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TESTING FOR INVERTIBILITY IN UNIVARIATE ARIMA PROCESSES

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ABSTRACT

We propose a test statistic for detecting whether a differenced time series follows an invertible ARIMA process. The test follows a χ_1^2 distribution, it is easy to compute and shows an excellent performance when compared with standard optimal tests for overdifferencing.

RESUMEN

En este trabajo se propone un contraste estadístico para detectar invertibilidad en un proceso ARIMA. El estadístico tiene una distribución estandar χ_1^2 , es fácil de calcular y presenta un excelente comportamiento al compararlo con los contrastes óptimos estandar de sobrediferenciación.

Keywords: Invertibility; Overdifferencing; Unit Root Tests.

JEL classification codes: C12, C22

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1. Introduction

In determining the degree of differencing of a time series two different test-based approaches compete.

First, the standard AR approach tests the null hypothesis of a unit root in the AR polynomial of the ARIMA representation of a potentially infradifferenced time series. The LBI (Local Best Invariant) statistic, LBIU (Local Best Invariant Unbiased) statistic, Dickey-Fuller (DF hereafter) test, von Neumann ratio, Durbin-Watson test or the modified Durbin-Watson test are some of its most outstanding representants [see Tanaka (1996) chapter 9, for a discussion on these tests]. Among them, Tanaka (1996, page 349) recommends the use of the DF test. Besides its better performance, DF is easy to compute and facilitates the analysis of the effects of misspecification errors.

Second, the MA approach tests the null hypothesis of a unit root in the MA polynomial of the ARIMA representation of a time series potentially overdifferenced. Some examples in this line are the tests of Arellano and Pantula (1990), Tanaka (1990), Tsay (1993) or Saikkonen and Luukkonen (1993). The later (SL hereafter) belongs to the LBI class of the MA approach. This type of tests performs better than any other into the MA approach and has two important advantages with respect to any test into the AR approach. First, when the data generating process is an univariate ARIMA model without deterministic components they have standard χ^2 null asymptotic distributions [see, Tanaka (1996), pages 376 and 384]. Second, if there is a root very close to the boundary of either the non-invertibility or non-stationarity region, the MA LBI tests perform better than any test into the AR approach, [see Tanaka (1996) , pages 385-388].

In this paper we propose a new test which shares some desirable features of both DF and MA LBI tests: (1) It has a standard χ^2 distribution, (2) it is easy to compute and (3) it performs like a LBI test when there is a root close to the boundary of non-invertibility, and much better than a LBI test when the root is far from that boundary.

The article is organized as follows. Section 2 describes the proposed test statistic. Section 3 illustrates the performance of this test in finite samples and it is compared with that of the powerfull tests, R1 and R2, proposed in SL. Finally, Section 4 presents the most important conclusions.

2. Three important results

Consider the stationary ARMA model for the non-seasonal time series $z_t \equiv \nabla y_t$, with $\nabla \equiv 1-B$:

$$\phi_p(B)\ z_t = \theta_q^z(B)\ a_t \tag{1}$$

where B is the lag operator and a_t follows a white noise process with variance σ^2.

The roots of $\Phi_p(B)=0$ are assumed to lie outside the unit circle, but the roots of $\theta_q^z(B)=0$ might have a factor ∇, i.e.: the process is stationary but it might be noninvertible. If (1) is

not invertible y_t will follow the stationary and invertible ARMA(p,q-1) process:

$$\phi_p(B)\ y_t = \theta_{q-1}^y(B)\ a_t$$

with

$$\theta_q^z(B) = \nabla\theta_{q-1}^y(B).$$

If $\theta_q^z(B)=0$ has not a ∇ factor, y_t will follow the ARIMA(p,1,q):

$$\phi_p(B)\nabla\ y_t = \theta_q^z(B)\ a_t$$

Result 1:

Whatever the process followed by y_t, it can be approximated by a long but finite AR(L) process:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_L y_{t-L} + u_t \tag{2}$$

if the following conditions hold:

$$\begin{aligned} &a)\ L\ is\ a\ function\ \ (n_T)\ \ of\ T \\ &b)\ n_T \rightarrow \infty\ \ as\ \ T \rightarrow \infty \\ &c)\ n_T^3 \rightarrow 0\ \ as\ \ T \rightarrow \infty \\ &d)\ \sqrt{T}\sum_{i=n_T-1}^{\infty} \|\phi_i\| \rightarrow 0\ \ as\ \ T \rightarrow \infty \end{aligned} \tag{3}$$

Least squares (LS) to model (2) yields consistent estimates of the coefficients ϕ_i, $i=1,2,\ldots,L$. See Lütkepohl (1993), pages 306-309.

