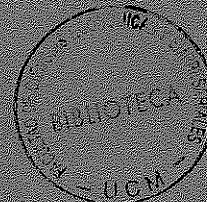


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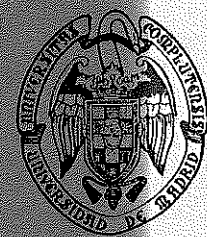
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HARMONIC ANALYSIS IN ECONOMICS

Nelson Julio Alvarez Vázquez



FACULTAD DE CIENCIAS ECONOMICAS Y EMPRESARIALES
UNIVERSIDAD COMPLUTENSE DE MADRID

Author(s):

alvarez vazquez, nelson julio


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Institute:

Departamento de Econometria de la
Universidad Complutense de Madrid

This paper reconsiders the positive statement that from the point of view of the applications, deterministic processes do not seem to have very great interest and that classical Fourier methods fail when applied to economic time series. The basic reason is in the transformation used to attain stationarity. A trend definition is proposed and empirical evidence in frequency domain is offered.



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ALVAREZ VÁZQUEZ, NELSON JULIO

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BY

NELSON JULIO ALVAREZ VÁZQUEZ

DEPARTAMENTO DE ECONOMETRÍA

DE LA UNIVERSIDAD COMPLUTENSE DE MADRID

I. INTRODUCTION

"For deterministic signals, the spectrum is the limit in the usual mathematical sense, of the sample spectrum $C_{xx}(f)$ as the record length tends to infinity. However, the white noise example above shows that when $C_{xx}(f)$ is used to analyze a time series its behavior is so erratic as to render it useless for estimation purposes". (Jenkins and Watts, pag. 213, 1.968).

This paper reconsiders the positive statement that from the point of view of the applications, deterministic processes do not seem to have a very great interest and the classical Fourier methods fail when applied to economic time series. The basic reason is in the transformation used to attain stationarity. A trend definition is proposed and empirical evidence in frequency domain is offered.

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The object of all science is the formulation of laws. While this is a constant fact in sciences such as Physics, it may be observed in this respect that in Economics there is a certain regression in relation to Classical Economics itself.

Although Econometrics appears in the thirties with the aim of applying the same methodological rigour as that found in Physics, at the same time the use of harmonic analysis is abandoned, arguing essential differences in context and managing to generate the sensation of finding oneself faced with a completely closed situation.

While it is true that numerous efforts were made during the 19th Century in the field of economics and that of time series in general, within the so-called unobserved component method, these trials were abandoned at the beginning of the present century to give way to autoregressive and moving-average models.

This means abandoning a deterministic conception of economic phenomena in favour of an interpretation of an stochastic nature.

In any case, the results caused a type of conclusion to be drawn which today takes the form of a maintained hypothesis upon which the economist bases his investigation work.

II. THE SOURCES OF HYPOTHESES IN ECONOMICS

From the moment when Physics becomes organized as a science, we can observe the use of a small number of theories, mainly the corpuscular and wave ones, which bears a marked contrast with the economic field.

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After our discipline had arisen within Philosophy, it was the latter which was to be a main source of hypotheses for a long time. It will suffice to mention the hypothetical case of the rational agent, subject to the principles of utilitarianism, who maximizes profits or satisfactions, still valid today in the hypothesis of rational expectancy, and in the theory of statistical decision itself.

Undoubtedly, a rational agent endowed with a free will favours a probabilistic interpretation of economic phenomena rather than a mechanistic explanation.

If on the other hand Economics is considered as a theory of choice, this means the optimization of linear functions, which evokes a second source of hypotheses in the available methods. But if it is accepted that the phenomena can approximate within a linear theory, this favours the opinion of those who hold that the main weight must be conceded to the trend, whilst the fluctuations must be limited to the role of mere accidental deviations from the trend line.

Both sources of hypotheses illustrate the difficulties of postulating a deterministic wave hypothesis to approximate economic phenomena.

III. PLAUSIBILITY OF A WAVE HYPOTHESIS IN ECONOMICS

Be that as it may, once in any science a determined corpus of hypotheses and theories have acquired a certain acceptance, any attempt at revision meets with difficulty. To confirm this statement, I shall call to mind one of the reasons that Copernicus adduced (page 100, 1982)

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to postulate the hypothesis of the roundness of the Earth: "It is also deduced because the waters crossed by sailors have this same form: since those who cannot make out the land from land from the ship, can see it from the top of the mast, from the land to those who remain on the shore, it seems that they are descending little by little as the ship advances, until it is finally hidden, as if setting like the sun".

In Economics also, it is possible to argue analogous considerations in favour of a wave theory. It is sufficient to observe the behaviour of different historical series and corresponding graphical representations.

The empirical evidence of fluctuations is previous to any deterministic or stochastic explanation; a logical antecedent of classical theories of the economic cycle, or of present-day probabilistic methods of time series analysis.

Consequently, in the very behaviour of historical series there is a presumption, in my opinion favourable to a wave theory rather than to a linear one.

What is more, the evidence of the existence of fluctuations in economic phenomena seems to be supporting the position of those who assign a substantial role to the determination of trend (sometimes described as the law which takes into account the long-term behaviour of a variable), which they try to approximate, not within a linear theory, but by means of polynomials of a higher degree than unity, exponentials, lo-

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gistics, etc. The acceptance of the fact that fluctuation exists is found in everyday language when we speak of observed change in trend.

But the convenience of postulating a wave hypothesis for economic phenomena can be taken as a natural consequence of the acceptance of the fact that the object of a science must be the formulation of laws. If it is not admitted at the same time that the phenomena considered in the said laws do not show a repetitive and recurring behaviour, such laws lose all significance.

In short, for the analysis of economic phenomena a deterministic wave hypothesis is being postulated.

IV. THE NATURE OF ECONOMIC FLUCTUATIONS

What I would venture to call an orthodox position at the moment defends the thesis that the fluctuations observable in economic phenomena lack the regularity demanded by a deterministic theory; rather, on the contrary, business cycles would be characterized by explosiveness, and so this constitutes the explanation of the inadequacy of harmonic analysis for economic science. Even those fluctuations which might appear to have a certain regularity can be explained by defects in the procedures used in the treatment of historical series: "The use of his method of analysis, often called harmonic analysis, was greatly discredited in later years when, applied indiscriminately, apparently discovered "cycles" even in series that were constructed explicitly for purposes of testing and thus contained no cycles." (Dhrymes, page 422,

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1.970).

V. HARMONIC ANALYSIS AND TIME SERIES

Thus the validity of harmonic analysis for the treatment of economic phenomena depends on the analysis of information contained in historical series, within the methodology of time series analysis.

In the unobserved components method, we can distinguish four classical components: trend, cycle, seasonality and an irregular component.

A deterministic wave hypothesis implies giving a substantial role to the cycle.

If it is to have a general character, it must be predicated from the four previous components. There is no special difficulty with respect to cycle and seasonality which are distinguished by the period, longer in the cycle than in seasonality, or in terms of harmonic analysis language, by its reciprocal, that is, the frequency, low and medium for the cycle, or high for seasonality.

With respect to the irregular component, a deterministic wave hypothesis maintains that only owing to limitations in knowledge, and to needs produced by the approximation of the function by means of the series of harmonics, can it be more operative to consider the rest of the components as one, and treated as a random component. A deterministic interpretation maintains that the goodness of the harmonic model allows us to consider the residual component as a purely random process (white noise), while those who

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defend a probabilistic position consider this component as autocorrelated noise, as a result of which it is necessary to try out models of the autoregressive-type. Of course, this residual component would be a result of the removal of trend and seasonality since they deny the existence of the cycle. In any case, its behaviour in the form of a fluctuation is accepted without difficulty, whatever the explanation may be.

