Radiative corrections in the Zeeman effect of $2\,^3P$ states of helium

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Radiative corrections using third-order perturbation theory are considered in a theoretical calculation of the Zeeman effect. The corrections have been calculated for the states $2\,^3P$ of helium, and found to be of the same order as other radiative corrections calculated by Anthony and Sebastian [Phys. Rev. A **48**, 3792 (1993)]. [S1050-2947(97)04411-9]

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I. INTRODUCTION

In order to resolve the long-standing discrepancy between theory and experiment for the Zeeman coupling factor g_L' of the atomic $2\,^3P_J$ states of helium [1], high-precision calculations, including relativistic corrections, have been done recently by several authors [2,3]. Their results for $\delta g_L = g_L' - g_L$ with $g_L = 1 - m_e/m_N$ are 10.6×10^{-6}, 8.838×10^{-6}, and 10.719×10^{-6} in Refs. [1], [2], and [3] respectively, which should be compared with the best experimental result [4] $\delta g_L = 4.9(2.9)\times10^{-6}$.

Anthony and Sebastian [2], extending the work of other authors [5,6], also included radiative corrections, and found by means of an accurate calculation that these corrections provide a contribution of 1.79×10^{-7} to the orbital g_L' factor, this contribution being too small to resolve the discrepancy. Such corrections come from the term proportional to $e^2\vec{B}_0 \times \vec{r}\cdot\vec{A}_{\rm vac}$, which is part of $e^2\vec{A}^2$ with $\vec{A} = (1/2)\vec{B}_0\times\vec{r} + \vec{A}_{\rm vac}$, where $\vec{B}_0$ is the external magnetic field and $\vec{A}_{\rm vac}$ is the potential vector of the vacuum field.

The purpose of this paper is to show that there are other radiative corrections, not considered in previous work, coming from terms containing $e\vec{p}\cdot\vec{A} = (1/2)e\vec{B}_0\times\vec{r}\cdot\vec{p} + e\vec{p}\cdot\vec{A}_{\rm vac}$, in third order of perturbation, but of the same order as those mentioned above [2]. It is also shown that the radiative corrections coming from $(-e/m)\vec{S}\cdot\vec{B}_{\rm vac}$ are zero or negligible.

II. THEORY

We consider the Zeeman effect of the helium atom adopting the LS coupling scheme. The Zeeman Hamiltonian is given by

$$H_Z = -\frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}_0, \tag{1}$$

where $e=-|e|$, $\vec{B}_0$ is the external magnetic field, and the values for the orbital and spin gyromagnetic factors are assumed, with small error for the present calculations, to be 1 and 2, respectively. We consider the total Hamiltonian

$$H = H_0 + H_{\rm rad} + H_Z + H_A + H_B, \tag{2}$$

where H_0 and $H_{\rm rad}$ are the Hamiltonians of the free atom and free radiation, respectively. The third term is the Zeeman Hamiltonian (1), and H_A and H_B account for the radiative corrections originating from the interaction of the two electrons of the atom with the vacuum field,

$$H_A \equiv -\frac{e}{m}\,\vec{p}\cdot\vec{A}_{\rm vac}, \quad H_B \equiv -\frac{e}{m}\,\vec{S}\cdot\vec{B}_{\rm vac}, \tag{3}$$

where the scalar products are in a six-dimensional space taking into account the two electrons denoted by the indexes 1, and 2, that is,

$$\vec{p}\cdot\vec{A}_{\rm vac} \equiv \vec{p}_1\cdot\vec{A}_{\rm vac}(\vec{r}_1) + \vec{p}_2\cdot\vec{A}_{\rm vac}(\vec{r}_2),$$

$$\vec{S}\cdot\vec{B}_{\rm vac} \equiv \vec{s}_1\cdot\vec{B}_{\rm vac}(\vec{r}_1) + \vec{s}_2\cdot\vec{B}_{vac}(\vec{r}_2), \tag{4}$$

where $\vec{B}_{\rm vac}(\vec{r}_i)$ is the vacuum magnetic field and $\vec{A}_{\rm vac}(\vec{r}_i)$ is the potential vector of the vacuum field,

$$\vec{A}_{\rm vac}(\vec{r}_i) = \sum_{\vec{k},\vec{\epsilon}} \left(\frac{\hbar}{2\omega\epsilon_0 V}\right)^{1/2} \vec{\epsilon}\,(e^{i\vec{k}\cdot\vec{r}_i}a_{\vec{k},\vec{\epsilon}} + e^{-i\vec{k}\cdot\vec{r}_i}a^{+}_{\vec{k},\vec{\epsilon}}), \tag{5}$$

expressed as a plane-wave expansion.

We use a perturbative treatment in which $H' \equiv H_Z + H_A + H_B$ is considered a perturbation to $H_0 + H_{\rm rad}$, and we calculate the third order-energy correction. If the unperturbed state is the atomic state ψ with energy E_0 and radiation state of zero photons, $|0\rangle$, the energy correction is

