Blow-up in nonlinear models of extended particles with confined constituents

A. Alvarez and A. F. Rañada
Departamento de Física Teórica, Universidad Complutense, 28040 Madrid, Spain
(Received 23 December 1987)

It is shown that the indefinite character of the charge in classical models of extended particles with confined constituents is a serious handicap since infinite amounts of positive and negative charge can be emitted in some solutions, causing a blow-up in finite time.

Finding a satisfactory mechanism for quark confinement is no doubt one of the most relevant problems in fundamental physics. One off-the-beaten-track view on this question considers confinement as a prequantum effect because of the way in which the constituent c-number fields are coupled. More precisely, with suitable interaction terms, each one of these fundamental fields can be a source to the rest of them in such a way that the set of solutions of the wave equations does not contain states in which only one of these constituent fields is excited. This parallels the nonexistence of isolated quarks. In the classical c-number models which realize this idea,[1] confinement is a classical effect previous to any second quantization. They make use of six constituent fields, three of them being charge conjugate to the other three; this has the advantage that antiparticles can be treated in the same way as particles, but, as a necessary price to pay, the charge has an indefinite character.

In this work we show that this indefinite character is a fundamental handicap for the validity of these models, even from a purely classical point of view, since it is the cause of the existence of solutions which blow-up in finite time.

For simplicity, we shall consider the version of the model presented in the last paper of Ref. 1. It refers to $(1+1)$-dimensional spacetime and makes use of six spinors ψ_a, ϕ_a, $a=1,2,3$, the field equations being

$$i\gamma^\mu\partial_\mu\psi_a - \psi_a + [v_\mu(\psi)+\sigma v_\mu(\phi)]\gamma^\mu\psi_a + g\sum_k (\bar{\chi}_{ak}\chi_{ak})\chi_{ak} = 0 , \quad (1a)$$

$$-i\gamma^\mu\partial_\mu\phi_a - \phi_a + [\sigma v_\mu(\psi)+v_\mu(\phi)]\gamma^\mu\phi_a + g\sum_k (\bar{\chi}_{ak}\chi_{ak})\gamma^5\chi_{ak} = 0 , \quad (1b)$$

where

$$v_\mu(\theta) = \sum_i \bar{\theta}_i\gamma_\mu\theta_i , \quad \chi_{ij} = \chi_i - \chi_j , \quad \chi_i = \psi_i + \gamma^5\phi_i ,$$

and $\mu=0,1$. Our γ matrices will be $\gamma^0=\sigma_1$, $\gamma^1=i\sigma_2$, and $\gamma^5=i\gamma^0\gamma^1$. The current

$$J^\mu = \sum_k (\bar{\psi}_k\gamma^\mu\psi_k - \bar{\phi}_k\gamma^\mu\phi_k)$$

is conserved, the corresponding constant quantity being

$$Q = \int_R \rho_Q(x,t)dx = \sum_k \int_R \psi_k^\dagger\psi_k dx - \sum_k \int_R \phi_k^\dagger\phi_k dx \equiv M - A ,$$

although the separate terms M and A, representing the contributions of matter and of antimatter, respectively, are not generally conserved. As in the case of the three-dimensional forms of the model, there are baryonic solutions with only the three ψ fields different from zero and mesonic solutions with one ψ and one ϕ as the only nonvanishing fields. Moreover, all the particlelike solutions have either the characteristic three-quark structure or the quark-antiquark pattern. This form of the model reduces

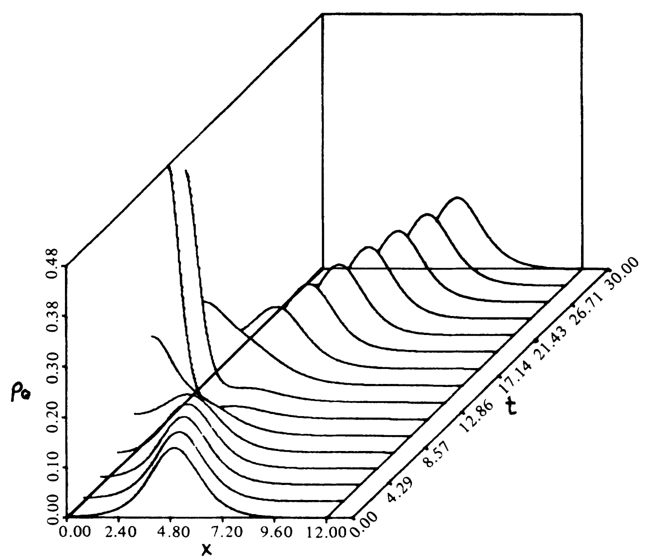

FIG. 1. Collision of two equal Thirring solitons with $\cos\delta=0.8$, $x_0=\pm 5$, initial velocity $v=\pm\tanh 0.3$. The diagram being symmetrical, only the positive x axis is shown.

to the massive Thirring one[2] for some particular initial conditions. For example, if

$$\psi_1=\psi_2=\psi_3=\psi \text{ and } \phi_a=0 \text{ at } t=0 ,$$

the field ψ is a solution of the massive Thirring model.

