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Título: **Monetary policy, Taylor's Rule and Endogenous Fluctuations**

Resumen:

The paper analyses the dynamic behaviour of an economy when the central bank (CB) implements an inflation targeting regime. For this purpose, a simple macroeconomic model is constructed and subjected to formal dynamic analysis. A first result is that, under certain conditions, the emergence of endogenous oscillations in the economy is a possible scenario. A second implication is that CBs face two main policy dilemmas: (i) the degree of gradualism adopted when setting the level of short-term interest rates and (ii) the desirability or otherwise of raising interest rates as soon as economic indicators suggest the emergence of inflationary pressures.

1.- Introduction

A conclusion that emerges from the economic literature on monetary policy in the 90s is the fact that central banks in most developed economies have been increasingly using short-term interest rates to target directly the rate of inflation. Growth rates for monetary or credit aggregates, if they were ever used at all, seem to have been abandoned in practice by most central banks (hereafter CBs). The reason whereby this shift in the monetary policy regime has taken place is usually ascribed to the structural instabilities embedded in money demand functions - allegedly the result of an intense process of financial innovation - which gradually became apparent throughout the 80s (Friedman, 1988). However, other kind of considerations have also been suggested, ranging from the inability of CBs to control the rates of growth of both the monetary base and the money supply (Kaldor, 1985; Moore, 1988) or their ability to control the monetary base but not money supply growth (Pollin, 1991; Palley, 1996).

Whatever the reasons central bankers around the world actually had in mind, the fact is that evidence has recently been mounting on a behaviour of CBs according to which short-term nominal interest rates act as the basic policy instrument with no monetary or credit aggregate performing the role of an intermediate-target variable. Therefore, it seems that CBs periodically change (or decide not to change if appropriate) short-term rates in order to affect the levels of aggregate demand and employment with a view to achieving a target value for the actual rate of inflation a number of periods ahead (Svensson, 1999). This type of behaviour has not only been well documented for that group of countries which have officially announced the adoption of inflation targeting (McCallum, 1996; Mishkin, 1999). There is also evidence that points into the direction that some other rather important countries - US, Japan and Germany - have behaved this way in the past (Taylor, 1993; Bernanke, 1996; Clarida, Galí and Gertler, 1998).

Alongside the aforementioned empirical evidence on recent central banking praxis, a burgeoning literature that explores the relative performance of both simple and optimal monetary policy rules has also flourished (Nota 1). Policy rules simulations have been performed using either large or small macroeconometric models. In Taylor's own words, *model simulations show that simple policy rules work remarkably well in a variety of situations; they seem to be surprisingly good approximations to fully optimal policy. Simulation results also show that simple policy rules are more robust than complex rules across a variety of models. Moreover, the basic results about simple policy rules designed for the United States seem to apply broadly to many countries* (Taylor, 1999, p.657).

No doubt, the simple policy rule that has by far attracted more interest is Taylor's rule which, in turn, fits surprisingly well the behaviour of several CBs (Nota 2) (Taylor, 1993; Clarida and Gertler, 1996). These two reasons, to which we can add its simplicity and amenability, have led us to look at it as a "reasonable" approximation to CB behaviour as long as interest rates management is concerned. Two additional justifications are the following. First, Taylor's rule implicitly assumes that a hypothetical CB's loss function has both the inflation and output gaps as its essential arguments. Second, it can be argued that in any simple model - including ours - used for dynamic analysis, the output gap - or equivalently the employment gap - is likely to contain all the relevant information the monetary authorities need to forecast inflation. In this last sense, implementation of what Svensson (1999) has defined as an "instrument-rule", such as Taylor's rule, can be seen as being akin to inflation targeting.

As we have stated above, the literature on monetary policy rules has focused so far in the simulation of policy rules with either large or small macroeconomic models. The purpose of these simulations has been to evaluate the relative performance of different policy rules under several types of exogenous shocks and in different stochastic models. In turn, the models on which such dynamic simulations have been performed were previously calibrated or estimated. Undoubtedly the former seems to be an appropriate way of obtaining useful information intended to solve or help solving practical policy problems. However we think that a policy analysis based upon simple deterministic dynamic models - an analysis that has not yet been pursued as far as we know - can also yield some policy insights. The relevance of such analysis is, according to us, enhanced if the underlying model has some built-in instabilities like: (i) a positive feedback mechanism linking the aggregate rates of profit and investment and (ii) an endogenously-determined money supply that prevents both the real-balance effects from acting as an stabilising force in the economy (Nota 3).

The first feature pays tribute to a long-standing tradition in economic analysis that started with Keynes (1936) and that emphasises the weakness, if not the non-existence, of self-regulation mechanisms in market-oriented economies. We believe that this old keynesian characterisation of market-oriented economies is still relevant in these days. The second feature can, somehow, be seen as the direct implication of the interest rate-based monetary policy regime that the CB implements. But it can also be looked at - and this is certainly our view - as an assumption which intends to capture the fact that, as Kaldor recognised, in a predominantly credit-money economy the money supply level is *credit-driven* and *demand-determined* and the only way a CB can affect the money supply level is by changing the level of interest rates (Kaldor, 1985).

The purpose of this paper is thus twofold. Firstly, it attempts to analyse at a theoretical level the different types of policy lags, constraints and dilemmas any CB faces when targeting the rate of inflation with no particular assumption about the CB's feedback policy rule being initially made. In the process we will set out a set of monetary policy implications. Secondly, it purports to show the conditions under which the implementation of an inflation targeting regime - where the impact of monetary policy upon aggregate demand over the business cycle provided the CB sets the level of short-term nominal interest rates according to Taylor's rule is measured by a specific policy reaction function - may lead to the emergence of endogenous fluctuations in the economy in the context of a model that exhibits a positive feedback mechanism between the rates of profit and investment.

2.- Monetary policy and Taylor's rule

As we stated above, a characterisation of CBs' behaviour based on Taylor's rule has recently received considerable empirical support. For instance, Taylor (1993) himself accompanied his policy proposal with evidence showing a surprisingly good fit of the *ex-post* predicted path of a short-term nominal interest rate derived from Taylor's rule when compared to the US federal funds rate actual path. A rather similar finding exists for the UK until its exit from the European Exchange Rate Mechanism (Stuart, 1996). The traditional claim that the Bundesbank is a money-targeter has received a sceptical response in recent empirical work. For example, Bernanke (1996) argues that the Bundesbank is better described as an inflation-targeter than as a money-targeter. According to Mishkin (1999, p.588), the Bundesbank missed its money targets fifty percent of the time. These findings have led to the claim that the Bundesbank strategy is better described as "inflation targeting in disguise" (Svensson, 1999, p.641). In addition, some authors have shown that a modified version of Taylor's rule predicts reasonably well the actual path followed by short-term nominal interest rates in Germany (Clarida and Gertler, 1996) and other industrialised economies (Clarida, Galí and Gertler, 1998; Gerlach and Smets, 1999). Although it is still too early to evaluate the strategy followed by the European Central Bank, there are reasons to expect that its behaviour will not diverge significantly from the Bundesbank's policy.

Alongside the evidence presented above, the formal adoption by a group of countries of inflation targeting strategies seems to be well documented. New Zealand was the first country to formally adopt inflation targeting in 1990. Canada, The United Kingdom and Sweden followed suit in 1991, 1992 and 1993 respectively. Finland also adopted inflation targeting as its official monetary policy strategy in 1993 whereas Australia and Spain did the same in 1994 (Bernanke and Mishkin, 1997; Mishkin, 1999, p.590-98). As Svensson has recently pointed out, *the policy reaction function under inflation targeting will, in general, not be a Taylor-type reaction function (where a Taylor-type reaction function denotes a reaction function rule, which is a linear function of current inflation and the output gap only), except in the special case when current inflation and the output gap are sufficient statistics for the state of the economy* (Svensson, 1999, p.628). However, for a simple theoretical model - like ours - this is indeed the case, since the number of variables the CB can use as a source of relevant information is limited. Therefore, we insist that, in a simple model, *setting interest rates according to Taylor's rule is akin to*

implementation of inflation targeting.