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$$\Delta E^{(3)}=\sum_{n,\vec{k},\vec{\epsilon}}\sum_{n',\vec{k}',\vec{\epsilon}'}\frac{\langle\psi;0|H'|n;\vec{k},\vec{\epsilon}\rangle\langle n;\vec{k},\vec{\epsilon}|H'|n';\vec{k}',\vec{\epsilon}'\rangle\langle n';\vec{k}',\vec{\epsilon}'|H'|\psi;0\rangle}{(E_n-E_0+\hbar\omega)(E_{n'}-E_0+\hbar\omega')}$$
$$-\sum_{n,\vec{k},\vec{\epsilon}}\frac{\langle\psi;0|H'|n;\vec{k},\vec{\epsilon}\rangle\langle n;\vec{k},\vec{\epsilon}|H'|\psi;0\rangle}{(E_n-E_0+\hbar\omega)^2}\langle\psi;0|H'|\psi;0\rangle, \quad (6)$$

where $|n\rangle$ and $|n'\rangle$ denote any atomic eigenstate of H_0, J^2, L^2, S^2, and J_z; $|\vec{k},\vec{\epsilon}\rangle$ and $|\vec{k}',\vec{\epsilon}'\rangle$ denote single photon states of wave vectors $\vec{k}$ and $\vec{k}'$ and polarizations $\vec{\epsilon}$ and $\vec{\epsilon}'$ with frequencies $\omega=ck$ and $\omega'=ck'$ respectively. Several contributions appear in Eq. (6) of the form

$$E_{ijl}\equiv\sum_{n,\vec{k},\vec{\epsilon}}\sum_{n',\vec{k}',\vec{\epsilon}'}\frac{\langle\psi;0|H_i|n;\vec{k},\vec{\epsilon}\rangle\langle n;\vec{k},\vec{\epsilon}|H_j|n';\vec{k}',\vec{\epsilon}'\rangle\langle n';\vec{k}',\vec{\epsilon}'|H_l|\psi;0\rangle}{(E_n-E_0+\hbar\omega)(E_{n'}-E_0+\hbar\omega')}$$
$$-\sum_{n,\vec{k},\vec{\epsilon}}\frac{\langle\psi;0|H_i|n;\vec{k},\vec{\epsilon}\rangle\langle n;\vec{k},\vec{\epsilon}|H_l|\psi;0\rangle}{(E_n-E_0+\hbar\omega)^2}\langle\psi;0|Hj|\psi;0\rangle, \quad (7)$$

where each H_i, H_j, and H_l may be H_Z, H_A, and H_B (note that the sum labeled $\vec{k},\vec{\epsilon}$, may eventually contain the vacuum state).

Retaining in Eq. (6) only the terms proportional to e^3 and linear in B_0, we obtain

$$\Delta E^{(3)}=E_{AZA}+E_{BZB}+2\,\mathrm{Re}E_{AZB}+2\,\mathrm{Re}E_{AAZ}+2\,\mathrm{Re}E_{BAZ}+2\,\mathrm{Re}E_{ABZ}+2\,\mathrm{Re}E_{BBZ}, \quad (8)$$

where we used $E_{BZA}=E^*_{AZB}$, $E_{ZAA}=E^*_{AAZ}$, $E_{ZAB}=E^*_{BAZ}$, $E_{ZBA}=E^*_{ABZ}$, and $E_{ZBB}=E^*_{BBZ}$. The corresponding third-order corrections of the gyromagnetic factors $g^{(3)}$ are then obtained from $\Delta E^{(3)}=(-e\hbar B_0/2m)(g_L^{(3)}M_L+g_S^{(3)}M_S)=|\mu_B|B_0(g_L^{(3)}M_L+g_S^{(3)}M_S)$, where $|\mu_B|\equiv|e|\hbar/(2m)$.

The states $|\psi\rangle$ here considered are the following atomic states of helium:

$$|\psi_a\rangle\equiv|2^3P_2,M_J=2\rangle=|v,L=1,S=1,M_L=1,M_S=1\rangle, \quad (9)$$

$$|\psi_b\rangle\equiv|2^3P_1,M_J=1\rangle=|v,L=1,S=1\rangle\frac{1}{\sqrt{2}}(|M_L=0,M_S=1\rangle-|M_L=1,M_S=0\rangle), \quad (10)$$

where v denotes the electronic configuration. The calculations of the different terms of (8) are grouped according to their similarity in different sections.

III. CORRECTION FROM THE TERM E_{AZA}

The first term of Eq. (8) is given by

$$E_{AZA}\equiv-\frac{e^3B_0}{2m^3}\left(\sum_{n,\vec{k},\vec{\epsilon}}\sum_{n',\vec{k}',\vec{\epsilon}'}\frac{\langle\psi;0|\vec{p}\cdot\vec{A}_{\mathrm{vac}}|n;\vec{k},\vec{\epsilon}\rangle\langle n;\vec{k},\vec{\epsilon}|L_z+2S_z|n';\vec{k}',\vec{\epsilon}'\rangle\langle n';\vec{k}',\vec{\epsilon}'|\vec{p}\cdot\vec{A}_{\mathrm{vac}}|\psi;0\rangle}{(E_n-E_0+\hbar\omega)(E_{n'}-E_0+\hbar\omega')}\right.$$
$$\left.-\sum_{n,\vec{k},\vec{\epsilon}}\frac{|\langle\psi;0|\vec{p}\cdot\vec{A}_{\mathrm{vac}}|n;\vec{k},\vec{\epsilon}\rangle|^2\langle\psi;0|L_z+2S_z|\psi;0\rangle}{(E_n-E_0+\hbar\omega)^2}\right). \quad (11)$$