We have simulated the time evolution of system (1) for arbitrary initial data with a code which uses a second-order implicit difference scheme in space and time discretization. Relative errors of the energy and charge greater than 0.05% were not allowed. The constant g was taken equal to 1 and, for the physical reason explained in the first two papers of Ref. 1, $\sigma=2$. However we can say that the conclusions do not depend on these particular numerical values.

The code was checked in the case of the interactions of solitons in the Thirring model, with the initial values

$$\psi_1=\psi_2=\psi_3=\begin{bmatrix} f(x-x_0,\delta)+f(x+x_0,\delta) \\ g(x-x_0,\delta)+g(x+x_0,\delta) \end{bmatrix} ,$$

$$\phi_1=\phi_2=\phi_3=0 ,$$

$$f(x,\delta)=(\tfrac{2}{3})^{1/2}\sin\delta \exp(\sin\delta x)[\exp(2\sin\delta x)+\exp(-i\delta)]^{-1} ,$$

$$g(x,\delta)=(\tfrac{2}{3})^{1/2}\sin\delta \exp(\sin\delta x+i\delta)[\exp(2\sin\delta x)+\exp(i\delta)]^{-1} ,$$

where $0<\delta<\pi$. The output of our code for the collision of solitons is displayed in Fig. 1, this numerical result being in complete agreement with the analytical prediction.[3]

We have used the code to ascertain to what extent Eq. (1) can be used to represent the phenomenon of confinement. Our results can be summarized as follows.

(i) Collisions of baryons and antibaryons. The baryon-like solutions (baryons for short) behave as solitons upon collisions between themselves, the corresponding analytical solutions being simply related to those of the Thirring model.[1] However, there are no explicit solutions for the baryon-antibaryon case. Therefore, we have studied the scattering of a baryon and an antibaryon, related to the Thirring soliton as

$$\psi_1=\psi_2=\psi_3=\begin{bmatrix} f(x-x_0,\delta) \\ g(x-x_0,\delta) \end{bmatrix} ,$$

$$\phi_1=\phi_2=\phi_3=\begin{bmatrix} f(x+x_0,\delta')^* \\ g(x+x_0,\delta')^* \end{bmatrix} .$$

It turns out that both bumps recover their shape and identity after the collision (see Fig. 2). It must be

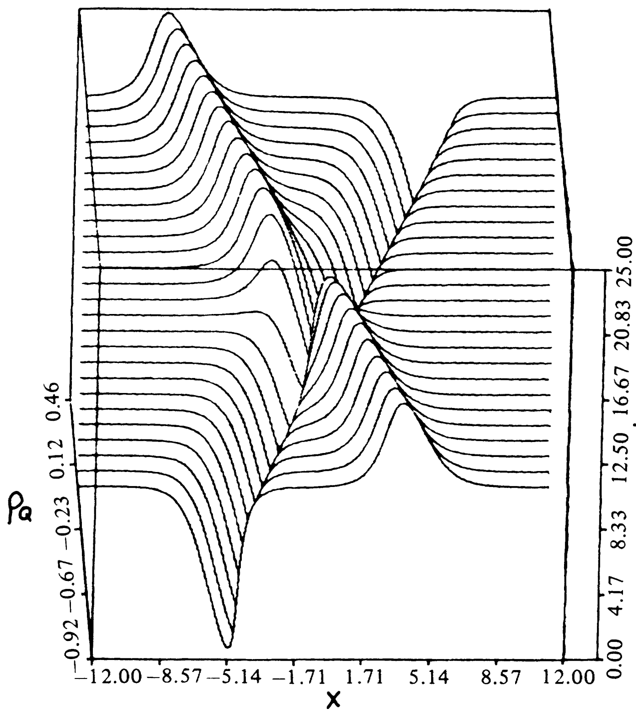

FIG. 2. Baryon-antibaryon collision in the c.m. system with $\cos\delta=0.8$, $\cos\delta'=0.6$, $x_0=\pm5$, initial baryon velocity $v=-\tanh 0.5$.

