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Los primeros tres segundos: de la inflación cosmológica al Big Bang

The first three seconds: From inflation to the hot Big Bang

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Resumen:

Este trabajo ofrece una introducción general a las teorías de Inflación Cosmológica. Comienza presentando un universo en expansión, en el que además se asume homogeneidad e isotropía. Se señalan las limitaciones del modelo estándar del Big Bang, motivando el planteamiento del paradigma inflacionario como solución a las mismas, a través de un campo real escalar: el inflatón. Posteriormente, se plantan las semillas para la formación de estructuras a gran escala a partir de la inclusión de inhomogeneidades perturbativas en el modelo. Estas perturbaciones son descritas mediante el formalismo ADM de la Relatividad General, y son finalmente cuantizadas. Este estudio culmina centrado en una clase particular de potenciales, para las que una simulación numérica ha sido llevada a cabo, siendo implementada en Python (github.com/santiherra/Inflation_Background). Finalmente, las "Effective Field Theories" en inflación son presentadas, remarcando la aparición de un bosón de Goldstone asociado a una ruptura espontánea en difeomorfismos temporales. Este formalismo se separa de los caminos dependientes de modelos concretos, y se estudia el efecto de la inclusión de operadores de mayor orden en los invariantes de la teoría.

Abstract:

This thesis provides an introduction to Cosmic Inflation theories. It begins by presenting an expanding Universe under the assumptions of homogeneity and isotropy. The limitations of the standard Big Bang model are outlined, motivating the emergence of inflation as a solution, for a real scalar matter field: the inflaton. Additionally, the seeds for small-scale structure formation are planted from a model in which perturbative inhomogeneities are included. Those perturbations are analyzed using the ADM formalism of General Relativity, and are subsequently quantized. This study is then focused on a class of potentials, for which a numerical simulation has been performed, being implemented in Python (github.com/santiherra/Inflation_Background). Finally, the Effective Field Theories are presented, highlighting the emergence of a Goldstone boson associated to a hidden symmetry on time diffeomorphisms. This formalism departs from the given model-dependence paths and studies the effect of introducing higher contributions from the invariant quantities of the theory.

Contents

INTRODUCTION

The inflationary paradigm is one of the most widely accepted frameworks beyond the Cosmological Standard Model. It was firstly introduced by Alan Guth in 1979 [1], and further developed by Linde in 1982 [2]. It posits a period of accelerated expansion in the Universe, driven by the presence of a dominating quantum field of matter. It is, however, not a single model, but rather a collection of families of models, most of which are recorded in an extensive catalogue [3]. Almost all of them take their matter component as a scalar field, as it is naturally linked to the isometries of the isotropic and homogeneous Universe. There are some other alternatives, with vector fields as exemplified in [4, 5, 6], or even spinor fields [7, 8], but they require some special treatment and a complicated relationship with regards to the aforementioned desired symmetries and will not be a subject of study here.

Cosmological inflation was introduced as a phenomenological solution to some of the ΛCDM unsatisfactory consequences, namely the flatness and horizon problems. We will introduce the isotropic and homogeneous Robertson-Walker Universe, and the most basic principles that motivated the inflationary schemes. Within this background, a real scalar field coupled to gravity can produce inflation, provided the conditions for a regime of slow variations in the field, known as "slow-roll".

Another of inflation's strengths as a predictive science appears when describing the small-scale structures of the Universe. For this task, we will introduce anisotropies and inhomogeneities in the metric, working in a perturbative scheme, following the tradition started by Mukhanov [9, 10]. We will need to work within a different mathematical formalism, developed by Arnowitt Deser and Misner [11], to describe this perturbations in a Hamiltonian picture of the General Relativity, and the dynamics will be conveniently solved relying on the bright choices of gauge in Maldacena's work [12]. The perturbations will be expanded in pseudo-Fourier modes, each of them evolving by their respective Mukhanov-Sasaki's equation. The quantization of these modes and the establishment of the notion of a quantum vacuum provided with a power spectrum, which can be linked to observations in the Cosmic Microwave Background.

In the recent years, a strong link between inflation and particle physics has been growing, due to the emergence of Effective Field Theories on the subject. In the language of EFT's, inflation can be produced from a process of spontaneous symmetry breaking, i.e, from a violation of a fundamental symmetry in the Lagrangian at ground level. We take a look into these mechanisms working with a Lagrangian compatible with the symmetries of the Universe with a spontaneously broken symmetry with respect to time diffeomorphisms, which leads to the production of a Goldstone boson. This boson will be linked to the perturbation fields introduced in the previous methodology, to reproduce similar results.

The structure of this work is decomposed in three main building blocks. In the first chapter, we review the basic concepts from Standard Cosmology and introduce the background dynamics of a single field in a slow-roll regime. The second chapter is dedicated to build higher-order inflation, beginning with a brief look on the most relevant objects from the ADM construction. We determine the perturbative quantities and constrain them, to end up with the equations of motion of scalar and tensor components and their respective power spectra. We end this section by giving a particular model in the potential, for which we provide an estimation of some cosmological observables such as the spectral tilt, and compare it to Planck results [13]. In the third chapter, we introduce the EFT formalism and the analogy with the Goldstone boson.

The simulations in the chosen potential have been carried out with a self-made code in Python, stored in the repository github.com/santiherra/Inflation_Background. The computations required for the power spectrum have been achieved with a modified version of the code provided by the SIGWAY project [14].

1. THE ROBERTSON-WALKER UNIVERSE

The Standard Cosmology relies on a homogeneous and isotropic depiction of the Universe. One of the most relevant empirical evidences is the Cosmic Microwave Background (CMB), with an averaged temperature of 2.72548 ± 00057 K [15], with relative deviations of just a part in 25000. Also, from Hubble's studies of relatively distant galaxies and their redshifts, it was determined that they are departing from the Milky Way, no matter the direction observed. The conclusion: the Universe is expanding. This highly symmetric Universe, in which the cosmological distances are growing with time, is well described by the Friedmann-Lemaître-Robertson-Walker (FLRW) set of metrics,

$$g_{\mu\nu} = \mathrm{diag}\left(-1,\ \frac{a^2(t)}{1-kr^2},\ a^2(t)r^2,\ a^2(t)r^2\sin^2(\theta)\right), \tag{1.1}$$

with a the comoving scale factor (it is a measure of distances evolving in time), and k the curvature of the spatial cross sections. The manifold

$$M_P^2\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right) = \ T_{\mu\nu}, \tag{1.2}$$

with M_P[1] the Planck's mass, $R_{\mu\nu}$ the Ricci tensor and $R = g^{\mu\nu}R_{\mu\nu}$ the Ricci scalar. The symmetries of the Universe also fix the type of matter that must be contained in it. For any component, we shall consider a perfect fluid, with an energy-momentum tensor

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu}, \tag{1.3}$$

ρ and p standing for the energy density and pressure of the fluid. They are related by an equation of state[2] $p = w\rho$, being w the equation of state constant. There are only two non-vanishing and independent equations of motion, coming from the 't-t' and any of the 'i-i' components of the Ricci tensor,

$$H^2 := \left(\frac{\dot a}{a}\right)^2 = \frac{\Lambda}{3} - \frac{k}{a^3} + \frac{1}{3M_P^2}\sum_i \rho_i, \qquad \frac{\ddot a}{a} = \frac{\Lambda}{3} - \frac{1}{6M_P^2}\sum_i(\rho_i + 3p_i), \tag{1.4}$$

with the index 'i' accounting for all possible components in the model, the "dot" notation being used for derivatives with respect to comoving time, and H is the Hubble parameter. These are the well-known Friedmann Equations, the latter being the acceleration equation, and the former the constraint equation. The system is fully integrable and a general solution of the form $F(a) = t$, with F some locally reversible function, can always be found. Conventionally, the contributions to the energy and pressure are given by four sources: matter (both luminous and dark, with $w = 0$ and $\rho \propto a^{-3}$, radiation (both photons and neutrinos) with $w = 1/3$ and $\rho \propto a^{-4}$, curvature and dark energy with $w = -1$ and $\rho = M_P^{-2}\Lambda = \text{const.}$. Defining the critical energy density $\rho_c := 3H^2M_P^2$, and the dimensionless density parameters $\Omega_i := \rho_i/\rho_c$ and $\Omega_k = -k\ a^{-2}$, the constraint equation turns into a balance equation

$$\Omega_M + \Omega_R + \Omega_\Lambda + \Omega_k = 1. \tag{1.5}$$

[1]Here we take the Planck's mass as $M_P = (8\pi G)^{-1/2} \approx 2.436 \cdot 10^{18}$ GeV.