Expanding $\vec{A}_{\mathrm{vac}}$ as in Eq. (5), and setting $|\vec{k},\vec{\epsilon}\rangle=|\vec{k}',\vec{\epsilon}'\rangle$ since L_z+2S_z does not connect different radiation states, we obtain

$$E_{AZA}=G\int_0^\infty\omega d\omega\int d\Omega\left(\sum_{n,n',\vec{\epsilon}}\frac{\langle\psi|\sum_j\vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\langle n|L_z+2S_z|n'\rangle\langle n'|\sum_t\vec{p}_t\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle}{(\omega_n+\omega)(\omega_{n'}+\omega)}\right.$$
$$\left.-\sum_{n,\vec{\epsilon}}\frac{\left|\langle\psi|\sum_j\vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\right|^2\langle\psi|L_z+2S_z|\psi\rangle}{(\omega_n+\omega)^2}\right), \quad (12)$$

where $G\equiv-e^3B_0/(32\pi^3\epsilon_0m^3\hbar c^3)=\alpha|\mu_B|B_0/(4\pi^2\hbar m^2c^2)$ with $\alpha\equiv e^2/(4\pi\epsilon_0\hbar c)\approx\frac{1}{137}$; $d\Omega$ is the differential of the solid angle, the indexes $j=1$ and 2 and $t=1$ and 2 denote the two electrons, $\omega_n\equiv(E_n-E_0)/\hbar$, and $\omega_{n'}\equiv(E_{n'}-E_0)/\hbar$.

A. Low frequencies

Let us first consider the region of frequencies where the dipole approximation holds ($e^{i\vec{k}\cdot\vec{r}_j}\simeq 1$); that is, $\omega\leqslant\omega_c\equiv\alpha mc^2/\hbar$. To simplify the calculation we decompose the integral as follows. The function of ω in the first term of Eq. (12) can be written

$$\frac{\omega}{(\omega_n+\omega)(\omega_{n'}+\omega)}=\frac{1}{\omega_n+\omega}-\frac{\omega_{n'}}{(\omega_n+\omega)(\omega_{n'}+\omega)}, \tag{13}$$

and similarly, in the second term of Eq. (12),

$$\frac{\omega}{(\omega_n+\omega)^2}=\frac{1}{\omega_n+\omega}-\frac{\omega_{n'}}{(\omega_n+\omega)^2}. \tag{14}$$

Expression (12) can then be written as

$$E_{AZA}=E^{I}_{AZA}+E^{II}_{AZA}, \tag{15}$$

where

$$E^{I}_{AZA}\equiv G\int_0^{\omega_c}d\omega\int d\Omega\sum_{n,\vec{\epsilon}}\left(\frac{\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|(L_z+2S_z)\vec{p}\cdot\vec{\epsilon}|\psi\rangle-\langle\psi|L_z+2S_z|\psi\rangle|\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle|^2}{\omega_n+\omega}\right), \tag{16}$$

(the summation over n' has been performed), and

$$E^{II}_{AZA}\equiv G\int_0^{\omega_c}d\omega\int d\Omega\sum_{n,n',\vec{\epsilon}}\left(-\frac{\omega_{n'}\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|L_z+2S_z|n'\rangle\langle n'|\vec{p}\cdot\vec{\epsilon}|\psi\rangle}{(\omega_n+\omega)(\omega_{n'}+\omega)}+\frac{\omega_n\langle\psi|L_z+2S_z|\psi\rangle|\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle|^2}{(\omega_n+\omega)^2}\right). \tag{17}$$

Let us now analyze E^{I}_{AZA}. In order that $L_z+2S_z=J_z+S_z$ acts on the state ψ, we use the commutator $[J_z+S_z,\vec{p}\cdot\vec{\epsilon}]=[L_{1z},\vec{p}_1\cdot\vec{\epsilon}]+[L_{2z},\vec{p}_2\cdot\vec{\epsilon}]=i\hbar(\vec{p}_1\times\vec{\epsilon})_z+i\hbar(\vec{p}_2\times\vec{\epsilon})_z\equiv i\hbar(\vec{p}\times\vec{\epsilon})_z$. Then Eq. (16) reads

$$E^{I}_{AZA}=G\int_0^{\omega_c}d\omega\int d\Omega\sum_{n,\epsilon}\left(\frac{\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|\vec{p}\cdot\vec{\epsilon}(J_z+S_z)|\psi\rangle-\langle\psi|J_z+S_z|\psi\rangle|\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle|^2}{\omega_n+\omega}+i\hbar\frac{\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|(\vec{p}\times\vec{\epsilon})_z|\psi\rangle}{\omega_n+\omega}\right). \tag{18}$$

In this expression, it is easy to see that there is no contribution from J_z since ψ is an eigenstate of J_z, so the terms with J_z are equal and cancel. Analogously there is no contribution from S_z if ψ is an eigenstate of S_z, which occurs for the state $|\psi_a\rangle\equiv|^3P_2,M_J=2\rangle$ given by Eq. (9).

For the state $|\psi_b\rangle\equiv|^3P_1,M_J=1\rangle$, given by Eq. (10), which is not an eigenstate of S_z, there is also no contribution from S_z, as can be shown inserting $\Sigma_{J',M_{J'}}|J'M_{J'}\rangle\langle J'M_{J'}|$ in Eq. (18) between $\vec{p}\cdot\vec{\epsilon}$ and S_z, such that the first term in Eq. (18) becomes

$$\sum_{J',M_{J'}}\langle\psi_b|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|\vec{p}\cdot\vec{\epsilon}|J'M_{J'}\rangle\langle J'M_{J'}|S_z|\psi_b\rangle, \tag{19}$$

where we must note that S_z connects only states with the same electronic configuration and the same quantum number M_J. In our case, with $M_J=1$, J' can be $J'=1$ and 2. For $J'=1$, the inserted state is the same as ψ_b and the first term in Eq. (18) cancels the second one. For $J'=2$, the product of the first two matrix elements in Eq. (19), summed over polarizations and integrated over angles, is