In the case of the trend component, what we are trying to elucidate is its oscillatory nature. The wave hypothesis states that it is simply another cyclic component, although through insufficient knowledge, that is, limitation in the size of the series, we do not know it in its totality; consequently the problem is centred on defining a criterion for approximation.

If, from a historical series, we remove trend (and for some theories, seasonality), there remains the empirical counterpart of a stationary process.

According to Wold's decomposition theorem, all stationary processes accept two components, deterministic and indeterministic, which would be brought together by a harmonic model and an autoregressive and/or moving average model respectively.

This is a qualitative proposition, and so to impute quantitatively the importance of one and the other is meaningful only if it refers to each historical series in particular. When it is stated today that the deterministic component is practically non-existent in time

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series, this is no more than a statement of particular value derived only from the empirical evidence considered, which cannot support any process of complete induction.

One of the relevant conclusions of this work in my opinion lies in the part played by the transformation process of each historical series in particular until it is considered to be the empirical counterpart of a stationary process. And within the transformation process, it depends on the concept of trend proposed.

VI. TREND

Within the ambiguity with which this concept is defined, there is a certain agreement to consider it as the component of the series which considers smooth movement over a long period of time.

In the classical method of decomposition, it has been approximated by low degree polynomial functions, exponential functions, logistic functions, moving averages, etc.

This interpretation identifies trend with the idea of average.

An example of this is produced when we estimate a linear trend by means of least squares; as regression one and two coincide, this means that the trend passes through the centre of gravity of the distribution, which determines that although at this point the residual is zero, at the extreme values of the series, the deviations with respect to the trend function will tend to present their maximum value; and so the cycle will show an explosive behaviour, a property that we been attributing to economic phenomena in themselves when it can be explained only by the series transformation procedure. This assertion may be

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generalized to semilogarithmical or doubly logarithmical functions, estimated by the least squares method. Of course, it depends on the historical-series nature.

But the latter as well as polynomials and moving averages have the drawback of not permitting a distinction between trend and cycle, since both are situated in the low frequency band.

In the probabilistic approach to time series, it is usually supposed that the trend takes the form of a polynomial, which because of its systematic character, constitutes a perturbation for the stationarity of the process, and because of this it is more helpful to remove it than to estimate it. This is based on the proposition: "...the differences of a polynomial of order k are represented by a polynomial of order $k-1$." (Kendall, page 47. 1.976). In the case of a linear trend, this latter would be removed by a first difference; a criterion for determining the degree of the polynomial would be given by the stabilization of the variance, given a determined order of differentiation.

Although the present work is based on a deterministic interpretation of economic fluctuations, the behaviour of this transformation will be examined in the light of harmonic analysis.

Since this procedure is to be applied to a trend-free series, a concept of trend which avoids the previous drawbacks is proposed in the present work. Since in fact we are concerned with an incomplete cycle of which we know only the ascending or descending segment, we can try to approximate it by means of the chord, (Myskis, pag. 181, 1975) which passes through the extreme values of the series.

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If this cycle is of long duration, the approximation ought to be relative satisfactory. The approximation will be worse when the trend is of shorter duration; in this case the size of the series, increasing as time passes, will enable us to discover it.

Even when none of the alternatives is entirely satisfactory, I believe that it is better than the concepts used so far, since whoever simply postulates a straight line is supposing that we are dealing with an infinite period component; the trend now proposed cannot solve the drawback of the insufficient size of the series, but it completely removes the explosiveness of the cycle, that is to say, we have a regular cycle which can be approximated by harmonic analysis.

VII. ESTIMATION OF THE TREND-FREE SERIES USING THE HARMONIC MODEL

The appropriate model for approximating the oscillatory processes contemplated in a wave theory, can be expressed by:

$$C_t = \sum_{p=0}^k (a_p \cos p \omega_0 t + b_p \sin p \omega_0 t) + V_t \quad (1)$$

where C_t indicates the trend-free historical series and the two terms of the second member represent the deterministic and indeterministic components.

The deterministic component is approximated by using a linear combination of k sinusoids, of frequencies $p\omega_0$, where

$$\omega_0 = 2\pi/T \quad (2)$$

indicates the angular frequency expressed in radians, of the cycle or fundamental harmonic, whose period T , coincides with the size of the

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series. In the interpretation of results the true frequency will be used, expressed in the units in which the information from the historical series is given, for example: months.

$$f_0 = \omega_0 / 2\pi = 1/T \quad (3)$$

which allows us to verify how the frequency is the reciprocal of the period. Consequently, given a size T of a given historical series, it is only possible to estimate cycles as big as the size of the series, as a result of which the larger cycles would be included inside the trend. For increasing values of p , the cycles will be of shorter duration, the minimum being, in the case of months, the two-month cycle; in fact,

$$\omega_t = T\omega_0 = \pi ; f = \omega_t / 2\pi = \pi / 2\pi = 1/2 \quad (4)$$

whose reciprocal is two.

The values a_p , b_p , Fourier coefficients, can be estimated by the least squares method, with $\cos p\omega_0 t$ and $\sin p\omega_0 t$ acting as independent variables.

Thus, model (1) can be interpreted as a multiple regression model, with the advantage that, as the independent variables are orthogonal, it does not meet with the problem of multicollinearity.

If the wave theory is correct, as k increases, that is to say, the number of sinusoids, the irregular residual component ought to be approximated to the counterpart of the purely random process. On the other hand, if k does not incorporate some of the important cycles, it will be far from such a process, in which case there may be evidence for trying out the probabilistic alternative using models like those considered in the Box-Jen-

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kins methodology.

The amplitude of the cycle is given by

$$R_p = \sqrt{a_p^2 + b_p^2} \quad (5)$$

given which we define the periodogram, which is defined in slightly different ways; here the following form will be adopted:

$$I(w_p) = TR_p^2/4T \quad (6)$$

which constitutes the series spectrum estimator. The term periodogram is used for the representation of the pair of values $(I(w_p), Pw_0)$. The spectrum is a representation in the frequency domain alternative to the autocorrelation function in the time domain.

The representation of Wold's decomposition theorem in the frequency domain corresponds to the existence of a discrete spectrum component (deterministic) and another continuous one (indeterministic).

In consequence, the negation of the validity of harmonic analysis implies the affirmation of the inexistence of the discrete component.

In statistical terms, it is said that the periodogram is an inconsistent estimator; that is, it does not converge to the population spectrum; as an example, the purely random process with constant spectrum is proposed, and nevertheless, the corresponding periodogram fluctuates in an erratic way.

To test the validity of these affirmations, we shall proceed to apply the criterion of convergence derived directly from harmonic analysis, that is, the convergence of Fourier coefficients, a_p , b_p ,

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(R_p) to zero, which must result in a succession of decreasing values of $I(w_p)$.

The degree of approximation reached by harmonic analysis, which is, in short, the measure of its validity, is expressed in Parseval's theorem.

$$(I/T) \sum_{t=1}^T (C_t - \bar{C})^2 = \sum_{p=1}^K R_p^2/2 + \sum_{t=1}^T d_{t/T}^2 \quad (7)$$

which permits the decomposition of the variance into harmonics. The goodness of the approximation will be shown by the high percentage of explanation of the variance, (in frequency analysis, power), with a small number of cycles.

At the opposite extreme we find the purely random process in which each harmonic contributes a proportion which is constant and equal for all frequencies; to explain the total variation, it would be necessary in theory to consider an infinite number of sinusoids, so that each harmonic would contribute a practically null quantity.

Although the convergence and the goodness of the variance explanation constitute the two criteria by which to judge the validity of harmonic analysis (and of course they are not independent), I shall use a third criterion, which is the comparison of the transformed series periodogram with that of the original series, which, in short, will allow the coherence of the transformation to be known.

To demonstrate the suitability of a wave hypothesis I have chosen the example of the series of registered applications for telephone lines in Spain in the period 1.972-1.982, in which I must point out

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the high number of fluctuations, the explanation of which will not be undertaken in the present work.