[2]In general, it might be an equation of state $\Phi(p, \rho, T, \dots) = 0$. For a barotropic fluid, $p = p(\rho)$

1.1. The Flatness Problem

The measurements of Baryon Acoustic Oscillations (BAO) in PLANCK 2018 [13] obtains an estimation in the curvature density parameter defined above, at present times as $\Omega_k = 0.0007 \pm 0.0019$ at 68% CL. It is compatible with 0, which corresponds to a flat Universe. Furthermore, stepping back in time, and regarding $\Omega_k \propto a^{-2}$, this density parameter would go to incredibly low values at early epochs such as Nucleosinthesis ($\Omega_k \sim 10^{-18}$). Indeed, the starting point $\Omega_{k,BB} = 0$ is an unstable stationary point of the dynamical equation for Ω_k coming from (1.5),[3] and any initial perturbation would quickly depart from it. In principle, one is forced to impose a fine tuning in the initial conditions of the Universe, in order to obtain the current observations.

1.2. The Horizon Problem

For the high level of thermal equilibrium observed in the CMB, there is a need for a causal structure between distantly located regions. The radiation that composes the CMB picture form a Last Scattering sphere, with a radius given by the comoving distance[4] $d = \Delta\tau$, where we have defined the conformal time τ which coincides with the conformal time

$$d\tau = \frac{dt}{a(t)}, \qquad \Delta\tau = \int \frac{dt}{a(t)}, \tag{1.6}$$

and so

$$R = \int_{t_{CMB}}^{t_0} \frac{dt}{a(t)} = \int_{a_{CMB}}^{1} \frac{d}{\mathcal{H}(a)}, \tag{1.7}$$

with $\mathcal{H} = aH$. The most distant points described on this sphere, separated by a physical distance $d = 2Ra_{CMB}$, still share a high level of thermalization. We can study their causal structure through the light cones, for which the particle Horizon distance, d_H, at the radiation decoupling instant

$$d_{H,CMB} = a_{CMB} \int_0^{t_{CMB}} \frac{dt}{a(t)} = a_{CMB} \int_0^{a_{CMB}} \frac{da}{\mathcal{H}(a)}. \tag{1.8}$$

In the ΛCDM model we have $\mathcal{H} = aH_0 \left[\Omega_{M,0}a^{-3} + \Omega_{R,0}a^{-4} + \Omega_{\Lambda,0}\right]^{1/2}$. For the values determined in [13], the distance $d_{CMB} \approx 24$ Mpc is much bigger than the radius of causal connection, $d_{H,CMB} \approx 0.42$ Mpc. This implies that there are regions in thermal equilibrium for which a null geodesic cannot be traced between the two, and consequently they cannot be connected in terms of causality.

1.3. The Inflationary Paradigm

The inflationary mechanism is set by considering an era, contained in an interval $[t_i, t_{end}]$, in which there exists a dominating matter component that produces a radical accelerated expansion. Taking (1.4) for a single component with equation of state w. We need to separate it into two cases; $w \neq -1$ and $w = -1$. The quation of state takes the latter value during the inflationary epoch, and a different value outside of it,

$$a(t) \propto \begin{cases} t^{\alpha}, & t < t_i \ \& \ t > t_{end} \\ e^{Ht}, & t_i < t < t_{end} \end{cases}, \text{ and so } \quad d_{H,CMB} \propto \begin{cases} a^{1/\alpha}, & t < t_i \ \& \ t > t_{end} \\ a, & t_i < t < t_{end} \end{cases}. \tag{1.9}$$

[3] More precisely, the Equation of Motion (EoM) for Ω_k is found from log-differentiating (1.5) with respect to the scale factor a.

[4] This comoving distance is obtained from the null radial geodesic condition, $\frac{dt}{a(t)} = \frac{dr}{\sqrt{1-kr^2}}$ or, equivalently, $d\tau = d\chi$.

The flatness problem is immediately solved. Let us define the number of e-folds as a time parametrization given by the logarithmic evolution of the comoving scale factor a,

$$dN = \frac{da}{a}, \qquad \text{or} \qquad N = \log\left(\frac{a_2}{a_1}\right). \tag{1.10}$$

This way, $\frac{\Omega_{k,end}}{\Omega_{k,i}} = e^{-2N}$, and so the small curvature result becomes an attractor.

Regarding the horizon problem, from (1.9) we can find a particle horizon that exits the sphere of last scattering, as far as the condition $\mathcal{H}_i^{-1} > \mathcal{H}_{end}^{-1}$. This gives a condition in the relation between the two scale factors, and for the conventional energy scales taken for inflation, it must take the order of 60 e-folds. For a proper duration of inflation, the radiation incident from the CMB is causally connected.

1.3.1. Single Field Inflation

There are several mechanisms that may produce the accelerated expansion scenario. In this work, we will present the inflationary process as a result of the dynamics of a real scalar field minimally coupled to gravity, the inflaton ϕ. The Einstein-Hilbert action and a canonical action for the matter sector are brought together giving

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_M = \frac{1}{2}\sqrt{-g}\Big(M_P^2 R - \partial_\mu\phi\partial^\mu\phi - 2V(\phi)\Big), \tag{1.11}$$

where $V(\phi)$ is a general potential yet to be fixed. The inflaton field is expanded around a purely isotropic and homogeneous solution, $\phi(x) = \phi_0(t) + \delta\phi(x,t)$. This field has an associated energy density and a pressure, $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$. The EoM for this field, obtained from varying the Lagrangian (1.11) with respect to the scalar field, giving a generalized Klein-Gordon equation

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \partial_\phi V = 0. \tag{1.12}$$

The equation of state constant w will be close to -1 in the *slow-roll* approximation, in which the background solution varies slowly ('*slowly rolls*') in an almost flat potential. This translates into $V \gg \dot{\phi}_0^2$, and $\partial_\phi V, H\dot{\phi}_0 \gg \ddot{\phi}_0$. In that case, the Hubble parameter is approximately constant, and the Klein-Gordon equation is reduced to a first order ODE, namely

$$H^2 \sim \frac{V}{3M_P^2} \sim \text{const.}, \qquad \text{and} \qquad \dot{\phi}_0 = -\frac{\partial_\phi V}{3H}. \tag{1.13}$$

We define a series of *Hubble flow-parameters*, by a recursive relation

$$\epsilon_{i+1} := \frac{1}{H}\frac{d\log\epsilon_i}{dt} = \frac{\dot{\epsilon}_i}{H\epsilon_i}, \qquad \text{with} \qquad \epsilon_1 = \epsilon = -\frac{\dot{H}}{H^2}, \tag{1.14}$$

which will play a crucial role in the characterization of the regime of validity of the slow-roll approximation, the time duration of inflation, and the effect of perturbations later on. In this work, we will be using the first two parameters, ϵ and $\epsilon_2 = \eta$.

$$\epsilon := \frac{1}{2M_P^2}\frac{\dot{\phi}_0^2}{H^2}, \qquad \eta := \frac{\dot{\epsilon}}{\epsilon H} = \frac{\ddot{H}}{H\dot{H}} - 2\epsilon. \tag{1.15}$$

They can be related to the kinetic energy or the relative variation of the potential in the slow-roll approximation as

$$\epsilon_{SR} = \frac{M_P^2}{2}\frac{(\partial_\phi V)^2}{V^2}, \qquad \eta_{SR} = 4\epsilon_{SR} - M_P^2\frac{\partial_\phi^2 V}{V}. \tag{1.16}$$

Inflation takes place as long as these parameters lay around the slow-roll regime, $\epsilon \ll 1$, for which the expansion is approximately exponential. Once it is close to the value of 1, the inflationary era is finished.