$$\begin{aligned}\int d\Omega\sum_{\vec{\epsilon}}\langle J=1,M_J=1|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|\vec{p}\cdot\vec{\epsilon}|J'=2,M_{J'}=1\rangle\\ =\frac{8\pi}{3}\langle J=1,M_J=1|\vec{p}|n\rangle\cdot\langle n|\vec{p}|J'=2,M_{J'}=1\rangle\\ =0,\end{aligned} \tag{20}$$

where the Wigner-Eckart theorem has been applied using $\vec{p}\cdot\vec{p}=p_0p_0+p_{-1}p_{-1}+p_1p_1$ and the orthogonality of the Clebsch-Gordan coefficients. Hence there is also no contribution from S_z for $|^3P_1,M_J=1\rangle$ in Eq. (18), which becomes

$$E^{I}_{AZA}=i\hbar G\int_0^{\omega_c}d\omega\int d\Omega\sum_{n,\vec{\epsilon}}\frac{\langle\psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|(\vec{p}\times\vec{\epsilon})_z|\psi\rangle}{\omega_n+\omega}, \tag{21}$$

contributing only to $g_L^{(3)}$, since it results from $[L_z,\vec{p}\cdot\vec{\epsilon}]$. Taking into account that $(\vec{p}\times\vec{\epsilon})_z=\vec{p}\times\vec{\epsilon}\cdot\vec{B}_0/B_0$, and rearranging the vector product, we obtain

$$\int d\Omega \sum_{\vec{\epsilon}} \langle \psi|\vec{p}\cdot\vec{\epsilon}|n\rangle\langle n|(\vec{p}\times\vec{\epsilon})_z|\psi\rangle = -(8\pi/3)(\langle\psi|p_x|n\rangle \times\langle n|p_y|\psi\rangle - \text{c.c.}),$$

where we recall that $p_x = p_{1x}+p_{2x}$, $p_y = p_{1y}+p_{2y}$ and ψ and $|n\rangle$ represent antisymmetrized functions. Due to the indistinguishability of the electrons, $\langle\psi|p_{1x}|n\rangle = \langle\psi|p_{2x}|n\rangle$, and there are then four equal terms, which allows us to write Eq. (21) as

$$E^{I}_{AZA} = -\frac{32\pi}{3} i\hbar G \int_0^{\omega_c} d\omega \sum_n \frac{\langle\psi|p_{1x}|n\rangle\langle n|p_{1y}|\psi\rangle - \text{c.c.}}{\omega_n+\omega}. \tag{22}$$

This expression is similar to that calculated by Anthony and Sebastian [2] for the radiative correction in the second order of perturbation. The integral in frequencies is $\int_0^{\omega_c} d\omega/(\omega_n+\omega) = \ln[(\omega_c+\omega_n)/\omega_n] \simeq \ln(\omega_c/\omega_n)$, since $\omega_n \ll \omega_c$ in our case [if we have $\omega_n \simeq \omega_c = \alpha mc^2$, then the electron is in the continuum and, by conservation of momentum $\omega_n \simeq \hbar k^2/(2m) \ll \omega$ so that $\omega \gg \omega_c$, contrary to our assumption]. Equation (22) then becomes

$$E^{I}_{AZA} = -\frac{32\pi}{3} i\hbar G \Bigg((\ln\,\omega_c)(\langle\psi|p_{1x}p_{1y}|\psi\rangle - \text{c.c.}) - \sum_n (\ln\,\omega_n)(\langle\psi|p_{1x}|n\rangle\langle n|p_{1y}|\psi\rangle - \text{c.c.})\Bigg), \tag{23}$$

where the sum over n has been made in the first term and its complex conjugate, which now cancel between them since $\langle p_{1x}p_{1y}\rangle_\psi = \langle p_{1y}p_{1x}\rangle_\psi$, leading to

$$E^{I}_{AZA} = \frac{32\pi}{3} i\hbar G \left(\sum_n (\ln\,\omega_n)(\langle\psi|p_{1x}|n\rangle\langle n|p_{1y}|\psi\rangle - \text{c.c.})\right), \tag{24}$$

which is independent of the cutoff frequency and equal to half of the quantity calculated by Anthony and Sebastian [2]. Therefore, the corresponding contribution to $g_L^{(3)}$, choosing $M_L = 1$, is

$$g^{(3)}_{AZA} = \frac{E^{I}_{AZA}}{|\mu_B|B_0 M_L} = \frac{1}{2} 2.39\times10^{-7}. \tag{25}$$

The mentioned authors multiply by $\frac{3}{4}$ the value they found to correct the self-energy contributions, obtaining the value $(\frac{3}{4})2.39\times10^{-7} = 1.79\times10^{-7}$ mentioned in Sec. I.