In addition, the conventional wisdom concerning the transmission mechanism of monetary policy that appears to grow increasingly dominant, points towards a fundamental role for interest rates and a negligible role for money (Taylor, 1997; Svensson, 1999). In this view, the transmission mechanism can be briefly described as follows. The monetary authorities closely control short-term nominal interest rates. Changes in short-term nominal rates, in turn, affect short-term *ex-ante* real interest rates as a result of rigidities in wages and good prices (Nota 4). Furthermore, current short-term interest rates and expectations about future short-term interest rates jointly determine long-term interest rates. According to Taylor (1995, p.14), the short-term interest rate is only one of many factors affecting long-term interest rates, and the effects of the former upon the latter are *uncertain* and *variable* over time. In any case, as far as we adhere to the expectations hypothesis of the term structure of interest rates we have that, everything else being constant, changes in short-term interest rates will lead to *less* than proportional changes in long-term interest rates (Nota 5). This way CBs are in a position to affect, albeit in a loose way, long-term real interest rates and, with a lag, determine the level of aggregate demand. Aggregate demand then affects inflation, with another lag, via an aggregate supply equation.

A combination of inflation and income level targeting is embodied in the so-called Taylor's feedback rule. According to it, the CBs' most controllable short-term interest rate (the federal funds rate in Taylor's original version) is the result of the sum of four components (Taylor, 1993, p.202):

- the rate of inflation over the previous four quarters
- an interest rate that is close to the steady-state growth rate and is thus compatible with a neutral long-term monetary policy
- a proportion of the difference between the current and target inflation rates (hereafter the inflation gap)
- a proportion of the economy output gap or the percent deviation of real GDP from its trend

As it has been presented above, Taylor's rule leads to some practical difficulties if inserted into a simple theoretical model. Therefore, if we want to use it as the basis for a formal analysis it is necessary to carry out some modifications. The first of these changes affects what Taylor himself calls the real interest rate compatible with a long-term neutral monetary policy. He assumed it was close to the steady-state growth rate. In the model below, such a role will be played by that "relevant" real rate of interest that keeps both the employment ratio (the ratio of total employment to total labour force) and the rate of investment (as well as the rate of capacity utilisation) constant and will hereafter be referred to as the long-term critical interest rate r^* or CRIR.

A second and more serious problem is that, in Taylor's rule, the interest rate that is actually set is a short-term nominal one whereas, for the purposes mentioned in the last paragraph, we are instead interested in how the "relevant" real interest rate is set. Since it is not the purpose of this paper to elucidate whether the "relevant" real interest rate is actually a short-term or a long-term one, it will be assumed that what we have just called the "relevant" real interest rate is, as far as our model is concerned, a weighted average of short and long-term real interest rates (Nota 6). Moreover, we can think of the different weights as being determined by the relative importance of each real interest rate in the monetary policy transmission mechanism. In practice, the actual value of these weights is *uncertain* and *variable* over time. However, at an initial stage of our policy analysis it will be assumed that the CB knows how this weighted average is actually determined.

The reason whereby we carry out this qualitative transformation in Taylor's rule stems from our decision to follow a two-stage strategy in the policy analysis. Firstly, it will be assumed that the CB *can* and actually *sets* the "relevant" real interest rate according to a modified version of Taylor's feedback rule. This will allow us to perform a dynamical analysis of the problem facing a CB in case it were able to both anticipate and appropriately offset all those factors pushing the relevant real interest rate in an undesired direction. Secondly, the former assumption will be left aside - the CB will only control short-term nominal interest rates - and the implications for monetary policy explored.

Therefore, the transformation of Taylor's rule along the lines scheduled above affects, in the first place, the rate of inflation over the previous four quarters that appears on it. Presumably, the insertion of this component in the feedback reaction function was originally intended to serve as a proxy for the expected rate of inflation. In any case, for the dynamical analysis below it will be initially assumed that the CB has enough knowledge about what the average expected future rate of inflation is. As a result of it, the CB will be in a position to set the short-term *ex-ante* real interest rate. Furthermore, as indicated above, the CB will initially know what the "relevant" real interest rate

is and it will be able to set its level according to the reaction function below. Thus, we have that, once all these assumptions are put together, the "relevant" real interest rate will be determined according to the following feedback rule:

$$r(t) = r^* + \alpha_1(\pi(t) - \pi^d) + \alpha_2(Y(t) / Y^*(t) - 1) \quad (1)$$

where r^* is the "relevant" real rate of interest, α_1 and α_2 are positive and constant reaction coefficients and π , π^d , Y and Y^* are current and target rates of inflation and current and target output levels respectively.

3.- A model of the economy

In order to explore the implications of the CBs' implementation of a monetary policy regime based on our modified version of Taylor's rule, let us begin with a model of a closed economy (Nota 7). The model will serve as the basis for the policy and formal dynamical analysis in the following sections. Let p be the general price level, w the money wage, Y the real value of GNP, N the employment level and m the average mark-up. Then according to "normal cost" pricing theory, the price level of the economy will be determined by the following relation

$$p(t) = m \cdot \frac{w(t) \cdot N(t)}{Y(t)} \quad (2)$$

Insofar as this model seeks to isolate the contribution of monetary factors, it will be assumed that m is constant, thus ignoring the behaviour of income shares over the cycle. By definition average labour productivity A is the ratio of the employment level to the real value of GNP. Thus expression (2) can be written as

$$p(t) = m \cdot \frac{w(t)}{A(t)} \quad (3)$$

Both average labour productivity and total labour force are assumed to grow over time and their law of motion is given by the two following first-order differential equations

$$A(t) = A_0 \cdot e^{at} \quad (4)$$

$$\text{and } L(t) = L_0 \cdot e^{bt} \quad (5)$$

where a and b are positive.

Following a bargaining approach to the determination of money wages, the money wage is determined by the product of a target real wage w^* and the expected price level p^e

$$w(t) = w^* \cdot p^e(t) \quad (6)$$

Let us assume that we are in a period when the inflation rate is trendless and perceived as a simple random walk

$$\pi = \pi_{-1} + \varepsilon \quad (7)$$

where π is the rate of inflation, ε is white noise and -1 means one period earlier. Then the rational forecast of the price level is

$$p^e = p_{-1} \cdot (1 + \pi_{-1}) \quad (8)$$

A target real wage for workers is assumed to exist and is given by

$$w^*(t) = \bar{w} \cdot e(t)^\phi \cdot e^{at} \quad (9)$$

where \bar{w} is the target real wage if there is full employment of labour (in the initial period), e is the employment ratio (the ratio of total employment to total labour force) and α is an autonomous component that reflects secular real target wage growth over time. For the matter of simplicity it is also assumed that α equals the growth rate of the productivity of labour. Combination of expressions (3), (4), (6), (7), (8) and (9) then yields the following expanded version of the general price level

$$p_{-1}(1 + \pi_t) = m \cdot \bar{w} \cdot e(t)^\phi \cdot p_{-1}(1 + \pi_{-1}) / A_0 \quad (10)$$