2. THE STRUCTURE OF THE UNIVERSE

A closer look to the Universe makes it necessary to take the inhomogeneities and anisotropies into account. The FLRW geometry will act as a background for a Universe with more complex details, being introduced in a perturbative manner. The ADM decomposition, introduced in [11], will prove to be very convenient for the task. We will present the most relevant concepts and the main geometric objects of interest, namely the 3-metric, the shift and lapse functions, and the curvature tensors. This formalism is also widely used in the context of Quantum Gravity and in numerical Relativity (e.g, see [16], chapter 21.7).

2.1. The Foliation of Space-Time: ADM Decomposition

The 4 dimensional spacetime, ${}^4\mathcal{M}$, is split into spatial sub-manifolds parametrized by each instant of time, ${}^3\mathcal{M}_t$, as ${}^4\mathcal{M} \to {}^3\mathcal{M}_t \times \mathbb{R}$. The line element is decomposed as

$$dx^\mu = \frac{\partial x^\mu}{\partial t}dt + dx^i = (Nn^\mu + N^i)dt + dx^i, \tag{2.1}$$

with n^μ a normal vector field to ${}^3\mathcal{M}_t$ and N, N^i the *lapse* and *shift* functions. Then, the decomposition of the metric follows as

$$ds^2 = -N^2dt^2 + \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j). \tag{2.2}$$

Here we have defined the 3-metric γ_{ij}, which is no longer diagonal and less symmetric than the FLRW ones. It will be very convenient for the study of the geometry of the spatial hypersurfaces and for the introduction of perturbations in the inhomogeneous Universe. In terms of the 4-metric and the normal vector field, it takes the form $\gamma_{ij} = g_{ij} + n_i n_j$

The curvature will also be encoded into two parts; an intrinsic and an extrinsic one. Both of them are settled by a 3-connection, compatible with the 3-metric and related to the 4-connection,

$$^{(3)}\nabla\gamma_{ij} = 0, \qquad ^{(3)}\nabla T^{i_1,\dots i_r}_{j_1,\dots j_s} = \gamma^{i_1}{}_{a_1}\dots\gamma^{i_r}{}_{a_r}\,\gamma_{j_1}{}^{b_1}\dots\gamma_{j_s}{}^{b_s}\,\gamma\left(\nabla T^{a_1,\dots a_r}_{b_1,\dots b_s}\right), \tag{2.3}$$

for any generic (r, s) rank tensor field well defined over the manifold. Then, the 3-Riemann tensor, ${}^{(3)}R_{ikjl}$, arises from the non-commutativity of this 3-connection. The contractions with the 3-metric give the 3-Ricci-curvature scalar

$$^{(3)}R = \gamma^{ij}\gamma^{kl}\,{}^{(3)}R_{ikjl}, \tag{2.4}$$

The extrinsic curvature tensor $K_{\mu\nu}$ is defined as the covariant derivative of the normal vector field,

$$K_{\mu\nu} := {}^{(3)}\nabla_\mu n_\nu. \tag{2.5}$$

This tensor, and its trace $K := g^{\mu\nu}K_{\mu\nu}$, capture the effect of curvature in the components that depart from the hypersurfaces Σ, and cannot be captured by intrinsic objects. From the projection of the Riemann tensor and contractions with normal vector fields, three key identities for the construction of a Lagrangian in the ADM formalism are obtained: the Gauss and Codazzi equations, which read

$$\gamma_\mu{}^\rho\gamma_\nu{}^\sigma\gamma_\lambda{}^\tau\ R^\alpha{}_{\rho\sigma\tau} = {}^{(3)}R^\alpha{}_{\mu\nu\lambda} + K_{\mu\lambda}K_\nu{}^\alpha - K_{\nu\lambda}K_\mu{}^\alpha, \tag{2.6}$$

$$\gamma_\mu{}^\rho\gamma_\nu{}^\sigma\gamma_\lambda{}^\tau\ R^\alpha{}_{\rho\sigma\tau}n_\alpha = 2\ {}^{(3)}\nabla_{[\mu}K_{\nu]\lambda}. \tag{2.7}$$

We can use this two identities in order to decompose the Ricci scalar in terms of the 3-Ricci scalar and the extrinsic curvature, as

$$R = {}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2, \tag{2.8}$$

up to the addition of a total derivative, which we shall omit.[5]

2.2. Perturbations Over Isotropic and Homogeneous Background

We will keep the study from the inflaton as a real scalar field. However, it is convenient to describe the geometry of gravity in terms of the ADM decomposition. Regarding the split of the metric (2.3) and that of the scalar curvature (2.8), the action (1.11) reads

$$S = \frac{1}{2}\int d^4x\sqrt{\gamma}N\Big[M_P^2\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right) + \dots$$
$$\dots + \left(\frac{1}{N^2}(\dot{\phi} - N^i\partial_i\phi)^2 - \partial^i\phi\partial_i\phi\right) - 2V(\phi)\Big]. \tag{2.9}$$

What is the strategy here? As it is the usual way in General Relativity using this formalism, the lapse and shift functions are treated as Lagrange multipliers. The variation of the action with respect to this quantities generates a system of (*energy* and *momentum*) constraint equations,

$${}^{(3)}R - K_{ij}K^{ij} + K^2 = 0\ , \qquad \nabla_i[K^i_j - \delta^i_j K] = 0. \tag{2.10}$$

The lapse and shift functions are expanded perturbatively around the background values $N^{(0)} = 1$, $N^{i(0)} = 0$, and so the constraint equations can be solved up to the desired order. Additionally, the shift function is split into irrotational, $\partial_i\Phi^{(n)}$, and divergence-less, $\hat{N}_i^{(n)}$, components,

$$N_i{}^{(n)} = \partial_i\Phi^{(n)} + \hat{N}_i{}^{(n)}. \tag{2.11}$$

The decomposition following Helmholtz's Theorem is generalized to the components of the 3-metric through two scalar functions ζ and E, a divergence-less vector F_i and a transverse, traceless tensor h_{ij};

$$\gamma_{ij} = a^2e^{2\zeta}\Big(\eta_{ij} + (\nabla_i\nabla_j - \frac{1}{3}\eta_{ij}\nabla^2)E + 2\nabla_{(i}F_{j)}\Big), \tag{2.12}$$

with η_{ij} the (spatial part of the) Minkowski metric.

[5] We wish to rewrite the action for the inflaton field in terms of this geometry. The addition of a total derivative, given the proper boundary conditions, produces no effect in the equations of motion.