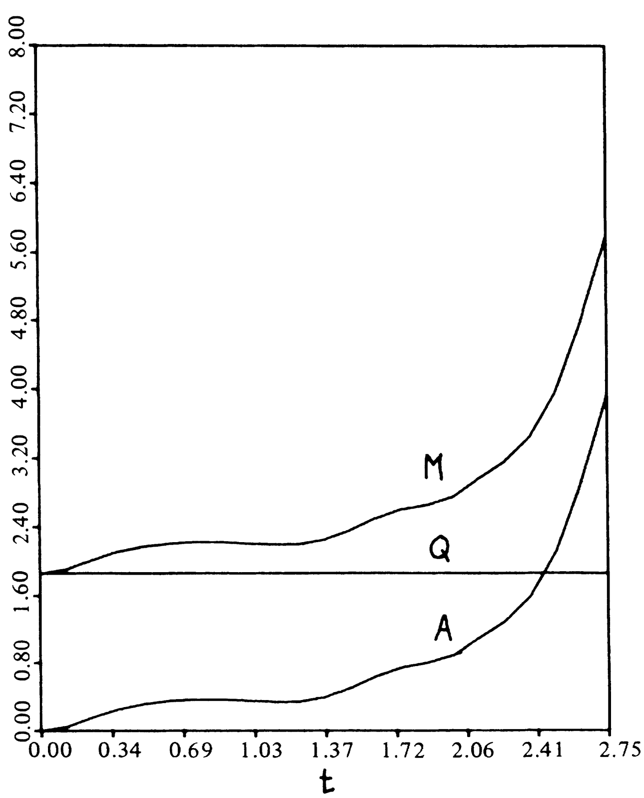

FIG. 3. M, A, and Q charges (see text) during the blow-up with $\cos\delta=0.8$ and $\alpha=3^{1/2}$. For the same δ, the solution goes to zero if $\alpha<0.5$.

remarked that in this type of solution the model reduces to a Thirring one with two different coupled spinors and consequently has a positive-definite conserved charge. Our result suggests the convenience of studying the integrability of the Thirring model with several coupled spinors by the inverse spectral method.

(ii) Evolution of a state with only a nonvanishing field at $t=0$. In this model the fields are confined in the sense that there are no solutions in which only one of them is nonvanishing for any value of time. However, if the picture thus obtained is to be quite satisfactory, this confining mechanism must work not only statically but also dynamically and this means that the zero-triality stationary solutions must be attractors. Or, in other words, it is necessary that, if at $t=0$ only one of the constituent fields is different from zero, it will act as a source to the rest of them up to reaching a zero-triality state. In order to test if this is actually the case, we have considered the following family of Cauchy data of (1):

$$\psi_1=\alpha\begin{bmatrix} f(x,\delta) \\ g(x,\delta)\end{bmatrix}, \quad \psi_2=\psi_3=\phi_1=\phi_2=\phi_3=0 \ ,$$

where α is a real parameter.

As expected, the fields ϕ_1, ϕ_2, and ϕ_3 do not keep vanishing, but begin also to grow. Although $M-A$ remains constant, M and A increase without bound if α is greater than a critical value α_c (see Fig. 3). On the other hand, if $\alpha<\alpha_c$, the solution goes to zero. Curiously enough, we have found the same shape, but not the same size, for the charge density of the spinors ψ_a and ϕ_a in the blow-up (see Fig. 4).