Result 2:

Only under invertibility, the process followed by z_t can be approximated by a finite AR process:

$$z_t = \pi_1 z_{t-1} + \pi_2 z_{t-2} + \ldots + \pi_{L-1} z_{t-(L-1)} + r_t \tag{4}$$

Note that condition (3d) holds only if the roots of $\theta_q^z(B)=0$ lie outside the unit circle. Thus, only under invertibility LS to (4) yields consistent estimates of π_i, $i=1,2,\ldots,L-1$.

Result 3:

Consider the quantity:

$$\tau_1 = \frac{u_{L+1}^2}{\sigma^2} = (T-2L-1)\,\frac{\left(\frac{T-L}{T-2L-1}\right)\frac{\sum_{t=L+1}^{T} u_t^2}{T-L} - \left(\frac{T-L-1}{T-2L-1}\right)\frac{\sum_{t=L+2}^{T} u_t^2}{T-L-1}}{\sigma^2} \tag{5}$$

where u_{L+1} and σ^2 are the one-step forecast error at period L+1 from the Gaussian AR model (2) and its variance, respectively. Under the assumption of normality τ_1 follows a standard χ_1^2 distribution [see Lütkepohl (1993), page 387]. Also, the ratio:

$$\hat{\tau}_1 = \frac{\hat{u}_{L+1}^2}{\hat{\sigma}^2}$$

follows a standard χ_1^2distribution [see Lütkepohl (1993), page 387]. Quantities with hat are LS estimates of the corresponding in the left hand side of (5).

Based on results 1-3 we propose the following statistic for testing the null hypothesis of invertibility:

$$\lambda = (T-2L-1)\left[\frac{\hat{\sigma}_{r,T-L}^2 - \hat{\sigma}_{u,T-L-1}^2}{\hat{\sigma}_{u,T-L-1}^2}\right] \tag{6}$$

where $\hat{\sigma}_{r,T-L}^2$ is the estimated residual variance in model (4) computed with T-L residuals and $\hat{\sigma}_{u,T-L-1}^2$ is the estimated residual variance in model (2) computed with T-L-1 residuals.

Under invertibility:

$$(a)\quad plim\left[\left(\frac{T-L}{T-2L-1}\right)\frac{\sum_{t=L+1}^{T} u_t^2}{T-L} - \hat{\sigma}_{r,T-L}^2\right] = 0$$

$$(b)\quad plim\left[\left(\frac{T-L-1}{T-2L-1}\right)\frac{\sum_{t=L+2}^{T} u_t^2}{T-L-1} - \hat{\sigma}_{u,T-L-1}^2\right] = 0 \tag{7}$$

then $plim\ (\tau_1 - \lambda) = 0$ implying that under invertibility λ and τ_1 have the same asymptotic χ_1^2 distribution. If invertibility does not hold (7a) will not hold and $plim\ (\tau_1 - \lambda) \neq 0$.

Note that λ can be approximated by:

$$\lambda = (T-2L-1)\ \ln\left(\frac{\hat{\sigma}_{r,T-L}^2}{\hat{\sigma}_{u,T-L-1}^2}\right) \tag{8}$$

Then, the null hypothesis of invertibility can be tested as follows:

(i) Apply LS to models (4) and (6).

(ii) Compute λ using (6) or (8) and compare its value with that of a χ_1^2.

It is important to mention that:

(i) A wrong choice of L will affect either the size or the power of our test. Next section contains some results about how this affects the performance of λ. The problem of correctly choosing L is analogous (but simpler) to the problem of correctly choosing the orders p and q of the ARIMA process when computing any LBI test; the advantage with respect to these tests is that ours simplifies the analysis of misspecification errors.

(ii) In finite samples λ can be negative, however if this occurs they are not expected to be large in absolute value and disappear as $T \to \infty$.

3. Simulation exercise

In this section we study the performance of our test in finite samples. Using simulations we compare the performance of λ with that of R1 (a LBI test) and R2 proposed in SL.