2.3. Scalar Perturbation

General Relativity is an invariant theory under time diffeomorphisms, $t \to t + \delta t$. However, the induced variations in the scalar fields, $\delta\zeta$ and $\delta\phi$, must satisfy a gauge relation in order to keep the invariance. In fact, the variation of the ζ field is given by the Hubble parameter, as $\zeta \to \zeta' = \zeta + \frac{\delta a}{a} = \zeta + H\delta t$. In this work we will resort to Maldacena's gauge, or comoving gauge [12]. In this gauge E, F_i and $\delta\phi$ are set to zero, so that the 3-metric is reduced to

$$\gamma_{ij} = a^2 e^{2\zeta}(\eta_{ij} + h_{ij}). \tag{2.13}$$

The main advantage of this gauge comes from the fact that the scalar and the tensor factors of the metric do not mix through the perturbative expansion, at first order. This way, the study of each factor can be performed separately. Let us begin with the scalar perturbations, for which we can set the 3-metric as

$$\gamma_{ij} = a^2 e^{2\zeta}\eta_{ij}. \tag{2.14}$$

After being placed in the constraint equations (2.10), and neglecting all quadratic terms in the expansion of the lapse and shift functions (2.11), the defined quantities are solved as

$$\Phi^{(1)} = -\frac{\zeta}{H} + a^2\frac{\dot{\phi}^2}{H^2}\psi \ , \qquad N^{(1)} = \frac{\dot{\zeta}}{H} \ , \qquad \hat{N}_i^{(1)} = 0, \tag{2.15}$$

in terms of the perturbation ζ, where the function ψ is the formal solution to $\partial^2\psi = \dot{\zeta}$. This classical treatment of the perturbations culminates after introducing these solutions back to the Lagrangian (2.9), always keeping the terms up to second order in the ζ field. Integration by parts, supplied by the usual vanishing boundary conditions, gives the quadratic action

$$S_\zeta = M_P^2 \int d^4x \ a^3\epsilon\left(\dot{\zeta}^2 - a^{-2}(\partial_i\zeta)^2\right). \tag{2.16}$$

The slow-roll approximation is implied in the ϵ suppressing factor. This action resembles that of a massive, free, real scalar field. In fact, it can be recast into a more familiar form, by using the conformal time (1.6), $dt \to a(\tau)d\tau$, and defining the auxiliary field $\varphi := z\zeta$, after which

$$S_\varphi = \int d\tau d^3x\Big((\partial_\tau\varphi)^2 - (\partial_i\varphi)^2 - m^2(\tau)\varphi^2\Big), \tag{2.17}$$

We have ended up with a time-dependent mass $m^2(\tau) := -\frac{1}{z}\frac{d^2z}{d\tau^2}$, being $z := a\sqrt{2\epsilon}$ the *Mukhanov-Sasaki variable*.

2.3.1. Quantization of Scalar Perturbations

We have built the scalar perturbations by adding slight variations to the homogeneous and isotropic picture of the Universe. So far, we have considered a classical field, which is now to be quantized, by promoting it to a quantum operator, and defining a vacuum state $|0\rangle$. The dynamics of the field occur near its vacuum state, with the creation and annihilation of particles around it. To the order considered in the theory, we have an almost free field, which behaves with Gaussian statistics. All the statistical correlations between distant points in space (or in the momentum configuration) can be obtained or are directly related to the power spectrum/ two-point functions.

The φ field is quantized in the canonical manner, by upgrading it to the corresponding operator $\hat{\varphi}$, with its quantized conjugated canonical momentum $\hat{\pi} = \dfrac{\partial \mathcal{L}}{\partial(\partial_\tau \varphi)}$ and the canonical commutation relation $[\hat{\varphi}(\tau, \vec{x}), \hat{\pi}(\tau, \vec{y})] = i\delta(\vec{x} - \vec{y})$. It is easier to work in the momentum space, by Fourier-expanding $\hat{\varphi}$,

$$\hat{\varphi}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left(e^{i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} \varphi_{\vec{k}}(\tau) + e^{-i\vec{k}\cdot\vec{x}} \hat{a}^\dagger_{\vec{k}} \varphi^*_{\vec{k}}(\tau) \right), \tag{2.18}$$

where $\hat{a}_{\vec{k}}$ and $\hat{a}^\dagger_{\vec{k}}$ are the bosonic annihilation and creation operators. In Minkowski spacetime, the Fourier modes contribute to form the plane waves $e^{ik^\mu x_\mu}$, but here they satisfy a time-dependent 'harmonic' oscillator equation, namely the Mukhanov-Sasaki equation [17, 18],

$$\left(\frac{d^2}{d\tau^2} + \omega_k^2(\tau) \right) \varphi_{\vec{k}} = 0, \quad \text{where} \quad \omega_k^2(\tau) = k^2 + m^2(\tau). \tag{2.19}$$

The vacuum state is the lowest energy solution to this equation. But we can build different basis of solutions, $\varphi_{\vec{k}}$ or $\tilde{\varphi}_{\vec{k}}$, and so the associated vacuum states are different. We will choose the *Bunch-Davis* solution, which takes subhorizon modes (locally Minkowski), $|k\tau| \to \infty, k \gg \mathcal{H}$, as plane waves $\varphi_{\vec{k}} \sim \dfrac{1}{\sqrt{2k}} e^{-ik\tau}$. Then, the solution of (2.19) is

$$\varphi_k = \frac{z_{dS} H}{2^{1/2} k^{3/2}} e^{ik\tau} (1 + ik\tau), \tag{2.20}$$

for a de Sitter case $z_{dS} \sim -(H\tau)^{-1}$. We can now compute the power spectrum, in momentum space, as

$$\langle 0|\varphi_k \varphi_{k'}|0\rangle = |\varphi_k|^2 \delta^3(\vec{k} + \vec{k}') = \frac{z^2 H^2}{2k^3} (1 + k^2\tau^2) \delta^3(\vec{k} + \vec{k}') := P_\varphi(\vec{k} + \vec{k}'). \tag{2.21}$$

being P_φ the power spectrum. It is taken for the (quantized) perturbation $\hat{\zeta}$ by undoing the change of variables,

$$P_\zeta = \frac{1}{z^2} P_\varphi = \frac{H^2}{4 M_P^2 \epsilon k^3} (1 + k^2\tau^2), \tag{2.22}$$

which is evaluated at the horizon-crossing scale $k \sim \mathcal{H}$. The quantum fluctuations in the field ζ produce a time delay in the inflation end condition given by the Hubble flow parameter, which depends on the location. Thus, different places will end inflation at different instances of time. This effect produces the fluctuations in density that, due to gravitational tendency, will develop the more detailed structures of the Universe.

Connecting it with the observations for the correlations in the CMB, the dependence of the power spectrum in momentum space is parametrized as $k^{-3+(n_s-1)}$, with n_s known as the *scalar spectral tilt*. Given in terms of a logarithmic scale,

$$n_s = 1 + \frac{d \log(H^4/\dot{\phi}^2)}{d \log k}. \tag{2.23}$$

This parameter measures the deviation from a scalar-invariant spectrum, which behaves as $\sim k^3$,[6] and would correspond to a pure de-Sitter scenario. It can be related to the

[6] It is called invariant since the k^3 factor is compensated by moving back to space configuration.

Hubble flow parameters, taking $\frac{d\log H}{d\log k} = \epsilon(\epsilon-1)$ and $\frac{d\log\epsilon}{d\log k} = \eta(\epsilon-1)$, from (1.14) and the chain rule, $\frac{d}{d\log k} = \left(\frac{d\log\mathcal{H}}{dt}\right)^{-1}\frac{d}{dt} = (1-\epsilon)\frac{d}{dt}$. Therefore,

$$n_s = 1 - \frac{2\epsilon+\eta}{1-\epsilon} \sim 1 - 2\epsilon - \eta, \tag{2.24}$$

where in the last equality, we are assuming slow-roll conditions. This way, from the evolution of the background dynamics, the Hubble flow parameters can be determined, and given that, the power spectrum can be fitted through the spectral tilt.

