, we present the data. The results of the comparison in statistical terms and in terms of VaR are presented in sections 4 and 5 respectively. The last section includes the main conclusions.

2. Methodology

According to Jorion (2001), VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. The VaR is thus a conditional quantile of the asset return distribution. Let $r_1, r_2, r_3, ..., r_n$ be identically distributed independent random variables representing the financial returns. Use $F(r)$ to denote the cumulative distribution function, $F(r) = \Pr(r_t < r \,|\Omega_{t-1})$, conditionally on the information set Ω_{t-1} that is available at time *t*-1. Assume that $\{r_t\}$ follows the stochastic process $r_t = \mu + \varepsilon_t$ where $\varepsilon_t = z_t\sigma_t \quad z_t \sim iid(0,1)$, μ is the conditional mean, σ_t the conditional standard deviation of returns. The VaR with a given probability $\alpha \in (0,1)$, denoted by $\mathrm{VaR}(\alpha)$, is defined as the α quantile of the probability distribution of financial returns: $F(VaR(\alpha)) = Pr(r_t < VaR(\alpha)) = \alpha$

Under the framework of the parametric techniques (see Jorion (2001)), the conditional VaR estimate can be calculated as $VaR_t = \mu_t + \hat{\sigma}_t k_\alpha$, where μ_t represents the conditional mean, which we assume is zero, $\hat{\sigma}_t$ sigma is the conditional standard deviation and k_α denotes the corresponding quantile of the distribution of the standardized returns at a given confidence level 1-α.[1]

Having obtained significant evidence from the Engle and Ng (1993) test on the fact that good and bad news have a different impact on conditional volatilities of asset returns, we use the Exponential GARCH model of Nelson (1991) to estimate σ_t needed for conditional VaR analysis[2]. Finally, once the variance has been calculated we estimate the distributions of the standardized returns under each of the considered distribution functions: normal, ST, SGT, SGED, SSD and IHS. Table 1 shows the density function of these skewed distributions.

In the first stage, before the calculation of the VaR, we compare the distributions in statistical terms. To do this, we use a likelihood test (to compare the fit of two models) and two goodness of fit tests KS and Chi2 (to determine whether a sample can be considered as a draw sample from a given specified distribution). The KS test is based on the maximum difference between an empirical and a hypothetical cumulative distribution function. The Chi2 test is based on the probability distribution function and performs by grouping the data into bins, calculating the observed and expected counts for those bins.

In the second stage, we calculate the VaR and test *the accuracy* of the VaR estimate under these distributions. We use four standard tests: unconditional and conditional coverage tests, the Back-Testing criterion and the dynamic quantile test. We have an exception when $r_{t+1} < VaR(\alpha)$ and then the exception indicator variable (I_{t+1}) is equal one (zero in other cases).

Kupiec (1995) shows that the *unconditional coverage test* has as a null hypothesis $\hat{\alpha} = \alpha$, with a likelihood ratio statistic ($LR_{UC} = 2\left[log\left(\hat{\alpha}^x(1-\hat{\alpha})^{N-x}\right) - log\left(\alpha^x(1-\alpha)^{N-x}\right)\right]$), which follows an asymptotic $\chi^2(1)$ distribution. A similar test for the significance of the deviation of $\hat{\alpha}$ from α is the *back-testing criterion* statistic $Z = (N\hat{\alpha} - N\alpha)/\sqrt{N\alpha(1-\alpha)}$ which follows an asymptotic N (0,1) distribution. The *conditional coverage test* (Christoffersen (1998)) jointly examines if the percentage of exceptions is statistically equal to the expected one and the serial independence of I_{t+1}. The likelihood ratio statistic of the conditional coverage test is $LR_{cc}=LR_{uc}+LR_{ind}$, which is asymptotically distributed $\chi^2(2)$, and the LR_{ind} statistic is the likelihood ratio statistic for the hypothesis of serial independence against first-order Markov dependence. Finally, the dynamic quantile test proposed by Engle and Manganelli (2004) examines if the exception indicator is uncorrelated with any variable that belongs to the information set Ω_{t-1} available when the VaR was calculated. This is a Wald test of the hypothesis that all slopes are zero in a regression of the exception indicator variable on a constant, 5 lags and the VaR.

Additionally, we evaluate the magnitude of the losses experienced. The model that minimizes the total loss is preferred to the other models. For this purpose, we have considered two

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[1] In case of the skewed distributions the k_α value is a function of the skewness and kurtosis parameters.
[2] The EGARCH models have been estimated below a ST distribution.

loss functions: the regulator loss function and the firm's loss function. Lopez (1998, 1999) proposed a loss function, which reflects the utility function of a regulator. In this specification, the magnitude loss function assigns a quadratic specification when the observed portfolio losses exceed the VaR estimate. Thus, we penalize only when an exception occurs according to the following quadratic specification:

$$RLF_t = \begin{cases} (VaR_t - r_t)^2 & if \quad r_t < VaR_t \\ 0 & otherwise \end{cases} \qquad (1)$$

This loss function gives higher scores when failures take place and considers the magnitude of the failure. In addition, the quadratic term ensures that large failures are penalized more than small failures.

But firms use VaR in internal risk management and, in this case, there is a conflict between the goal of safety and the goal of profit maximization. A too high VaR forces the firm to hold too much capital, imposing the opportunity cost of capital upon the firm. Taking this into account, Sarma et al. (2003) define the firm's loss function as follows:

$$FLF_t = \begin{cases} (VaR_t - r_t)^2 & if \quad r_t < VaR_t \\ -\beta .VaR_t & otherwise \end{cases} \qquad (2)$$

β being the opportunity cost of capital.

3. *Data*

The data consist of closing daily returns on nine composite indexes from 1/1/2000 to 11/30/2012 (around 3250 observations). The indexes are: Japanese Nikkei, Hong Kong Hang Seng, Israeli Tel Aviv (100), Argentine Merval, US S&P 500 and Dow Jones, UK FTSE100, the French CAC40 and the Spanish IBEX-35. The data were extracted from the Bloomberg database. The computation of the indexes returns (r_t) is based on the formula, r_t=ln(I_t)-ln(I_{t-1}) where I_t is the value of the stock market index for period t.

Figure 1 shows the daily returns and Table 2 provides basic descriptive statistics of the data. For each index, the unconditional mean of daily return is very close to zero. The unconditional standard deviation is especially high for Merval (2.14). For the rest of stock index returns the standard deviation moves between 1.27 Dow Jones and 1.63 Hang Seng. Going back to Figure 1, we can see that the range fluctuation of the returns is not constant, which means that the variance of these returns changes over time.

In order to gain some intuition, we adopt the volatility measure proposed by Franses and van Dijk (2000), wherein the volatility of returns is defined as:

$$V_t = \left(r_t - E\left(r_t^2 \middle| \Omega_{t-1} \right) \right)^2 \qquad (3)$$

where Ω_{t-1} is the information set at time t-1. Figure 2 presents V_t as "volatilities". The volatility of the series was high during the early 2000s, especially in the Merval index. From 2001 to 2002 the conditional volatility of MERVAL was almost 1 point higher than the whole period, even greater than those showed from 2008 to 2009. This corresponds to the Argentine economic crisis (1999–2002) which was the major downturn in Argentine´s economy[3]. The period from 2003 to early 2007 was very quiet. In August 2007 the financial market tensions started and they were followed by a global financial and economic crisis leading to a significant rise in the volatility of returns. This increase was especially important after August 2008 coinciding with the fall of Lehman Brothers. From 2008 to 2009, the volatility of the S&P500, Nikkei and IBEX35, measured by the standard deviation of returns was 2.42, 2.20, and 2.10 respectively. In the case of S&P500, the standard deviation was almost 1 point higher than the standard deviation of the whole period 2000-2012 (1.57). A similar increase is observed in all indexes. In the last two years of the sample, we observe a period that is more stable than during the financial crisis.

The skewness statistic is negative and significant for all the indexes considered except in the case of the CAC40 and the IBEX35. This means that the distribution of those returns is skewed to the left. When considering the CAC40 and the IBEX35 the skewness statistic is positive, implying that these distributions are skewed to the right but but jonly in the case of IBEX35 this statistic is statistically significant at 1% level.