As in SL the time series y_t is generated according to:

$$y_1 = u_1$$
$$\nabla y_t = u_t - u_{t-1} \quad for\ t = 2,\dots,T$$

There is not a constant and u_t is assumed to be an unobservable zero mean error term with a stationary Gaussian ARMA representation given by either:

$$u_t = \epsilon_t + \beta\epsilon_{t-1}$$

or

$$u_t + \alpha u_{t-1} = \epsilon_t$$

where $\epsilon_t \sim NID(0,1)$. The assumed values for θ are : .6, .8, .9, .95 and 1.0; the nuisance parameters α and β are chosen as 0, ±.5 and ±.8. For each combination of (θ,α) or (θ,β) we consider the sample sizes T = 100, 200 and 300. Finally, we analyzed five lag lengths (L=3 $T^{1/4}$, 4 $T^{1/4}$, 5 $T^{1/4}$ and 6 $T^{1/4}$) for each T. The nominal 5% significance level is used throughout and the number of replications is 1000. Negative values of the test will be assumed to be zero, i.e.: they will not be considered a rejection of the null hypothesis.

Table 1 illustrates the performance of the test for $\beta=\alpha=0$. Tables 2-5 illustrate the performance of λ for each value of β. Except for those appearing in the row of $\theta=1$, figures represent empirical sizes related to a particular value of θ. Figures in the row of $\theta=1$ represent empirical powers, i.e.: the probability of rejecting invertibility being false, or in other words, the percentage of success in detecting noninvertibility (PNI, hereafter).

[Insert Tables 1-5]

Tables 6-9 illustrate the performance of λ for each value of α. As in Tables 1-5 figures represent empirical sizes, except when $\theta=1$ which represent empirical powers.

[Insert Tables 6-9]

From the inspection of Tables 1-9 can be concluded that: (1) The larger α (or the smaller β) the larger the number of lags needed to get the 5% nominal size. (2) The larger L the lower the power (or PNI). In these circumstances two alternative strategies could be adopted, (i) to identify the orders p and q of the ARMA generating process for z_t or (ii) to run the test for different choices of L.

Tables 10 and 11 compare the performance of λ against R1 and R2 proposed in SL. Figures in this Table represent PNI. As the null hypothesis tested by R1 and R2 is noninvertibility an empirical size of 5% implies a PNI = 95%. In order to make a fair comparison with λ, the choice of L was made trying to keep a PNI = 95%.

[Insert Tables 10-11]

Tables 10-11 show a clear superiority of λ in detecting invertibility. While for $\theta=.95$ both λ and R1 perform very similar, when the true parameter value is $\theta\leq.9$, λ performs better. This result hods for all sample sizes and values of nuissance parameters α and β.

4. Conclusions

In this paper we propose a test for invertibility in an ARIMA(p,1,q) process. This test has some desirable features: (1) it is easy to compute throughout a LS routine, (2) it has a standard asymptotic distribution (χ_1^2), (3) when a root is close to the boundary of noninvertibility it behaves as the local best invariant R1 test of SL and (4) when the root is far from the boundary of noninvertibility it behaves, not only better than R1, but better than R2 of SL. Finally, our simulation exercises indicate that the choice of L affects both the power and size of the test, the larger L the lower the distortions in size but also the lower power. Hence, an adequate choice of L is important and clearly depends on the structure of autocorrelation of the time series. More research about this point is in progress.

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Table 1: Empirical power and size of LR test for the process $z_t = (1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	5.0	6.4	5.3	5.0
	.8	8.6	6.6	4.1	4.6
	.9	19.4	9.7	6.2	5.0
	.95	44.2	23.8	15.6	9.1
	1.0	91.8	62.3	44.4	30.3
200	.6	4.9	4.8	5.3	5.4
	.8	7.1	5.4	5.2	4.2
	.9	21.8	12.2	8.1	7.3
	.95	49.2	31.4	19.4	13.3
	1.0	100.0	97.9	87.5	75.1
300	.6	6.00	5.7	5.7	5.5
	.8	8.7	7.8	6.6	6.3
	.9	20.8	10.7	8.9	6.4
	.95	53.7	30.2	20.9	15.7
	1.0	100.0	99.9	99.0	94.3

Table 2: Empirical power and size of LR test for the process $z_t = (1 - .8B)(1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	15.6	6.2	5.4	6.0
	.8	35.2	12.3	7.0	5.3
	.9	67.3	30.0	13.4	8.4
	.95	93.2	57.8	35.0	18.2
	1.0	99.9	96.2	77.9	53.1
200	.6	12.1	7.8	6.3	6.3
	.8	26.4	10.6	6.8	5.4
	.9	63.8	35.1	17.4	9.6
	.95	91.9	68.0	42.0	26.4
	1.0	100.0	100.0	99.4	95.6
300	.6	11.6	6.0	4.3	5.1
	.8	22.1	9.0	6.3	5.6
	.9	60.2	28.3	16.8	10.9
	.95	91.0	62.7	41.2	27.7
	1.0	100.0	100.0	100.0	100.0