2.4. Tensor Perturbations

It is now time to work with the other part of the perturbation in the 3-metric. Taking it as

$$\gamma_{ij} = a^2(\tau)(\eta_{ij} + h_{ij}), \tag{2.25}$$

we obtain a conformally flat metric, except from the addition of the perturbative term, which leads to the production of gravitational waves. The procedure is pretty similar to the scalar case, but now it is not even required to solve the constraint equations. Introducing the metric in the action (2.9), and keeping only the quadratic term (the gravitational wave action), we find

$$S_h = \frac{1}{2}\int d\tau d^3x\; a^2(\tau)\Big(\dot{h}_{nm}^2 - a^{-2}(\partial_i h_{nm})^2\Big). \tag{2.26}$$

This field is expanded in Fourier modes too, regarding two possible polarization states,

$$h_{nm}(x) = \sum_{s=1,2}\int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\; \varepsilon_{nm}^{(s)} h_{\vec{k}}^{(s)}(t). \tag{2.27}$$

with ε_{nm} the usual basis of polarization states[7] in gravitational wave physics. The EoM for the Fourier modes is similar to that of $\varphi_{\vec{k}}$, (2.19). And the power spectrum distribution is also analogous, but taking into account the correlation in the polarization too,

$$\langle 0|h_{\vec{k}}^{(s)} h_{\vec{k}'}^{(s')}|0\rangle = \frac{H^2}{2M_P^2 k^3}\delta^3(\vec{k}+\vec{k}')\delta_{s,s'} := P_h\delta^3(\vec{k}+\vec{k}')\delta_{s,s'}, \tag{2.28}$$

at subhorizon scales. A tilt in the gravitational wave spectrum can also be defined, in a similar manner as in the scalar case,

$$n_h = \frac{d\log H^2}{d\log k}, \tag{2.29}$$

which can again be rewritten by using the flow parameter ϵ as

$$n_t = \frac{-2\epsilon}{1-\epsilon} \sim -2\epsilon. \tag{2.30}$$

By convention, a scale invariant spectrum would correspond here to $n_t = 0$, contrasting to the scalar counterpart. This spectrum of primordial gravitational waves is, however, yet to be discovered.

Furthermore, from the two spectral tilts defined, we can build a well known observational parameter, the *scalar-to-tensor* ratio, which, as its name indicates, is the rate between the two power spectra

$$r = \frac{P_\zeta}{P_h} \sim 16\epsilon. \tag{2.31}$$

[7] The s index correspond to 1 for $(+)$ and 2 for $(\times)$ polarization.

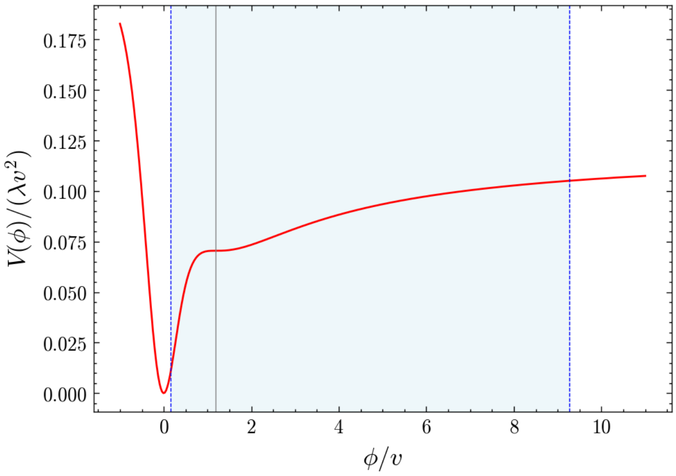

Figure 1: Single field scalar potential, in terms of the dimensionless field, and rescaled in terms of the parameters λ and v. The inflationary era takes place in the blue region. The gray line corresponds to a pseudo-inflection point.

2.5. Particular Model

Previously, we have worked with the slow-roll approximation, with a perturbative scheme so to solve the background equations of motion, and the second order Mukhanov-Sasaki equation in the scalar perturbation. In this section, we will present a given model for the inflaton potential. We will solve numerically the equations of motion, controlling the magnitude of the flow parameters, and solve the modes of the perturbation, which will give the power spectrum.

The first truly working model was designed and proposed by Starobinsky [19]. Ever since, a vast world of different proposals is available, awaiting for further observational constraints to discard all but, presumably, one of them. There even exists a huge Encyclopedia of inflationary models [3]. We will present here a potential which is highly recurrent in the literature dedicated to the search of Primordial Black Holes as a candidate of Dark Matter, in several mechanisms, such as Higgs inflation [20, 21]. It also appears in the context of a non-minimally coupled field with a potential in which higher order operators are included, which is redefined in terms of a canonical field in the Einstein frame (see [22]).

Let us present the simple family of rational potentials

$$V(\phi) = \frac{\alpha\phi^2 + \beta\phi^3 + \gamma\phi^4}{(1+\xi\phi^2)^2}, \tag{2.32}$$

represented in Figure 1. It can be reparametrized in terms of a dimensionless field $x = \phi/v$

$$V(x) = \frac{\lambda v^4}{12}\frac{6x^2 - 4ax^3 + 3x^4}{(1+bx^2)^2}, \tag{2.33}$$

with a redefinition $\alpha = \lambda v^2/2$, $\beta = -\lambda v a/3$, $\gamma = \lambda/4$, and $b = \xi v^2$. The main features of this potential come from its asymptotic flatness in the large field regime, and the appearance of an inflection point for some choices in the parameters. This inflection point will lead a period of evolution for the inflaton field where the slow-roll conditions are no longer satisfied, since the curvature of the potential is changing notably. The first and second derivatives of the potential V,

$$\partial_x V(x) = \frac{\lambda v^2}{3} \frac{3x - 3ax^2 + 3(1-b)x^3 + ab^2x^4}{(1+bx^2)^3},$$

$$\partial_x^2 V(x) = \frac{\lambda v^4}{3} \frac{3 + (3-8b)x^2 - 9(1-b)bx^4 - 2ax(3 - 8bx^2 + b^2x^4)}{(1+bx^2)^2}, \tag{2.34}$$

are involved when finding the inflection point, and must also be used to compute the flow parameters in the slow-roll approximation (1.16),

$$\epsilon_{SR} = \frac{M_P^2}{2} \frac{\partial_x V}{V} = \frac{8M_P^2}{v^2} \frac{(3 - 3ax + 3(1-b)x^2 + abx^3)^2}{x^2(6 - 4ax + 3x^2)^2(1+bx^2)^2},$$

$$\eta_{SR} = \epsilon_{SR} + \frac{M_P^2}{v^2} \frac{\partial_x^2 V}{V}. \tag{2.35}$$

They will be compared with the exact values obtained from the numerical integration of the equations of motion. It is convenient to rewrite them in terms of the number of e-folds instead of the comoving time. On the one hand, the exact background system is

$$\frac{d^2x}{dN^2} + 3\frac{dx}{dN} - \frac{v^2}{2M_P^2}\left(\frac{dx}{dN}\right)^3 + \left[3\frac{M_P^2}{v^2} - \left(\frac{dx}{dN}\right)^2\right] \frac{d\log V}{dx} = 0,$$

$$\frac{dH}{dN} = -\frac{v^2}{2M_P^2} H \left(\frac{dx}{dN}\right)^2, \tag{2.36}$$

with the constraint equation (the Friedman Equation)

$$H^2 = \frac{V(x)/v^2}{3\frac{M_P^2}{v^2} - \frac{1}{2}\left(\frac{dx}{dN}\right)^2}. \tag{2.37}$$

On the other hand, the SR evolution of the inflaton field can be solved in an integral form, giving the implicit relation

$$\int_{x_i}^{x} d\tilde{x}\, \frac{V(\tilde{x})}{\partial_x V(\tilde{x})} = \frac{M_P^2}{v^2} N. \tag{2.38}$$

We will take a similar choice in the parameters as in [23], with $\lambda = 3 \cdot 10^{-7}$, $v = \sqrt{0.108} M_P$, $a = 1$ and $b = b_c(1) - 10^{-4}$. The critical value $b_c(a)$ is that for which the potential V has an inflection point,

$$b_c(a) = 1 + \frac{a^2}{2}\left(\left(\frac{9}{2a^2} - 1\right)^{2/3} - 1\right). \tag{2.39}$$