Concerning the term E^{II}_{AZA} given by Eq. (17), it can be seen that the integral in frequencies is convergent, which suggests small contribution from high frequencies. We can also suppose here that $\omega_n \ll \omega_c$. The integral in frequencies in the first term of Eq. (17) is then

$$\int_0^{\omega_c} - \frac{\omega_{n'}\,d\omega}{(\omega_n+\omega)(\omega_{n'}+\omega)} = \frac{\omega_{n'}}{\omega_n-\omega_{n'}}\left[\ln\frac{\omega+\omega_n}{\omega+\omega_{n'}}\right]_0^{\omega_c} \simeq -\frac{\omega_{n'}}{\omega_n-\omega_{n'}}\ln\frac{\omega_n}{\omega_{n'}}. \tag{26}$$

It must be noted that $|n\rangle$ and $|n'\rangle$, connected by $L_z+2S_z = J_z+S_z$ in the first term of Eq. (17), must have the same configuration, their maximum energetic difference being only due to the spin-orbit interaction, which is small. Hence, if we consider that $\epsilon \equiv (\omega_n/\omega_{n'}-1)$ is much smaller than unity, we can write $\ln(\omega_n/\omega_{n'}) \equiv \ln(1+\epsilon) \simeq \epsilon + \epsilon^2/2 + \cdots$. Retaining only the first term of this expansion, expression (26) results equal to -1. The integral over frequencies in the second term of E^{II}_{AZA} in Eq. (17) can be easily calculated, and it is close to 1 (assuming that $\omega_n \ll \omega_c$). These results permit us to sum over n and n' in Eq. (17), which becomes

$$E^{II}_{AZA} = G\int d\Omega \sum_{\vec{\epsilon}} [\langle\psi|J_z+S_z|\psi\rangle\langle\psi|(\vec{p}\cdot\vec{\epsilon})^2|\psi\rangle - \langle\psi|(\vec{p}\cdot\vec{\epsilon})^2(J_z+S_z)|\psi\rangle], \tag{27}$$

where we have taken into account that the commutator $[J_z+S_z, \vec{p}\cdot\vec{\epsilon}] = i\hbar(\vec{p}\times\vec{\epsilon})_z$ and that the sum over polarizations and the angular integration of the expression $(\vec{p}\cdot\vec{\epsilon})(\vec{p}\times\vec{\epsilon}\cdot\vec{B}_0/B_0)$ is zero. Since ψ is an eigenstate of J_z, it is obvious that the contribution from J_z in Eq. (27) cancels. It can be seen that the contribution from S_z in the same expression is zero. This is obvious for the state $|\psi_a\rangle \equiv |^3P_2, M_J = 2\rangle$, which is an eigenstate of S_z. For the state $|\psi_b\rangle \equiv |^3P_1, M_J = 1\rangle$, given by Eq. (10), it can be seen in Eq. (27) that, once the polarizations are summed and the angular integration performed, taking into account that $\langle p^2\rangle_{M_L=0} = \langle p^2\rangle_{M_L=1}$, the expression cancels. Therefore the part of order ϵ of E^{II}_{AZA} is zero.

If we consider the second term of the expansion $\ln(1+\epsilon) \simeq \epsilon + \epsilon^2/2 + \cdots$, i.e., $\epsilon^2/2 \equiv (\omega_n-\omega_{n'})^2/(2\omega_{n'}^2)$, the corresponding contribution to E^{I}_{AZA} is negligible since the maximum value of $(\omega_n-\omega_{n'})$ corresponds to spin-orbit interaction. We then have $\epsilon \equiv (\omega_n-\omega_{n'})/\omega_{n'} \approx \alpha^2$ (as is known from the fine structure theory) and $\epsilon^2 \approx \alpha^4$, which would lead to a correction term much lower than E^{I}_{AZA}, and is therefore negligible. Terms of order ϵ^3 and higher will be even smaller.

B. High frequencies

Let us now analyze the contribution from high frequencies (above ω_c) to the term E_{AZA} without using the dipole approximation. We write Eq. (13) as

$$\frac{\omega}{(\omega_n+\omega)(\omega_{n'}+\omega)} = \frac{1}{\omega} - \left(\frac{\omega_n}{(\omega_n+\omega)\omega} + \frac{\omega_{n'}}{(\omega_n+\omega)(\omega_{n'}+\omega)}\right), \tag{28}$$

and, similarly, Eq. (14) as

$$\frac{\omega}{(\omega_n+\omega)^2}=\frac{1}{\omega}-\left(\frac{\omega_n}{(\omega_n+\omega)\omega}+\frac{\omega_{n'}}{(\omega_n+\omega)^2}\right). \qquad (29)$$

We have now that $\omega \geqslant mc^2\alpha/\hbar$ and then $\omega_n, \omega_{n'} \sim mc^2\alpha^2/\hbar \ll \omega$. (For very high frequencies $\omega \sim mc^2/\hbar$, this is not true, but in this regime the electron becomes relativistic and could not be treated with our methods. However, such frequencies are effectively cut off due to the rapid convergence of the integral in frequencies which will be considered). It is then easy to see that the terms in parentheses in Eqs. (28) and (29) will give a contribution of order α with respect to the previous one. We retain only the first term, $1/\omega$, which is the only one that may make a non-negligible contribution for our purpose. In this case, the sum over n and n' in Eq. (12) can be performed, and we obtain

$$\begin{aligned}E_{AZA}=G\int_{\omega_c}^{\infty}\frac{d\omega}{\omega}\int d\Omega\sum_{\vec{\epsilon}}\Bigg(&\langle\psi|\sum_j \vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}(L_z+2S_z)\\&\times\sum_t \vec{p}_t\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle\\&-\langle\psi|\sum_j \vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}\sum_t \vec{p}_t\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle\\&\times\langle\psi|L_z+2S_z|\psi\rangle\Bigg). \qquad (30)\end{aligned}$$