VIII. FREQUENCY ANALYSIS OF THE SERIES OF REGISTERED APPLICATIONS FOR TELEPHONE LINES IN SPAIN (1.972-1.982): ORIGINAL SERIES AND TREND-FREE SERIES

The results of the estimation of the harmonic model appear in Table I. As there are 132 observations in the historical series, for an exact estimation it would suffice to include 66 pairs of coefficients; if it is taken into account that $b_0 = 0$ since the sine function is odd, it only remains to consider the coefficient a_p corresponding to the frequency π .

In accordance with the previous criteria, we observe the convergence of Fourier coefficients to zero, which is seen in the succession of decreasing values in the periodogram, in which the major peaks correspond to low and seasonal frequencies.

This conclusion is confirmed when we consider the contribution to the variance of the different harmonics; the first four harmonics account for 55% of the variance, whereas from twelve months onwards (eleventh harmonic), only five out of fifty-three harmonics contribute one per cent or more, precisely in the region of the six-, four- and three-month cycles.

The fact that the succession converges means that from a given value of p upwards, disregard of the remaining harmonics will cea-

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TABLE I
FREQUENCY ANALYSIS OF THE ORIGINAL SERIES OF APPLICATIONS FOR
TELEPHONE LINES

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w1	-9.617,328	-3.068,083	0,445	1.070.445.057,000
w2	358,031	2.631,661	0,475	74.094.982,260
w3	1.750,082	103,359	0,489	32.284.425,180
w4	1.762,051	-3.218,311	0,547	141.411.564,100
w5	-270,797	-672,448	0,550	5.520.152,908
w6	1.673,981	-822,161	0,564	34.463.141,290
w7	-390,603	801,002	0,567	8.342.192,907
w8	-52,096	-162,134	0,568	304.637,552
w9	2.765,616	-1.403,007	0,610	101.019.779,300
w10	-935,812	-587,083	0,615	12.819.468,510
w11	6.371,890	-637,540	0,794	430.751.421,400
w12	351,099	-943,340	0,798	10.642.470,930
w13	-307,842	-9,616	0,799	996.422,124
w14	109,041	-519,638	0,800	2.961.284,138
w15	39,571	-40,481	0,800	33.661,585
w16	891,411	-1.195,943	0,810	23.370.781,860
w17	168,019	-834,196	0,813	7.606.250,463
w18	-541,094	-511,037	0,815	5.818.727,180
w19	388,825	-534,605	0,817	4.590.214,375
w20	156,278	-314,393	0,818	1.294.811,554
w21	629,251	88,505	0,819	4.241.501,053
w22	-783,486	-2.907,379	0,859	95.238.699,200
w23	681,254	130,821	0,861	5.054.855,797
w24	591,772	526,059	0,864	6.585.437,369
w25	222,609	33,185	0,864	532.102,201
w26	83,008	-609,614	0,866	3.976.055,065
w27	-482,071	-514,968	0,868	5.226.740,059
w28	214,126	-757,026	0,870	6.501.468,024
w29	517,664	-299,600	0,872	3.757.741,738
w30	98,487	-163,400	0,872	382.345,948
w31	32,094	-372,240	0,873	1.466.312,698
w32	1.128,447	-1.451,315	0,888	35.501.216,010
w33	-2.686,759	216,758	0,919	76.320.114,970
w34	1.032,480	404,258	0,925	12.914.310,460
w35	880,628	99,777	0,928	8.250.661,354
w36	-665,273	358,744	0,930	6.000.911,323
w37	-72,737	27,863	0,931	63.729,329
w38	377,921	-661,553	0,933	6.097.458,104
w39	-205,665	56,149	0,933	477.425,512
w40	680,590	-248,569	0,936	5.514.606,339
w41	-307,761	-456,915	0,937	3.187.908,194
w42	-135,146	-536,229	0,938	3.212.255,219
w43	1.027,298	-778,489	0,945	17.451.577,620
w44	-1.058,088	-1.040,540	0,955	23.133.181,270
w45	761,209	541,814	0,959	9.170.205,817
w46	365,649	-151,818	0,960	1.646.515,366

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TABLE I

FREQUENCY ANALYSIS OF THE ORIGINAL SERIES OF APPLICATIONS FOR
TELEPHONE LINES

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
	-92,719	179,804	0,960	429.899,024
w47	-753,986	-178,668	0,962	6.303.917,505
w48	352,790	-214,816	0,963	1.792.091,354
w49	-134,946	-459,371	0,964	2.407.906,246
w50	-271,032	167,369	0,965	1.065.871,476
w51	-91,404	-367,132	0,965	1.503.581,212
w52	71,346	-157,202	0,965	313.054,519
w53	933,713	-26,556	0,969	9.165.201,973
w54	32,319	-1.202,979	0,975	15.212.251,880
w55	-96,488	960,920	0,979	9.797.052,117
w56	-344,208	-246,461	0,980	1.882.590,240
w57	-869,685	434,042	0,984	9.923.809,519
w58	527,673	-402,320	0,986	4.625.012,678
w59	-294,176	199,244	0,987	1.326.029,262
w60	-46,063	310,426	0,987	1.034.520,291
w61	-583,952	-791,579	0,991	10.163.860,440
w62	400,505	99,203	0,992	1.788.297,146
w63	-420,078	449,413	0,994	3.975.193,861
w64	-28,557	-926,830	0,998	9.031.842,036
w65				

se to affect significantly the approximation of the function by using the trigonometrical series.

The importance of the cycles of a duration longer than the year is confirmed (61.5% of the power), as well as seasonal ones. Nonetheless, if a trend is supposed to exist, the series cannot be suitably approximated by a harmonic model.

In Table II are offered the results of the estimation of the trend-free series, according to the first difference transformation.

The first thing that can be pointed out is the non-convergence. The power has disappeared from the low frequencies, although it is retained in the seasonal cycles, but now the maximum ordinate is found in the four-month cycle when in the original series, apart from the eleven-year cycle, it was in the twelve-month one. Or expressed in terms of contribution to the variance, the twelve-month cycle changes from contributing 17.9% to 9.9%. Where the greatest alteration is produced is in the low frequencies, since the first four harmonics now account for 0.7%.

Consequently, the transformed series shows little coherence with the original series, making it evident that the effect of the transformation into differences displaces the power not only of the low and medium frequencies, but also of already high frequencies such as that which corresponds to the twelve-month cycle. Whereas in the original series the cycles larger than the year account for 61.5%, in the transformed series their contribution to the variance is reduced to

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TABLE II
FREQUENCY ANALYSIS OF THE TREND-FREE SERIES (FIRST DIFFERENCE) OF
APPLICATIONS FOR TELEPHONE LINES