For all the indexes considered, the excess kurtosis statistic is very large and significant at 1% level implying that the distributions of those returns have much thicker tails than the normal distribution. Similarly, the Jarque-Bera statistic is statistically significant rejecting the assumption of normality. These results are in line with those obtained by Bollerslev (1987), Bali and Theodossiou (2007), and Bali et al (2008), among others. All of them find evidence that the empirical distribution of the financial return is asymmetric and exhibits a significantly excess of kurtosis (fat tails and peakness).

In order to capture the non-normal characteristics observed in our data set, we fit several skewed distributions: SGT, SGED, SSD and IHS. In this comparison we also include the normal and symmetric ST distributions. In Table 3 we present the estimated parameters of these distributions. This Table provides the estimates for the mean (μ) and the standard deviation (σ) of log-returns and its standard errors in brackets. As expected, these estimates are quite similar across distributions and do not differ much from the simple arithmetic means and standard deviations of log-returns presented in Table 2. The unconditional mean is close to zero for all the indexes and the unconditional standard deviation moves around 1.5 (in percentage) except Merval (2.14). As expected from the previous analysis, the Merval index is the most volatile index.

The skewness parameter λ, for all indexes considered is negative and significant at the 1% level, which means that the distributions of these returns are skewed to the left. This result is in opposition to the preliminary evidence that suggested a symmetric distribution for CAC40 and a skewed distribution to the right for IBEX35.

[3]It began in 1999 with a decrease of the real Gross Domestic Product (GDP). The crisis caused the fall of the government, default on the country's foreign debt, widespread unemployment, riots, the rise of alternative currencies and the end of the peso's fixed exchange rate to the US dollar.

On the other hand, the kurtosis parameters η and κ, in the case of SGT, the parameter κ controls mainly the peakness of the distribution around the mode, while the parameter η controls mainly the tails of the distribution (adjusting the tails to the extreme values). The parameter η has the degrees of freedom interpretation as in ST. For all the series and all distributions considered, the kurtosis parameters are highly significant. For the SGT, the value of κ is around 1.5, except for Nikkei and Tel Aviv which are 1.89 and 1.78 respectively. The value of η is around 4.5 for Nikkei, Merval, DJ, FTSE and CAC40. For the rest of the indexes it is a little bit higher. These estimates are quite different from those of the normal distribution ($\kappa = 2$ and $\eta = \infty$), which indicates that this set of returns is characterized by excess kurtosis.

4. Comparison of the distributions in statistical terms

In this section we want to answer the following question: Which distribution is the best one for fitting asset returns?

The above results provide strong support to the hypothesis that stock returns are not normal. As the normal distribution is nested within the SGT, SGED and SSD distributions we can use the log-likelihood ratio for testing the null hypothesis of normality against that of SGT, SGED or SSD. For all the indexes considered, this statistic is quite large and statistically significant at the 1% level, providing evidence against the normality hypothesis (see Table 4). Additional evidence against the normality hypothesis can be found in Figure 3 where we present the histogram and the density functions of several skewed distributions for the Nikkei index. We can see that all of these distributions provide a better fit than the normal ones[4].

To evaluate which is the most adequate, we perform several kinds of tests. First, as the SGT nets all the distributions considered in this paper (except IHS), we use the likelihood ratio test to evaluate which distribution is best for fitting the data[5]. Overall, for all the indexes considered, the likelihood statistics indicate rejection of the SGED, the SSD, and the ST in favour of the SGT (see Table 4). As the IHS is not nested in the SGT distribution, we cannot conclude that the SGT distribution is the best. So, to ensure the robustness of the results several alternative tests have been used: Chi2 and KS tests. Unlike the likelihood ratio test used to compare two distributions, the Chi2 and the KS tests are used to examine if the asset returns' empirical distribution follows a particular theoretical distribution. The theoretical distributions we have considered are: normal, ST, SGT, SSD, SGED and IHS. The Chi2 statistic (see Table 4) suggests that the empirical distributions of the returns considered in this paper can be adequately characterized using two of the distributions we have considered: SGT and IHS. Both distributions seem to fit the data well in 8 of the 9 indexes considered. For the Hang Seng, Tel Aviv and S&P 500 indexes, the SGED distribution cannot be refused either. On the other hand, the ST and the normal distributions do not fit any index. The KS test provides similar results (see Table 4). According to this test, the

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[4] The qualitative results of the remaining indexes are similar. We only represent the results for one index in order to free space.

[5] Specifically, it gives for $\eta = \infty$ the SGED, for $\kappa = 2$ the SSD, for $\lambda=0$ and $\kappa = 2$ the ST and for $\lambda=0$, $n = \infty$ and $k = 2$ the normal distribution (see Hansen, McDonald and Theodossiou (2001) for a comprehensive survey on the skewed fat-tailed distributions).

empirical distribution of all the indexes considered (except Nikkei) follows a SGT distribution. The IHS fits the data well in only five of the indexes (Merval, CAC40, IBEX35, Tel Aviv and Nikkei). According to this test, the SSD distribution fits the data well in four of the considered indexes (Merval, CAC40, IBEX35 and FTSIE) and the SGED distribution fits the data well in four indexes (Nikkei, Merval, IBEX35 and Hang Seng). The ST distribution only fits well in three of the nine indexes while the normal distribution does not do well in any index.

Taking into account the results described in this section, we can conclude that the symmetric distributions (normal and ST) do not fit financial returns well. This is in line with the previous results shown in the above sections. Among the set of skewed distributions considered in this paper, the SGT distribution seems to be the best in fitting the data, followed closely by the IHS distribution.

5 Evaluating the performance in terms of VaR

In this section we compare the normal, the ST and the skewed distributions in terms of VaR. The comparison is carried out evaluating (i) the accuracy of the VaR estimates and (ii) the losses that VaR produces. For each distribution, we use parametric approaches to forecast the VaR out-of-the-sample one-step-ahead at 1% and 0.25% confidence level. The analysis period runs from the first of January 2008 to the end of December 2009. We choose this period because it is characterized by a high volatility all over the world so that it is known in financial literature as the Financial Global Crisis period. In Figure 1, we highlight in black the period analyzed.

5.1 Back Testing

The results of the accuracy test are presented in Tables 5 and 6. In Table 5 we show the results of the accuracy test at 1% confidence level and Table 6 reports the results at 0.25% confidence level. In both tables, we present the percentage of exceptions obtained with each distribution: normal, ST, SSD, SGED, SGT and IHS together with Riskmetric. Below these percentages, we present the five statistics used to test the accuracy of the VaR estimates. When the null hypothesis that "*the VaR estimate is accurate*" has not been rejected by any test, we have shaded the area (the percentage of exceptions).

In the analyzed period the VaR estimates obtained under a normal distribution are very poor. For almost all the indexes considered, the parametric approach under a normal distribution underestimates risk at the 1% and 0.25% confidence levels. This result does not depend on the volatility model we have used to forecast the VaR, EGARCH or MME (Riskmetrics).

At the 1% confidence level, the VaR estimate provided by the skewed distributions and the ST distribution is quite accurate. At this confidence level, the SGT and the HIS perform well in eight of the nine indexes considered, only failing in the IBEX35. The ST, the SSD and the SGED distributions provide accurate VaR estimates in seven of the 9 cases considered. At the 0.25% confidence level, all the skewed distributions provide accurate VaR estimates in eight of the nine indexes considered, except the IHS that fails in two cases. At this confidence level, the ST distribution performs well in five of the nine indexes considered: Nikkei, S&P500, DJ, CAC40 and IBEX35. In the case of Merval, Hang Seng and Tel Aviv, this distributions overestimate risk. Then, at the higher confidence level the evidence in favor of the skewed distributions related to the ST one is more obvious.

5.2 Loss Functions

In this section we evaluate the VaR estimate in terms of the regulator loss function (Table 7) and the firm's loss function (Table 8). The results in Table 7 have been multiplied by 1000 given the small value obtained. The data marked in bold type represents the minimum value for this function in each case.

