Volatility Spillovers from the US to Australia and China across the GFC

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Abstract

This paper features an analysis of volatility spillover effects from the US market, represented by the S&P500 index to the Australian capital market as represented by the Australian S&P200 for a period running from 12th September 2002 to 9th September 2012. This captures the impact of the Global Financial Crisis (GFC). The GARCH analysis features an exploration of whether there are any spillover effects in the mean equations as well as in the variance equations. We adopt a bi-mean equation to model the conditional mean in the Australian markets plus an ARMA model to capture volatility spillovers from the US. We also apply a Markov Switching GARCH model to explore the existence of regime changes during this period and we also explore the non-constancy of correlations between the markets and apply a moving window of 120 days of daily observations to explore time-varying conditional and fitted correlations. There appears to be strong evidence of regime switching behaviour in the Australian market and changes in correlations between the two markets particularly in the period of the GFC. We also apply a tri-variate Cholesky-GARCH model to include potential effects from the Chinese market, as represented by the Hang Seng Index

Keywords: Volatility spillovers, Markov-switching GARCH, Cholesky-GARCH, Time-varying correlations.

1. Introduction

The Global Financial Crisis (GFC) had a major impact on the world's financial markets. This paper examines whether there is evidence of spillovers of volatility from the US stock market to the Australia stock market, as represented by the S&P500 index and the Australian S&P200 index . The paper features an application of regime switching model to assess the impact of the GFC on Australian market volatility and then a number of multivariate analyses are applied to explore the impact of spillovers from the US market. We apply a Cholesky-GARCH trivariate model and include the influence of China in the system, as represented by the Hang Seng index.The analyses include a bi-mean equation to model the conditional mean in the individual markets plus an ARMA

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model to capture volatility spillovers from the US to the other five markets. The non-constancy of correlations between the markets is explored using a moving window of 120 days of daily observations to explore time-varying conditional and fitted correlations.

The recent GFC crisis commencing in 2007 and continuing through to the European sovereign debt crisis. Alan Greenspan (2010) takes the view that: "The bubble started to unravel in the Summer of 2007. But unlike the debt-like deflation of the earlier dotcom boom, heavy leveraging set off serial defaults, culminating in what is likely to be viewed as the most virulent financial crisis ever. The major failure of both private risk management and official regulation was to significantly misjudge the size of tail risks that were exposed in the aftermath of the Lehman default."

The U.S. subprime mortgage and credit crisis was characterized by turbulence that spread from subprime mortgage markets to credit markets more generally, and then to short-term interbank markets as liquidity evaporated, particularly in structured credit then on to stock markets globally.

Gorton (2010) suggested that the GFC was not particularly different from previous crises except that, prior to 2007, most investors had never heard of the markets that were involved. Concepts such as subprime mortgages, asset-backed commercial paper conduits, structured investment vehicles, credit derivatives, securitization, or repo markets were not common knowledge. Gorton (2010) suggests that the securitized banking system is a real banking system that is still vulnerable to a panic. He argues that the crisis beginning in August 2007 can best be understood as a wholesale panic involving institutions, where large financial firms "ran" on other financial firms, making the system insolvent.

In this paper we focus on how the GFC impacted on volatility spillovers across the world to the Australian equity market. Even though the Australian financial markets were spared the major effects of the GFC in terms of distress to major financial institutions the Australian financial market was still impacted by the major global events. The degree to which the Australian market is influenced by extreme events in the US has implications for portfolio optimization by investors and fund managers alike and effects the degree to which it is possible to hedge risk during times of financial turbulence. We examine how volatility spillovers and correlations changed between the Australian market and the US during the financial crisis. We also include the impact of the Chinese market in some of our analyses, though the main focus of the paper is on the influence of the US market on the Australian one.

2. Research Method

2.1. Data set and econometric models

The data set includes daily data for each index from 12th September 2002, until 9th September 2012. The indexes are total market indexes, based on market capitalizations and are taken from Datastream. Daily returns are calculated as follows:

$$y_{it} = ln(p_{it}) - ln(p_{it-1}) \tag{1}$$

The data sets used are shown in Table 1.

Country	Symbol	Abbreviation for
USA	USRET	AMERICAN market
AUSTRALIA	AUSRET	AUSTRALIAN market
CHINA	HANGSENGRET	CHINESE market

Table 1: **List of countries and indices**

There are a variety of models used to test for the existence of time-varying volatility and for spillover effects in returns and volatility across markets. Manganelli and Engle [24] claim that the main difference between models is how they deal with the return distribution, and classify these models into three distinct groups:

- Parametric, such as RiskMetrics and GARCH;
- Nonparametric, such as Historical simulation and the Hybrid Model;
- Semiparametric, such as CAViaR, Extreme Value Theory, and Quasi-Maximum Likelihood GARCH.

In this paper, we adopt a variety of parametric techniques. We commence our analysis with a vanilla GARCH model, before moving on to a regime switching model, as developed by Gray (1996). We also use various multivariate models including a Cholesky-GARCH model for the empirical analysis.

2.2. Univariate conditional volatility models

Engle (1982) developed the Autoregressive Conditional Heteroskedasticity (ARCH) model that incorporates all past error terms. It was generalised to GARCH by Bollerslev (1986) to include lagged term conditional volatility. In other words, GARCH predicts that the best indicator of future variance is the weighted average of long-run variance, the predicted variance for the current period, and any new information in this period, as captured by the squared residuals (Engle, (2001)).