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w1	-14,061	461,086	0,002	2.235.278,453
w2	369,566	-45,949	0,003	1.456.834,698
w3	117,928	-250,116	0,004	803.205,940
w4	-519,882	-275,314	0,007	3.635.251,069
w5	-29,875	82,796	0,007	81.383,523
w6	-175,981	-421,407	0,009	2.190.689,703
w7	404,493	83,704	0,011	1.792.241,026
w8	64,505	30,982	0,011	53.789,825
w9	-711,717	-1.022,089	0,025	16.294.229,400
w10	-43,952	494,073	0,027	2.584.458,855
w11	-1.051,410	-3.100,538	0,125	112.592.688,400
w12	-444,718	-40,073	0,127	2.094.332,049
w13	172,525	180,347	0,127	654.307,371
w14	-223,526	43,777	0,128	544.962,392
w15	84,855	-16,016	0,128	78.328,789
w16	-950,525	-284,738	0,137	10.342.142,900
w17	-534,767	136,934	0,140	3.200.917,828
w18	-78,425	585,301	0,143	3.663.115,091
w19	-447,668	-101,518	0,145	2.213.372,219
w20	-200,705	4,719	0,145	423.370,381
w21	-93,582	-570,011	0,148	3.504.994,446
w22	-2.005,098	2.132,221	0,226	89.987.441,330
w23	-131,769	-676,387	0,231	4.988.062,537
w24	253,626	-845,824	0,238	8.190.433,758
w25	11,979	-227,524	0,238	545.281,391
w26	-510,911	331,768	0,242	3.898.118,945
w27	-26,832	832,413	0,248	7.286.060,723
w28	-778,318	370,456	0,255	7.804.814,018
w29	-592,869	-265,400	0,258	4.432.056,700
w30	-125,182	42,680	0,259	183.741,139
w31	-278,555	304,922	0,260	1.791.709,170
w32	-2.403,375	255,097	0,313	61.358.188,330
w33	3.024,562	2.469,984	0,453	160.176.805,700
w34	-556,778	-1.454,823	0,475	25.488.627,900
w35	-743,998	-985,918	0,489	16.024.904,350
w36	1.236,052	248,709	0,503	16.698.365,860
w37	234,880	38,306	0,504	594.916,980
w38	-988,886	450,275	0,514	12.401.742,200
w39	438,533	125,379	0,516	2.185.205,590
w40	-1.017,033	-313,297	0,527	11.896.153,160
w41	119,003	912,426	0,534	8.893.748,943
w42	-175,464	881,894	0,542	8.492.925,769
w43	-2.068,977	222,106	0,581	45.483.267,090
w44	807,010	2.477,160	0,643	71.298.347,450
w45	-595,917	-1.475,090	0,666	26.586.276,230
w46	-580,361	-57,951	0,669	3.573.298,363
w47	412,429	-218,064	0,671	2.286.240,516

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TABLE II
FREQUENCY ANALYSIS OF THE TREND-FREE SERIES (FIRST DIFFERENCE) OF
APPLICATIONS FOR TELEPHONE LINES

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w48	1.233,764	865,470	0,692	23.857.323,940
w49	-630,678	107,699	0,696	4.299.945,079
w50	36,618	884,934	0,703	8.240.030,427
w51	706,473	-116,370	0,708	5.384.950,372
w52	57,313	712,249	0,712	5.363.283,739
w53	-99,610	243,897	0,713	729.076,271
w54	-1.612,509	-455.899	0,739	29.496.173,730
w55	-540,735	2.228,614	0,787	55.242.930,660
w56	743,602	-1.770,861	0,820	38.748.962,620
w57	675,969	613,587	0,828	8.754.464,752
w58	1.959,389	-513,775	0,865	43.100.626,040
w59	-1.036,895	609,919	0,878	15.201.216,500
w60	753,595	-307,489	0,885	6.958.576,242
w61	285,081	-601,184	0,889	4.650.151,404
w62	1.128,608	1.679,377	0,926	43.004.965,330
w63	-661,771	-254,413	0,931	5.280.126,118
w64	916,574	936,698	0,946	18.041.124,540
w65	133,990	1.853,955	0,978	36.293.177,950

2.7% which supports the hypothesis that seeks to explain economic phenomena within probabilistic theory.

Table III presents the results corresponding to the trend-free series, in which the trend is interpreted as the straight line that passes through the extreme values of the series.

The first thing that stands out is the convergence of Fourier coefficients and consequently that of the periodogram, so that it presents a structure entirely analogous to the original series, with the most important peaks in the low-frequency and seasonal cycles.

The effect of the removal of the trend principally affects the five-and-a-half-year cycle (second harmonic), whose contribution rises from 3.1% in the original series to 10.5% in the transformed series. The alterations are minimal in the rest of the harmonics.

As an illustration the results have been used to make Table IV, in which we compare the six most important harmonics in relation to each transformation with the original series.

Part A considers the transformation into differences; there is not a single cycle with a period larger than the year, and in all of them except the twelve-month one, the power that the original series showed is considerably amplified; on the other hand, the differences between the straight line and the former do not meet at a point. As a whole, the cycles that accounted for 27.3% now find their power amplified to 46.8%.

As Part B of Table IV shows, among the most important cycles

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TABLE III

FREQUENCY ANALYSIS OF THE TREND-FREE SERIES (STRAIGHT LINE) OF
APPLICATIONS FOR TELEPHONE LINES

w_p	FOURIER a_p	COEFFICIENTS b_p	ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
w1	-9.728,110	1.462,052	0,421	1.016.533.052,000
w2	259,066	4.907,068	0,526	253.639.580,000
w3	1.636,243	1.612,604	0,549	55.439.024,760
w4	1.649,514	-2.073,680	0,579	73.750.647,030
w5	-388,180	224,775	0,580	2.113.529,239
w6	1.491,881	-66,478	0,590	23.425.771,610
w7	-500,688	1.429,978	0,600	24.112.719,770
w8	-170,058	392,488	0,601	1.921.922,073
w9	2.668,177	-916,696	0,635	83.608.389,420
w10	-1.045,323	-149,956	0,640	11.714.176,300
w11	6.280,123	-232,856	0,812	414.855.663,500
w12	247,840	-579,006	0,813	4.166.738,878
w13	-404,134	344,154	0,814	2.959.736,603
w14	5,005	-203,081	0,814	433.477,301
w15	-72,791	270,739	0,815	825.612,601
w16	779,731	-921,814	0,821	15.312.236,260
w17	41,045	-571,942	0,823	3.453.814,205
w18	-653,790	-277,978	0,825	5.301.620,933
w19	263,069	-319,157	0,826	1.796.921,108
w20	51,200	-111,958	0,826	159.202,408
w21	516,120	269,341	0,827	3.560.138,879
w22	-880,448	-2.723,951	0,963	86.083.160,660
w23	583,726	293,072	0,865	4.481.389,054
w24	497,392	701,663	0,868	7.770.288,834
w25	132,365	191,909	0,868	570.990,048
w26	-21,667	-440,252	0,869	2.040.879,588
w27	-579,874	-360,441	0,871	4.896.771,644
w28	94,531	-607,222	0,873	3.966.970,067
w29	409,932	-159,388	0,874	2.032.029,743
w30	-25,923	-40,702	0,874	24.460,716
w31	-79,863	-249,693	0,874	721.899,724
w32	1.011,383	-1.353,178	0,886	29.978.917,500
w33	-2.796,408	322,167	0,921	83.232.224,870
w34	931,379	488,166	0,926	11.615.288,560
w35	776,862	196,056	0,928	6.743.214,531
w36	-754,705	445,049	0,932	8.063.550,888
w37	-178,041	121,736	0,932	488.638,237
w38	285,220	-569,525	0,934	4.261.660,953
w39	-318,331	140,763	0,934	1.272.574,906
w40	577,188	-160,029	0,936	3.768.446,503
w41	-424,040	-387,568	0,937	3.466.592,941
w42	-248,452	-458,643	0,938	2.858.008,760
w43	914,503	-724,851	0,944	14.303.864,430
w44	-1.173,842	-979,224	0,955	24.546.115,000
w45	660,398	585,377	0,958	8.180.604,745
w46	255,957	-101,858	0,958	797.155,625
w47	-183,774	227,326	0,959	897.586,063