A small deviation from b_c is set in order to avoid numerical instabilities. Taking $x(0) = 9.27$ as the starting point of inflation, the total duration of this era goes to $N_{end} = 56$ e-folds, when the condition $\epsilon \sim O(1)$ is satisfied. In Figure 2, the evolution of the Hubble parameters is displayed, while the dynamics of the field x is represented in Figure 3. Most of the trajectory, the field evolves with very low values in the ϵ parameter. However, the concavities in the potential makes it deviate from the flatness assumptions, and so the ϵ_{SR} deviates significantly from the exact value. We can see how the field spends much longer around the pseudo-inflection point $x_{ip} = 1.19$ in the slow-roll approximation. This discrepancy is even more noticeable in the η parameter, which departs from the slow-roll

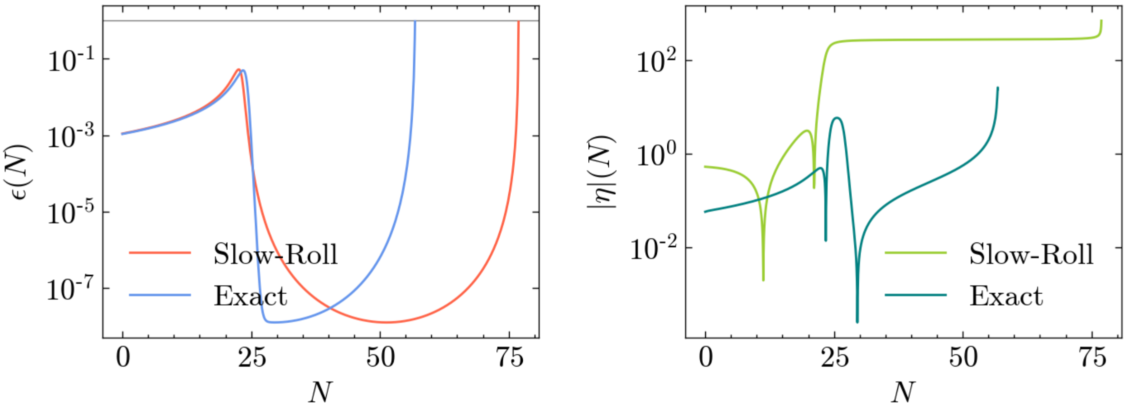

Figure 2: Left: first Hubble flow parameter. The dark gray line corresponds to the value of 1. Right: second Hubble flow parameter, in absolute value. Along the central region, which is separated by two non-smooth points, it takes negative values.

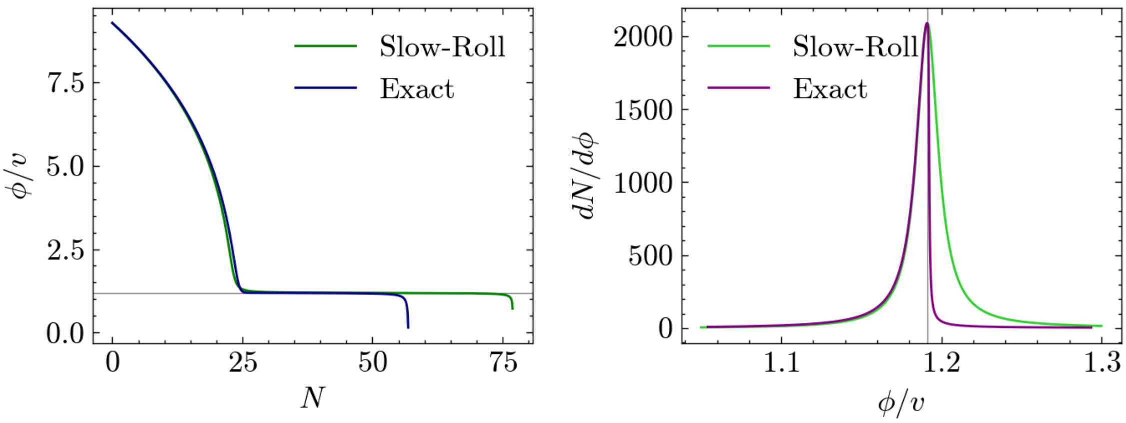

Figure 3: Left: evolution of the inflaton units in units of v, with respect to the number of e-folds. The slow-roll result is compared with the exact numerical computation. Right: Inverse of the derivative of the field vs. the field itself. This gives the amount of e-folds required to advance a unit in the field. In both figures, the dark gray line corresponds to the pseudo-inflection point.

value from the very beginning. The end of slow-roll, $\epsilon_{SR} \sim O(1)$, is met much later than in the exact approach. This means that either the inflationary era is overextended with this method, or it should start at a later point in the field.

Once the time (e-folds) dependence on the lowest order solution and also the $H(N)$, $\epsilon(N)$ and $\eta(N)$ functions are established, the Mukhanov-Sasaki equation is completely determined for each of the perturbation modes φ_k. Moving to e-folds parametrization,

$$\frac{d^2\varphi_k}{dN^2} + (1-\epsilon)\frac{d\varphi_k}{dN} + \left[\frac{k^2}{\mathcal{H}^2} - \left(2 - \epsilon + \frac{1}{2}\eta\right)\left(1 + \frac{1}{2}\eta\right) + \frac{d\eta}{dN}\right]\varphi_k = 0. \tag{2.40}$$

This equation is in fact a system of equations, for the real and imaginary part of each field mode φ_k. The equations are identical, but the imposition of the Bunch-Davies vacuum sets a difference in the initial conditions,

$$\varphi_{k,r}(0) = \frac{1}{\sqrt{2k}}, \qquad \frac{d\varphi_{k,r}}{dN}(0) = 0, \qquad \varphi_{k,i}(0) = 0, \qquad \frac{d\varphi_{k,i}}{dN}(0) = -\sqrt{\frac{k}{2}}, \tag{2.41}$$

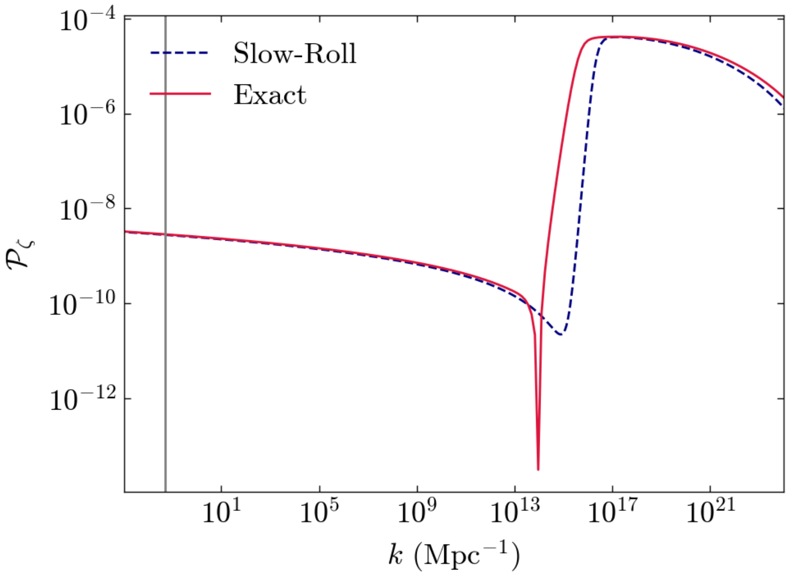

Figure 4: Power spectrum of the scalar perturbations, in momentum space. The slow-roll approximation is compared to the result from the numerical running of the Mukhanov-Sasaki equation. The gray vertical line marks the pivot value k_*.

where, obviously, we have set $\varphi_k = \varphi_{k,r} + i\varphi_{k,i}$. The equations begin with the subhorizon scales, $k \gg \mathcal{H}$, which for practical purposes will be set for $k = 100\mathcal{H}$. Let us follow the algorithm carefully detailed in [24]. The motion for the modes will end once the field have submerged into superhorizon scales $k \ll \mathcal{H}$, which will be taken in our numerical procedure as $k \sim 0.1\mathcal{H}$. The starting point for the running of the Mukhanov-Sasaki equation for each mode is set by solving the transcendental equation relating the wavenumbers and the comoving Hubble parameter. In order to determine a reference value in the scale factor $a(N)$, a pivot scale k_* is required, taking $k_* = 0.002$ Mpc^{-1} for Planck + BICEP/Keck.