In this expression, the terms coming from a process where the photon is emitted and absorbed by the same electron (i.e., $j=t$) make a quadratically divergent contribution, which means that the nonrelativistic approximation fails. Actually these terms, in which only one electron is involved, contribute to the anomalous magnetic moment of the electron, and we think that they should not be considered in our calculation. Taking only the terms where different electrons contribute, the integral in ω is convergent. Rearranging the terms remembering that $L_z=l_{1z}+l_{2z}$ and $S_z=s_{1z}+s_{2z}$, for the operator in the first matrix element of Eq. (30) we obtain

$$\begin{aligned}&(\vec{p}_1\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_1})(\vec{p}_2\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_2})(l_{1z}+2s_{1z})+(\vec{p}_2\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_2})\\&\times(\vec{p}_1\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_1})(l_{2z}+2s_{2z})+(l_{1z}+2s_{1z})(\vec{p}_2\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_2})\\&\times(\vec{p}_1\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_1})+(l_{2z}+2s_{2z})(\vec{p}_1\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_1})(\vec{p}_2\cdot\vec{\epsilon}e^{-i\vec{k}\cdot\vec{r}_2}). \qquad (31)\end{aligned}$$

In the above expression, only the even part in $\vec{k}$ [proportional to $\cos(\vec{k}\cdot\vec{r}_{12})$ with $\vec{r}_{12}\equiv\vec{r}_1-\vec{r}_2$] contributes to the angular integral. Let us call $N\equiv\int d\Omega\sum_{\vec{\epsilon}}(\vec{p}_1\cdot\vec{\epsilon})(\vec{p}_2\cdot\vec{\epsilon})\cos(\vec{k}\cdot\vec{r}_{12})$. Expression (30) can then be written as

$$\begin{aligned}E_{AZA}=G\int_{\omega_c}^{\infty}\frac{d\omega}{\omega}[&\langle\psi|N(L_z+2S_z)|\psi\rangle+\langle\psi|(L_z+2S_z)N|\psi\rangle\\&-\langle\psi|2N|\psi\rangle\langle\psi|L_z+2S_z|\psi\rangle], \qquad (32)\end{aligned}$$

which is zero when ψ is an eigenstate of $L_z+2S_z=J_z+S_z$, which occurs for the state $|\psi_a\rangle\equiv|{}^3P_2, M_J=2\rangle$. For the state $|\psi_b\rangle\equiv|{}^3P_1, M_J=1\rangle$ given by Eq. (10), an eigenstate of J_z but not of S_z, the contribution—to $g_S^{(3)}$—is also zero due to the rotational invariance of N and its independence on the spin, which leads to $\langle N\rangle_{M_L=1}=\langle N\rangle_{M_L=0}$, and, in consequence, the cancellation of Eq. (32). Therefore, we can conclude that the contribution from high frequencies to the term E_{AZA} is zero or, at least, negligible.

IV. CORRECTIONS FROM THE TERMS $E_{BZB}, E_{AZB}, E_{BZA}$

The term E_{BZB} of Eq. (8) is obtained in a way analogous to E_{AZA}, but substituting $\vec{S}\cdot\vec{B}_{\rm vac}$ for $\vec{p}\cdot\vec{A}_{\rm vac}$ in Eq. (11). We have, $\vec{S}\cdot\vec{B}_{\rm vac}\equiv\vec{s}_1\cdot\vec{\nabla}_1\times\vec{A}_{\rm vac}(r_1)+\vec{s}_2\cdot\vec{\nabla}_2\times\vec{A}_{\rm vac}(r_2)$. Once $\vec{A}_{\rm vac}$ has acted on the photon states, using $\vec{\nabla}\times\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}=i\vec{k}\times\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}$ we obtain [compare with Eq. (12)]

$$\begin{aligned}E_{BZB}=\frac{G}{c^2}\int_0^{\infty}\omega^3 d\omega\int d\Omega\Bigg(&\sum_{n,n',\vec{\epsilon}}\frac{\langle\psi|\sum_j \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_j e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\langle n|L_z+2S_z|n'\rangle\langle n'|\sum_t \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_t e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle}{(\omega_n+\omega)(\omega_{n'}+\omega)}\\&-\sum_{n,\vec{\epsilon}}\frac{\left|\langle\psi|\sum_j \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_j e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\right|^2\langle\psi|L_z+2S_z|\psi\rangle}{(\omega_n+\omega)^2}\Bigg). \qquad (33)\end{aligned}$$

At low frequencies ($e^{i\vec{k}\cdot\vec{r}_j}\approx 1$), $\omega\leqslant\omega_c\equiv mc^2\alpha/\hbar$, the integral in ω, from 0 to ω_c, is quadratic in ω_c and therefore in α. As a consequence, this contribution is of order α^2 smaller than E_{AZA}^I (that is, of order α^5 in $g_L^{(3)}$) and may be neglected.

In order to calculate the high-frequency part, $\omega>\omega_c$, we shall use the identity

$$\begin{aligned}\frac{\omega^3}{(\omega_n+\omega)(\omega_{n'}+\omega)}=&\ \omega-(\omega_n+\omega_{n'})+\omega\frac{\omega_n^2+\omega_{n'}^2+\omega_n\omega_{n'}}{(\omega_n+\omega)(\omega_{n'}+\omega)}\\&+\frac{\omega_n\omega_{n'}(\omega_n+\omega_{n'})}{(\omega_n+\omega)(\omega_{n'}+\omega)}, \qquad (34)\end{aligned}$$

and a similar one for $\omega^3/(\omega_n+\omega)^2$. As stated below Eq.