If the expected value of the rate of inflation is realised, we have

$$m \cdot \bar{w} \cdot e^\phi(t) / A_0 = 1 \quad (11)$$

Thus the non-accelerating inflation employment ratio (NAIER hereafter) is

$$\bar{e} = (A_0 / m \cdot \bar{w})^{1/\phi} \quad (12)$$

Expression (12) tells us that the NAIER - provided that secular target real wage growth tracks average productivity growth - is a function of the initial level of the average productivity of labour, the average mark-up set by firms and the value of the target real wage when there is full employment of labour. The former means that, if *any* of the components above changes, so will the NAIER. Following the method of previous sections of introducing assumptions aimed at simplifying the policy analysis, it will be initially assumed that an estimate of the true value of the NAIER is currently available for the CB. This assumption will be removed at a later stage. We may further simplify equation (10) with a view to obtaining a dynamic expression of the rate of inflation

$$(1 + \pi(t)) / (1 + \pi_{-1}) = m \cdot \bar{w} \cdot e(t)^\phi / A_0 \quad (13)$$

Taking natural logarithms gives

$$d\pi(t) / dt \cong \ln m + \ln \bar{w} - \ln A_0 + \phi \cdot \ln e(t) \quad (14)$$

When $e(t) = \bar{e}$ in (14), we have that $d\pi / dt = 0$. Therefore, the change in the inflation rate can be written as

$$d\pi(t) / dt \cong \phi \cdot (\ln e(t) - \ln \bar{e}) \quad (15)$$

As it stands, expression (15) is not an adequate representation of inflation dynamics. Further transformation yields

$$d\pi(t) / dt \cong \phi \cdot \ln(e(t) / \bar{e}) \quad (16)$$

A final simplification allows us to express the change in the rate of inflation as (Nota 8)

$$d\pi(t) / dt \approx \phi \cdot (e(t) - \bar{e}) \quad (17)$$

Therefore, the change in the inflation rate is expressed as a linear function of the difference between the actual and the non-accelerating inflation employment ratios. Insofar as it's been assumed that the average mark-up is constant, we have that labour's share in real GNP will also be constant. If we further assume that workers spend fully their wage income and there is no consumption demand by capitalists, equilibrium in the goods market can be written as

$$uY(t) / K(t) = g(t) \quad (18)$$

where u is the profit share in GNP, K is the real value of the capital stock and g is the net rate of accumulation. The profit rate will thus be equal to the accumulation rate. In turn, the desired rate of investment will be expressed as a linear function of the difference between the profit rate b and the relevant real interest rate r

$$g^d(t) = b_1 \cdot b(t) - b_2 r(t) \quad (19)$$

with b_1 and b_2 being positive constants (NOTA 9) As indicated by expression (18) the profit rate must always be equal to the net rate of

accumulation. Therefore we may represent the desired accumulation rate by

$$g^d(t) = b_1 \cdot g(t) - b_2 r(t) \quad (20)$$

To determine the dynamics of g we will use a partial adjustment framework to allow for possible order and construction lags (Jarsulic, 1989, p.40)

$$(dg(t)/dt) / g(t) = d \cdot (g^d(t) - g(t)) \quad (21)$$

where $d \neq 0$.

Combining (20) and (21) gives the dynamics of g in the form

$$dg(t)/dt = d b_3 g(t)^2 - d b_2 r(t) g(t) \quad (22)$$

where $b_3 = b_1 - 1 \neq 0$.

By definition, average labour productivity is the ratio of real income to total employment. Therefore, the employment ratio can be expressed as the ratio of real income to the product of the average productivity of labour and the total labour force

$$e(t) = Y(t) / (A(t)L(t)) \quad (23)$$

Differentiation of (23) gives

$$(de(t)/dt) / e(t) = (dY(t)/dt) / Y(t) - g_n \quad (24)$$

with g_n being Harrod's natural rate of growth.

Similarly, differentiation of (18) gives

$$(dY(t)/dt) / Y(t) = (dg(t)/dt) / g(t) + g(t) \quad (25)$$

Combining expressions (22), (24) and (25) yields

$$de(t)/dt = (d b_3 + 1) g(t) e(t) - d b_2 r(t) e(t) - g_n e(t) \quad (26)$$

Expression (26) describes the employment ratio law of motion. It is the result of the joint action of two counteracting feedback mechanisms. The first feedback mechanism is a positive one and links the profit and investment rates of the economy. A higher/lower profit rate leads - everything else being constant - to a higher/lower rate of investment which translates into a higher/lower rate of growth of aggregate demand. In turn, a higher/lower rate of growth of aggregate demand leads to a higher/lower level of capacity utilisation. Finally, a higher/lower rate of capacity utilisation leads - since income distribution has been assumed to be constant - to a higher/lower profit rate and a new feedback round starts again. The negative feedback mechanism is the result of changes in the relevant real interest rate and it has been assumed to determine - along with the profit rate - the desired rate of investment. Each time a new feedback round sets off, the level of the relevant real interest rate - in conjunction with the profit rate - determines the acceleration/deceleration of the rate of investment. Therefore, the actual path the economy follows can be seen as the net result of these two countervailing forces.

By setting $de(t)/dt = 0$, we obtain the expression of what we will be referred to as the *short-term critical real interest rate* (SCRIR hereafter) and that we define as that relevant real interest rate that guarantees $de(t)/dt = 0$ for a given rate of capital accumulation (Nota 10). Its formal expression is

$$r_s^*(t) = (d b_3 + 1) g(t) / d b_2 - (g_n / d b_2) \quad (27)$$

By setting $de(t)/dt = 0$ and $dg(t)/dt = 0$ and then combining the resulting expressions, we obtain the real interest rate compatible with a long-term neutral monetary policy or what we will hereafter refer to as the *critical real interest rate* (CRIR hereafter) and whose expression is

$$r^* = b_3 g_n / b_2 \quad (28)$$

As can be seen above, when the actual investment rate is equal to the natural growth rate, the SCRIR will be equal to the CRIR. The former means that when $g(t) \neq g_n$, setting the interest rate equal to r^* will lead to the employment ratio having a positive growth rate and vice versa. Substituting r^* into our feedback rule gives

$$r(t) = (b_3 g_n / b_2) + \alpha_1 (\pi(t) - \pi^d) + \alpha_2 (Y(t) / Y^*(t) - 1) \quad (29)$$

where both α_1 and α_2 are exogenous and positive.

It will be further assumed that the NAIER is equivalent to the ratio of trend output to the product of average labour productivity and total labour force

$$\bar{e} \equiv (Y^*(t) / A(t) L(t)) \quad (30)$$

The former assumption seems to be justified since our feedback policy rule indicates that the CBs' actions aim at hitting an inflation target. Therefore, trend output must be equivalent to that level of output compatible with an approximately constant inflation rate in the long-run. Thus, this assumption allows us to further simplify the feedback policy rule and express it as

$$r(t) \equiv (b_3 g_n / b_2) + \alpha_1 (\pi(t) - \pi^d) + \alpha_2 (e(t) - \bar{e}) \quad (31)$$

Finally, substituting (31) into differential equations (22) and (26) gives

$$dg(t)/dt \equiv d(b_3 g(t)^2 + c_1 g(t) - c_2 \pi(t) g(t) - c_3 e(t) g(t)) \quad (32) \text{ and}$$

$$de(t)/dt \equiv (d b_3 + 1) g(t) e(t) + d c_1 e(t) - d c_2 \pi(t) e(t) - d c_3 e(t)^2 - g_n e(t) \quad (33)$$

where $c_1 = b_2 \alpha_1 \pi^d + b_2 \alpha_2 \bar{e} - b_3 g_n$; $c_2 = b_2 \alpha_1$ and $c_3 = b_2 \alpha_2$.