The framework is developed as follows: consider a time series $y_t = E_{t-1}(y_t)+\varepsilon_t$, where $E_{t-1}(y_t)$is the conditional expectation of y_t at time $t-1$ and ε_t is the error term. The GARCH model has the following specification:

$$\varepsilon_t = \sqrt{h_t \eta_t}\,,\ \ \eta_t \sim N(0,1) \tag{2}$$

$$h_t = \omega + \sum_{j=1}^{p} \alpha \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{3}$$

in which $\omega > 0$, $\alpha_j \geq 0$ and $\beta_j \geq 0$, are sufficient conditions to ensure a positive conditional variance, $h_t \geq 0$. The ARCH effect is captured by the parameter α_j, which represents the short run persistence of shocks to returns. β_j captures the GARCH effect, and $\alpha_j + \beta_j$ measures the persistence of the impact of shocks to returns to long-run persistence. A GARCH(1,1) process is weakly stationary if $\alpha_j + \beta_j \leq 1$.

Ling and McAleer (2003) and Harris, Stoja and Tucker (2007) claim that the GARCH model is "perhaps the most widely used approach to modeling the conditional covariance matrix of returns", and Engle (2001) states it has been successful, even in its simplest form, in predicting conditional variance. The main advantage of this model is that it allows "a complete characterization of the distribution of returns and there may be spacefor improving their performance by avoiding the normality assumption" (Manganelli and Engle, (2001, p.9)). However, Engle (2001) , Nelson (1991) , Zhang and Li (2008) and Harris, Stoja and Tucker (2007) also outline some of the disadvantages of the GARCH model as follows:

- GARCH can be computationally burdensome and can involve simultaneous estimation of a large number of parameters.
- GARCH tends to underestimate risk (when applied to Value-at-Risk, VaR) as the normality assumption of the standardized residual does not always hold with the behaviour of financial returns.
- The specification of the conditional variance equation and the distribution used to construct the log-likelihood may be incorrect.
- GARCH rules out, by assumption, the negative leverage relationship between current returns and future volatilities, despite some empirical evidence to the contrary.
- GARCH assumes that the magnitude of excess returns determines future volatility, but not the sign (positive or negative returns), as it is a symmetric model. This is a significant problem as research by Nelson (1991) and Glosten, Jagannathan and Runkle (GJR) (1993) shows that asset returns and volatility do not react in the same way for negative information, or 'bad news', as they do for positive information, or 'good news', of equal magnitude.

In order to deal with these problems, a large number of variations on the basic GARCH model have been created, each one dealing with different issues. Bollerslev (1990) developed a multivariate GARCH (MGARCH) model that asumes Constant Conditional Correlation (CCC). In other words, it assumes independence of asset returns' conditional variance. Multivariate GARCH (MGARCH) models have recently been used widely in risk management and sensitivity analysis.

Bauwens, Laurent and Rombouts (2003) suggest that the most appropriate use of multivariate GARCH models is to model the volatility of one market with regard to the co-volatility of other markets. In other words, these models are used to see if the volatility of one market leads the volatility of other markets (the 'Spillover Effect'). They also assert that these models can be used to model the tangible effects of volatility, such as the impact of changes in volatility on exports and output growth rates. Bauwens, Laurent and Rombouts (2003) suggest that these models are also efficient in determining whether volatility is transmitted between markets through the conditional variance (directly) or conditional covariances (indirectly), whether shocks to one market increase the volatility of another market, and the magnitude of that increase, and whether negative information has the same impact as positive information of equal magnitude.

Nelson (1991) developed the Exponential GARCH (EGARCH) model. This model uses logarithms to ensure that the conditional variance is non-negative, and captures both the size and sign effects of shocks, capturing the effect of asymmetric returns on conditional volatility. This model was the first to capture the asymmetric impact of information. A second model, which is computationally less burdensome then Nelson's EGARCH, is the Glosten, Jagannathan and Runkle GJR model (1993). They found significant evidence of seasonal effects on the conditional variance in the NYSE Value-Weighted Index. Engle and Ng (1993) claim that the GJR forecasts of volatility are more accurate than those of the EGARCH model.

The GJR model is specified as:

$$h_t = \omega + \sum_{j=1}^{r}(\alpha_j + \gamma_j I(\varepsilon_{t-j}^2))\varepsilon_{t-j}^2 + \sum_{j=1}^{s}\beta_j h_{t-j} \tag{4}$$

where

$$I_{it} = \left\{ \begin{array}{ll} 0, & \varepsilon_{it} \geq 0 \\ 1, & \varepsilon_{it} < 0 \end{array} \right\}$$

where I_{it} is an indicator function that distinguishes between positive and negative shocks of equal magnitude. In this model, when there is only one lag, that is, when $r = s = 1$, the sufficient conditions to ensure that the conditional variance is positive ($h_t > 0$) are that $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \gamma_1 \geq 0$,and $\beta_1 \geq 0$; where α_1 and $(\alpha_1 + \gamma_1)$ measure the short run persistence of positive and negative shocks, respectively. These models can be estimated by maximum likelihood techniques when the errors follow a joint normal distribution. If this is not the case, quasi-maximum likelihood estimation (QMLE) can be used.

Necessary and sufficient conditions for the second order stationarity of the GARCH model are $\sum\limits_{i=1}^{r} \alpha_i + \sum\limits_{i=1}^{s} \beta_i < 1$, as demonstrated by Bollerslev (1986). The necessary and sufficient conditions for the GJR (1,1) model were developed by Ling and McAleer (2003), who showed that $E(\varepsilon_t^2) < \infty$ if $\alpha_1 + \frac{\gamma_1}{2} + \beta_1 < 1$. Subsequently, McAleer et al. (2007) demonstrated the log-moment condition for the GJR(1,1) model, which is sufficient for consistency and asymptotic normality of the QMLE, namely $E(log(\alpha_1 + \gamma_1 I(\eta_t)\eta_t^2 + \beta_1)) < 0$.