From the regulator loss function (see Table 7), we find that the parametric approach under a normal distribution joined to Riskmetrics provide the highest losses while the ST distribution provides the lowest losses followed by the IHS and the SGT distributions. Among the skewed distributions, the SSD gives the worst outcome in all cases. According to this result, we can conclude that from the point of view of the regulator the best distribution is the ST, as this distribution is the most conservative.

The problem associated with the regulator loss function is that this function does not take into account the firms' opportunity cost. So that one model that overestimates the risk, as the ST distribution does in three of the cases, may be considered the most appropriate. Taking this into account we calculate the losses from a firm´s point of view.[6]

In terms of the firm's loss function (see Table 8), the normal distribution provides the lowest losses while the ST distribution shows the highest losses. This result is coherent since it is well known that the normal distribution underestimates risk providing the lowest capital opportunity cost. Since the ST distribution tends to overestimate risk, the capital opportunity cost with this distribution is the highest. The magnitudes of losses obtained by all the skewed distribution are very similar. In terms of this loss function, the best skewed distribution is the SSD. This distribution obtains the lowest losses in seven of the nine cases. The SGT distribution, although it is not the best, works out well giving lower losses than the ST does.

On the whole, following this selection process in two stages, where first we ensure that the distributions provide accurate VaR estimate and then focusing in the firm's loss function, we can conclude that the skewed and fat tail distributions outperformed the normal and the ST distribution. From a point of view of the regulator, the superiority of the skewed distributions related to the ST is not so clear.

6. Conclusion

[6] In order to calculate the firm's loss function we need to know the cost of capital. For this purpose, we have used the daily data of the interest rate of the Eurosystem monetary policy operations for the European indexes. For the rest of the indexes, we took the interest rate of the open market operations used by the Federal Reserve in the implementation of its monetary policy.

This paper evaluates the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and in forecasting VaR. The skewed distributions considered are: (i) the skewed Student-*t* distribution of Hansen (1994); (ii) the skewed error generalised distribution of Theodossiou (2001); (iii) the skewed generalised-*t* distribution of Theodossiou (1998) and (iv) the inverse hyperbolic sign of Johnson (1949). The symmetric distributions are the normal and the Student-*t* ones.

For this study we have used daily returns on nine composite indexes: the Japanese Nikkei, Hong Kong Hang Seng, Israeli Tel Aviv (100), Argentine Merval, US S&P 500 and Dow Jones, UK's FTSE100, the French CAC40 and the Spanish IBEX-35. The sample used for the statistical analysis runs from January 2000 to the end of November 2012. The analysis period for forecasting VaR runs from 2008 to 2009, which is known as the Global Financial Crisis period.

From the results presented in the paper, we can conclude that the skewness and fat tail distributions outperform the normal one in fitting financial returns and forecasting VaR. Among all the skewed distributions considered in this paper, the skewed generalised-*t* distribution of Theodossiou (1998) is the best one in fitting data. However, in terms of their ability to forecast the VaR, we do not find significant differences as all of them provide accurate VaR estimates for a high number of indexes and produce similar losses.

Finally, we find evidence in favor of the skewed distributions compared to the ST distribution. In statistical terms, most of them fit the data better than the ST. In terms of value at risk, the accuracy VaR test indicates that the skewed distributions outperform the ST. On the other hand, with regards to the loss function, the result depends on the kind of function we use to measure the losses. From a point of view of the regulator, ST distribution is the best in forecasting VaR as this distribution provides the more conservative VaR estimate. However, from the point of view of the firm, the skewed distributions outperform the ST distribution, since the latter distribution tends to raise the firm´s capital cost. As companies are free to choose the VaR model they use to forecast VaR, it is clear that they will prefer the skewed distributions.

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Table 1. Density functions

	Formulations		Restrictions
SSD of Hansen (1994)	$f(z_t \| v,\eta) = \begin{cases} bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1-\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t < -\left(\frac{a}{b}\right) \\ bc\left[1+\frac{1}{\eta-2}\left(\frac{bz_t+\alpha}{1+\eta}\right)^2\right]^{-(\eta+1)/2} & if \quad z_t \geq -\left(\frac{a}{b}\right) \end{cases}$	$a = 4\lambda c\left(\frac{\eta-2}{\eta-1}\right) \qquad b^2 = 1+3\lambda^2-a^2$ $c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)\Gamma\left(\frac{\eta}{2}\right)}} \qquad z_t = (r_t - \mu_t)/\sigma_t$	$\|\lambda\| < 1$ $\eta > 2$
SGED of Theodossiou (2001)	$f(z_t \| \lambda,k) = \frac{C}{\sigma} exp\left(-\frac{\|z_t+\delta\|^k}{\left(1+sign(z_t+\delta)\lambda\right)^k \theta^k}\right)$	$z_t = (r_t - \mu_t)/\sigma_t$ $C = k/\left(2\theta\Gamma(1/k)\right) \qquad \delta = 2\lambda AS(\lambda)^{-1}$ $\theta = \Gamma(1/k)^{0.5}\Gamma(3/k)^{-0.5}S(\lambda)^{-1}$ $\delta = 2\lambda AS(\lambda)^{-1} \qquad S(\lambda) = \sqrt{1+3\lambda^2-4A^2\lambda^2}$	$\|\lambda\| < 1$ *skewed parameter* *k=kurtosis parameter*
SGT of Theodossiou (1998)	$f(z_t \| \lambda,\eta,k) = C\left(1+\frac{\|z_t+\delta\|^k}{\left((\eta+1)/k\right)\left(1+sign(z_t+\delta)\lambda\right)^k \theta^k}\right)^{-\frac{\eta+1}{k}}$	$C = 0{,}5k\left(\frac{\eta+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\theta^{-1} \qquad \theta = \frac{1}{\sqrt{g-\rho^2}}$ $\rho = 2\lambda B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}} B\left(\frac{\eta-1}{k},\frac{2}{k}\right)$ $g = (1+3\lambda^2)B\left(\frac{\eta}{k},\frac{1}{k}\right)^{-1}\left(\frac{\eta+1}{k}\right)^{\frac{1}{k}} B\left(\frac{\eta-1}{k},\frac{3}{k}\right) \qquad \delta = \rho\theta$	$\|\lambda\| < 1$ *skewed parameter* $\eta > 2$ *tail-thickness parameter* $k > 0$ *peakedness parameter* δ *Pearson's skewness* $z_t = (r_t - \mu_t)/\sigma_t$
IHS of Johnson (1949)	$IHS(z_t \| \lambda,k) = -\frac{k}{\sqrt{2\pi\left(\theta^2+(z_t+\delta)^2\right)}} \times exp\left(-\frac{k^2}{2}\left(ln\left((z_t+\delta)+\sqrt{\theta^2+(z_t+\delta)^2}\right)-(\lambda+ln(\theta))\right)^2\right)$	$\theta = 1/\sigma_w \qquad \delta = \mu_w/\sigma_w$ $\sigma_w = 0.5(e^{2\lambda+k^{-2}}+e^{-2\lambda+k^{-2}}+2)^{0.5}(e^{k^{-2}}-1)$	μ_w *mean* σ_w *standard deviation* $w = sinh(\lambda + x/k)$ *x standard normal variable*

Note: In all these distributions *z* represents the standardized returns.