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4.- Formal analysis

The purpose of this section is to obtain the requisite formal conditions for the emergence of endogenous oscillations in the economy. The proofs are based on a truncated version of the interpretation of the Hopf bifurcation theorem by Guckenheimer and Holmes (1983, pp.151-4) that can be found in Lorenz (1993, pp.96-8). However, their details have been moved to the mathematical appendix. Thus, equations (17), (32) and (33) make up a dynamical system involving three variables g , e and π , whose critical points can be found by setting $\dot{g} = \dot{e} = \dot{\pi} = 0$. Linearizing the dynamical system at the critical point $P_{g,e,\pi}^* = (g_n, \bar{e}, \pi^d)$ gives

$$\begin{pmatrix} \dot{g} \\ \dot{e} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} d b_3 g_n & -d b_2 \alpha_2 g_n & -d b_2 \alpha_1 g_n \\ (d b_3 + 1) \bar{e} & -d b_2 \alpha_2 \bar{e} & -d b_2 \alpha_1 \bar{e} \\ 0 & \phi & 0 \end{pmatrix} \cdot \begin{pmatrix} g - g_n \\ e - \bar{e} \\ \pi - \pi^d \end{pmatrix} \quad (34)$$

where all the variables are evaluated at the appropriate critical point. Let us define

$$\Delta_1 = d(b_3 g_n - a_2 b_2 \bar{e}) \quad (35)$$

$$\Delta_2 = -a_1 b_2 d g_n \bar{e} \phi \quad (36)$$

$$\Delta_3 = db_2 \bar{e} (a_1 \phi + a_2 g_n) \quad (37)$$

$$\Delta_4 = \bar{e} db_2 \left(-a_1 \phi g_n (d b_3 + 1) - a_2 b_3 g_n^2 d + a_2 b_2 \bar{e} d (a_1 \phi + a_2 g_n) \right) \quad (38)$$

where $\Delta_1 = Tr(J)$, $\Delta_2 = Det(J)$, $\Delta_3 = J_{11} + J_{22} + J_{33}$ and $\Delta_4 = -\Delta_1 \Delta_3 + \Delta_2$, the J_{ii} being the principal minors (of order 2) of the jacobian matrix J, that is, the determinants of the matrices that are obtained after deleting the i-th row and the i-th column. Then, it is well known that a *necessary* and *sufficient* condition for local stability of the linear system (34) is (Nota 11)

$$\Delta_1 < 0, \Delta_2 < 0, \Delta_3 > 0 \text{ y } \Delta_4 > 0 \quad (39)$$

Therefore, as $\Delta_2 < 0$ y $\Delta_3 > 0$ for any value of the parameters, the critical point in (34) is locally asymptotically stable if the two following conditions are satisfied

$$b_3 g_n - a_2 b_2 \bar{e} \pi > 0 \quad (40)$$

$$-a_1 \phi g_n (d b_3 + 1) - a_2 b_3 g_n^2 d + a_2 b_2 \bar{e} d (a_1 \phi + a_2 g_n) > 0 \quad (41)$$

It is important to ascertain how the stability of the dynamical system is affected when the value of certain parameters changes. For example, we have that:

$$\frac{\partial \Delta_1}{\partial a_1} = 0 \quad \text{and} \quad \frac{\partial \Delta_1}{\partial a_2} = -b_2 \bar{e} \pi > 0 \quad (42)$$

$$\frac{\partial \Delta_4}{\partial a_1} = \phi (a_2 b_2 \bar{e} d - g_n (d b_3 + 1)) > 0 \quad \text{and} \quad \frac{\partial \Delta_4}{\partial a_2} = 2 a_2 g_n d (\bar{e} b_2 - b_3) + b_2 \bar{e} d a_1 \phi > 0$$

Therefore, an increase in the value of the two reaction coefficients a_1 or a_2 increases the stability of the dynamical system. In order to explore an additional insight of this paper, let us go beyond the local stability analysis and examine what can happen if the equilibrium becomes unstable due to certain changes in parameter values. The stability condition (41) indicates that increasing, for example, the importance of construction and order investment lags (reducing d) beyond a certain value destabilises the model. Then, let d be the chosen bifurcation parameter and assume an initial value of d such that the Routh-Hurwitz conditions are initially fulfilled. To the extent that the sign of Δ_1 , Δ_2 and Δ_3 does not depend upon d and since the roots of the Jacobian vary continuously with d , there must be a value of d , say d^* , where either a real root or the real part of a pair of complex roots is equal to zero (see section A in the appendix). Therefore, the existence of a pair of pure imaginary eigenvalues requires the following condition to be fulfilled

$$-a_1 \phi g_n (d b_3 + 1) - a_2 b_3 g_n^2 d + a_2 b_2 \bar{e} d (a_1 \phi + a_2 g_n) = 0 \quad (43)$$

Therefore, the critical value d^* indicates the borderline case where the dynamical system changes from being locally convergent to locally divergent. In order to show the emergence of closed orbits around the critical point we can make use of the Hopf bifurcation theorem. The first condition of the theorem has already been proved, as we have just shown the existence of a pair of pure imaginary roots in (34). The second condition that has to be fulfilled is (see section B in the appendix)

$$a_2 b_2 \bar{e} d (2 a_1 \phi + a_2 g_n (1 + b_2)) - 2 b_3 d g_n (a_1 \phi + a_2 g_n) - a_1 \phi g_n > 0 \quad (44)$$

Thus if conditions (43) and (44) hold, we can assure the existence of at least one closed orbit in a neighbourhood of the critical point. The determination of the stability of the closed orbit requires a more complex analysis, but it is clear from the Hopf bifurcation Theorem that the orbits will exist either in a left (sub-critical) or a right (super-critical) neighbourhood of d^* . Given that the real root is negative (since Δ_1 and

Δ_2 have not changed their sign), the bifurcating orbit will be locally attracting if it is super-critical and locally repelling if it is sub-critical. Whether the super-critical or the sub-critical case holds will depend on the higher-order non-linear terms in the Taylor expansion of the dynamical system at the critical point. In any case, the subcritical case may also be of interest. It corresponds to the "corridor stability" concept introduced by Leijonhufvud according to which, the economy will be locally stable for small perturbations around the stationary point (Benhabib and Miyao, 1981, p.593).

5.- Policy analysis

This section contains the monetary policy analysis. It has been divided into three subsections. The first subsection describes the three types of policy lag a CB faces in the absence of any particular reaction function for the real interest rate. The second subsection contains a verbal description of the process by which endogenous fluctuations may emerge. It thus represents an extension of the formal results of section 4, with the setting of the "relevant" real interest rate by the CB based on equation (31). Finally, the third subsection sets out several policy implications.

5.1.- Policy lags

The purpose of this subsection is to analyse at a theoretical level the policy lags a CB will have to deal with if it implements an inflation targeting regime given a set of simplifying assumptions. These policy lags are the result of the interaction of the model of the economy with the reaction function of the CB and they lie at the core of the emergence of endogenous oscillations, so that their assimilation is essential for an understanding of the arguments in the following sections. The policy analysis contained in this section does "not" make any particular assumption about the reaction function that guides the CB's behaviour so that its conclusions have, in principle, general validity in the context of inflation targeting. However, as we noted above, it is assumed that the CB *can* and actually *sets*, albeit with a lag, the appropriate level of the "relevant" real interest rate in order to hit its inflation target. Therefore, the problem at hand is what types of policy lags a CB faces when setting short-term nominal interest rates.

The model we set up in the previous section serves as the benchmark against which policy lags will be studied. In particular, equations (17), (27) and (28) provide all the information we need. As shown above, equation (17) describes the motion of the rate of inflation as a linear function of the employment gap. The true value of the NAIER is not known, although it is implicitly assumed that an estimate is currently available and that the true value does not experience significant changes in the short-run. This assumption is to be kept throughout the analysis. Finally, equations (27) and (28) represent the SCRIR and CRIR respectively.