2.3. Markov switching model

We presume that the asset return r_t follows a simple two-state Markov switching model with different risk premiums and different GARCH dynamics:

$$r_t = \left\{ \begin{array}{ll} \beta_1\sqrt{h_t} + \sqrt{h_t}\epsilon_t, & h_t = \alpha_{10} + \alpha_{11}h_{t-1} + \alpha_{12}a_{t-1}^2 \;\; if\, s_t = 1 \\ \beta_2\sqrt{h_t} + \sqrt{h_t}\epsilon_t, & h_t = \alpha_{20} + \alpha_{12}h_{t-1} + \alpha_{22}a_{t-1}^2 \;\; if\, s_t = 2, \end{array} \right\} \tag{5}$$

where $a_t = \sqrt{h_t}$,$\{\epsilon_t\}$is an i.i.d. Gaussian sequence with mean zero and variance 1, and the parameters of α_{ij}satisfy regularity conditions so that the unconditional variance of $a_t$exists. The transition from one state to another is governed by the following probability:

$$P(s_t = 2 \mid s_{t-1} = 1) = e_1, \quad P(s_t = 1 \mid s_{t-1} = 2) = e_2 \tag{6}$$

where $0 < e_i < 1$. A small value of $e_i$means that the return series has a tendency to stay in the ith state with an expected duration $1/e_i$. To identify the model it is frequently assumed that one state is associated with greater risk, i.e $\beta_2 > \beta_1$. The model in expression (2) is a Markov-switching GARCH-M model. In the next subsection of the paper we will briefly consider the properties of GARCH models before proceeding to a discussion of the multivariate GARCH models utilised.

2.4. Multivariate conditional volatility models

We adopt a bi-mean equation to model the conditional mean in the individual markets plus an ARMA model to capture volatility spillovers from the US to the five markets considered. We commence by adopting a vector ARMA structure with exogenous variables for the conditional mean equation μ_t as shown below:

$$u_t = \Upsilon x_t + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \Theta_i a_{t-i} \tag{7}$$

where x_t denotes an m-dimensional matrix of explanatory variables, $\Upsilon$is a $k \times m$ matrix and p and q are nonnegative integers.

We have considered univariate models of single assets in the previous section. However, in finance the behaviour of portfolios of assets is of primary interest. If we want to forecast the returns of portfolios of assets, we must consider the correlations and covariances between individual assets. A common approach adopted to the specification of multivariate conditional means and conditional variances of returns is as follows:

$$y_t = E(y_t \mid F_{t-1}) + \varepsilon_t \tag{8}$$

$$\varepsilon_t = D_t \eta_t$$

In (5) above, $y_t = (y_{1t}, \ldots.., y_{mt})^{'}$, $\eta_t = (\eta_{it}, \ldots..., \eta_{mt})^{'}$, a sequence of (i.i.d) random vectors, F_t is a vector of past information available at time t, $D_t = diag(h_1^{1/2}, \ldots...., h_m^{1/2})$, m is the number of returns, and $t = 1, ..., n$. (For a full exposition, see Li, Ling and McAleer (2003), McAleer (2005) and Bauwens et al (2003). The Bollerslev (1990) constant conditional correlation (CCC) model assumes that the conditional variance of each return, h_{it}, $i = 1, ..., m$, follows a univariate GARCH process:

$$h_{it} = \omega + \sum_{j=1}^{r} \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^{s} \beta_{ij} h_{i.t-j} \tag{9}$$

In (6) above, α_{ij} represents the ARCH effect, or the short run persistence of shocks to return i, and β_{ij} captures the GARCH effect; the impact of shocks to return i on long run persistence, given by:

$$\sum_{j=1}^{r} \alpha_{ij} + \sum_{j=1}^{s} \beta_{ij}.$$

It follows that the conditional correlation matrix of CCC is $\Gamma = E(\eta_t \eta_t^{'} \mid F_{t-1}) = E(\eta_t \eta_t^{'})$, where $\Gamma = \{\rho_{it}\}$ for $i,\ j = 1, ..., m$. From (5), $\varepsilon_t \epsilon_t^{'} = D_t \eta_t \eta_t^{'} D_t$, $D_t = (diag Q_t)^{1/2}$, and $E(\varepsilon_t \varepsilon_t^{'} \mid F_{t-1}) = Q_t = D_t \Gamma D_t$, where Q_t is the conditional covariance matrix, $\Gamma = D_t^{-1} Q_t D_t^{-1}$is the conditional correlation matrix and the individual conditional correlation coefficients are calculated from the standardised residuals in equations (5) and (6). This means that there is no multivariate estimation required in CCC, which involves m univariate GARCH models, except in the case of the calculation of conditional correlations.

2.5. Model specifications

Our goal in this paper is to model spillover effects. In the context of measuring asymmetric shocks and spillover effects, the following models have been proposed:

1. We begin with simple univariate models before advancing to more complex multivariate ones. In the context of measuring asymmetric shocks and spillover effects, the following models are adopted. The GARCH model is estimated with an auxiliary term added to capture spillover effects:

$$h_{AUS,t} = \omega + \alpha\epsilon^2_{AUS,t-1} + \beta_1 h_{AUS.t-1} + \beta_2\epsilon^2_{US,t-1} \tag{10}$$

The null hypothesis is that there is no conditional volatility or a spillover effect. The alpha and first beta test for GARCH effects in Australia. The second beta is an additional term used to capture the effect of the lagged squared residuals of a GARCH (1,1) on the US S&P500 index, and added to the Australian market equation to test for spillover effects, as suggested by Hamao, Masulis and Ng (1990). If the coefficients are statistically significant then there is a spillover effect of volatility from the US to Australia.