TABLE III

FREQUENCY ANALYSIS OF THE TREND-FRE SERIES (STRAIGHT LINE) OF APPLICATIONS FOR TELEPHONE LINES

w_p	FOURIER a_p	COEFFICIENTS b_p	ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
w48	-861,180	-131,001	0,962	7.970.525,512
w49	257,459	-159,031	0,962	961.934,977
w50	-245,622	-415,950	0,963	2.451.104,150
w51	-378,712	222,286	0,964	2.025.570,333
w52	-205,254	-332,853	0,965	1.606.309,682
w53	-49,395	-113,482	0,965	160.904,056
w54	819,943	-4,466	0,968	7.062.269,334
w55	-91,989	-1.178,842	0,974	14.686.278,430
w56	-200,606	972,897	0,978	10.365.269,400
w57	-457,802	-237,258	0,979	2.792.800,961
w58	-962,456	448,674	0,984	11.844.879,700
w59	424,679	-394,474	0,986	3.529.020,749
w60	-388,235	223,825	0,987	2.109.501,213
w61	-146,638	322,062	0,987	1.315.408,905
w62	-689,697	-762,611	0,992	11.105.671,860
w63	295,415	112,477	0,992	1.049.593,817
w64	-542,535	-428,001	0,994	5.016.163,435
w65	-142,243	-920,214	0,998	9.107.446,474

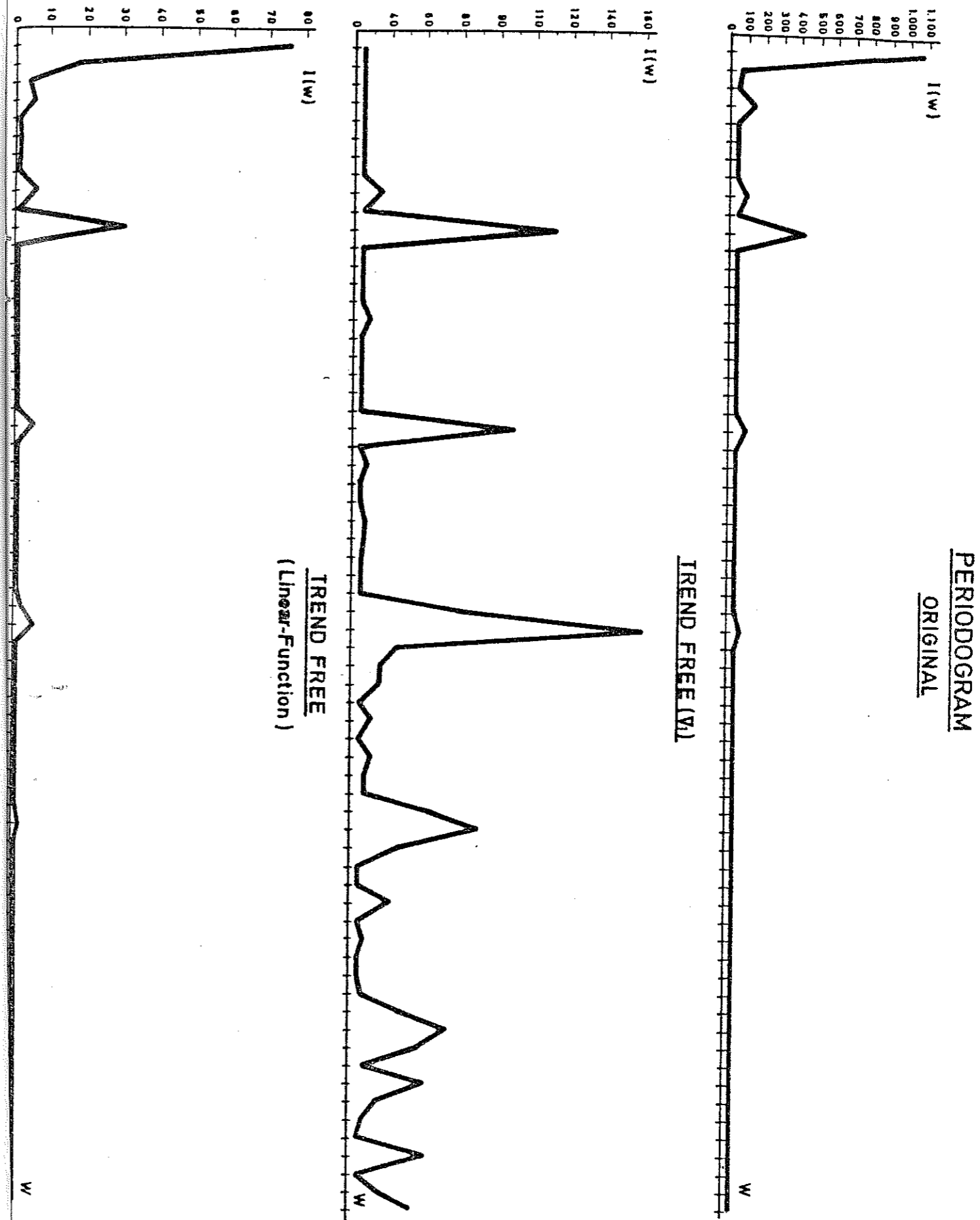


TABLE IV

COMPARISON OF RESULTS

A				
P	Months	First Difference	STRAIGHT LINE	ORIGINAL
33	4	14,0	3,4	3,2
11	12	9,9	17,2	17,9
22	6	7,9	3,6	3,9
44	3	6,2	1,0	1,0
55	2,4	4,8	0,6	0,6
43	3,1	4,0	0,6	0,7
TOTAL		46,8	26,4	27,3

B				
P	Months	STRAIGHT LINE	First Difference	ORIGINAL
1	132	42,1	0,2	44,3
11	12	17,2	9,9	17,9
2	66	10,5	0,1	3,1
22	6	3,6	7,9	3,9
9	15	3,5	1,4	4,2
4	33	3,1	0,3	5,9
TOTAL.....		80,0	19,8	79,3

of the transformation in relation to the straight line, there exist only two seasonal ones, the twelve-month and six-month ones, and in all of them contribution to the variance in relative terms is slightly lower than that of the original series, except in the five-and-a-half-year cycle, reaching, with only six cycles, 80%, equivalent to 79.3% in the original series; the figure of 19.8% from the transformation into differences produces a substantial alteration in the structure of the spectrum of the original series, difficult to justify since also in the case of the straight line the stationarity of the process is reached.

The fact that the transformation of the straight line observes a high level of coherence with the spectrum of the original series, and that with only six harmonics we attain an explanation of the 80% variance, constitutes in my opinion valuable empirical evidence for postulating the validity of harmonic analysis as an approximation of economic phenomena and therefore constitutes a solid basis for postulating a deterministic wave theory in Economics, in the same way as occurs in Physics.

IX. FREQUENCY ANALYSIS OF THE SERIES OF REGISTERED APPLICATIONS FOR TELEPHONE LINES IN SPAIN: TREND-FREE AND SEASONALITY-FREE

Since the harmonic model, in certain conditions is stationary, and the seasonal cycles constitute a particular case different from the rest of the harmonics only in what refers to frequency, the

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TABLE V

FREQUENCY ANALYSIS OF THE TREND-FREE AND SEASONALITY-FREE SERIES OF APPLICATIONS FOR TELEPHONE LINES ($\nabla_1 \nabla_{12}$)