In figure (4) we can compare the power spectrum predicted from the SR approximation and that from solving Mukhanov Sasaki. It can be seen that for the lowest modes the SR values are replicated, but for higher wavenumbers the exact power spectrum departs from it, and also presents a resonance at around $k \sim O(10^{14})$, which is a scale in which the inflaton is near the inflection point. Clearly, this behavior is absent in the slow-roll curve. After that, the exact power spectrum adopts larger values than in the slow-roll alternative. The scalar perturbations are then enhanced with respect to the slow-roll predictions in the larger scales (smaller distances).

Finally, we wish to connect our results to the observational evidence. The main two parameters are the spectral tilt n_s and the scalar-to-tensor ratio, as defined in (2.23 and (2.31). From the determination of the two Hubble-flow parameters, we have obtained a value of $n_s = 0.9621$, and $r = 0.0173$. The observational results from Planck's mission are depicted in Figure 5, and the point obtained here is also present. We can see that the numerical result obtained from our model fits well within the most likely region.

3. EFFECTIVE FIELD THEORY

The previous single-field description is highly dependent on the model for the potential, including the slow-roll suppression and validity, or the duration of inflation itself. Thus, it is necessary to check the compatibility of each model with observations, which may narrow

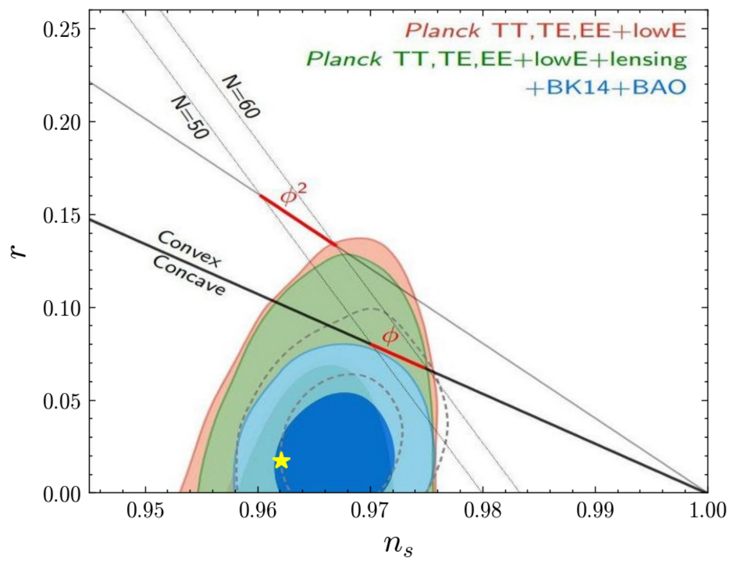

Figure 5: Scalar to tensor ratio against spectral tilt, to 1σ (68% CL) and 2σ(95% CL). The starred dot corresponds to our estimation. Base: Figure 28 of Planck 2018 results [13]. The inflation limits between 50 and 60 e-folds for a polynomial potential are included.

down the possibilities, one by one. The aim of this work is to introduce a more general framework which may allow to discard entire families of models which do not predict some ranges in the cosmological observables. This framework is an example of an *Effective Field Theory* (EFT). There are several ways to introduce this kind of theories in the context of inflation. We will be focusing in Goldstone Equivalence proposals, started with [25] and with further treatment in [26, 27, 28]. It proves to be very convenient so to expand the Lagrangian of the theory to third order and extract from it the contributions to the three point functions, and then finding the general behaviour of the non-Gaussianities (particular examples are provided in DBI and Ghost inflation). For the purpose of illustration, we will only be interested on setting the bridge between this theories and the single field approaches.

3.1. Most General Action

As we have witnessed in the previous sections, the inflation mechanism is a result of a time asymmetry in the end of inflation. In order to mimic this, we are interested on an EFT that works with an action invariant under space diffeomorphisms, and experiences a spontaneous symmetry breaking in time diffeomorphisms. This action is originated from the Einstein-Hilbert action, adding all terms consistent with the symmetries. The main building blocks of the Lagrangian density are the scalar quantities from g^{00} and the extrinsic curvature $K_{\mu\nu}$. Indeed, the action is expanded around their background values $(g^{00})^{(0)} = -1$ and $K^{(0)}_{\mu\nu} = a^2 H \gamma_{\mu\nu}$,

$$S_{EFT} = \int d^4x \sqrt{-g} \Big(\frac{M_P^2}{2} R + \sum_{n=0}^{\infty} \frac{M_n^4(t)}{n!} (\delta g^{00})^n - \frac{\tilde{M}_1^2(t)}{2} \delta g^{00} \delta K^\mu{}_\mu - \dots$$
$$\dots - \frac{\tilde{M}_2^2(t)}{2} (\delta K^\mu{}_\mu)^2 - \frac{\tilde{M}_3^2(t)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \dots \Big), \tag{3.1}$$

where the 'delta' quantities are the deviations in g^{00} and $K_{\mu\nu}$ with respect to their background values, respectively; and the $M_n(t)$, $\bar{M}_n$ are general smooth functions in time. The first two terms in the expansion with respect to g^{00} are involved in the background dynamical equations. We would like to recover the Friedmann Equations (1.4) with flat spatial sections as the lowest order theory. For that purpose, it is necessary to redefine the coefficients $M_0^4(t)$ and $M_1^4(t)$ in the form

$$M_0^4(t) = -M_P^2(3H^2 + 2\dot{H}), \qquad M_1^4(t) = M_P^2\dot{H}. \tag{3.2}$$

as can be checked from the explicit variation of (3.1), keeping the lowest order quantities. Thus, the action is rewritten as follows,

$$S_{EFT} = \int d^4x\sqrt{-g}\Big(\frac{M_P^2}{2}R + M_P^2H^2g^{00} - M_P^2(\dot{H} + 3H^2) + \sum_{n=2}^{\infty}\frac{M_n^4(t)}{n!}(\delta g^{00})^n - \dots$$
$$\dots - \frac{\tilde{M}_1^2(t)}{2}\delta g^{00}\delta K^{\mu}{}_{\mu} - \frac{\tilde{M}_2^2(t)}{2}(\delta K^{\mu}{}_{\mu})^2 - \frac{\tilde{M}_3^2(t)}{2}\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} + \dots\Big). \tag{3.3}$$

No further conditions are applied over the rest of the coefficients so far. This Lagrangian also includes a power expansion in contractions of the variation term $\delta R_{\mu\nu\lambda\sigma}$, which is that of the Riemann tensor with respect to its background solution, and is invariant under space-time diffeomorphisms too.