2. We then apply a Markov switching model as considered in section 2.3. This is followed by a variety of multivariate specifications.

3. The first multivariate model we apply is an exponentially weighted moving average model to estimate the covariance matrix as shown below:

Given the innovations $F_{t-1} = \{a_1, ...' a_{t-1}\}$, the (unconditional) covariance matrix of the innovation can be estimated as:

$$\hat{\sum} = \frac{1}{t-1}\sum_{j=1}^{t-1} a_j a_j',$$

where it is assumed that the mean of a_j is zero. To accommodate a time-varying covariance matrix and to put greater weight on recent innovations we can use exponential smoothing and estimate the covariance matrix of a_t as shown below:

$$\hat{\sum} = \frac{1-\lambda}{1-\lambda^{t-1}}\sum_{j=1}^{t-1} \lambda^{j-1} a_{t-j} a_{t-j}' \tag{11}$$

where $0 < \lambda < 1$ and the weights $(1-\lambda)\lambda^{j-1}/(1-\lambda^{t-1})$ sum to 1.

4. We utilise Cholesky decompositions to build a higher dimensional GARCH model. We write the vector return series as $r_t = \mu_t + \alpha_t$ and use a vector AR model for modelling the behaviour of the mean. We then proceed in stages.

- First we build a univariate GARCH model of the US S&P500 index series.
- Then we add the Australian S&P200 index series to the system,perform an orthogonal transformation on the shock process of the Australian return series, and build a bivariate volatility model for the system. The parameter estimates for the US model developed in step one can be used as staring values in the bivariate estimation.
- Given that Australia is a major trading partner of China it is possible that links with the Chinese markets also impact upon volatility. A third component of the system is a Chinese index, in this case the Hang Seng index. The shock process for this third return series is subjected to an orthogonal transformation and a three-dimensional volatility model is then constructed. Once again the parameter values from the bivariate system can be used as starting values.

The application of Cholesky decompositions to GARCH models is discussed in Tsay (2005), Chang and Tsay (2010) and Dellaportos and Pourahmadi (2011). This type of model is closely related to factor models; see for example, the discussion of orthogonal GARCH models in Alexander (2001).

The advantage of the approach is that the multivariate conditional covariance estimation can be reduced to estimating the $3N$ parameters of univariate GARCH models and a few 'dependence' parameters. The advantage of this approach is that the Cholesky-GARCH models have correlation matrices that are time-varying, and can be more flexible than Bollerslev's (1990) constant-correlation GARCH models. The main disadvantage of the approach is that the stocks have to be ordered to construct the model.

The results from the empirical application of these three models are presented in the next section together with some further analysis using a moving window to examine time-varying correlations.

5. We commence again by adopting a vector ARMA structure with exogenous variables for the conditional mean equation μ_t as shown below:

$$u_t = \Upsilon x_t + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \Theta_i a_{t-i} \tag{12}$$

where x_t denotes an m-dimensional matrix of explanatory variables, $\Upsilon$is a $k \times m$ matrix and p and q are nonnegative integers. We then adopt a constant-correlation GARCH(1,1) for the mean equation and a VARMA format for the variances to produce a time-varying correlation model. In the tables below the first four row entries refer to the mean equation, the next eleven rows to the variance equation and the final row to the correlation between the two series.

3. Empirical results

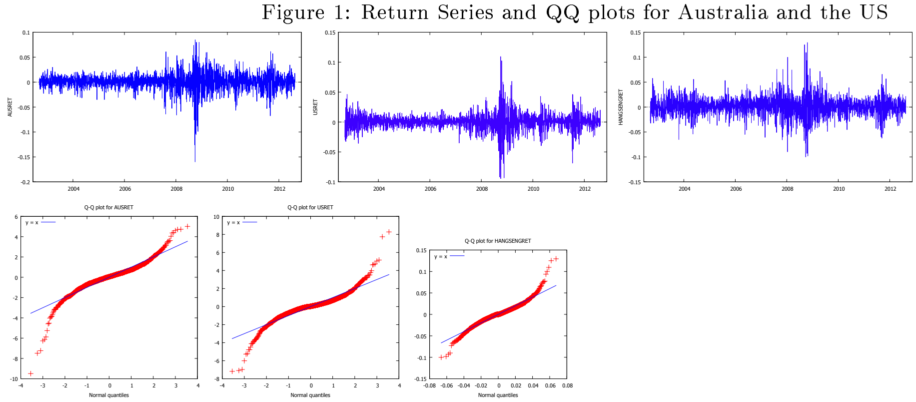
The characteristics of the basic index series used in our data set presented in Table 2 suggest the existence of non-normality and fat tails. The Jarque-Bera Lagrange Multiplier test rejects the null hypothesis that the data are normally distributed: the p-values for all indexes above are zero. This is also evident from the skewness and excess kurtosis of the data. In order to estimate the parameters in the GARCH models, the Quasi-Maximum Likelihood Estimator (QMLE) will be used.

	USRET	AUSRET	HANGSENGRET
Mean	0.00017557	0.00037129	0.00049530
Median	0.00051042	0.0011360	0.00016735
Maximum	0.10957	0.085091	0.12959
Minimum	-0.094695	-0.16002	-0.10018
Skewness	-0.24979	-0.92324	0.13466
Excess Kurtosis	9.6286	9.0803	4.3952
Standard Deviation	0.013216	0.016900	0.018857
Coefficient of Variation	75.275	45.517	38.072
Jarque-Bera	10039.7	9273.05	2094.12
Probability	0.000000	0.000000	0.000000

Table 2: **Descriptive statistics**

The two returns series are clearly non-normal as reflected in their QQ plots shown in Figure I which also shows their return plots.