Table 2. Descriptive Statistics

	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque Bera
Nikkei	-0.022	0.004	13.234	-12.111	1.568	-0.393** (0.044)	9.686** (0.087)	5996 (0.001)
Hang Seng	0.008	0.044	13.407	-13.582	1.632	-0.065 (0.043)	10.386** (0.087)	7253 (0.001)
Tel Aviv	0.024	0.055	9.782	-8.425	1.338	-0.311** (0.044)	6.945** (0.087)	2107 (0.001)
Merval	0.047	0.090	16.117	-12.952	2.140	-0.093* (0.043)	7.944** (0.087)	3243 (0.001)
S&P 500	-0.001	0.050	10.957	-9.47	1.354	-0.158** (0.043)	10.293** (0.086)	7212 (0.001)
Dow Jones	0.010	0.049	10.089	-8.7	1.265	-0.185** (0.043)	9.372** (0.086)	5515 (0.001)
Ftsie100	-0.004	0.025	9.384	-9.266	1.301	-0.135** (0.043)	8.692** (0.086)	4416 (0.001)
CAC40	-0.015	0.019	10.595	-9.472	1.572	0.038 (0.043)	7.494** (0.085)	2782 (0.001)
IBEX35	-0.012	0.060	13.484	-9.5858	1.576	0.1227** (0.043)	7.8219** (0.086)	3177 (0.001)

Note: This table presents the descriptive statistics of the daily percentage returns of Nikkei, Hang Seng, Tel Aviv 100, Merval, S&P 500, Dow Jones, Ftsie 100, CAC-40 and IBEX-35. The sample period is from January 2nd, 2000 to November 30th, 2012. The index return is calculated as $R_t=100(\ln(I_t)-\ln(I_{t-1}))$ where I_t is the index level for period t. Standard errors of the skewness and excess kurtosis are calculated as $\sqrt{6/n}$ and $\sqrt{24/n}$ respectively. The JB statistic is distributed as the Chi-square with two degrees of freedom. *, ** denote significance at the 5% and 1% level respectively.

Table 3. Maximum likelihood estimates of alternative distribution functions

Nikkei	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.047*	(0.021)	4.766**	(0.282)	1.896**	(0.078)
SGED	0.000	(0.000)	0.015**	(0.000)	-0.041**	(0.004)			1.133**	(0.033)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.048*	(0.021)	4.442**	(0.236)		
IHS	0.000	(0.000)	0.015**	(0.000)	-0.086	(0.032)			1.472**	(0.054)
ST	0.000	(0.000)	0.016**	(0.001)			4.404**	(0.232)		
Normal	0.000	(0.000)	0.016**	(0.000)						
Hang Seng	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.034**	(0.014)	6.328**	(0.547)	1.338**	(0.044)
SGED	0.000	(0.000)	0.016**	(0.000)	-0.031	(--)			0.977**	(0.028)
SSD	0.000	(0.000)	0.017**	(0.000)	-0.041*	(0.018)	3.314**	(0.100)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.067*	(0.027)			1.21	(0.033)
ST	0.000	(0.000)	0.017**	(0.001)			3.297**	(0.100)		
Normal	0.000	(0.000)	0.016**	(0.000)						
Tel Aviv	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.060**	(0.021)	5.247**	(0.365)	1.785**	(0.068)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.052**	(0.016)			1.175**	(0.035)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.062**	(0.021)	4.381**	(0.232)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.102**	(0.032)			1.463**	(0.054)
ST	0.001**	(0.000)	0.014**	(0.001)			4.331**	(0.228)		
Normal	0.000	(0.000)	0.013**	(0.000)						
Merval	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.022**	(0.001)	-0.043*	(0.018)	4.456**	(0.241)	1.531**	(0.051)
SGED	0.000	(0.000)	0.021**	(0.000)	-0.033**	(0.002)			0.998**	(0.028)
SSD	0.000	(0.000)	0.023**	(0.000)	-0.047**	(0.018)	3.083**	(0.075)		
IHS	0.000	(0.000)	0.022**	(0.000)	-0.068*	(0.027)			1.171**	(0.029)
ST	0.001*	(0.000)	0.023**	(0.001)			3.088**	(0.078)		
Normal	0.000	(0.000)	0.021**	(0.000)						
S&P 500	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.014**	(0.001)	-0.064**	(0.013)	5.735**	(0.430)	1.239**	(0.038)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.062	(--)			0.902**	(0.008)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.069**	(0.016)	2.760**	(0.046)		
IHS	0.000	(0.000)	0.014**	(0.000)	-0.087**	(0.024)			1.079**	(0.023)
ST	0.000	(0.000)	0.015**	(0.001)			2.770**	(0.049)		
Normal	0.000	(0.000)	0.014**	(0.000)						
Dow Jones	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.058**	(0.017)	4.496**	(0.241)	1.524**	(0.051)
SGED	0.000	(0.000)	0.012**	(0.000)	-0.057**	(0.002)			0.983**	(0.027)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.059**	(0.018)	3.122**	(0.078)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.088**	(0.026)			1.178**	(0.029)
ST	0.000	(0.000)	0.014**	(0.001)			3.122**	(0.080)		
Normal	0.000	(0.000)	0.013**	(0.000)						
Ftsie100	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.013**	(0.001)	-0.054**	(0.018)	4.273**	(0.212)	1.623**	(0.055)
SGED	0.000	(0.000)	0.013**	(0.000)	-0.049**	(0.003)			1.015**	(0.028)
SSD	0.000	(0.000)	0.014**	(0.000)	-0.056**	(0.018)	3.237**	(0.089)		
IHS	0.000	(0.000)	0.013**	(0.000)	-0.083**	(0.027)			1.208**	(0.031)
ST	0.000	(0.000)	0.014**	(0.001)			3.231**	(0.091)		
Normal	0.000	(0.000)	0.013**	(0.000)						
CAC40	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.062**	(0.018)	4.545**	(0.249)	1.673**	(0.059)
SGED	0.000	(0.000)	0.015**	(0.000)	-0.044*	(0.021)			1.065**	(0.030)
SSD	0.000	(0.000)	0.016**	(0.000)	-0.066**	(0.019)	3.540**	(0.120)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.094**	(0.028)			1.277**	(0.036)
ST	0.000	(0.000)	0.016**	(0.001)			3.533**	(0.122)		
Normal	0.000	(0.000)	0.016**	(0.000)						
IBEX35	**μ**	**S.E**	**σ**	**S.E**	**λ**	**S.E**	**η**	**S.E**	**κ**	**S.E**
SGT	0.000	(0.000)	0.016**	(0.001)	-0.073**	(0.017)	7.127**	(0.717)	1.380**	(0.045)
SGED	0.000	(0.000)	0.016**	(0.000)	-0.068	(--)			1.050**	(0.030)
SSD	0.000	(0.000)	0.017**	(0.000)	-0.069**	(0.018)	3.548**	(0.125)		
IHS	0.000	(0.000)	0.016**	(0.000)	-0.092**	(0.028)			1.270**	(0.037)
ST	0.000	(0.000)	0.016**	(0.001)			3.584**	(0.132)		
Normal	0.000	(0.000)	0.016**	(0.000)						

Note: Parameter estimates of the Normal, SGT, SGED, SSD, IHS and ST. S.E. denotes standard errors (in parentheses). Nine stock market returns in the period 1/1/2000-11/30/2012. μ, σ, λ and η are the estimated mean, standard deviation, skewness parameter, and tail-tickness parameter; κ represents the peakness parameter. An $*$ ($**$) denotes significance at the 5% (1%) level.