(29), $\omega > mc^2\alpha/\hbar$ and $\omega_n,\omega_n' \sim mc^2\alpha^2/\hbar$, hence each term in Eq. (34) makes a contribution of order α with respect to the previous one. The main contribution to Eq. (33) can then be calculated by retaining only the first term of the right-hand side of expression (34). In this case we can perform the sum over n and n', and obtain

$$E_{BZB}=\frac{G}{c^2}\int_0^\infty \omega d\omega \int d\Omega \Bigg(\sum_{\vec{\epsilon}} \langle \psi | \sum_j \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_j e^{i\vec{k}\cdot\vec{r}_j} | L_z + 2S_z | \sum_t \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_t e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle - \sum_{\vec{\epsilon}} \langle\psi|\sum_j \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_j e^{i\vec{k}\cdot\vec{r}_j}\sum_t \frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_t e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle \times\langle\psi|L_z+2S_z|\psi\rangle\Bigg). \tag{35}$$

The contribution from the interaction of one electron with itself will be not considered here for the reasons explained above Eq. (31). The terms in Eq. (35) where different electrons contribute lead to convergent integrals. Rearranging the terms in a way analogous to that in Eq. (31), for the operator in the first matrix element of Eq. (35) we obtain

$$\begin{aligned}&(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_1 e^{i\vec{k}\cdot\vec{r}_1})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_2 e^{-i\vec{k}\cdot\vec{r}_2})(l_{1z}+2s_{1z})\\ &+(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_2 e^{i\vec{k}\cdot\vec{r}_2})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_1 e^{-i\vec{k}\cdot\vec{r}_1})(l_{2z}+2s_{2z})\\ &+(l_{1z}+2s_{1z})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_2 e^{i\vec{k}\cdot\vec{r}_2})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_1 e^{-i\vec{k}\cdot\vec{r}_1})\\ &+(l_{2z}+2s_{2z})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_1 e^{i\vec{k}\cdot\vec{r}_1})(\vec{k}\times\vec{\epsilon}\cdot\vec{s}_2 e^{-i\vec{k}\cdot\vec{r}_2}),\end{aligned} \tag{36}$$

where only the even part in $\vec{k}$ contributes to the angular integral. The integral in frequencies (or k) is straightforward extending the lower limit to zero [which gives a small error of order $\alpha^2 E^I_{AZA}$, as explained below Eq. (33)]. We obtain

$$E_{BZB}=G[\langle\psi|R(L_z+2S_z)|\psi\rangle+\langle\psi|(L_z+2S_z)R|\psi\rangle -2\langle\psi|R|\psi\rangle\langle\psi|L_z+2S_z|\psi\rangle], \tag{37}$$

where

$$R\equiv\int_0^\infty kdk\int d\Omega\sum_{\vec{\epsilon}}\left(\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_1\right)\left(\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_2\right)\cos(\vec{k}\cdot\vec{r}_{12}) =2\,\frac{(\vec{s}_1\cdot\vec{r}_{12})(\vec{s}_2\cdot\vec{r}_{12})}{\hbar^2 r_{12}^4}, \tag{38}$$

which has rotational invariance.

We shall show that the term E_{BZB} is zero, but, before doing that, it is important to realize that it is of the same order as E^I_{AZA} (i.e., a contribution of order α^3 to $g_L^{(3)}$). In fact, r_{12} is of the order of the Bohr radius, i.e., $\hbar/(mc\alpha)$, so that R, given by Eq. (38), is proportional to α^2. We recall that G is proportional to α. This implies that the error of replacing the left-hand side of (29) by the first term ω is negligible (it would make a contribution of order α^4 to $g_L^{(3)}$).

We see in Eq. (37) that E_{BZB} is zero when ψ is an eigenstate of $L_z+2S_z=J_z+S_z$, which occurs for the state $|\psi_a\rangle \equiv|^3P_2,M_J=2\rangle$. For the state $|\psi_b\rangle\equiv|^3P_1,M_J=1\rangle$, an eigenstate of J_z but not of S_z, the contribution is also zero, as can be shown by inserting the identity $\Sigma_{J'M_{J'}}|LSJ'M_{J'}\rangle\langle M_{J'}J'SL|$ between R and S_z in Eq. (37), applying the Wigner-Eckart theorem, and taking into account that S_z can neither change the configuration nor M_J and that R cannot change J because of its rotational invariance. Then $E_{BZB}=0$, or is at least negligible.

We now analyze the term E_{AZB} together with E_{BZA} because the sum $E_{AZB}+E_{BZA}$ allows an easier rearrangement of the operators. The term E_{AZB} is given by

$$E_{AZB}=\frac{G}{c}\int_0^\infty\omega^2 d\omega\int d\Omega\Bigg(\sum_{n,n',\vec{\epsilon}}\frac{\langle\psi|\sum_j \vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\langle n|J_z+S_z|n'\rangle\langle n'|\sum_t -i\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_t e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle}{(\omega_n+\omega)(\omega_{n'}+\omega)} -\sum_{n,\vec{\epsilon}}\frac{\langle\psi|\sum_j \vec{p}_j\cdot\vec{\epsilon}e^{i\vec{k}\cdot\vec{r}_j}|n\rangle\langle n|\sum_t -i\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_t e^{-i\vec{k}\cdot\vec{r}_t}|\psi\rangle\langle\psi|L_z+2S_z|\psi\rangle}{(\omega_n+\omega)^2}\Bigg), \tag{39}$$

and E_{BZA} is given by a similar expression but with the appropriate change of the operators.