The essential argument is that any CB basically faces "three" types of policy lags and the potential emergence of endogenous oscillations. The first type of policy lag recognises the fact that the policy instrument that the CB directly controls is a short-term nominal interest rate whereas the transmission mechanism of monetary policy referred to above operates through a vector of short and long-term real interest rates, i.e., the "relevant" real interest rate. It also recognises the fact that CBs do *not* change the value of the nominal short-term interest rate at the highest possible speed because of: (i) model uncertainty considerations and (ii) concern about output-gap volatility (Nota 12). In practice all CBs adopt a gradual approach to short-term interest rate setting, so that sharp and sudden changes in short-term interest rates tend to be avoided (Svensson, 1999, p.625).

Therefore, it will take some time before the CB actually sets the nominal short-term interest rate at that level requisite to make the "relevant" real interest rate reach the desired level, i.e., the level that derives from equation (31). For example, if the actual employment ratio were above/below the NAIER, the CB would have to bring the actual employment ratio down/up. In order to do so, it should first set the short-term nominal interest rate at such a level that the actual "relevant" real interest rate goes above/below the SCRIR (the real interest rate that makes the growth rate of the employment ratio equal to zero for a given value of the actual investment rate and the rest of parameters of the model). The time elapsed between the detection of the existence of an employment gap and the precise point in time when the "relevant" real interest rate reaches the SCRIR will depend on a complex set of factors among which we may include the degree of commitment to price stability by the CB and its concern with output-gap volatility, the degree of model uncertainty, the fragility of the financial sector and - in the context of an open economy with a flexible exchange rate regime - the degree of capital mobility

Once the "relevant" real interest rate gets above/below the SCRIR, a *second* policy lag will come into action. The initial employment ratio is not likely to be equal to the NAIER. The real issue is that, according to equation (33), the rate of growth of the employment ratio is a function of both the rate of investment and the real rate of interest. As a result of it, by setting the real interest rate above/below the SCRIR, we can *only* affect the rate of growth of the employment ratio. Of course, as long as the former is different from zero, the level of the actual employment ratio will change in subsequent periods. Thus, if the initial employment gap is not equal to zero, a second policy lag will consist of that additional time requisite to push the level of the employment ratio below/above - depending on whether the inflation rate was initially above or below the target rate - the NAIER.

Equation (33) actually merges into one what are actually two different though closely related policy lags. If split into two, a first policy lag connects the real interest rate to the rate of growth of aggregate demand and a second policy lag connects the latter to the rate of growth of the employment ratio. Insofar as the natural rate of growth is not constant over time, the CB can never know how fast real income must grow in order to attain a given growth rate for the employment ratio. In any case, a CB can observe the evolution of the employment ratio in the short-run, so that "all" it has to do is to set a time path for the real interest rate so that the actual employment ratio approaches its target value at approximately the desired pace. The third and final policy lag is a consequence of the fact that - as shown in equation (17) in our model - it is the *change* in the rate of inflation and *not* its level that is a function of the employment gap. Once the actual employment ratio is below/above the NAIER - depending on whether the initial rate of inflation was above or below its target level - a *uncertain* number of periods will elapse until the rate of inflation actually converges to its target value.

However, it may well be the case that, once the inflation gap is actually equal to zero, the employment gap is not, thereby making the maintenance of a zero inflation gap a difficult task even in the absence of external shocks. To put it in a different way, since the employment gap was necessarily different from zero - otherwise the inflation gap would have remained constant and different from zero all this time - it may occur that, once the inflation rate has actually converged to its target level, the employment gap is, however, sufficiently high as to be able to push again the rate of inflation away from its target. If this were the case, an oscillation may arise. Therefore, the three policy lags identified above plus the oscillation problem we have just mentioned constitute the theoretical basis upon which an account of self-sustained oscillations in the level of economic activity induced by monetary policy is to be built in next section.

5.2- Monetary policy and endogenous oscillations

In this subsection we aim at answering the following question: under what circumstances the implementation of an inflation targeting regime by a CB, where the latter is assumed to set the "relevant" real interest rate according to equation (31) will lead to the emergence of self-sustained or endogenous oscillations in the level of economic activity?. Before we proceed with this problem, two points need to be further clarified.

First, as we noted above, the policy reaction function under inflation targeting will, in general, not be a Taylor-type reaction function (Svensson, 1999, p.628). However, as we emphasised above, for a simple macroeconomic model - like ours -, and provided all the empirical evidence referred to in section 2 is taken into account, our modified version of Taylor's rule may be a reasonable approximation to the actual behaviour of CBs. We think that an abstraction of the CB's pattern of behaviour is indispensable if we want to reach some conclusions about the problem at hand and, therefore, we address the problem in the most amenable way we found. Second, and related to what we have just said, it is implicitly assumed that the CB is in a position to know and set, with a lag, the level of short-term nominal interest rates requisite to make the "relevant" real interest rate attain the level dictated by equation (31) above.

Having made clear these two points, we then move on to the dynamics. The explanation of the circumstances under which oscillations in the economy may arise makes use of an example based on figure 1 below. For simplicity, it is assumed that the CRIR has the same numerical value as target inflation. Both the real interest and the inflation rates are represented in the vertical axis whereas time is represented in the horizontal axis. It is also assumed for convenience that the point in the vertical axis crossed by the horizontal axis corresponds to the actual value of both the CRIR and target inflation. The variables plotted in figure 1 are the inflation rate, the SCRIR and the actual real interest rate set by the CB. The explanation for the emergence of oscillations draws heavily on the policy lags we analysed in section 5.1. The additional element was also scheduled in the last section, i.e., an oscillation problem that comes about because the

employment gap may be significantly different from zero when the inflation rate has reached its target value.

In figure 1 below it's been assumed that the inflation rate is initially equal to its target value, the SCRIR is equal to the CRIR and the employment ratio is above the NAIER. As a result of it, the rate of inflation will rise from the start. As soon as both a positive inflation and employment gaps are detected, the real interest rate will rise according to equation (31). The actual interest rate will, after some periods, rise above the SCRIR and the employment gap will thus start narrowing. However, the inflation rate will not stop rising until the employment ratio has actually fallen below the NAIER. In turn, the time elapsed until the employment gap becomes zero will depend upon how forceful changes in real interest rates induced by the CB are. However, as we emphasised above, the ability of the CB to induce sharp changes in interest rates in the short-run is somehow constrained.