Table 4. Goodness-of-fit tests

	Log-L	LR_Normal	LR_SGT	Chi2		KS	
Nikkei							
SGT	8920.4	463.2**	--	5.239	(0.022)**	0.031	(0.004)
SGED	8897.4	417.2**	46.0**	7.715	(0.006)	0.027	(0.021)**
SSD	8920.3	463.0**	0.2	13.448	(0.001)	0.034	(0.001)
IHS	8918.6	--	--	3.453	(0.063)*	0.029	(0.011)**
ST	8918.2	--	4.4	20.958	(0.000)	0.029	(0.008)
Normal	8688.8		--	124.218	(0.000)	0.058	(0.000)
Merval							
SGT	8016.9	612.6**	--	8.164	(0.017)**	0.019	(0.197)*
SGED	8003	584.8**	27.8**	12.318	(0.002)	0.027	(0.021)**
SSD	8012.5	603.8**	8.8*	15.965	(0.003)	0.020	(0.147)*
IHS	8017	--	--	6.005	(0.111)*	0.018	(0.260)*
ST	8010.4	--	13.0**	18.687	(0.000)	0.024	(0.053)*
Normal	7710.6	--	--	253.700	(0.000)	0.072	(0.000)
S&P 500							
SGT	9777.7	824.2**	--	14.092	(0.001)	0.028	(0.013)*
SGED	9769.2	807.2**	17.0**	8.761	(0.013)**	0.033	(0.002)
SSD	9762.2	793.2**	31.0**	35.861	(0.000)	0.038	(0.000)
IHS	9769.2	--	--	22.316	(0.000)	0.035	(0.001)
ST	9757.1	--	41.2**	33.963	(0.000)	0.037	(0.000)
Normal	9365.6	--	--	266.854	(0.000)	0.080	(0.000)
Dow Jones							
SGT	9929.7	682.6**	--	6.333	(0.042)**	0.028	(0.011)**
SGED	9914.2	651.6**	31.0**	24.553	(0.000)	0.032	(0.002)
SSD	9925.1	673.4**	9.2**	21.875	(0.000)	0.034	(0.001)
IHS	9928.4	--	--	8.647	(0.034)**	0.029	(0.007)
ST	9921.6	--	16..2**	30.360	(0.000)	0.030	(0.007)
Normal	9588.4	--	--	256.272	(0.000)	0.071	(0.000)
CAC40							
SGT	9297.4	523.6**	--	3.209	(0.201)*	0.023	(0.067)*
SGED	9281	490.8**	32.8**	17.858	(0.000)	0.033	(0.002)
SSD	9295.3	519.4**	4.2*	7.248	(0.027)**	0.027	(0.018)**
IHS	9297.4	--	--	2.761	(0.430)*	0.022	(0.079)*
ST	9291.1	--	12.6**	38.232	(0.000)	0.025	(0.030)**
Normal	9035.6	--	--	191.314	(0.000)	0.064	(0.000)
IBEX35							
SGT	9176.8	484.2**	--	3.767	(0.052)*	0.027	(0.018)**
SGED	9169.8	470.2**	14.0**	11.509	(0.001)	0.028	(0.011)**
SSD	9167.1	464.8**	19.4**	13.293	(0.001)	0.028	(0.011)**
IHS	9170.9	--	--	7.174	(0.067)*	0.029	(0.010)**
ST	9162.4	--	28.8**	25.413	(0.000)	0.034	(0.001)
Normal	8934.7	--	--	118.562	(0.000)	0.065	(0.000)
Hang Seng							
SGT	8927.5	649.0**	--	1.543	(0.214)*	0.027	(0.020)**
SGED	8918.4	630.8**	18.2**	5.519	(0.063)*	0.029	(0.010)**
SSD	8916.3	626.6**	22.4**	9.290	(0.002)	0.037	(0.000)
IHS	8920.4	--	--	1.873	(0.392)*	0.034	(0.001)
ST	8914.6	--	25.8**	15.599	(0.000)	0.035	(0.001)
Normal	8603	--	--	23.434	(0.000)	0.072	(0.000)
Tel Aviv							
SGT	9358.2	316.8**	--	5.721	(0.057)*	0.027	(0.023)**
SGED	9343.6	332.6**	29.2**	4.288	(0.039)**	0.034	(0.002)
SSD	9357.3	360.0**	1.8	11.097	(0.004)	0.029	(0.008)
IHS	9358.6	--	--	5.878	(0.053)*	0.026	(0.024)**
ST	9354	--	8.4*	33.459	(0.000)	0.025	(0.041)**
Normal	9177.3	--	--	106.813	(0.000)	0.058	(0.000)
Ftsie100							
SGT	9857	628.2**	--	3.311	(0.191)*	0.025	(0.037)**
SGED	9839.1	592.4**	35.8**	10.540	(0.005)	0.034	(0.001)
SSD	9854.2	622.6**	5.6*	16.291	(0.000)	0.027	(0.018)**
IHS	9857.3	--	--	4.518	(0.211)*	0.027	(0.015)**
ST	9851.2	--	11.6**	25.173	(0.000)	0.029	(0.007)
Normal	9542.9	--	--	203.848	(0.000)	0.072	(0.000)

Note: Log-L is the maximum likelihood value. LR_{Normal} is the LR statistic from testing the null hypothesis that the daily returns are distributed as Normal against they are distributed as SGT, SGED or SSD. LR_{SGT} is the LR statistic from testing the null hypothesis of alternative distribution against the SGT. Chi2 and KS denote Chi-square and Kolmogorov Smirnov tests. Figures in brackets denote p-value. An *(**) denotes significance at the 5%(1%) level.

Table 5. Accuracy test, 1% level

	Nikkei	Merval	S&P500	DJ	CAC40	IBEX35	Hang	Tel Aviv	Ftsie100
VaR_Normal	**2.87%**	**2.24%**	**3.56%**	**2.77%**	**2.34%**	**2.17%**	**1.62%**	**2.64%**	**3.55%**
LR_{UC}	4.970*	2.450	8.770**	4.696*	2.943	2.270	0.700	3.997*	8.725**
BTC	4.149**	2.762**	5.792**	4.003**	3.056**	2.640**	1.384	3.653**	5.771**
LR_{IND}	0.310	0.219	0.579	0.348	0.251	0.212	1.105	0.388	0.577
LRcc	5.280	2.670	9.349**	5.043	3.193	2.482	1.805	4.386	9.301**
DQ	1.969	2.770	0.362	0.578	1.053	2.477	2.906	0.655	1.484
VaR_MME	**2.05%**	**2.85%**	**2.38%**	**1.58%**	**1.17%**	**1.77%**	**1.82%**	**2.23%**	**2.37%**
LR_{UC}	1.808	4.920*	3.027	0.642	0.063	1.079	1.177	2.429	3.003
BTC	2.329*	4.123**	3.108**	1.319	0.391	1.748	1.836	2.748**	3.093**
LR_{IND}	0.807	0.358	0.254	0.112	0.062	0.141	0.914	0.219	0.253
LRcc	2.615	5.278	3.281	0.754	0.125	1.221	2.092	2.647	3.256
DQ	5.132*	3.668	1.331	0.295	2.339	4.004*	2.289	3.758*	2.067
VaR_T	**1.64%**	**0.61%**	**1.19%**	**0.99%**	**1.17%**	**1.18%**	**0.61%**	**0.61%**	**2.17%**
LR_{UC}	0.734	0.379	0.074	0.000	0.063	0.069	0.389	0.385	2.280
BTC	1.420	-0.866	0.425	-0.022	0.391	0.410	-0.877	-0.874	2.647**
LR_{IND}	0.089	0.016	0.063	0.044	0.062	0.062	0.016	0.016	0.212
LRcc	0.822	0.395	0.137	0.044	0.125	0.132	0.405	0.401	2.493
DQ	3.652	0.047	2.423	0.253	0.145	9.879**	0.136	0.071	2.406
VaR_SGT	**1.84%**	**1.43%**	**1.78%**	**1.39%**	**1.17%**	**1.57%**	**1.01%**	**1.01%**	**1.78%**
LR_{UC}	1.222	0.345	1.100	0.295	0.063	0.627	0.000	0.000	1.086
BTC	1.874	0.948	1.767	0.872	0.391	1.302	0.027	0.032	1.754
LR_{IND}	0.116	0.088	0.142	0.086	0.062	0.111	0.045	0.045	0.142
LRcc	1.338	0.433	1.242	0.381	0.125	0.738	0.045	0.045	1.228
DQ	2.689	0.238	0.940	0.142	0.145	5.068*	3.156	0.185	0.229
VaR_IHS	**1.84%**	**1.43%**	**1.78%**	**1.19%**	**1.17%**	**1.57%**	**1.01%**	**1.01%**	**1.58%**
LR_{UC}	1.222	0.345	1.100	0.074	0.063	0.627	0.000	0.000	0.632
BTC	1.874	0.948	1.767	0.425	0.391	1.302	0.027	0.032	1.308
LR_{IND}	0.116	0.088	0.142	0.063	0.062	0.111	0.045	0.045	0.112
LRcc	1.338	0.433	1.242	0.137	0.125	0.738	0.045	0.045	0.743
DQ	2.688	0.237	0.942	0.121	0.145	5.069*	3.153	0.185	0.134
VaR_SSD	**1.84%**	**1.83%**	**2.18%**	**1.39%**	**1.17%**	**1.57%**	**1.21%**	**1.62%**	**1.97%**
LR_{UC}	1.222	1.199	2.301	0.295	0.063	0.627	0.093	0.706	1.639
BTC	1.874	1.855	2.661**	0.872	0.391	1.302	0.479	1.390	2.201*
LR_{IND}	0.116	0.146	0.213	0.086	0.062	0.111	0.064	0.115	0.175
LRcc	1.338	1.346	2.514	0.380	0.125	0.738	0.158	0.821	1.814
DQ	2.689	1.608	0.928	0.142	0.145	5.067*	2.134	0.254	0.824
VaR_SGED	**1.84%**	**1.43%**	**1.78%**	**1.39%**	**1.17%**	**1.57%**	**1.01%**	**1.22%**	**1.97%**
LR_{UC}	1.222	0.345	1.100	0.295	0.063	0.627	0.000	0.095	1.639
BTC	1.874	0.948	1.767	0.872	0.391	1.302	0.027	0.484	2.201*
LR_{IND}	0.116	0.088	0.142	0.086	0.062	0.111	0.045	0.064	0.175
LRcc	1.338	0.433	1.242	0.381	0.125	0.738	0.045	0.160	1.814
DQ	2.689	0.237	0.941	0.142	0.145	5.067*	3.156	0.067	0.825

