Dynamic programming: an alternative approach to light propagation in arbitrary optical media

Calvo, Maria L.

Maria L. Calvo, "Dynamic programming: an alternative approach to light propagation in arbitrary optical media," Proc. SPIE 2778, 17th Congress of the International Commission for Optics: Optics for Science and New Technology, 277841 (1 September 1996); doi: 10.1117/12.2299027

Event: 17th Congress of the International Commission for Optics: Optics for Science and New Technology, 1996, Taejon, Korea, Republic of

DYNAMIC PROGRAMMING: AN ALTERNATIVE APPROACH TO LIGHT PROPAGATION IN ARBITRARY OPTICAL MEDIA

Maria L. Calvo* and Vasudevan Lakshminarayanan**

*: Departamento de Optica, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain.

**: School of Optometry and Dept. of Physics & Astronomy, University of Missouri-St.Louis, St. Louis, MO 63121, USA.

ABSTRACT

We apply techniques of optimal control theory namely, the dynamic programming to the problem of light propagation in optical waveguides. The formulation is equivalent to the resolution of an eikonal equation. We illustrate this optimization technique for a gradient profile optical waveguide, i.e., an ideal parabolic refractive index profile distribution. We discuss the possibility of extending this procedure to other type of optical waveguides and optical media and implications in optical design.

1. INTRODUCTION

Optimization techniques applied to light propagation phenomena in optical media are at present a relatively new area of investigation. The methodology concerns optimized design of arbitrary optical media, and, "a priori" definition of significant parameters. The problem of characterizing the output of a system from input partial data is defined as a control theory . It involves a large field of applications including optical system design, refractive index distribution, laser cavities, laser beam modification, aberrations simplified models among the most relevant. Currently, optimization techniques applied to waveguide design are based upon variational analysis connected with the Sturme-Liouville theory and the Rayleigh-Ritz method. They are alternative simpler procedures to the well known ray tracing technique. Searching of new simplified methods leads to establishing light propagation phenomena in inhomogeneous media as a mechanical model. The dynamic properties in refraction phenomena are given in terms of two Hamiltonian equations. The solutions give both the vector position and the tangent to the trajectory. With the aim of finding simpler alternative models we have applied the concept of dynamic programming to light propagation in inhomogeneous media[1]. There is a former idea of this subject in the earlier paper due to Kalaba[2]. In it he demostrated that the eikonal equation may be derived from Fermat's principle of least time using Bellman's principle of optimality[3].

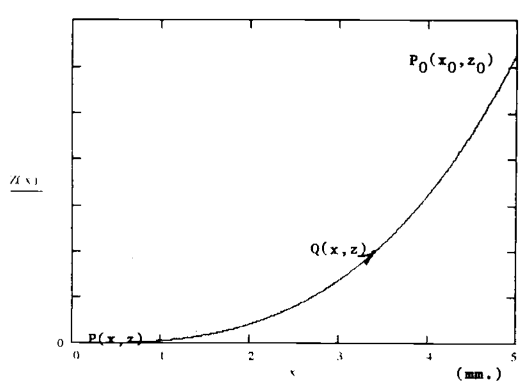

Fig.1. Arbitrary trajectory z = z(x) from initial point P(x,z) up to final $P(x_0, z_0)$

2. BASIC IDEAS ON DYNAMIC PROGRAMMING

The mathematical framework to solve control process deals, in general, with the minimization of a quadratic fonctional of the form:

$$J(u,v) = \int^{T} dt\left[u^2(t) + v^2(t)\right] \qquad (1)$$

where: u'(t) = au(t) + v(t). a:const. u(0) = constant. Solutions for Eq.(1) require defining a minimum pathway. Application of Fermat's principle gives eikonal equation.

17th Congress of the International Commission for Optics: Optics for Science and New Technology, edited by Joon-Sung Chang, Jai-Hyung Lee, ChangHee Nam, Proc. of SPIE Vol. 2778, 277841
© (1996) 2017 SPIE · CCC code: 0277-786X/17/$18 · doi: 10.1117/12.2299027

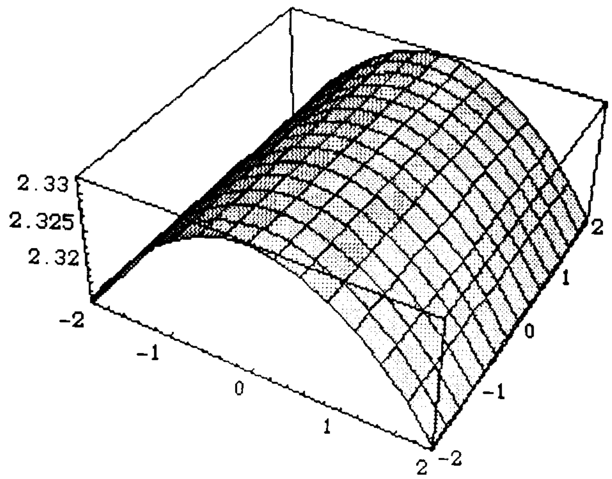

Fig. 2. Refractive index distribution as Eq.(3): p=2. n_0=1.527. Δ=0.0078.ρ=3mm.