3.2. The Goldstone Boson

The broken symmetry in (3.3) is recovered by introducing a Goldstone boson field π with a gauge transformation. For a diffeomorphism $t \to t' = t + \xi^0(t,\vec{x})$, $\vec{x} \to \vec{x}' = \vec{x}$, we have the common tensor transformation in g^{00},

$$g^{00} \to g'^{00} = \frac{\partial x'^0}{\partial x^\mu}\frac{\partial x'^0}{\partial x^\nu}g^{\mu\nu}, \quad \text{and so} \quad g^{00} = \frac{\partial x^0}{\partial x'^\mu}\frac{\partial x^0}{\partial x'^\nu}g'^{\mu\nu}, \tag{3.4}$$

with $x^0 = t = t' - \xi^0$. Then, the Goldstone boson is introduced via the change $\xi^0(x) \to \xi'^0 = -\pi(x)$. This field must obey the gauge transformation rule $\pi \to \pi'(x'(x)) = \pi(x) - \xi^0(x)$. Up to this point, we will only be interested in the expansion terms for the '0-th' metric component in 3.3. We discard those for extrinsic curvature, and after having considered the time diffeomorphism and the Goldstone pion π, the action takes the form

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}R - M_P^2\left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \dots$$
$$\dots + M_P^2\dot{H}(t+\pi)\left((1+\dot{\pi})^2g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + \partial_i\pi\partial^i\pi\right) + \dots$$
$$\dots + \sum_{n=2}^{\infty}\frac{M_n^4(t+\pi)}{n!}\left((1+\dot{\pi})^2g^{00} + 2(1+\dot{\pi})\partial_i\pi g^{0i} + \partial_i\pi\partial^i\pi + 1\right)^n\Big]. \tag{3.5}$$

The tilde notation has been dropped as the element of volume is invariant under all diffeomorphisms, and the variables of integration become dummy.

Not even the whole displayed action will play in the analysis. The aim is to study low energy perturbations, with respect to the high energy scales provided by the Planck's Mass. In this regime, we can work with a finite number of terms, depending on the highest order required for each task. The precise suppressing parameters are given by a ratio between the coefficients M_n^4 and Planck's Mass, being $\dfrac{M^2(t)}{M_P}$ the leading coefficient.

A decreasing behavior in the higher order coefficients is natural, and thus the scales $\frac{M_n^2(t)}{M_P}$ will be less and less relevant for increasing n. Stopping the expansion after second order terms in derivatives of π, the remaining contributions to the action (3.5) are left as

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}R - M_P^2(3H^2+\dot{H}) + M_P^2\dot{H}\left(a^{-2}(\partial_i\pi)^2 - 1 - 2\dot{\pi} - \dot{\pi}^2\right) + 2M_2^4\dot{\pi}^2\Big]. \tag{3.6}$$

The use of the background values for the metric components, $(g^{00})^{(0)} = -1$, $(g^{0i})^{(0)} = 0$, $(g^{ii})^{(0)} = a^{-2}$, is implicit in the presented form of the action. Now that we have expressed the action for the Goldstone boson to desired order,it will allow to compute the cosmological observables. Furthermore, it will be connected to the single field procedure presented in the previous sections.

3.3. Effective Field Theory and Single Field Inflation

As we have seen, in the infrared limits of the theory, the bosonic field π is decoupled from gravity. This is analogous to the decomposition in the 3-metric carried out in the single-field framework, into two components of the perturbation, which were treated separately. Under time diffeomorphisms, the scalar perturbation changes as $\zeta \to \zeta' = \zeta + \frac{da}{a} = \zeta - H\delta t$. Thus, we naturally associate the two fields through

$$\zeta(x) = -H(t)\pi(x). \tag{3.7}$$

This relation fixes the relation with the Goldstone particle. We are now interested on computing the correlations in the Goldstone field. Using, again, a Fourier decomposition, with modes $\pi_{\vec{k}}$, the power spectrum is then

$$\langle 0|\pi_{\vec{k}}\pi_{\vec{k}'}|0\rangle = \frac{H^2}{2M_P^2\dot{H}k^3}\delta^3(\vec{k}+\vec{k}'). \tag{3.8}$$

The Hubble parameter H and its derivative are evaluated at superhorizon scale, as was done before. It is also clear that the form for the two-point function is similar to that in (2.22). Indeed, from (3.7), the power spectrum of the slow-roll scalar perturbations is recovered.

$$\langle 0|\zeta_{\vec{k}}\zeta_{\vec{k}'}|0\rangle = \frac{H^4}{2M_P^2\dot{H}k^3}\delta^3(\vec{k}+\vec{k}') = \frac{H^2}{4M_P^2\epsilon k^3}\delta^3(\vec{k}+\vec{k}'). \tag{3.9}$$

It can be connected again with observations through the tilt n_s defined in (2.23). Then, the deviation from a scale-invariant scale is obtained as

$$n_s = 1 + \frac{d\log(H^4/\dot{H})}{d\log(k)}. \tag{3.10}$$

which takes the same form as in (2.23) in the slow-roll regime.

3.4. Effect of the Extrinsic Curvature

We have witnessed the link between the scalar perturbations from single field formalism and those coming from the low order dynamics of the Goldstone boson. Until this point, the terms in (3.1) involving the extrinsic curvature have been ignored. However, they shall take part in the production of gravitational waves. The ADM decomposition of the

Einstein-Hilbert action leads to the quadratic action for the gravitational waves recorded in (2.26). But including now the contributions from the operators in $\delta K^{\mu}_{\ \nu}$ a correction is induced in the kinetic part of h_{ij}, so that the quadratic action reads

$$S_h = \frac{M_P^2}{8} \int d^4x \sqrt{-g} \left[\left(1 - \frac{\hat{M}_3^4}{M_P^2} \right) \dot{h}_{nm}^2 - a^{-2} (\partial_i h_{nm})^2 \right]. \tag{3.11}$$

This correction is small, as far as the energy regime is far from the Planck's scale, i.e, $M_P \gg \hat{M}_n^2$. Thus, the single field wave fluctuations are also recovered, adding a small deviation coming from the effect of higher order operators from the building of a more general action. From this, a momentum power spectrum for h_{nm} is computed exactly the same way, but defining an auxiliary field $\hat{h}_{\vec{k}} = \sqrt{1 - \frac{\hat{M}_3^2}{M_P^2}} h_{\vec{k}}$, and where the scale factor appeared in (2.28), it is now modified by the same multiplying factor, thus leaving

$$\langle 0 | h_{\vec{k}}^{(s)} h_{\vec{k}'}^{(s')} | 0 \rangle = \left(1 - \frac{\hat{M}_3^2}{M_P^2} \right)^{-1} \frac{H^2}{2M_P^2 k^3} \delta^3(\vec{k} + \vec{k}') \delta_{s,s'} \sim \left(1 + \frac{\hat{M}_3^2}{M_P^2} \right) \frac{H^2}{2M_P^2 k^3} \delta^3(\vec{k} + \vec{k}') \delta_{s,s'}. \tag{3.12}$$

It is then clear that the gravitational wave correlations are recovered with a small correction with a suppressing factor $\frac{\hat{M}_3^2}{M_P^2} \ll 1$, again in the infrared regime.

4. CONCLUSION

Summarizing, we have shown that inflation sets an attractor solution towards the conditions for the Universe that can be observed, both regarding its seemingly (not if inflation took place) unnatural flatness and thermally connected background radiation. If the exponential expansion of the length scales is sufficiently rapid, and the inflationary era lasts long enough (around 60 e-folds), the considered problems in the ΛCDM are solved.

We have constructed the perturbative scheme for both the scalar and tensor higher order factors in the metric, making use of the strong tools that have been developed in the literature for this goal. These perturbations have been quantized to yield the primordial density fluctuations which are expanded and imprinted in the CMB patterns. Using a particular potential, the slow-roll dynamics have been tested against a more precise calculation. We have compared the computed observational parameters to the actual values found in extensive observational research, being a success for a good choice in the parameters of the model. A 56 e-folds duration of inflation has been found, which is close to the 60 e-folds expectation.

Lastly, we have set the production of a Goldstone boson in an Effective Field Theory, using the most general operators. This Goldstone boson is linked to the scalar perturbation field, reaching equivalent conclusions in the second order dynamics as found in the Mukhanov-Sasaki schematics. Furthermore, we have obtained an analogous spectrum for gravitational waves from the higher order operators in the EFT.

Overall, this thesis provides a solid foundation for understanding the theoretical basis of cosmological inflation and motivates further study in the field. The richness of current research developments and the abundance of open questions make it appealing for a continued exploration in the early Universe cosmology.

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