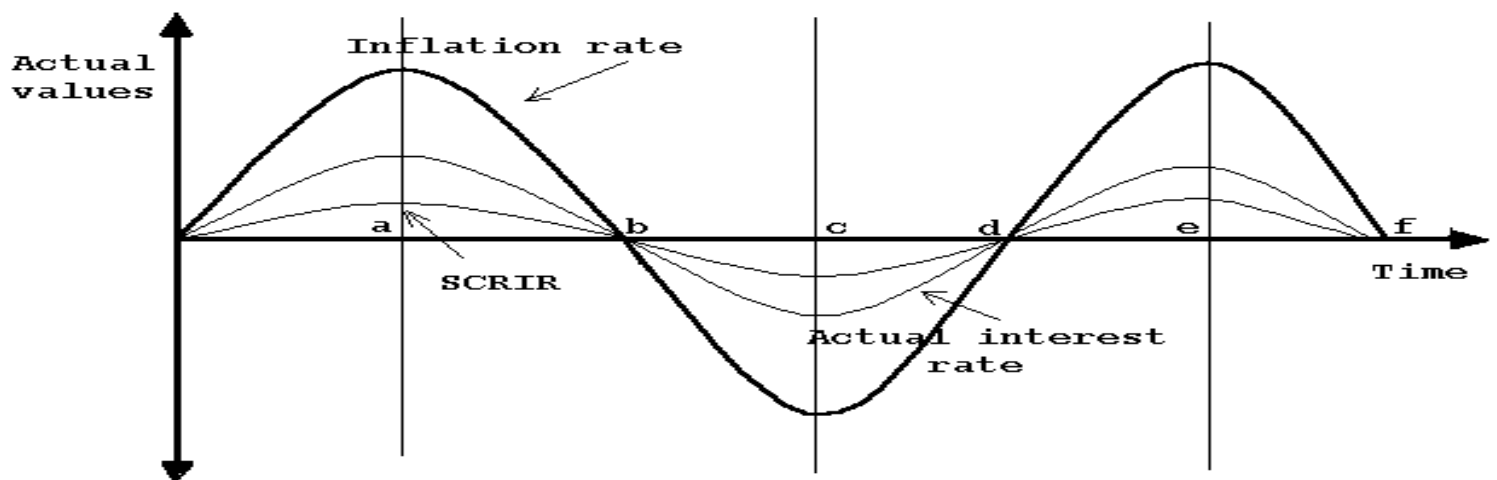


Figure 1

At some point, after the rise in the real interest rate and the resulting fall in the employment gap level, the inflation rate will stop rising (this corresponds to point a in the horizontal axis of figure 1). As soon as the employment ratio has changed its sign and turned negative, the inflation rate will start approaching its target value. Alongside the convergence of the inflation rate towards its target value, both the SCRIR and the actual interest rate will start approaching the CRIR. In the case of the SCRIR, this will occur as long as the rate of investment converges towards the natural rate. In the case of the actual interest rate, the convergence is partially determined by the relative weight of the inflation gap reaction coefficient (compared to the relative weight of the employment gap coefficient). If the value of these two coefficients were approximately the same, then the actual interest rate would cross the horizontal axis somewhere left of point b in figure 1. If the value of the employment gap reaction coefficient were equal to zero, then the actual interest rate would cross the horizontal axis exactly at point b. The latter case is precisely the assumption we have made in order to simplify the graphical exposition (Nota 13).

Therefore, at point b in figure 1 above, we have that the rate of investment is equal to the natural growth rate, the rate of inflation is equal to its target rate, the rate of interest is equal to both the SCRIR and CRIR and the employment ratio is below its non-accelerating inflation value. As a result, the inflation rate will keep on falling and the inflation gap will start widening again. In turn, the interest rate will fall below the SCRIR and the employment ratio will start rising. However, it will take some time before the employment ratio gets back to the NAIER. Before this occurs, the inflation gap will continue growing. Eventually, the actual employment ratio will reach the NAIER again (point c in figure 1). By the time this occurs, the rate of interest will necessarily be below the SCRIR and the inflation gap will start narrowing again. However, it will take again some time before the inflation gap becomes zero. The former occurs at point d, where it has been assumed that the interest rate is again equal to both the CRIR and SCRIR - the rate of investment is therefore assumed to be equal to the natural rate -, the inflation gap is equal to zero and the employment gap is positive.

Now the problem the CB faces is equivalent to the problem it faced at the initial period. As long as the employment gap remains positive the inflation gap will start growing again. However, the CB is not in a position to change this scenario immediately. Firstly, it will rise the interest rate above the SCRIR, which itself will require some time. Then, after some time, during which the employment ratio will have been falling, the employment gap will turn negative (point e in figure 1) and the inflation gap will start narrowing again. When the inflation rate reaches its target value (point f in figure 1), the employment gap is likely to be negative so that the rate of inflation will continue falling and, therefore, the whole process will repeat again.

Therefore, the emergence of self-sustained oscillations has to be seen as the result of the interaction of positive and negative feedback mechanisms, with monetary policy lags playing an essential role. In terms of the model we presented in section 3, the emergence of self-sustained oscillations requires a set of conditions to be fulfilled. These conditions affect, first of all, the relative values of the model parameters including the reaction coefficients. Second, these results crucially depend on the likelihood of equation (31) being a valid approximation to the impact of monetary policy upon aggregate demand over the business cycle if the CB sets short-term nominal interest rates according to Taylor's rule.

5.3.- Policy implications

The monetary policy implications of the analysis developed in this paper are, in our opinion, the following. First, a crucial problem for monetary policy that has emerged above is that, in theory, the CB can only affect real aggregate demand growth and, therefore, the rate of inflation through changes in a set of short and long-term real interest rates. However, strictly speaking, the CB has only close control over short-term nominal interest rates and, in addition, the ability of the CB to induce sudden and large enough changes in the former is in fact constrained by the CB's own concern about output volatility and model uncertainty. The expectations hypothesis to the term structure of interest rates is not very helpful here since it attaches a crucial role to expectations about future short-term nominal interest rates which, for obvious reasons, can not be known by the CB in advance. As a result of it, it is very unlikely that a certain change in short-term nominal interest rates in periods t and $t + n$ (however short n is) will have the same effect upon a set of different long-term real rates in any two such periods. In addition, it is also unlikely that the relative importance of the credit channel in transmission mechanism will be invariant. Insofar as this were the case, the actual path of the "relevant" real interest rate would depart from equation (31), even if the short-term nominal interest rate were set according to Taylor's rule,

As a result, the *first kind of uncertainty facing the CB is thus how a certain change in the level of short-term nominal interest rates will affect short and long-term real interest rates and, in turn, how the latter will affect aggregate demand*. To this, we can add the uncertainty surrounding the current value of the natural rate of growth or equivalently, the uncertainty about the impact that a certain rate of growth of real income will have upon the rate of growth of total employment.

Moreover, it can also be ascertained from our analysis above that changes in short-term nominal interest rates will have to be large enough as to: (i) push the "relevant" real interest rate above the SCRIR to stop the employment gap from widening and (ii) keep the "relevant" real interest rate above the SCRIR long enough as to let the employment gap narrow in the first place, and then push it in the opposite direction in order to gradually close the inflation gap. In this same line, the mathematical analysis indicated (Nota 14) that, were the CB to set the level of the relevant real interest rate according to a policy reaction function like equation (31), the higher the value of the reaction coefficients, the more stable the economy would be in a neighbourhood of the stationary point.

However, this sort of formal dynamical analysis does not recognise the constraints - the main constraints in a closed economy being model uncertainty and concern for output volatility - the CB faces when setting short-term nominal interest rates. Thus, as a first policy implication, it can be argued that, *when setting the short-term nominal interest rate, the CB will have to strike a balance between the need to induce large and fast enough changes in the former in order to be able to offset the impact of the destabilising forces at work in the economy - especially the positive feedback mechanism linking the rates of profit and investment - and hit its inflation target, and the need to keep the degree of output volatility below a certain desired level*. Again, the former will require gradual changes in short-term nominal interest rates. This is, in our opinion, the first policy dilemma facing CBs.

A second policy problem, not explicitly analysed in this paper, refers to the consequences of the uncertainty surrounding the value of the NAIRU, better known as NAIRU. It has been recently argued that the NAIRU plays a limited role in forecasting inflation. For instance, it has been argued that *precise knowledge of the NAIRU is not very important from the perspective of forecasting inflation. Forecasts of inflation based on the deviation of unemployment from the NAIRU are similar whether the NAIRU is assumed to be 4.5, 5.5 or 6.5 percent. The difficulty in estimating the NAIRU and its limited role in forecasting inflation are, of course, interrelated; after all, if the NAIRU played a more important role in forecasting inflation, then its value could be pinned down with greater precision from the data* (Staiger, Stock and Watson, 1997).