Note: The statistics are as follows: (i) the unconditional coverage test (LRuc); (ii) the back-testing criterion (BTC); (iii) statistics for serial independence (LRind); (iv) the Conditional Coverage test (LRcc) and (v) the Dynamic Quantile test (DQ). An **, (*) denotes rejection at 1% (5%) level. The shaded cells indicate that the null hypothesis that the VaR estimate is accurate is not rejected by any test.

Table 6. Accuracy test, 0.25% level

	Nikkei	Merval	S&P 500	Dow Jones	CAC40	IBEX35	Hang Seng	Tel Aviv	Ftsie100
					Panel A: 2008-09				
VaR_Normal	**0.82%**	**0.81%**	**1.19%**	**0.59%**	**0.98%**	**1.18%**	**0.61%**	**0.61%**	**1.18%**
LR_{UC}	1.718	1.703	4.028	0.749	2.698	4.003*	0.782	0.786	4.011*
BTC	2.520*	2.506*	4.222**	1.548	3.292**	4.201**	1.590	1.594	4.209**
LR_{IND}	0.016	0.029	0.063	0.016	0.043	0.062	0.016	0.016	0.063
LRcc	1.734	1.732	4.090	0.764	2.741	4.065	0.798	0.802	4.074
DQ	0.023	0.044	0.080	0.089	0.080	9.833**	0.127	0.071	0.142
VaR_MME	**1.23%**	**1.63%**	**0.99%**	**0.40%**	**0.59%**	**0.59%**	**1.42%**	**0.61%**	**1.38%**
LR_{UC}	4.170*	1.629	2.743	0.159	0.728	0.740	5.570*	0.786	5.439*
BTC	4.333**	6.120**	3.331**	0.657	1.522	1.537	5.194**	1.594	5.098**
LR_{IND}	0.045	0.115	0.044	0.007	0.015	0.016	0.088	0.016	0.085
LRcc	4.215	7.299*	2.787	0.166	0.743	0.755	5.658	0.802	5.525
DQ	3.301	1.100	0.275	0.037	10.336**	10.309**	1.484	0.329	6.901**
VaR_ST	**0.20%**	**0.00%**	**0.40%**	**0.20%**	**0.59%**	**0.20%**	**0.00%**	**0.00%**	**0.99%**
LR_{UC}	0.018	1.068	0.159	0.026	0.728	0.027	1.074	1.072	2.730
BTC	-0.199	-1.109	0.657	-0.234	1.522	-0.240	-1.113	-1.112	3.320**
LR_{IND}	0.002	NaN	0.007	0.002	0.015	0.002	NaN	NaN	0.043
LRcc	0.020	NaN	0.166	0.027	0.743	0.029	NaN	NaN	2.774
DQ	0.163	NaN	0.038	0.131	0.016	0.034	NaN	NaN	0.077
VaR_SGT	**0.20%**	**0.20%**	**0.59%**	**0.40%**	**0.59%**	**0.20%**	**0.40%**	**0.20%**	**0.99%**
LR_{UC}	0.018	0.020	0.749	0.159	0.728	0.027	0.174	0.020	2.730
BTC	-0.199	-0.206	1.548	0.657	1.522	-0.240	0.689	-0.210	3.320**
LR_{IND}	0.002	0.002	0.016	0.007	0.015	0.002	0.007	0.002	0.043
LRcc	0.020	0.021	0.764	0.166	0.743	0.029	0.181	0.022	2.774
DQ	0.166	0.102	0.059	0.036	0.015	0.027	0.027	0.013	0.073
VaR_IHS	**0.20%**	**0.20%**	**0.59%**	**0.40%**	**0.59%**	**0.20%**	**0.00%**	**0.20%**	**0.79%**
LR_{UC}	0.018	0.020	0.749	0.159	0.728	0.027	1.074	0.020	1.626
BTC	-0.199	-0.206	1.548	0.657	1.522	-0.240	-1.113	-0.210	2.430*
LR_{IND}	0.002	0.002	0.016	0.007	0.015	0.002	NaN	0.002	0.028
LRcc	0.020	0.021	0.764	0.166	0.743	0.029	NaN	0.022	1.654
DQ	0.162	0.100	0.061	0.036	0.015	0.027	NaN	0.012	0.068
VaR_SSD	**0.20%**	**0.61%**	**0.59%**	**0.40%**	**0.59%**	**0.20%**	**0.40%**	**0.41%**	**0.99%**
LR_{UC}	0.018	0.792	0.749	0.159	0.728	0.027	0.173	0.175	2.730
BTC	-0.199	1.602	1.548	0.657	1.522	-0.240	0.689	0.692	3.319**
LR_{IND}	0.002	0.016	0.016	0.007	0.015	0.002	0.007	0.007	0.043
LRcc	0.020	0.808	0.764	0.166	0.743	0.029	0.181	0.182	2.774
DQ	0.169	0.050	0.058	0.036	0.015	0.024	0.025	0.043	0.072
VaR_SGED	**0.20%**	**0.41%**	**0.59%**	**0.40%**	**0.59%**	**0.20%**	**0.40%**	**0.20%**	**0.99%**
LR_{UC}	0.018	0.178	0.749	0.159	0.728	0.027	0.174	0.020	2.730
BTC	-0.199	0.698	1.548	0.657	1.522	-0.240	0.689	-0.210	3.320**
LR_{IND}	0.002	0.007	0.016	0.007	0.015	0.002	0.007	0.002	0.043
LRcc	0.020	0.185	0.764	0.166	0.743	0.029	0.181	0.022	2.774
DQ	0.169	0.135	0.058	0.036	0.015	0.024	0.027	0.012	0.073

Note: The statistics are as follows: (i) the unconditional coverage test (LRuc); (ii) the back-testing criterion (BTC); (iii) statistics for serial independence (LRind); (iv) the Conditional Coverage test (LRcc) and (v) the Dynamic Quantile test (DQ). An **, (*) denotes rejection at 1% (5%) level. The shaded cells indicate that the null hypothesis that the VaR estimate is accurate is not rejected by any test.