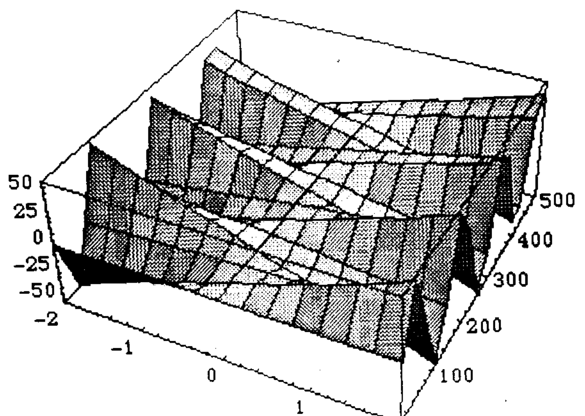

Fig.3. The 3-D representation of the trajectory as given by Eqs.(2)-(3).

4. CONCLUSIONS

Dynamic programming is a powerful technique for solving optimization problems. We demonstrate that the mathematical procedure is equivalent to solving an eikonal-type equation. It seems to be useful for optimizing optical design.

5. REFERENCES

1. M.L. Calvo and V. Lakshminarayanan, "Light propagation in optical waveguides: a dynamic approach", submitted to J.Opt.Soc.Am.A (1996)

2. R. Kalaba, "Dynamic programming, Fermat's principle, and the eikonal equation", J. Opt. Soc. Am., **51**, 1150-1151 (1961).

3. R.E. Bellman, *Dynamic Programming*, (Princeton University Press, Princeton, New Jersey, 1975).

This one is obtained applying the condition that the slope of the trajectory be a minimum:

$$\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial z}\right)^2 = \frac{n^2(x,z)}{c^2} \qquad (2)$$

where n(x,z): arbitrary refractive index distribution. c: speed of light in free space. Eq.(2) is the key equation. In general it has no explicit solution. This one depends on n(x,z). Solutions t = t(x,z) are "geometrical time surfaces".

3. GRADIENT INDEX OPTICAL WAVEGUIDES

Consider an arbitrary refractive index profile:

$$n^2(x) = n_0^2\left\{1 - 2\Delta\left(\frac{x}{\rho}\right)^p\right\} \qquad (3)$$

with $p \geq 1$, Δ: height of the profile, ρ: half-widht, n_0: refractive index of the core. Fig.2 gives the profile. Eq.(2) according to Eq.(3) is a Euler-Lagrange equation. Solution gives ray trajectory z = z(x), (see Fig 3) and the time minimizing the longitudinal trajectory: t = t(z). This one turns out to coincide with the ray transit time as predicted by geometrical optics approximation. Similarly one obtains: t = t(x). The slope $\partial t / \partial x$ fits the optimum profile for the transit time or equalization. This result is given in Fig.4.

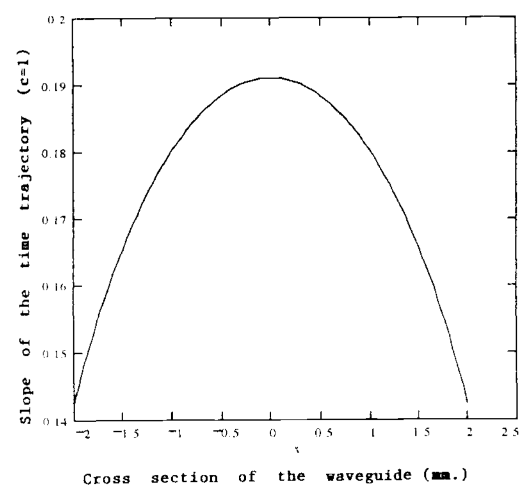

Fig.4. Slope of the time variation. It indicates how the time compensates across transverse section. Minimum equalization corresponds to waveguide axis. Maximum equalization to waveguide boundary (maximum local speed).