Figure 1: Return Series and QQ plots for Australia and the US



Before we conduct the GARCH tests we test for the existence of ARCH effects in the data sets. The results are shown below in Table 3 and display clear evidence of significant ARCH effects in all of the index series.

Market	Test Statistic (Chi-Square)	p value	ARCH effect
USRET	977.931	0.000000	Yes
AUSRET	961.123	0.000000	Yes
HANGSENGRET	131.512	0.0019	Yes

Table 3: **Test results for ARCH effects**

The results in Table 3 mean we can proceed with confidence to the GARCH analysis; which is broken down into several parts. First we apply the vanilla GARCH(1,1) model and then augment the results with shocks to the US GARCH(1,1) model and lagged forecasts of US volatility. The results are shown in Table 4.

Table 4: GARCH(1,1) model of Australian returns augmented with spillovers from US GARCH(1,1) model

	Coefficients	t statistic	probability
ω	0.00000328	5.71	0.0000
$\epsilon^2_{t-1,AUS}$	0.063053	8.77	0.0000
$h_{t-1,AUS}$	0.921555	109.0255	0.0000
$\epsilon^2_{t-1,US}$	-0.0000847	-7.571791	0.0000
Durbin Watson	1.9498		
ARCH Test F-Statistic	0.118628		0.7306

The results are quite satisfactory in that shocks to the US index returns do have a significant influence on Australian volatility, though when lagged forecasts of US volatility were added to this vanilla GARCH specification they had no significance and are not reported. The model appears to perform satisfactorily in terms of the Durbin Watson statistic and the absence of any significant ARCH effects in the residuals. However, this simple model assumes that the average volatility is constant throughout the entire period which is not probable given the massive shocks to returns experienced during the GFC. We explored this issue suing a Markov-switching GARCH Model. The results of which are presented in the next section.

3.1. Markov switching GARCH model to the Australian returns.

Table 4 presents the results of the Regime Switching GARCH model applied to the Australian returns.

Table 5: Parameter estimates and related statistics for single regime and regime-switching GARCH Models

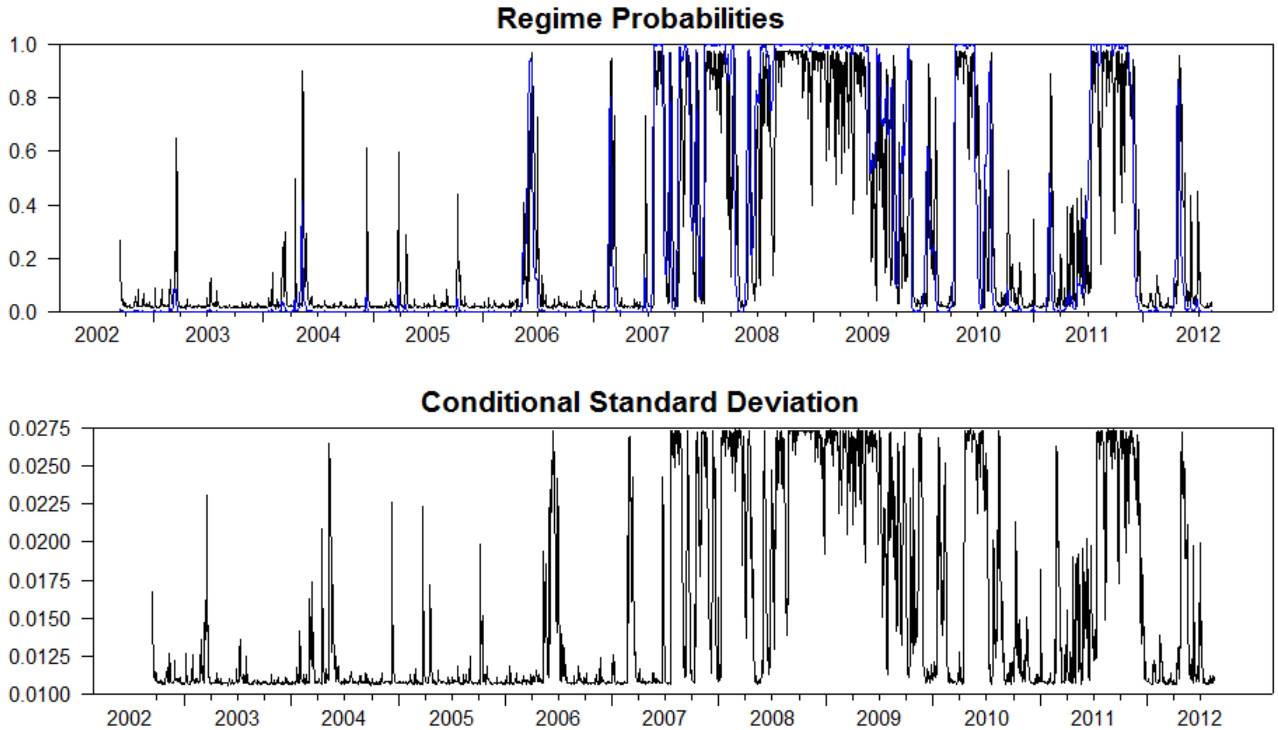
	Single regime			Regime switching		
Parameter	Estimate	t-statistic	p-value	Estimate	t-statistic	p-value
a_{01}	0.00089	4.1108	0.0000	0.0001784	3.83009	0.0001
a_{02}				-0.0003732	-0.69961	0.48417
a_{11}	0.04474	2.13329	0.0329	-0.0340	-0.83748	0.402324
a_{12}				0.1214	3.23636	0.001210
b_{01}	0.00000304	3.77095	0.0001	0.00000484	1.57742	0.114699
b_{11}	0.0874	7.42459	0.0000	-0.0180	-0.94672	0.34378
b_{21}	0.89872	66.9935	0.0000	0.9512	24.84656	0.0000
b_{02}				-0.0000014	-0.43980	0.66008
b_{12}				0.1510	4.72127	0.00000
b_{22}				0.8709	20.9993	0.00000

The first column of Table 4 shows the estimates for the single regime version of the model. The mean reversion parameter (a_1) is positive and significant. The implied long run mean is 0.0199. The GARCH terms are significant.