The same considerations concerning the term E_{BZB} apply here, and we discard ω_n and $\omega_{n'}$ compared with ω, and then sum in n and n'. We rearrange the operators in $E_{AZB}+E_{BZA}$ following the same procedure as in Eqs. (31) and (36), obtaining

$$E_{AZB}+E_{BZA}=G[\langle\psi|T(L_z+S_z)|\psi\rangle+\langle\psi|(L_z+S_z)T|\psi\rangle -2\langle\psi|T|\psi\rangle\langle\psi|L_z+S_z|\psi\rangle], \tag{40}$$

where

$$T\equiv\int_0^\infty dk\int d\Omega\sum_{\vec{\epsilon}}\left[(\vec{p}_1\cdot\vec{\epsilon})\left(\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_2\right)-(\vec{p}_2\cdot\vec{\epsilon})\left(\frac{\vec{k}}{k}\times\vec{\epsilon}\cdot\vec{s}_1\right)\sin(\vec{k}\cdot\vec{r}_{12})\right]=\frac{\hat{l}_1\cdot\vec{s}_2+\vec{l}_2\cdot\vec{s}_1}{\hbar^2 r_{12}^2}. \tag{41}$$

The operator T has rotational invariance, and we proceed in the same way as in E_{BZB} to show that $E_{AZB}+E_{BZA}=0$ or at least negligible.

V. CORRECTIONS FROM THE TERMS $E_{AAZ}, E_{BAZ}, E_{ABZ}, E_{BBZ}$

The terms analyzed here are given by expression (7), where H_l is H_Z (which does not connect different radiation states) and H_i and H_j are H_A or (and) H_B, which verify $\langle\psi;0|H_A|\psi;0\rangle=0$, and $\langle\psi;0|H_B|\psi;0\rangle=0$.

It is now more convenient to consider the operator $\vec{A}_{\text{vac}}$ expanded in spherical waves instead of plane waves as in Eq. (5). The radiation states are then characterized by $|l,m_l,\vec{k},\vec{\epsilon}\rangle$, where l and m_l are the photon angular momentum and its projection, respectively. The expression of E_{AAZ}, for example, is given by

$$E_{AAZ}=\frac{e^2}{m^2}\sum_{n,\vec{k},\vec{\epsilon},l,m_l}\sum_{n'}\frac{\langle\psi;0|\vec{p}\cdot\vec{A}_{\text{vac}}|n;l,m_l,\vec{k},\vec{\epsilon}\rangle\langle n;l,m_l,\vec{k},\vec{\epsilon}|\vec{p}\cdot\vec{A}_{\text{vac}}|n';0\rangle\langle n';0|J_z+S_z|\psi;0\rangle}{(E_n-E_0+\hbar\omega)(E_{n'}-E_0)}, \tag{42}$$

where the state $|n\rangle$ can be any atomic eigenstate, including ψ, while $|n'\rangle$ must be different from ψ. We see in Eq. (42) that E_{AAZ} is zero if ψ is an eigenstate of J_z+S_z, since we obtain $\langle n'|\psi\rangle=0$. This is the case for $|\psi_a\rangle\equiv|^3P_2,M_J=2\rangle$.

For $|\psi_b\rangle\equiv|^3P_1,M_J=1\rangle$, since S_z can neither connect different configurations nor different M_J, there is only one state $|n'\rangle$, with the same configuration as ψ_b and having $J=2$ and $M_J=1$. The sum over n' then consists of only one term. We denote by $|J,M_J\rangle$ the angular part of the state $|n\rangle$, and note that $|J,M_J,l,m_l\rangle=\Sigma_{J',M_{J'}}C'|J,l,J',M_{J'}\rangle$ (C' is a Clebsch-Gordan coefficient). Taking into account the rotational invariance of $\vec{p}\cdot\vec{A}_{\text{vac}}$, which cannot connect different J, and applying the Wigner-Eckart theorem, we find that a product of two Clebsch-Gordan coefficients appears to come from the first and second matrix elements of Eq. (42). For each $\vec{k}$ and each atomic configuration of $|n\rangle$, we must calculate the sum

$$\sum_{J,M_J,l,m_l}\langle J=1,M_J=1|J,M_J,l,m_l\rangle\times\langle J,M_J,l-m_l|J=2,M_J=1\rangle, \tag{43}$$

which is zero due to the orthogonality of the Clebsch-Gordan coefficients, and then $E_{AAZ}=0$.

The operator $\vec{S}\cdot\vec{B}_{\text{vac}}$ also has rotational invariance, and we can follow exactly the same procedure as before to show that $E_{BAZ}=0$, $E_{ABZ}=0$, and $E_{BBZ}=0$.

VI. CONCLUSIONS

We have analyzed those radiative corrections in the Zeeman effect of helium, which have not been considered in previous work. The atomic states are 2^3P_1 and 2^3P_2.

These corrections are the third-order perturbative contributions from the Zeeman Hamiltonian and vacuum radiative interaction $(-e/m)(\vec{p}\cdot\vec{A}_{\text{vac}}+\vec{S}\cdot\vec{B}_{\text{vac}})$, retaining the terms linear in the external magnetic field and proportional to e^3.

We have found that the greatest correction comes from terms proportional to $\vec{L}\cdot\vec{B}_0$ and $\vec{p}\cdot\vec{A}_{\text{vac}}$, contributing to the orbital gyromagnetic factor with a correction of the same order ($\sim\alpha^3$) as that calculated by Anthony and Sebastian [2] in the second order of perturbation. Specifically, we have found that the correction is just one half that calculated by these authors, i.e., $\frac{1}{2}\times2.39\times10^{-7}$ [see Eq. (25)]. Therefore, the result does not resolve the discrepancy between theory and experiment. We have also shown that the third-order corrections coming from terms involving $\vec{S}\cdot\vec{B}_{\text{vac}}$, are zero or negligible (of order α^4 or lower).

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