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w1	735,567	9,650	0,006	5.167.620,315
w2	572,076	-501,652	0,012	5.528.333,009
w3	269,480	-675,085	0,017	5.045.458,968
w4	-62,650	591,851	0,021	3.382.481,466
w5	837,483	69,191	0,028	6.743.380,644
w6	-358,940	742,537	0,035	6.495.423,134
w7	254,460	-333,726	0,037	1.681.850,139
w8	1.141,416	-924,167	0,060	20.597.022,270
w9	-308,403	-109,143	0,061	1.022.009,694
w10	108,861	-88,351	0,061	187.706,861
w11	-2,501	7,827	0,061	644,739
w12	366,959	-582,644	0,066	4.527.636,050
w13	207,160	-176,793	0,067	708.281,168
w14	374,793	-1.545,122	0,093	24.139.397,430
w15	-536,874	542,777	0,099	5.565.095,462
w16	-672,275	776,886	0,109	9.223.467,067
w17	876,359	-150,289	0,118	7.549.596,344
w18	231,346	339,717	0,119	1.613.149,416
w19	605,435	111,117	0,123	3.618.214,409
w20	315,425	257,513	0,125	1.583.329,484
w21	-215,850	123,825	0,126	591.329,239
w22	-768,947	-400,393	0,134	7.177.194,431
w23	-973,768	-469,708	0,146	11.161.692,670
w24	-484,164	239,016	0,149	2.784.034,653
w25	1.103,373	-829,791	0,169	18.200.816,820
w26	1.253,241	221,378	0,186	15.466.243,410
w27	408,710	764,782	0,193	7.180.453,877
w28	-5,500	422,691	0,195	1.706.439,547
w29	279,634	-954,290	0,206	9.442.961,139
w30	578,050	652,950	0,214	7.262.101,636
w31	-551,148	715,254	0,222	7.786.041,912
w32	-881,168	1.565,578	0,256	30.820.273,730
w33	98,479	-1.188,002	0,271	13.569.997,980
w34	-357,449	-1.585,101	0,298	25.213.150,700
w35	799,038	1.797,586	0,339	36.536.649,750
w36	541,222	-1.344,918	0,360	20.070.001,860
w37	-1.036,745	1.149,775	0,385	22.887.971,260
w38	-252,664	555,321	0,389	3.554.444,045
w39	436,378	-729,384	0,397	6.898.667,568
w40	643,793	753,370	0,407	9.377.750,957
w41	-761,313	958,517	0,423	14.308.210,720
w42	145,585	1.411,927	0,444	19.239.281,480
w43	-891,144	1.035,163	0,463	17.816.123,260
w44	979,810	-2.192,485	0,524	55.070.966,350
w45	-203,987	1.160,351	0,538	13.254.663,720
w46	1.394,333	1.410,224	0,579	37.556.392,680
w47	-12,509	-379,444	0,581	1.376.380,455
w48	855,316	184,014	0,589	7.309.285,733

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TABLE V

FREQUENCY ANALYSIS OF THE TREND-FREE AND SEASONALITY-FREE SERIES OF APPLICATIONS FOR TELEPHONE LINES ($\nabla_1 \nabla_{12}$)

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w49	996,243	-730,408	0,605	14.572.188,020
w50	964,089	661,249	0,619	13.051.194,000
w51	-684,039	441,964	0,626	6.333.490,086
w52	1.350,330	199,641	0,645	17.792.704,300
w53	-449,038	-2.723,291	0,725	72.746.054,300
w54	1.484,880	20.451,407	0,792	61.006.239,010
w55	798,075	-18,655	0,799	6.085.496,609
w56	-2.736,844	1.168,040	0,891	84.555.512,100
w57	2.020,777	-2.022,208	0,976	78.045.110,680
w58	-41,496	1.099,507	0,789	11.560.737,120
w59	-70,829	-311,315	0,990	973.395,864

removal of this component is not determined for methodological reasons. In the Box-Jenkins methodology, this component is usually seen as having a systematic kind of nature, and for this reason it is usually interpreted as an obstacle to knowledge of the stationary stochastic process, the generator of the series.

The procedure usually makes use of the application of a twelfth difference; consequently to the trend-free series according to a first difference, there has been applied a seasonal twelfth difference to which frequency analysis has been applied even when now the size of the series is reduced to 120 values and the number of frequencies considered to 59.

The transformation is effective for the removal of the power in the seasonal frequencies between 12 and 2.4 months, a consequence of the subsequent displacement of the power towards even higher frequencies: in the original series the cycles shorter than three months contributed 5% to the variance, in $\sqrt{1}$, 40%, and in $\sqrt{12}$, 60%.

As convergence does not exist either, it can be understood that this type of transformation favours the stochastic approximation of economic phenomena.

To test the effect of one of the most controversial procedures (Slutzky effect) on the trend-free series I have applied A. Alcaide's method of deseasonalization of the Annual Movement figures, which is based on the use of moving totals (in this case, twelve-month ones), dated in the last month of the total. They behave in the same

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TABLE VI

FREQUENCY ANALYSIS OF THE TREND-FREE AND SEASONALITY-FREE SERIES OF APPLICATIONS FOR TELEPHONE LINES (C.M.A.)

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w1	-115.878,1	-7.769,0	0,791	1.288.017.854(11)
w2	-12.006,9	45.075,5	0,919	2.077.894.802(10)
w3	5.950,8	27.491,3	0,966	7.555.246.814,0
w4	11.625,7	-13.824,2	0,985	3.115.605.128,0
w5	-5.943,4	-400,3	0,987	338.849.566,7
w6	7.658,2	-7.890,4	0,994	1.154.571.449,0
w7	1.510,2	2.451,4	0,994	79.164.298,9
w8	-4.349,7	5.417,3	0,997	460.916.178,8
w9	3.579,2	297,3	0,998	123.176.948,9
w10	1.312,5	593,9	0,998	19.818.356,7
w11	1.109,1	-119,3	0,998	11.882.526,8
w12	272,8	1.557,2	0,998	23.866.477,5
w13	355,2	478,8	0,998	3.393.974,4
w14	193,6	2.840,1	0,999	77.384.147,4
w15	1.725,2	-769,0	0,999	34.068.796,6
w16	1.496,3	-1.099,5	0,999	32.924.197,9
w17	-514,1	248,3	0,999	3.112.609,5
w18	125,8	-337,8	0,999	1.240.782,9
w19	-195,8	-257,8	0,999	1.000.751,8
w20	158,0	-136,0	0,999	415.012,4
w21	462,9	-108,7	0,999	2.159.020,5
w22	1.082,9	357,4	0,999	12.417.973,7
w23	982,7	428,2	0,999	10.972.662,5
w24	603,5	-273,6	0,999	4.192.802,1
w25	-421,3	651,7	1,000	5.750.649,2
w26	-630,6	-148,6	1,000	4.008.205,7
w27	59,4	-467,6	1,000	2.121.644,7
w28	136,2	-175,9	1,000	472.606,6
w29	8,7	427,7	1,000	1.747.549,7
w30	-41,9	-272,3	1,000	724.819,3
w31	357,0	-372,4	1,000	2.540.650,4
w32	615,6	-748,3	1,000	8.965.989,7
w33	61,2	570,6	1,000	3.144.867,9
w34	263,0	582,7	1,000	3.902.876,7
w35	-115,1	-695,9	1,000	4.751.012,1
w36	-153,8	472,0	1,000	2.353.313,8
w37	555,6	-474,8	1,000	5.100.531,4
w38	196,2	-156,4	1,000	601.179,8
w39	-53,4	169,7	1,000	302.231,9
w40	-37,4	-256,7	1,000	642.607,0
w41	280,9	-343,3	1,000	1.878.916,7
w42	115,5	-502,9	1,000	2.542.487,4
w43	392,1	-297,2	1,000	2.311.600,6
w44	-207,0	571,5	1,000	3.532.462,5
w45	228,0	-348,9	1,000	1.658.858,1
w46	-355,7	-417,6	1,000	2.873.500,3
w47	165,0	41,1	1,000	276.110,4