Table 7. Magnitude of the regulatory loss function

	level	NORMAL	MME	ST	SGT	IHS	SSD	SGED
Nikkei	1.00%	0.00338	0.00860	**0.00134**	0.00186	0.00176	0.00212	0.00186
	0.25%	0.00065	0.00397	**0.00004**	0.00015	0.00008	0.00020	0.00015
Merval	1.00%	0.00667	0.00833	**0.00053**	0.00256	0.00244	0.00340	0.00251
	0.25%	0.00191	0.00307	**0.00000**	0.00013	0.00009	0.00039	0.00022
S&P 500	1.00%	0.00617	0.00343	**0.00337**	0.00352	0.00362	0.00393	0.00349
	0.25%	0.00293	0.00145	**0.00121**	0.00133	0.00130	0.00167	0.00137
Dow Jones	1.00%	0.00220	0.00078	**0.00056**	0.00073	0.00065	0.00080	0.00067
	0.25%	0.00044	0.00012	**0.00003**	0.00004	0.00003	0.00008	0.00006
CAC40	1.00%	0.00568	0.00602	0.00462	0.00445	**0.00427**	0.00445	0.00443
	0.25%	0.00282	0.00375	0.00185	0.00158	**0.00148**	0.00175	0.00178
IBEX35	1.00%	0.00554	0.00742	**0.00308**	0.00355	0.00336	0.00366	0.00350
	0.25%	0.00274	0.00516	**0.00152**	0.00161	0.00158	0.00186	0.00182
Hang Seng	1.00%	0.00333	0.00581	**0.00048**	0.00124	0.00127	0.00165	0.00125
	0.25%	0.00062	0.00128	**0.00000**	0.00001	**0.00000**	0.00006	0.00001
Tel Aviv	1.00%	0.00150	0.00270	**0.00024**	0.00060	0.00054	0.00069	0.00062
	0.25%	0.00030	0.00153	**0.00000**	0.00000	0.00000	0.00004	0.00003
Ftsie100	1.00%	0.00376	0.00399	0.00254	0.00227	**0.00205**	0.00228	0.00228
	0.25%	0.00126	0.00131	0.00056	0.00036	**0.00029**	0.00047	0.00048

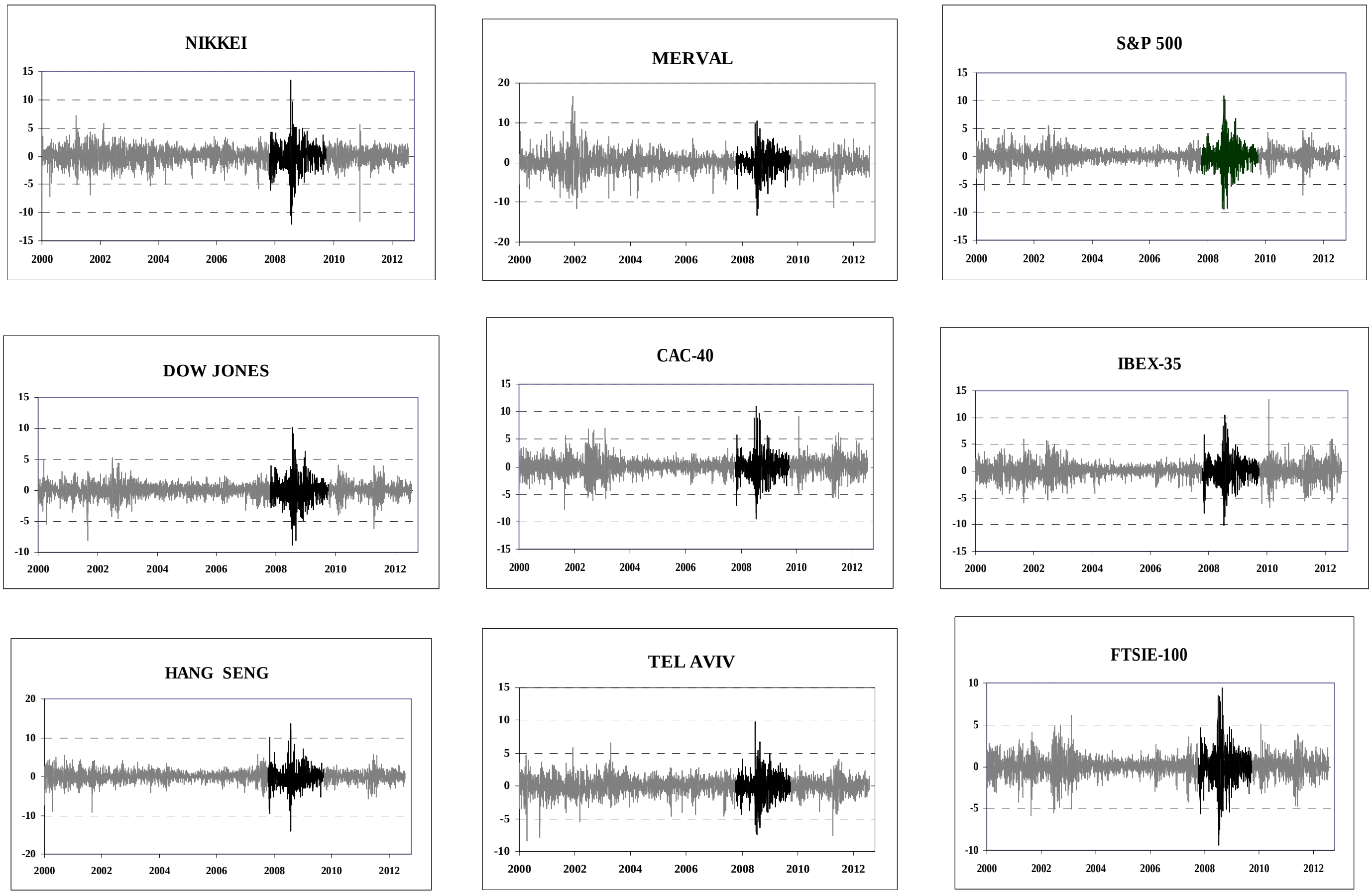
Note: This table reports the average of the loss function of each VaR model in both confidence levels. The average was multiplied by 1,000. Boldface figures denote the minimum value for the average of the loss function for each index.

Table 8. Magnitude of the firm's loss function

	level	NORMAL	MME	ST	SGT	IHS	SSD	SGED
Nikkei	1.00%	**0.00054**	0.00056	0.00062	0.00059	0.00059	**0.00058**	0.00059
	0.25%	**0.00066**	0.00068	0.00080	0.00076	0.00077	**0.00074**	0.00075
Merval	1.00%	0.00056	**0.00052**	0.00079	0.00065	0.00066	**0.00062**	0.00066
	0.25%	0.00068	**0.00063**	0.00112	0.00090	0.00092	**0.00081**	0.00085
S&P 500	1.00%	**0.00044**	0.00046	0.00052	0.00051	0.00050	**0.00049**	0.00051
	0.25%	**0.00054**	0.00056	0.00066	0.00065	0.00065	**0.00062**	0.00064
Dow Jones	1.00%	**0.00040**	0.00044	0.00048	0.00046	0.00047	**0.00045**	0.00046
	0.25%	**0.00050**	0.00054	0.00062	0.00060	0.00061	**0.00058**	0.00059
CAC40	1.00%	**0.00111**	0.00120	**0.00121**	0.00122	0.00123	0.00122	0.00122
	0.25%	**0.00136**	0.00144	0.00150	0.00153	0.00154	0.00150	**0.00150**
IBEX35	1.00%	**0.00109**	0.00118	0.00132	0.00125	0.00127	**0.00124**	0.00125
	0.25%	**0.00132**	0.00144	0.00173	0.00167	0.00168	**0.00158**	0.00159
Hang Seng	1.00%	**0.00062**	0.00067	0.00080	0.00072	0.00071	**0.00069**	0.00071
	0.25%	**0.00077**	0.00081	0.00107	0.00092	0.00096	**0.00089**	0.00092
Tel Aviv	1.00%	**0.00040**	0.00041	0.00052	0.00046	0.00047	**0.00045**	0.00046
	0.25%	**0.00050**	0.00051	0.00069	0.00062	0.00062	**0.00058**	0.00059
Ftsie100	1.00%	**0.00099**	0.00110	**0.00108**	0.00111	0.00113	0.00110	0.00110
	0.25%	**0.00122**	0.00133	**0.00135**	0.00140	0.00143	0.00137	0.00136