Table 3: Empirical power and size of LR test for the process $z_t = (1 - .5B)(1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	6.4	4.8	5.1	3.7
	.8	10.7	5.6	5.2	4.1
	.9	28.8	12.1	8.1	5.9
	.95	58.4	25.8	16.0	11.7
	1.0	97.2	72.5	48.0	33.4
200	.6	5.7	5.2	5.0	5.6
	.8	7.8	5.9	5.5	5.3
	.9	26.4	14.7	8.5	6.2
	.95	62.9	40.1	24.8	16.2
	1.0	100.0	98.7	94.0	78.8
300	.6	7.5	5.5	5.9	4.9
	.8	9.1	5.7	5.7	6.0
	.9	26.3	13.2	9.6	6.9
	.95	62.0	36.7	25.2	17.6
	1.0	100.0	100.0	99.6	97.3

Table 4: Empirical power and size of LR test for the process $z_t = (1 + .5B)(1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	6.0	5.6	5.0	3.7
	.8	6.3	5.5	4.8	5.4
	.9	18.7	9.6	6.3	5.1
	.95	39.4	19.5	14.7	9.2
	1.0	87.7	58.5	39.4	25.8
200	.6	5.4	5.8	5.4	6.1
	.8	6.8	6.4	5.5	6.3
	.9	17.8	11.7	7.2	6.9
	.95	45.9	29.4	18.4	12.5
	1.0	99.9	96.3	88.0	73.6
300	.6	5.9	5.2	6.6	5.8
	.8	8.7	6.5	5.1	5.2
	.9	17.5	9.1	6.8	6.6
	.95	48.6	28.9	17.7	13.1
	1.0	100.0	99.9	98.7	94.1

Table 5: Empirical power and size of LR test for the process $z_t = (1 + .8B)(1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	5.5	5.9	4.9	4.1
	.8	7.2	5.6	4.3	4.1
	.9	16.9	11.0	6.5	4.6
	.95	36.5	16.9	13.1	9.5
	1.0	84.0	56.5	42.8	29.6
200	.6	5.6	5.3	4.0	4.2
	.8	6.9	6.3	6.6	6.2
	.9	18.4	10.2	8.3	5.9
	.95	45.6	27.6	17.3	12.5
	1.0	99.8	96.8	86.8	72.7
300	.6	6.3	6.3	5.4	6.1
	.8	7.1	5.8	5.4	5.9
	.9	15.6	8.7	7.7	4.5
	.95	46.2	24.2	17.8	12.5
	1.0	100.0	99.7	98.8	95.8

Table 6: Empirical power and size of LR test for the process $(1 - .8B)z_t = (1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	5.0	5.8	5.5	4.6
	.8	5.6	5.4	4.6	4.9
	.9	8.9	6.3	5.8	5.1
	.95	16.7	11.8	7.2	7.7
	1.0	49.3	36.0	25.1	19.2
200	.6	6.6	6.3	6.1	5.7
	.8	5.1	4.2	5.5	5.4
	.9	8.3	8.1	7.3	7.1
	.95	23.3	16.4	12.6	10.6
	1.0	92.3	81.2	66.6	53.5
300	.6	4.8	4.9	6.3	6.0
	.8	6.9	5.4	7.1	5.2
	.9	8.3	6.2	6.7	6.4
	.95	27.1	18.2	12.8	10.7
	1.0	99.6	95.9	91.0	81.9

Table 7: Empirical power and size of LR test for the process $(1 - .5B)z_t = (1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	6.3	5.3	5.2	4.0
	.8	7.6	6.8	6.4	4.7
	.9	13.9	7.1	5.5	5.8
	.95	30.4	14.4	11.1	7.0
	1.0	80.1	53.0	37.3	24.6
200	.6	6.5	6.4	6.8	5.6
	.8	6.8	5.8	5.6	5.2
	.9	14.9	8.9	7.0	6.7
	.95	4.7	25.5	16.5	11.8
	1.0	99.5	93.7	82.7	69.0
300	.6	6.5	5.9	6.1	6.6
	.8	9.2	7.4	7.1	6.0
	.9	16.1	9.3	6.8	6.1
	.95	41.7	26.5	18.8	12.8
	1.0	100.0	99.6	98.6	93.0