Nevertheless, the authors just quoted also reject the argument that the limited role of the NAIRU in forecasting inflation involves that a NAIRU does not exist. According to them, *this argument could either be based on a belief that the NAIRU has shifted, or on the wide confidence intervals surrounding the estimates*. A theoretical justification for such a position could be that the hysteresis that has been proposed as a possible description of European unemployment is also present in the U.S. economy. These authors, however, believe that the empirical evidence does not support this view and argue that *although there is evidence that the NAIRU has shifted, the shifts have been relatively minor over the past three decades* (Staiger, Stock and Watson, 1997, p.47). What emerges from their discussion is that, despite the relatively wide confidence intervals surrounding the estimates and its limited role in forecasting inflation, the existence of a NAIRU, at least in the U.S. economy, can not be denied. Therefore we believe that it is legitimate to keep the notion of the NAIRU provided some degree of uncertainty about its true value is retained in the policy analysis.

How does such uncertainty affect our results?. We guess that the practical implication of the uncertainty surrounding the value of the NAIRU is likely to be a high degree of conservatism in the actions of the CB for fear of fuelling inflation. In turn, this behaviour may lead to self-inflicted recessions as long as the CB assumes that the NAIRU to be approximately constant and exogenous at a particular value. For instance, it may raise interest rates before it is justified (if the NAIRU has actually fallen or its estimator is upward biased), that is, *it may make the business cycle peak endogenous to policy* (Galbraith, 1997, p.99). In the opposite case, authorities' concern about unemployment and its related social costs might well make them keep interest rates constant as long as there is no consistent evidence of an overheating economy. *This is, according to us, the second policy dilemma*. In addition, uncertainty surrounding the value of the NAIRU or of any other model parameter, as we have already noted, is likely to translate into a more gradual approach when setting interest rates. The former, in turn, means that, other things being the same, the ability of the CB to offset the positive feedback mechanisms at work in the economy will be further weakened.

6.- Conclusions

This paper has analysed the dynamics of an economy that is inherently unstable when the CB - as the only stabilising force in the economy - implements an inflation targeting regime. Our first result has been that, under certain conditions - where an admittedly strong assumption concerning the impact of monetary policy upon aggregate demand over the business cycle when the CB actually sets short-term nominal interest rates according to Taylor's rule was made -, the emergence of endogenous fluctuations in the economy is an scenario that can not be ruled out. Its likelihood depends on the extent to which the impact of monetary policy over aggregate demand over the business cycle is well described by the reaction function for real interest rates we have used and in the values the model parameters actually take. It has also been argued that at the core of the emergence of endogenous oscillations lie a set of policy lags that we have subsequently identified. A second implication of the analysis above is that the two main policy dilemmas any CB faces are: (i) the degree of gradualism adopted when setting the level of short-term nominal interest rates and (ii) the desirability or otherwise of raising interest rates as soon as economic indicators suggest the emergence of inflationary pressures in the economy.

An aspect of our analysis to be explored in the future is the explicit inclusion of open economy considerations. As it was scheduled above, this should affect the impact interest rate changes have upon aggregate demand - presumably raising the short-term nominal interest rate sensitivity of aggregate demand - and the ability of CB - limiting the range of possible values for short-term nominal interest rates when exchange rates are flexible - to set interest rates. Therefore, as far as our results are concerned, the net outcome deriving from the explicit inclusion of open economy considerations can not be devised until a formal treatment of it is properly undertaken.

Finally, these results suggest that the flourishing literature on inflation targeting should also pay attention to the analysis of the stabilising properties of different interest rate reaction functions in models where the self-stabilising mechanisms of the economy work slowly, to say the least. This way, we think we can help avert some undesired policy scenarios - for example the inability of CBs to stimulate aggregate demand growth by lowering short-term nominal interest rates when the former are close enough to the zero floor and a deflationary process has set in - which might come about in the foreseeable future when inflation targeting regimes are more widespread as well as gain some additional understanding about the workings of monetary policy.

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Appendix

A) Proof of the existence of a pair of pure imaginary eigenvalues

By Orlando's formula (See Gantmacher, 1954, p.197),

$$-(\lambda_1 + \lambda_2)(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3) = -\Delta_1\Delta_3 + \Delta_2 = \Delta_4 \quad (45)$$

must always hold. Since

$$\Delta_1 = \lambda_1 + \lambda_2 + \lambda_3 \quad (46) \quad \Delta_2 = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \quad \text{and} \quad (47) \quad \Delta_3 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \quad (48)$$

where the λ s are the eigenvalues of the linearized system, it can be shown that the real parts of the complex conjugate eigenvalues are zero and that there is no other eigenvalue which equals zero if (46), (47) and (48) are different from zero and the following condition is fulfilled

$$\Delta_4 = -a_1 \phi g_x (d b_3 + 1) - a_2 b_3 g_x^2 d + a_2 b_2 \bar{e} d (a_1 \phi + a_2 g_x) = 0 \quad (43)$$

Having chosen d as the bifurcation parameter, we have that an increase in d implies

$$\partial \Delta_4 / \partial d = (a_2 b_2 \bar{e} - b_3 g_x) \cdot (a_1 \phi + a_2 g_x) \neq 0 \quad (49)$$

which is equivalent to saying that a decrease in d lowers Δ_4 and destabilises the model. The fact that a pair of pure imaginary eigenvalues and a non-zero real eigenvalue exist at the bifurcation value d^* can be seen from Orlando's formula (45). As the product of all three eigenvalues is initially negative ($\Delta_2 < 0$), and the determinant of the Jacobian matrix doesn't vanish as d is reduced, it is impossible to encounter a real zero eigenvalue. When the case of a saddle point is excluded (Nota 15), a pair of real eigenvalues can not come with opposite signs. It follows that $\Delta_4 = 0$ can only be fulfilled when a pair of eigenvalues is purely imaginary.

B) Proof of the emergence of closed orbits

At this point we may establish the emergence of closed orbits by means of the Hopf bifurcation theorem. Before this is done, a truncated version of Guckenheimer and Holmes (1983, pp. 151ff.) that can be found in Lorenz (1993, pp.96-8) will be shown. Consider the continuous-time system

$$\dot{x} = f(x, \mu), \quad x \in \mathbb{R}^n \quad \text{y} \quad \mu \in \mathbb{R}. \quad (50)$$

Assume that (50) possesses a unique fixed point x_0^* at the value μ_0 of the parameter, i.e.,

$$F(\mu) = 0 = f(x_0^*, \mu_0). \quad (51)$$

Furthermore, assume that the determinant of the Jacobian matrix J of (50) differs from zero for all possible fixed points (x, μ) . Consider a neighbourhood $B_r(\mu_0) \in \mathbb{R}$ of the parameter value $\mu = \mu_0$. Then the implicit function theorem ensures the existence of a smooth function $x^* = x^*(\mu)$ for $\mu \in B_r(\mu_0)$; i.e., for every μ in the neighbourhood there exists a unique fixed point x^* . Assume that this fixed point is stable for large values of the parameter μ . The Hopf bifurcation theorem establishes the existence of closed orbits in a neighbourhood of a fixed point for appropriate values of the parameter μ . Suppose that the system (50) has a fixed point (x_0^*, μ_0) at which the following properties are satisfied:

- i) The Jacobian of (50), evaluated at point (x_0^*, μ_0) has a pair of pure imaginary eigenvalues and no other eigenvalue with zero real parts.
- ii) $\partial \text{Re} \lambda(\mu) / \partial \mu \neq 0$

then there exists some periodic solutions bifurcating from $x^*(\mu_0)$ at $\mu = \mu_0$. The first of these properties has already been proved. As for the second property, we have that

$$\Delta_4 = -2\alpha(\alpha^2 + \beta^2 + 2\alpha\lambda_1 + \lambda_1^2) = -\Delta_1 \cdot \Delta_3 + \Delta_2 \quad (52)$$

Differentiating both sides with respect to d at the bifurcating point $d = d^*$ yields

$$-2 \cdot (\alpha^2 + \lambda_1^2) \cdot \frac{\partial \alpha(d^*)}{\partial d} = \Delta_3 \cdot \frac{-\partial \Delta_1}{\partial d} - \Delta_1 \cdot \frac{\partial \Delta_3}{\partial d} + \frac{\partial \Delta_2}{\partial d} \quad (53)$$

Differentiating (35), (36) and (37) with respect to d gives

$$\partial \Delta_1 / \partial d = b_3 g_n - a_2 b_2 \bar{e} \pi \quad (54)$$

$$\partial \Delta_2 / \partial d = -a_1 b_2 \phi g_n \bar{e} \pi \quad (55)$$

$$\partial \Delta_3 / \partial d = b_2 \bar{e} (a_1 \phi + a_2 g_n) \phi \quad (56)$$

As $(\alpha^2 + \lambda_1^2)$ is always positive and we are seeking to prove that $\partial \alpha(d^*) / \partial d \neq 0$ (as d crosses d^* from right to left) the condition to be fulfilled is

$$\Delta / \partial d = b_3 g_n - a_2 b_2 \bar{e} \pi \quad (57)$$

After computing (57), we have that system (34) will possess closed orbits in a neighbourhood of the bifurcating point if condition (43) is fulfilled and

$$a_2 b_2 \bar{e} d (2 a_1 \phi + a_2 g_n (1 + b_2)) - 2 b_3 d g_n (a_1 \phi + a_2 g_n) - a_1 \phi g_n \phi \neq 0 \quad (44)$$

and therefore our proof ends up here.

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Notas a pie de página

1. Pioneering work in this area is due to Bryant, Hooper and Mann (1993). A recent and comprehensive survey of this type of literature appears in Taylor (1999).
2. A forward looking version of the simple backward looking reaction function proposed by Taylor (1993) and that Clarida and Gertler (1996) argued to have been a good description of Bundesbank monetary policy in the past, is found to do quite a good job of characterising monetary policy after 1979 for a group of countries which includes the US, Germany, Japan, UK, France and Italy (Clarida, Galí and Gertler, 1998). According to Gerlach and Smets (1999), a Taylor rule accounts for recent movements in interest rates in the European Monetary Union area.
3. Tobin (1993) analyses the stability of a market-oriented economy that has suffered an aggregate demand shock. He focuses on two stabilising mechanisms. On the one hand, there is what he labels the "Keynes" effect and the "Pigou-Patinkin" or "real-balance" effect. On the other hand there is what he calls the "rate of change effect", or the effect that changes in the rate of inflation (or deflation) have upon the *ex-ante* real interest rate. The first two effects, however, concern the behaviour of the level (and not the rate of change) of prices and arise when, for example, a deflationary process pushes down the price level thereby rising the real value of the monetary base. In turn, the rise in the real value of the monetary base increases the demand for interest-bearing financial assets and real assets respectively. In an economy where the CB controls short-term interest rates, the Keynes effect is non-existent. However, in the case of the real balance effect, an increase in aggregate demand may occur as long as consumers see the marginal utility of money balances fall relative to the marginal utility of a wide spectrum of service-yielding consumer products and real assets. Leaving aside the theoretical importance of what Tobin calls the "reverse Pigou-Patinkin effect" or "Fisher wealth redistribution effect", it can be argued that the practical relevance of this effect will be undermined by acceptance of the "endogenous money hypothesis". For instance, as the price level falls and the real value of the monetary base rises, individuals may well devote their increased real holdings of currency to write off outstanding debt thus preventing aggregate demand from rising.
4. See Taylor (1995, p.13).
5. Taylor also argues that changes in short-term interest rates are empirically *significant* in explaining movements in long-term interest rates (Taylor, 1995, p.18).
6. Two different approaches to this problem. First, there are those ones for whom, presumably, the "relevant" real interest rate is likely to be a long-term interest rate (Taylor, 1995, p.17). In particular, he argues that *it is difficult to determine on theoretical grounds whether the short-term interest rate or the long-term interest rate has a greater effect on consumption and investment; changes in the form of the debt instrument - for example, the introduction of variable rate mortgages - are likely to change the relative importance of long versus short rates. However, there is surely some a priori reason to believe that for long-term decisions like buying a house or investing in plant and equipment, the long-term interest rate should be the variable of greater interest.* Second, a somewhat different approach emerges from the literature on the credit channel of monetary policy, where changes in short-term interest rates are having a significant impact on expenditure decisions (e.g. Bernanke and Gertler, 1995, p.28). To these two different views, it could be added that when the monetary policy transmission mechanism is analysed in the context of an open economy with a flexible exchange rate regime and high capital mobility, the relative importance of changes in short-term real interest rates *vis a vis* long-term rates is revised upwards, since the sensitivity of aggregate demand to short-term real interest rate changes is likely to increase as a result of the changes in the real exchange rate that movements in the former bring about. However, in such a context, it will also be the case that the margin for setting the level of short-term nominal interest rates is very limited.

7. Although the model is representative of a closed economy, we think it can be easily amended to account for some of the features of an open economy provided it is implicitly assumed that a flexible exchange rate regime is in place. As we noted above, if this were the case, the effectiveness of monetary policy would increase. The former could be incorporated into our model by assuming: (i) a higher sensitivity of aggregate demand to short-term nominal interest rate changes and (ii) that there is no significant effect of nominal exchange rate changes on input prices.
8. The difference between the actual and the non-accelerating inflation employment ratios will hereafter be referred to as the "employment gap".
9. We have allowed for the possibility that the sensitivity of the desired investment rate with respect to the profit rate be different to its sensitivity with respect to the relevant real interest rate because it is possible that changes in capacity utilisation have an impact on investment plans that is not fully captured by its impact upon expected profitability (Rowthorn, 1981). In any case, policy analysis is not significantly affected by the type of assumption made as for the relative value of these two parameters.
10. As a clarification, the short-term critical real interest rate is not necessarily a short-term interest rate. In fact, it might well be a long-term real interest rate since, following our previous discussion on the monetary policy transmission mechanism, for some types of spending decisions long-term interest rates are likely to have a greater impact on aggregate demand than short-term interest rates.
11. See Routh-Hurwitz conditions for the stability of dynamical systems (Gandolfo, 1980, p.250).
12. We reckon that in the context of a closed economy, the upper and lower limits to the value of the short-term *ex-ante* real interest rate are given by the level of unemployment and the rate of inflation the CB is willing to tolerate respectively. In the context of an open economy with a flexible exchange rate regime and a high degree of capital mobility, the upper and lower limits to the level of the short-term *ex-ante* real interest rate are given by a relatively small neighbourhood of the real interest rate that results from the sum of the *ex-ante* short-term real interest rate prevailing in foreign financial markets plus - provided forward foreign currency markets exist - the country (default and political) risk premium.
13. As the formal analysis showed -expression (42) - a positive value for both α_1 and α_2 is a necessary condition for both the stability of the dynamical system and the emergence of oscillations in the proximity of the stationary point. Therefore, the simplifying assumption that $\alpha_2 = 0$ has to be interpreted just as a graphical device.
14. See expression (42) in page 23.
15. Unfortunately, a negative Δ_2 is a necessary but not sufficient condition for excluding a saddle point.