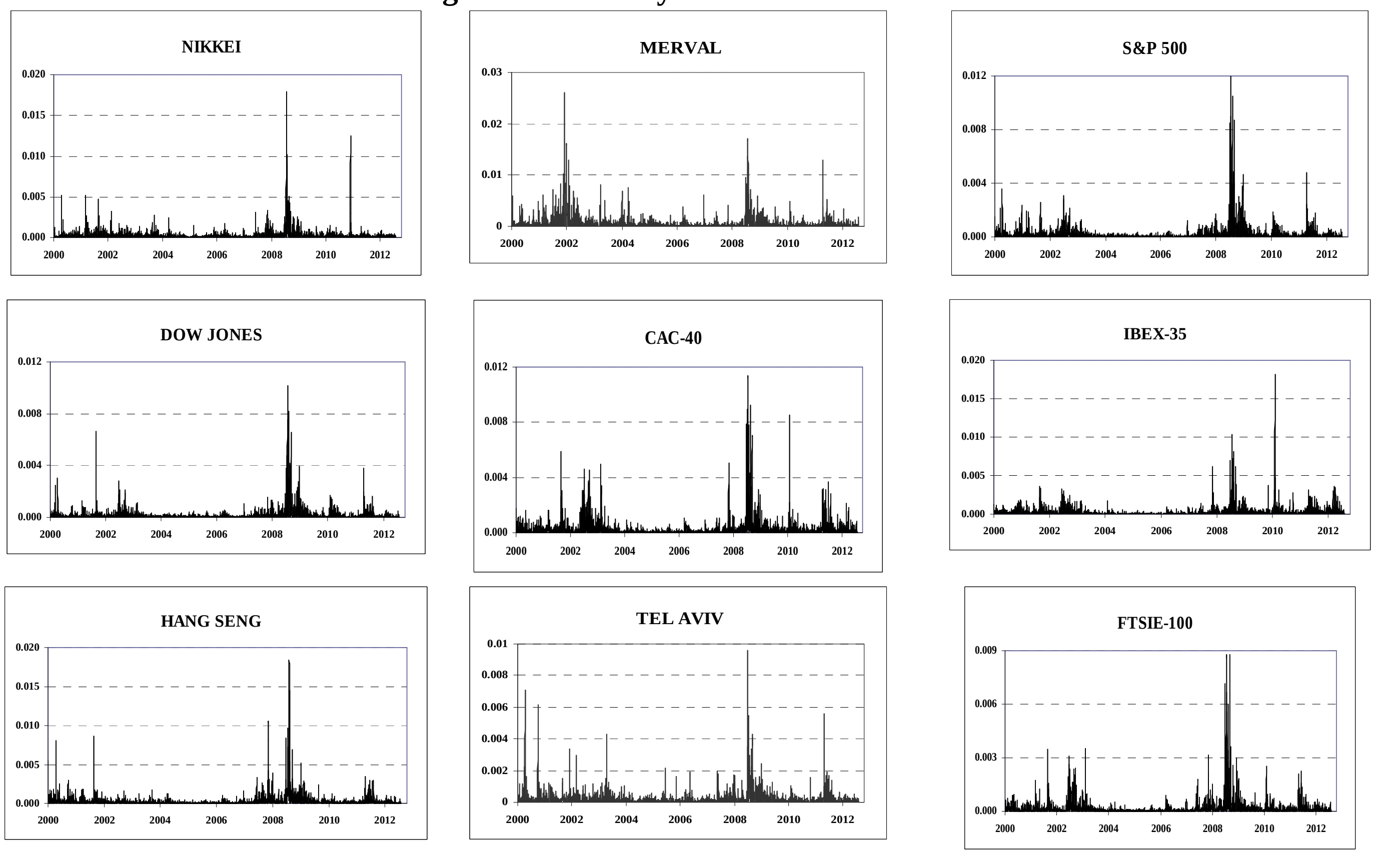
Note: This table reports the average of the loss function of each VaR model in both confidence levels. Boldface figures denote the minimum value for the average of the loss function for each index.

Figure 1. Stock index returns



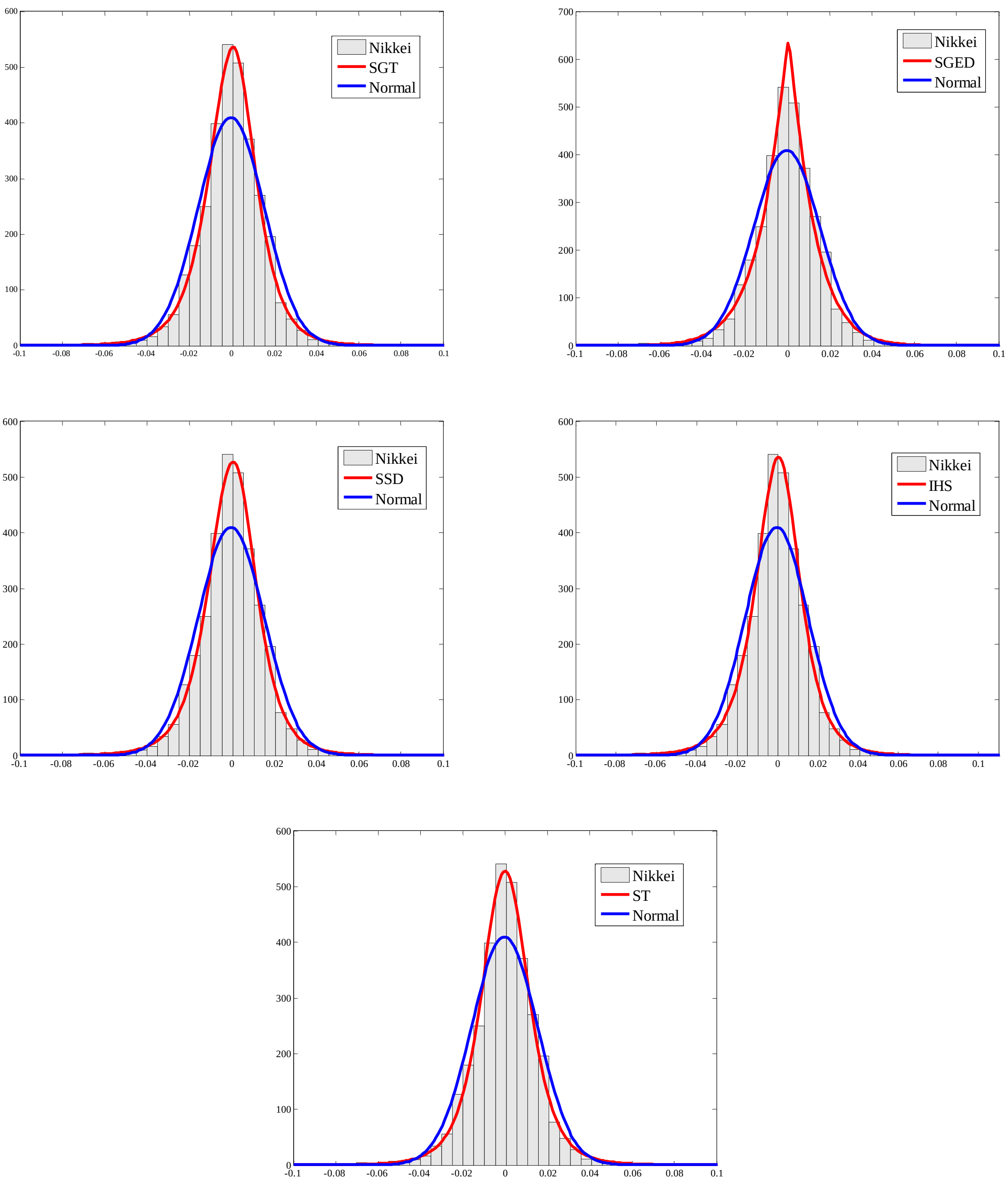
This figure illustrates the daily evolution of returns of nine indexes (Nikkei, Merval, S&P 500, Dow Jones Industrial Average, CAC40, IBEX35, Hang Seng, Telaviv and Ftsie-100.) from January 3rd 2000 to November 30th, 2012. Source: Bloomberg.

Figure 2. Volatility of the returns



Note: This figure illustrates the conditional volatility of daily returns. The volatility was estimated using the approach proposed by Franses and van Dijk (1999). Sample runs from January 3rd 2000 to November 30th, 2012. Source: Bloomberg.

Figure 3. Histograms, Normal versus other skewed distributions



Note: These figures illustrate the histograms, Normal distribution (blue line) versus the rest of considered distributions (red line). The data used in the graphs are those obtained from the Nikkei Index and the sample runs from January 3, 2000 to November 30, 2012.