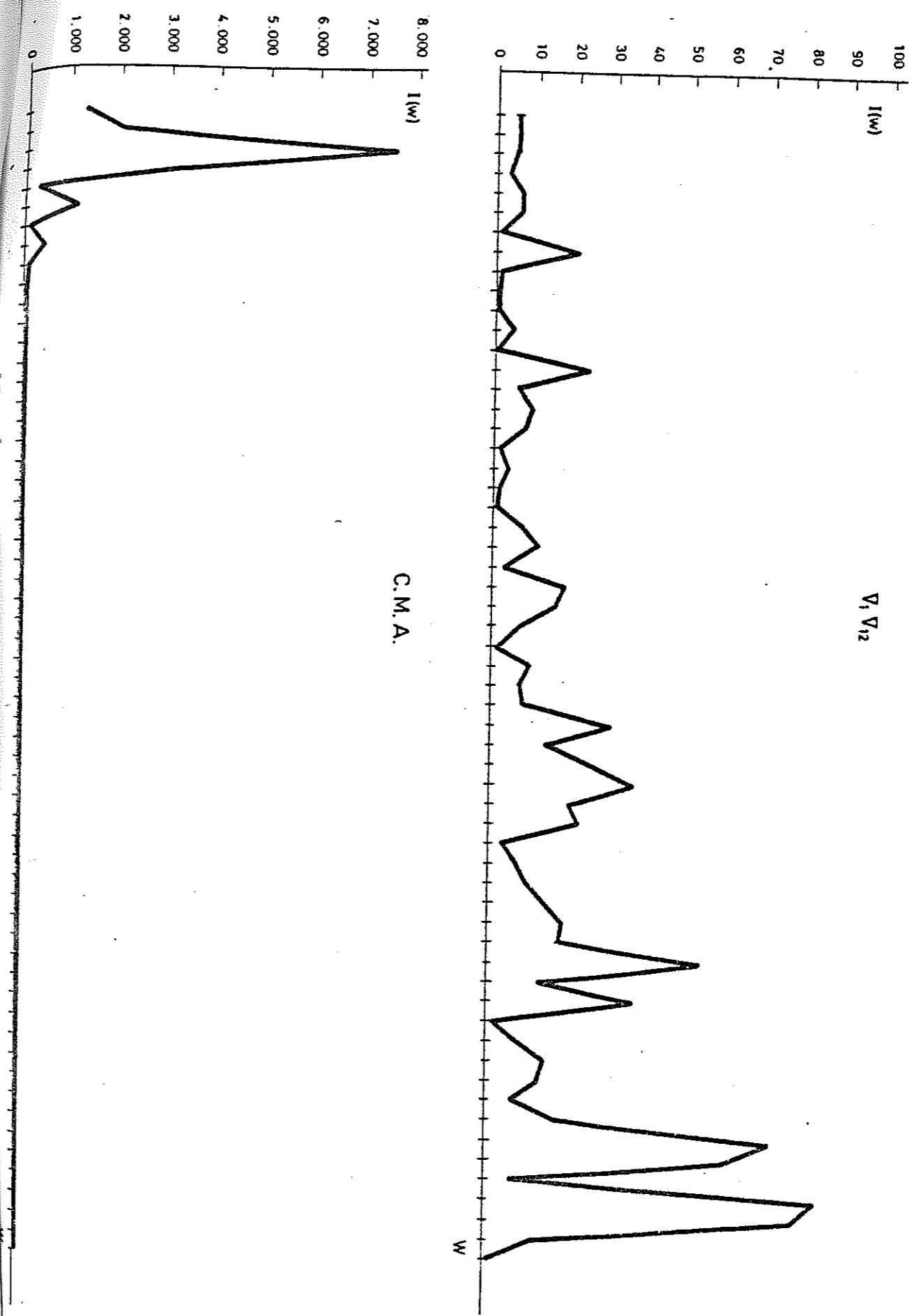
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TABLE VI

FREQUENCY ANALYSIS OF THE TREND-FREE AND SEASONALITY-FREE SERIES OF APPLICATIONS FOR TELEPHONE LINES (C.M.A.)

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$
	a_p	b_p		
w48	-109,2	-42,5	1,000	131.120,3
w49	-192,7	124,6	1,000	502.851,2
w50	-77,5	-176,6	1,000	355.174,7
w51	246,9	-115,9	1,000	710.395,3
w52	-196,1	-117,2	1,000	498.387,8
w53	247,2	717,1	1,000	5.494.094,7
w54	-303,6	-573,1	1,000	4.016.592,4
w55	-2,8	10,5	1,000	1.127,7
w56	771,7	-281,2	1,000	6.441.901,3
w57	-362,3	456,5	1,000	3.243.452,4
w58	142,8	-238,2	1,000	736.547,6
w59	100,8	53,9	1,000	124.769,7

PERIODOGRAM: ANALYSIS OF TREND-FREE AND SEASONALLY ADJUSTED DATA



way as a moving average, with the unique advantage that they do not need centring operations.

The results in Table VI make clear in the first place the intensification of the convergence process: - none of the harmonics from twelve months onwards contributes 0.1% to the explanation of the variance, while the ten - and five - year cycles account for 91.9%.

The goodness of the deseasonalization method is brought out even in multiples of the seasonal cycles such as the 24-month one finds its power reduced in relation to those of contiguous frequencies. On the other hand, the procedure amplifies the power in those harmonics in which this was important in the original series, and in the trend-free series according to the straight line, as are first two (ten and five years) which must not be confused with possibilistic interpretations of the Slutsky effect, according to which it would be inducing nonexistent cycles. This amplifying effect for the long cycles and attenuating effect for the short cycles corresponds with the fundamental idea that is sought with the deseasonalization procedure, which is to make long-term movement evident. It seems logical that the amplifying effect should be important, given the great importance of seasonality in the original series.

On the other hand, the removal of seasonality by means of the variate-difference method, generates a series in which the greatest contribution to the variance is found in the cycles of least duration (2.7, 2.6, 2.3, 2.2, and 2.1 months) and so it can be explained that

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the fluctuations produce the appearance of irregularity, and as a result, constitute empirical evidence favourable to a probabilistic hypothesis for economic phenomena.

The proposed transformation enables us to confirm how the inconsistency is not imputable to the periodogram but to the type of transformation; the periodogram simply estimates a sinusoid by least squares in the same way as a straight line would be estimated. We cannot affirm that the estimated cycles exist in reality, but they do exist in the data.

The periodogram correctly represents both the original series and the transformed one.

Our hypothesis evaluated in the light of the periodogram enables us to confirm that although transformation into differences is sufficient to reach stationarity, it is not necessary; it falls into the error of identifying the trend with a polynomial, it is not coherent with the frequency decomposition of the original series, and in short, it has only randomness as a hypothesis for formulating economic laws, which I believe must be the unrenounceable aim of any science project.

The analysis of the empirical evidence resulting from the straight-line transformation undergone in this work, enables us to have at our disposal a method and a hypothesis whose success in Physics is an incontrovertible fact.

The generality of a wave theory demands that the irregular component also should be considered as a particular case of the same

../..

theory.

X. FREQUENCY ANALYSIS OF THE IRREGULAR COMPONENT RESULTING FROM THE CMA TRANSFORMATION

The irregular component constitutes the empirical counterpart of the purely random process, whose representation would require an infinite number of sinusoids to be considered, and it is characterized by a constant population spectrum, which in the case of considering only positive frequencies, takes the value

$$I(w_p) = \sigma^2 / \pi \quad (8)$$

which indicates that all the frequencies contribute equally to explain the variance.

Those who object to harmonic analysis maintain that the periodogram fluctuates erratically, when it ought to observe the constancy of the population spectrum, all of which verifies the inconsistency.

I think it is far from being conclusive since it would be necessary to justify previously that the residual corresponds to a purely random process. The previous evidence suggests that if the periodogram shows peaks at certain frequencies, this means that the residual is not purely random.

In Table VII we have carried out the estimation of the residuals from the CMA transformation, after removing the 59 known harmonics.

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TABLE VII

FREQUENCY ANALYSIS OF THE IRREGULAR COMPONENT SERIES OF APPLICATIONS FOR TELEPHONE LINES (C.M.A.)