Table 8: Empirical power and size of LR test for the process $(1 + .5B)z_t = (1 - \theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	4.9	5.8	4.0	3.7
	.8	7.1	5.2	5.6	4.1
	.9	21.7	11.9	8.2	5.9
	.95	47.8	24.8	15.8	10.4
	1.0	93.1	64.5	44.9	29.0
200	.6	5.8	6.3	5.8	6.0
	.8	8.1	6.6	5.5	5.3
	.9	22.5	11.7	9.1	7.0
	.95	50.8	33.6	22.1	13.4
	1.0	100.0	98.4	92.1	75.2
300	.6	7.3	5.3	6.5	4.6
	.8	7.5	5.5	5.8	6.0
	.9	22.5	11.8	8.8	6.5
	.95	53.9	33.2	22.3	16.5
	1.0	100.0	100.0	99.1	96.4

Table 9: Empirical power and size of LR test for the process $(1+.8B)z_t=(1-\theta B)a_t$

T	θ	L			
		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
100	.6	6.1	5.6	4.9	5.1
	.8	8.7	6.1	4.9	5.1
	.9	22.8	9.4	8.0	6.3
	.95	51.2	21.9	14.0	10.6
	1.0	94.7	68.1	52.0	34.4
200	.6	5.1	5.0	4.2	5.0
	.8	7.9	6.3	5.0	5.1
	.9	22.7	14.8	8.4	6.8
	.95	55.6	35.1	21.5	14.8
	1.0	100.0	99.0	92.0	77.0
300	.6	6.1	5.2	7.0	4.7
	.8	9.4	5.2	5.3	7.1
	.9	22.1	11.9	8.4	7.9
	.95	54.7	34.4	23.6	16.8
	1.0	100.0	99.9	99.1	96.2

Table 10: Percentage of success in detecting invertibility†

	α =	-.8	-.5	0	.5	.8
θ =	Test	N=100				
.95	λ‡	62 (1)	60 (2)	56 (3)	52 (3)	49 (5)
	R1	62	61	57	59	60
	R2	43	38	32	32	32
.90	λ	82	81	81	78	77
	R1	72	77	73	76	75
	R2	61	62	56	60	60
.80	λ	91	90	91	93	91
	R1	76	84	83	84	83
	R2	78	82	81	86	86
θ =	Test	N=200				
.95	λ‡	77 (3)	75 (4)	69 (4)	78 (5)	79 (5)
	R1	77	76	74	77	76
	R2	62	61	58	59	59
.90	λ	92	91	88	91	92
	R1	84	86	85	86	86
	R2	80	85	82	84	85
.80	λ	95	94	95	95	95
	R1	85	91	90	92	91
	R2	88	95	96	98	98

† The stochastic process is defined by $z_t=u_t-\theta u_{t-1}$; $u_t+\alpha u_{t-1}=\varepsilon_t$.
‡ The number of lags (L) is $(T)^{1/4}$ times the figure in parentheses. It has been selected so that the probability of detecting $\theta=1$ is 95%.

Table 11: Percentage of success in detecting invertibility†

	β =	-.8	-.5	0	.5	.8
θ =	Test	N=100				
.95	λ‡	42 (4)	42 (3)	56 (3)	61 (3)	64 (3)
	R1	51	54	57	59	59
	R2	13	28	32	34	34
.90	λ	70	70	81	81	83
	R1	67	70	73	75	76
	R2	36	50	56	60	62
.80	λ	88	89	91	94	93
	R1	74	78	83	86	86
	R2	53	70	81	85	86
θ =	Test	N=200				
.95	λ‡	74 (6)	75 (5)	69 (4)	71 (4)	72 (4)
	R1	72	74	74	75	75
	R2	43	53	58	59	59
.90	λ	90	92	88	88	90
	R1	82	82	85	86	87
	R2	64	76	82	84	85
.80	λ	95	95	95	94	94
	R1	87	87	90	91	91
	R2	79	91	96	97	97

† The stochastic process is defined by $z_t = u_t - \theta u_{t-1}$; $u_t = \varepsilon_{t-1} + \beta \varepsilon_{t-1}$

‡ The number of lags (L) is $(T)^{1/4}$ times the figure in parentheses. It has been selected so that the probability of detecting $\theta = 1$ is 95%.