The second set of columns report estimates of the regime-switching GARCH model. The conditional mean is positive and significant in regime 1, the high volatility regime. The implied long-run mean in this regime is 0.52% and in the low volatility regime is 0.31%. The high volatility regime appears to be less sensitive to recent shocks ($b_{11} < b_{12}$) and shows evidence of greater persistence ($b_{21} > b_{22}$) than the low volatility regime. It seems like the high volatility regime is non-stationary with explosive variance whilst this is not the cas for the low volatility regime ($to\, be\, stationary\, the\, requirement\, is\, (b_{1i} + b_{2i} < 1)$).

Figure 2 shows the regime probabilities and the conditional volatility for the two states for the Australian set of returns.

Figure 2: Regime switching GARCH Australian returns



It is apparent that Australia, despite faring relatively well in comparison to other developed nations during the GFC, has not been immune from its effects and has spent much of the time since 2007 in the 'high' volatility regime.

3.2. EWMA

The EWMA model is set up as follows.

3.3. VARMA-GARCH models

In the next subsection we present the results from the VARMA-GARCH models. Table 4 presents the results for the VARMA-GARCH model for the full period.

the mean equation:

$r_{AUS_t} = 0.09359 + 0.0301 r_{AUS_{t-1}} + a_{1t}$

$r_{US_t} = 0.054032 + 0.00230 r_{AUS_{t-1}} + 0.02176 r_{US_{t-1}} + a_{2t}$

The t statistics for the first equation above are 3.93 and 3.65, whilst for the second equation are 3.44, 0.41 and 5.17 respectively.

The ARCH effects are as follows"

$$\begin{matrix} \sigma_{AUS_t} \\ \sigma_{US_t} \end{matrix} = \begin{bmatrix} 0.0895 \\ (4.11) \\ -0.0290 \\ (-1.121) \end{bmatrix} + \begin{bmatrix} -0.057 & -0.0109 \\ (-0.0357) & (-1.492) \\ -0.0126 & 0.1191 \\ (-0.644) & (8.631) \end{bmatrix} \begin{bmatrix} a^2_{1,t-1} \\ a^2_{2,t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} 0.8999 & \\ (63.82) & \\ 0.694 & 0.8736 \\ (2.041) & (68.79) \end{bmatrix}$$

where the numbers in parenthesis are the t-statistics. The Ljung-Box tests on the two residual series cannot reject their independence for up to 40 lagged terms. There is evidence of significant impacts of both US shocks and lagged variance terms on the Australian market in the coefficients reported above.

3.3.1. Trivariate model based on Cholesky decompositions.

We estimate a univariate GARCH model for the US S&P500 index return series. We then add the Australian S&P 200 index return series to the system, perform orthogonal transformation on the shock process for the Australian return series, and then build a bivariate volatility model for the system. We then augment the system further and add in a return series for the Hang Seng Index to capture co-dependencies with China. The system then becomes a trivariate one.The parameter estimates for the GARCH model of the US return series are used as the commencement values in the bivariate estimation, and the estimation is augmented in a stepwise fashion, first adding in the Australian index and then the Hang Seng index. The components of the return series are ordered as $r_t = (USRETL_t, AUSRET_t, HANGSENGRET_t)$. The sample means, standard errors and correlation matrix of the data are:

$$\hat{\mu} = \begin{bmatrix} 0.00017 \\ 0.00037 \\ 0.00049 \end{bmatrix} . \begin{bmatrix} \hat{\sigma}_1 \\ \hat{\sigma}_2 \\ \hat{\sigma}_3 \end{bmatrix} = \begin{bmatrix} 0.0132 \\ 0.0169 \\ 0.0188 \end{bmatrix} . \hat{\rho} = \begin{bmatrix} 1.00 & -0.0337 & -0.0278 \\ -0.0337 & 1.00 & 0.6074 \\ -0.0278 & 0.6074 & 1.00 \end{bmatrix} .$$

Tests of serial correlation in the three return series applying Ljung-Box statistics we obtain $Q_3(1) = 223.61037$, $Q_3(4) = 1075.17373$, $and Q_3(8) = 1177.05173$,and all are highly significant with p values close gto zero in terms of chi-squared distributions with 9, 36, and 72 degrees of freedom respectively. There is also significant evidence of dependencies in cross-correlation matrices of returns up to six lags.

The initial estimate of the GARCH model for the US S&P 500 index return series yields the mean equation $r_{1t} = 0.000568(0.0005) + -0.0781 r_{t-1}(0.0002) + a_{1t}$ with significance levels in parenthese. The GARCH equation for the US S&P 500 index return series is $h_{1t} = 0.0000014(0.000) + 0.0796\alpha^2_{1,t-1}(0.000) + 0.9087 h_{1,t-1} + e_{1t}$. The system is then augmented by adding in the Australian S&P 200 index returns series. The model is re-estimated and finally the Hang Seng Index return series is added to the system. Our final model is estimated as shown in Table 6.