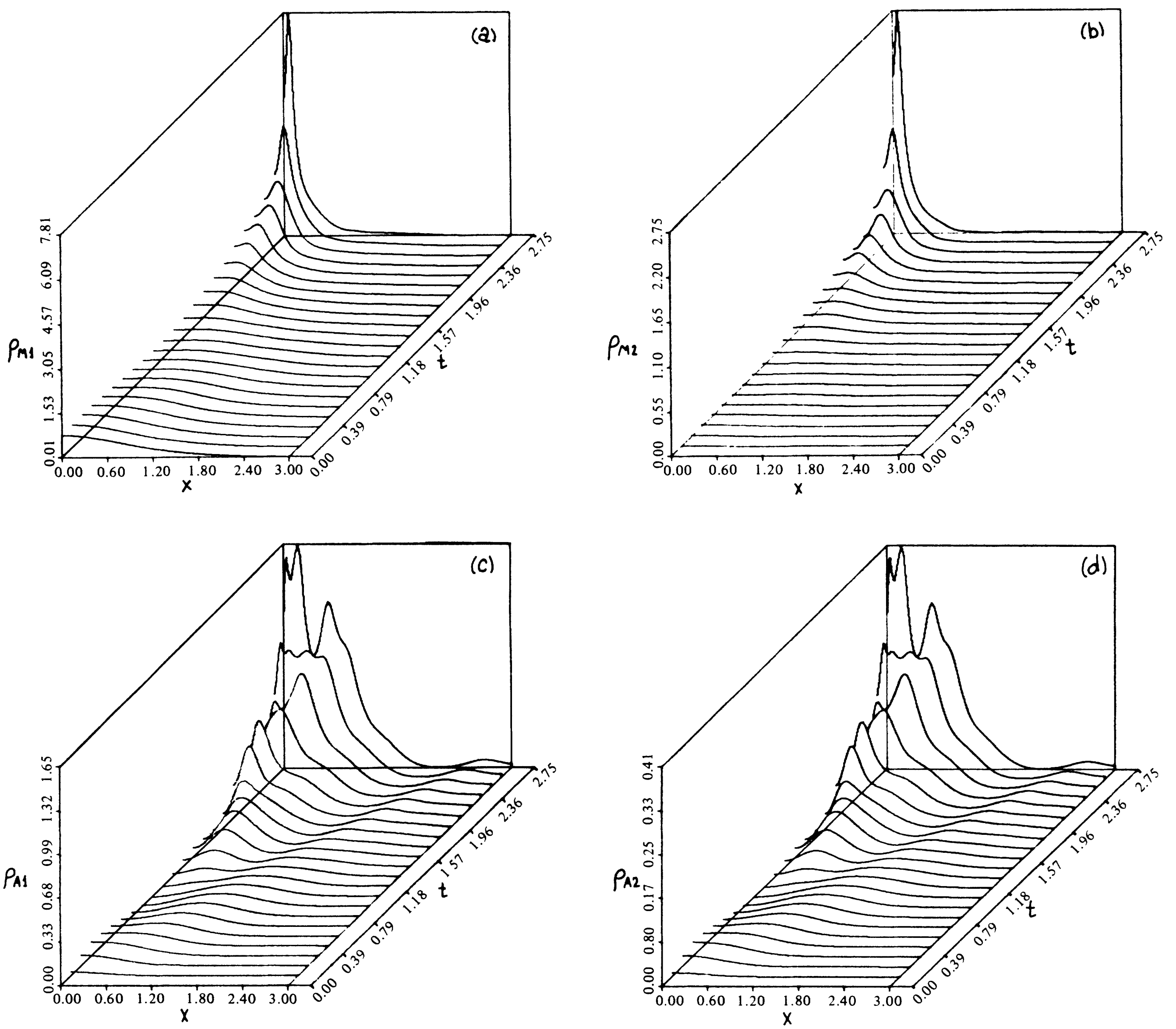

FIG. 4. Charge density of the spinors (a) ψ_1, (b) $\psi_2=\psi_3$, (c) ϕ_1, and (d) $\phi_2=\phi_3$ during the blow-up shown in Fig. 3.

(iii) Collisions of baryons and mesons. The localized solutions do not behave as solitons upon mixed collisions of this type, although we have observed no blow-up.

The explosive instability (ii) is a real handicap for this approach in its present form. Nothing similar has been observed in nonlinear spinorial models of extended fundamental particle without structure, which have positive-definite charge in $1+1$ as well as in $1+3$ dimensions.[3,4] We conclude, therefore, that the indefinite character of the charge, introduced in the attempt to model extended objects with confined constituents at the classical level, is the reason for the blow-up.

This work has been supported by Comisión Asesora de Investigación Científica y Técnica (CAICYT) under Grant No. 2140/83. We are also grateful to Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas (CIEMAT) for use of computer facilities.

[1]A. F. Rañada and M. F. Rañada, Physica D **9**, 251 (1983); Phys. Rev. D **29**, 985 (1984); J. Werle, in *Quantum Theory of Particle and Fields,* edited by B. Jancevicz and J. Lukierski (World Scientific, Singapore, 1983); M. F. Rañada, Phys. Rev. D **30**, 1830 (1984); A. F. Rañada, *ibid.* **33**, 1714 (1986).

[2]W. E. Thirring, Ann. Phys. (N.Y.) **3**, 91 (1958).

[3]E. A. Kuznetsov and A. V. Mikhailov, Teor. Mat. Fiz. **30**, 303 (1977); D. J. Kaup and A. C. Newell, Lett. Nuovo Cimento **20**, 325 (1977).

[4]A. Alvarez and B. Carreras, Phys. Lett. **86A**, 327 (1981); A. Alvarez, Kuo Pen-Yu, and L. Vázquez, Appl. Math. Comput. **13**, 1 (1983); A. Alvarez and M. Soler, Phys. Rev. Lett. **50**, 1230 (1983); A. Alvarez, Phys. Rev. D **31**, 2701 (1985); A. Alvarez and M. Soler, *ibid.* **34**, 644 (1986).