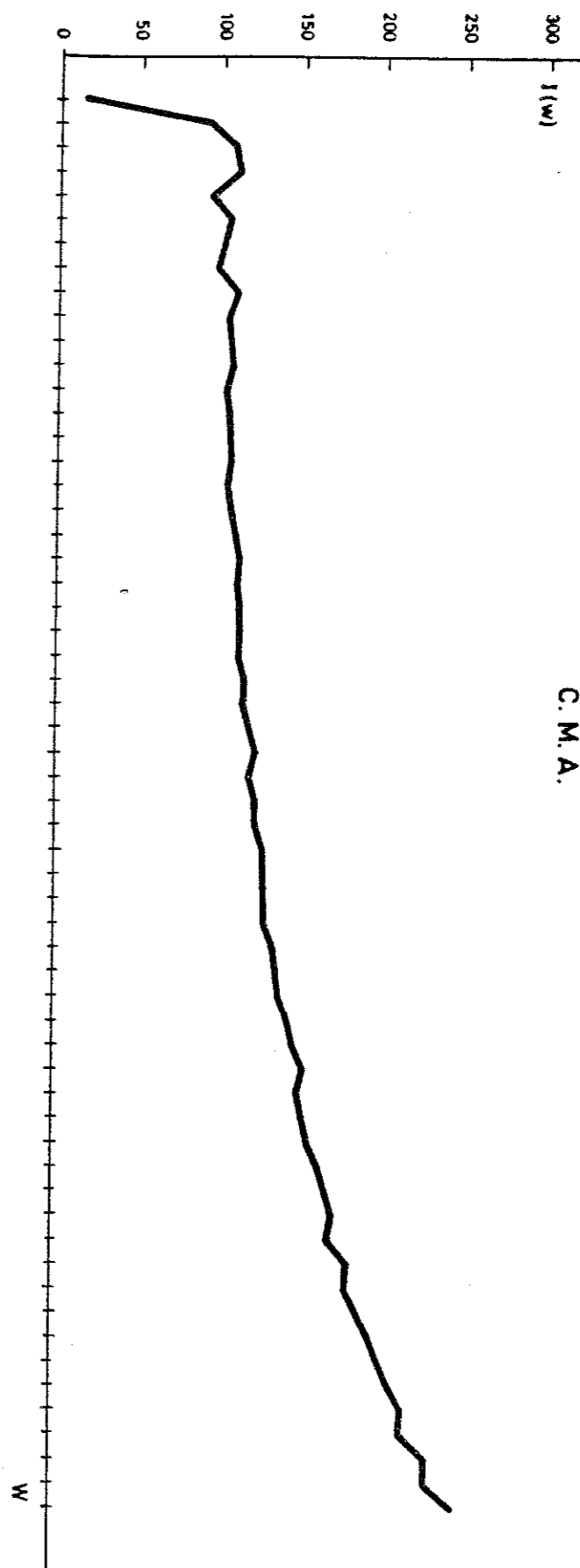
w_p	FOURIER COEFFICIENTS		ACCUMULATED	PERIODOGRAM	$LI(w_p)$
	a_p	b_p	EXPLAINED VARIATION	$I(w_p)$	
w1	-1,3	0,0	0,000	16,138	2,781
w2	-3,0	-0,9	0,001	93,679	4,540
w3	-3,3	-0,7	0,001	108,671	4,688
w4	-3,4	-0,1	0,001	110,485	4,705
w5	-3,1	-0,4	0,001	93,297	4,536
w6	-3,3	-0,4	0,001	105,520	4,659
w7	-3,2	-0,6	0,001	101,223	4,617
w8	-3,1	-0,8	0,001	97,880	4,584
w9	-3,3	-0,8	0,001	110,103	4,701
w10	-3,2	-0,9	0,001	105,520	4,659
w11	-3,2	-0,9	0,001	105,520	4,659
w12	-3,2	-1,0	0,001	107,334	4,676
w13	-3,1	-1,1	0,001	103,323	4,658
w14	-3,1	-1,2	0,001	105,520	4,659
w15	-3,1	-1,2	0,001	105,520	4,659
w16	-3,1	-1,3	0,001	107,907	4,681
w17	-3,0	-1,4	0,001	104,660	4,651
w18	-3,0	-1,5	0,001	107,430	4,677
w19	-3,0	-1,6	0,001	110,390	4,704
w20	-3,0	-1,7	0,001	113,541	4,732
w21	-2,9	-1,8	0,001	111,249	4,712
w22	-2,9	-1,9	0,001	114,783	4,743
w23	-2,9	-1,9	0,001	114,783	4,743
w24	-2,8	-2,0	0,001	113,064	4,728
w25	-2,8	-2,1	0,001	116,979	4,762
w26	-2,7	-2,2	0,001	115,833	4,752
w27	-2,7	-2,3	0,001	120,130	4,789
w28	-2,7	-2,4	0,001	124,618	4,825
w29	-2,6	-2,4	0,001	119,557	4,784
w30	-2,6	-2,5	0,001	124,236	4,822
w31	-2,5	-2,6	0,001	124,236	4,822
w32	-2,5	-2,7	0,001	129,297	4,862
w33	-2,4	-2,8	0,001	129,870	4,867
w34	-2,4	-2,8	0,001	129,870	4,867
w35	-2,3	-2,9	0,001	130,825	4,874
w36	-2,3	-3,0	0,001	136,459	4,916
w37	-2,2	-3,1	0,001	137,987	4,927
w38	-2,1	-3,2	0,001	139,897	4,941
w39	-2,1	-3,3	0,001	146,104	4,984
w40	-2,0	-3,4	0,001	148,587	5,001
w41	-2,0	-3,5	0,001	155,176	5,045
w42	-1,9	-3,5	0,001	151,452	5,020
w43	-1,8	-3,6	0,001	154,699	5,041
w44	-1,7	-3,7	0,001	158,327	5,065
w45	-1,7	-3,8	0,001	165,489	5,109
w46	-1,6	-3,9	0,001	169,691	5,134
w47	-1,5	-4,0	0,001	174,275	5,161

.../..

TABLE VII

FREQUENCY ANALYSIS OF THE IRREGULAR COMPONENT SERIES OF APPLICATIONS FOR TELEPHONE LINES (C.M.A.)

w_p	FOURIER COEFFICIENTS		ACCUMULATED EXPLAINED VARIATION	PERIODOGRAM $I(w_p)$	$LI(w_p)$
	a_p	b_p			
w48	-1,4	-4,0	0,001	171,505	5,145
w49	-1,3	-4,2	0,001	184,588	5,218
w50	-1,2	-4,2	0,001	182,201	5,205
w51	-1,2	-4,3	0,002	190,317	5,249
w52	-1,1	-4,4	0,002	196,429	5,280
w53	-1,0	-4,5	0,002	202,920	5,313
w54	-0,9	-4,6	0,002	209,798	5,346
w55	-0,8	-4,7	0,002	217,056	5,380
w56	-0,7	-4,7	0,002	215,623	5,374
w57	-0,6	-4,9	0,002	232,716	5,450
w58	-0,5	-4,9	0,002	231,666	5,445
w59	-0,4	-5,1	0,002	249,905	5,521



PERIODOGRAM: ANALYSIS OF A RESIDUAL COMPONENT

C.M.A.

The results make it possible to verify how the contribution is 0.1%, or 0.2% at all frequencies, except at w_1 where it takes the value of 0.

The succession of Fourier coefficients, as well as the last two columns allow us to confirm that the periodogram is similar to the mentioned constancy of the population spectrum, so that in this case also, the empirical evidence is favourable to the validity of harmonic analysis in Economics.

XI. CONCLUSION

The previous evidence enables us to try out a wave theory for the interpretation of economic phenomena. The method detects cycles in the data, but evidently this does not allow us to infer that this is transferable to the phenomena.

I believe that it would be a considerable advance to prove the permanence of the cycles independently of the size of the series.

Then, or simultaneously, it would be the moment to begin to think about the search for the "physical causes", which explain the results that allow us to arrive at the formulation of economic laws.

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