Our final mean equations are shown below:

$$\begin{aligned} r_{USRETL,t} &= C_1 - P_3 USRETL_{t-1} + a_{1t} \\ r_{AUSRET,t} &= C_2 + P_{21} USRETL_{t-1} - P_{22} AUSRET_{t-1} + a_{2t} \\ r_{HANGSENGRET,t} &= C_3 + P_{31} USRETL_{t-1} - P_{33} HANGSENGRET_{t-1} + a_{3t} \end{aligned} \tag{13}$$

It can be seen in Table 6 that all coefficients on lagged returns in the US market are significant in all three mean equations but the lagged terms on the Australian and Chinese markets are not significant in the mean equations. Manipulation of the above equations provides the three residual series a_{1t}, a_{2t}, a_{3t}.

The three dimensional time-varying volatility model can be obtained as follows:

$$\begin{aligned} g_{11,t} &= A_0 + A_1 b_{1,t-1}^2 + A_2 g_{11,t-1} \\ q_{21,t} &= T_0 + T_1 q_{21,t-1} - T_2 a_{2,t-1} \\ g_{22,t} &= B_0 + B_1 b_{2,t-1}^2 + B_2 g_{22,t-1} \\ q_{31,t} &= U_0 + U_1 q_{31,t-1} + U_2 a_{3,t-1} \\ q_{32,t} &= W_0 + W_1 q_{31,t-1} + W_2 a_{2,t-1} \\ g_{33,t} &= D_0 + D_1 b_{3,t-1}^2 + D_2 g_{33,t-1} + D_5 g_{22,t-1} \end{aligned} \tag{14}$$

Where $b_{1t} = a_{1t}$, $b_{2t} = a_{2t} - q_{21,t} b_{1t}$, $b_{3,t} = a_{3,t} - q_{31,t} b_{1t,} - q_{32,t} b_{2t}$.

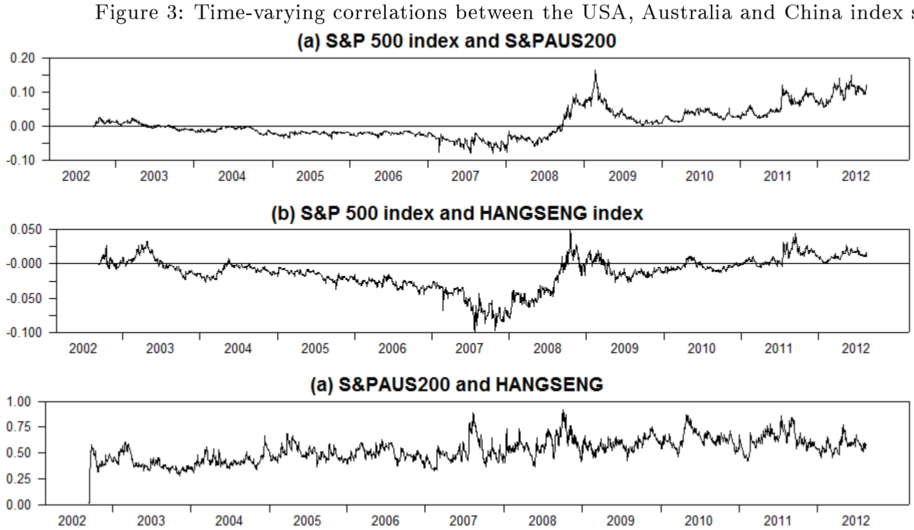
It can be seen in Table 6 that all terms except P_{22},P_{33},D_5, and $U_0$are significant.

Table 6: Tri-variate GARCH model based on Cholesky decompositions, US, Australia, and China

Variable	Coefficient	t statistic	significance
C_1	0.000572238	3.35732	0.00078702
P_3	-0.084374441	-3.57811	0.00034609
C_2	0.000748640	3.09502	0.00196797
P_{21}	0.230690975	10.07234	0.00000
P_{22}	-0.009400782	-0.52871	0.59700942
C_3	0.000659586	2.21778	0.02657005
P_{31}	0.151604359	5.51762	0.00000003
P_{33}	0.004369145	0.24146	0.80919677
A_0	0.000001407	6.50968	0.00000000
A_1	0.079632189	10.52727	0.00000000
A_2	0.908621578	110.17086	0.00000000
B_0	0.000003361	4.65290	0.00000327
B_1	0.086424831	8.56456	0.00000000
B_2	0.897410480	75.97968	0.00000000
D_0	0.000001370	2.55333	0.01066987
D_1	0.043078355	6.83477	0.00000000
D_2	0.947017611	131.73776	0.00000000
D_5	0.002884550	1.33700	0.18122414
T_0	0.000048328	2.34697	0.01892678
T_1	1.000860811	674.33104	0.00000000
T_2	-0.176761984	-1.90135	0.05725663
U_0	-0.000051104	-0.44351	0.65739525
U_1	0.997767261	415.29082	0.00000000
U_2	-0.184299002	-1.63769	0.10148659
W_0	0.276093430	2.04615	0.04074154
W_1	0.568365592	2.72744	0.00638286
W_2	-1.617510386	-2.30356	0.02124744

The model diagnostics appear to be reasonably satisfactory, the Ljung-Box Q statistics for the three sets of residual series are insignificant for series RES1, RES2 and RES3 for 4, 8 and 12 lags respectively. There is evidence of an increased degree of correlation between the markets during and after the financial crisis, as shown in Figure 3 below.

Figure 3: Time-varying correlations between the USA, Australia and China index series



3.4. Further analysis

Given that the evidence of spillover effects during the period of the GFC was fairly weak in the context of the VARMA-GARCH and VARMA-AGARCH specifications we decided to augment the analysis further. We commence again by adopting a vector ARMA structure with exogenous variables for the conditional mean equation μ_t as shown below:

$$u_t = \Upsilon x_t + \sum_{i=1}^{p} \Phi_i r_{t-i} - \sum_{i=1}^{q} \Theta_i a_{t-i} \tag{15}$$

where x_t denotes an m-dimensional matrix of explanatory variables, $\Upsilon$is a $k \times m$ matrix and p and q are nonnegative integers.

4. Conclusion

[1] Alexander, C. (2001) Market Models: A Guide to Financial Data Analysis. Wiley, New York.

[2] Barth, J. R., R. Koepp, and Z. Zhou, Banking Reform in China: Catalyzing the Nation's Financial Future, (2004) Available at SSRN: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=548405

[3] Bauwens, L., S. Laurent, and J. V. K. Rombouts, Multivariate GARCH Models: A Survey, CORE Discussion Paper No. 2003/31. (2003) Available at SSRN: http://ssrn.com/abstract=411062

[4] Bollerslev, T. Generalized Autoregressive Conditional Heteroscedasticity, Journal of Econometrics, 31, (1986) 307-327.

[5] Bollerslev, T. Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model, Review of Economics and Statistics, 72, (1990) 498-505.

[6] Chang, C., Tsay, R.S. (2010) Estimation of covariance matrix via sparse Cholesky factor with Lasso. J. Stat. Plan. Inference 40, 3858– 3873

[7] Dellaportos, P. and M. Pourahmadi, (2012) Cholesky-GARCH models with applications to finance, Statistical Computing, 22: 849-855.

[8] Engle, R.F. Autoregressive Conditional Heteroskedasticity With Estimates of the Variance of UK Inflation, Econometrica 50 (1982): 987-1008.

[9] Engle, R. F. "GARCH 101: An Introduction to the Use of Arch/Garch Models in Applied Econometrics." Journal of Economic Perspectives,15, 4 (2001) 157–168

[10] Engle, R. F. and V. K. Ng, Time-Varying Volatility and the Dynamic Behavior of the Term Structure, Journal of Money, Credit and Banking 25(3): (1993) 336-349.

[11] Gokcan, S. Forecasting volatility of emerging stock markets: linear versus non-linear GARCH models, Journal of Forecasting 19, (2000) 499-504.

[12] Glosten, L.R., R. Jagannathan and D. Runkle, On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, Journal of Finance, 48, (1993) 1779-1801.

[13] Hamao, Y., Masulis, R.W., Ng, V. Correlations in Price Changes and Volatility across International Stock Markets, The Review of Financial Studies 3(2) (1990) 281-307.

[14] Harris, R. D. F., E. Stoja, and J. Tucker. A simplified approach to modeling the co-movement of asset returns, Journal of Futures Markets 27(6) (2007) 575-598.

[15] Harvey, C. R. and A. R. Siddique, Autoregressive Conditional Skewness, Fuqua School of Business Working Paper No. 9604. (1999) Available at SSRN: http://ssrn.com/abstract=61332 or doi:10.2139/ssrn.61332

[16] Johansson, A. C. and C. Ljungwall, Spillover Effects among the Greater China Stock Markets, World Development, 37, 4, (2009) 839-851.

[17] Lima, L. R. and B. P. Neri (2007). "Comparing Value-at-Risk Methodologies." Brazilian Review of Econometrics 27(1): 1-25.

[18] Ling, S., and M. Mcaleer, Asymptotic Theory For A Vector ARMA-GARCH Model, Econometric Theory, 19(2), (2003) 280-310.

[19] McAleer, M. Automated inference and learning in modeling financial volatility, Econometric Theory, 21, (2005) 232-261.

[20] McAleer, M., Chan, F., Hoti, S., and Lieberman, O. (2008). Generalized Autoregressive Conditional Correlation, Econometric Theory, 24(6), 1554-1583.

[21] McAleer, M., F. Chan and D. Marinova, An econometric analysis of asymmetric volatility: theory and application to patents, Journal of Econometrics, 139, (2007) 259-284.

[22] Manera, M., M. McAleer, and M. Grasso, Modelling time-varying conditional correlations in the volatility of Tapis oil spot and forward returns, Applied Financial Economics 16(7) (2006) 525 - 533.

[23] Manganelli , S. and R. F. Engle, Value at Risk Models in Finance, ECB Working Paper No. 75. (2001) Available at SSRN: http://ssrn.com/abstract=356220

[24] McAleer, M., S. Hoti, and F. Chan, (2009). "Structure and Asymptotic Theory for Multivariate Asymmetric Conditional Volatility." Econometric Reviews 28(5) (2009) 422 - 440.

[25] Moon G.H and W.C. Yu, Volatility Spillovers between the US and the China Stock Markets: Structural Break Test with Symmetric andAsymmetric GARCH Approaches," Glob. Econ. Review, 39(2) (2010) 129-149.

[26] Nelson, D. B. Conditional Heteroskedasticity in Asset Returns: A New Approach, Econometrica 59(2) (1991) 347-370.

[27] So, M. K. and A. S. L. Tse, Dynamic Modeling of Tail Risk: Applications to China, Hong Kong and Other Asian Markets, Asia-Pacific Financial Markets, 16 (2009) 183-210.

[28] Tsay, R.: Analysis of Financial Time Series, 3rd edn.Wiley, New York (2005)

[29] Yao, Y. Perspective of China Stock Market Under Global Financial Uncertainty, (2002) Available at SSRN: http://ssrn.com/abstract=354540

[30] Yi, Z., H. Chen, Sr., and W.K Wong, China s Stock Market Integration with a Leading Power and a Close Neighbor, (2010) Available at SSRN: http://ssrn.com/abstract=1547665

[31] Yilmaz, K. Return and Volatility Spillovers Among the East Asian Equity Markets, Journal of Asian Economics, 21, 3, (2010) 304-313.

[32] Zhang, B. and X. Li, "The asymmetric behaviour of stock returns and volatilities: evidence from Chinese stock market, Applied Economics Letters, 15 (2008) 959-962.