



ELSEVIER

1 January 1998

PHYSICS LETTERS B

Physics Letters B 416 (1998) 163–168

Study of the Coulomb-Higgs transition in the abelian Higgs model

A. Cruz ^a, D. Iñiguez ^a, L.A. Fernández ^b, A. Muñoz-Sudupe ^b, A. Tarancón ^a

^a *Departamento de Física Teórica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain*

^b *Departamento de Física Teórica I, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain*

Received 5 September 1997

Editor: L. Alvarez-Gaumé

Abstract

The order of the Coulomb-Higgs transition in the U(1)-Higgs model with unfrozen modulus of the scalar field is studied. Large lattices (up to 24^4 in one case) and high statistics are used. We fix $\beta = 1.15$ and explore specially a region of λ -values where metastability is observed. We study the thermodynamical limit of several observables, in particular, the latent heat, the specific heat, the decrement of the free energy between the maxima and the central minimum of the two-peaked histogram, the Binder cumulant and the displacement of the critical coupling with the lattice size. The results point towards a second order transition for $\lambda \geq 0.005$, while for smaller values of λ the strong metastability growing with the lattice size seems to derive from a first order character. © 1998 Elsevier Science B.V.

PACS: 14.80.Bn; 11.15.Ha

Keywords: Lattice; Monte Carlo; U(1); Higgs; Phase transition

1. Introduction

The Higgs mechanism is an essential part of the present day formulation of the Standard Model. The U(1) gauge model coupled to scalars is a simplification of the gauge-Higgs sector of the SM, which keeps the unsolved problem of defining non perturbatively a non-asymptotically free field theory.

The U(1)-Higgs model has been widely studied previously. One of the main objectives has been to determine the order of the Coulomb-Higgs transition, both in the cases of frozen and unfrozen modulus of the scalar field, by using analytical (mean field, one-loop effective potential, etc.) and numerical techniques (see [1,2] and references therein).

From those analyses the situation could be defined in the following way. For large values of λ the transition seems to be second order, while for small values it looks first order. These results are based on Monte Carlo studies of the system searching for metastabilities or single/double peaked histograms. Due to computational limitations, these calculations had been made with small lattices and short statistics. Here we carry out a study with much larger lattices and high statistics in order to approach the thermodynamical limit in a more reliable way, obtaining results qualitatively consistent with the previous ones.

However, in those works the conclusion that the transition is first order has been obtained by consid-

ering the presence of a double peak for a given volume V (or observing metastability). As we will show this is not correct because even in this case, when V is increased, both peaks approach, and the latent heat disappears in the thermodynamical limit, obtaining in this way a second order transition for λ values much smaller than previously considered.

2. The model and the Coulomb-Higgs transition

The three parameter U(1)-Higgs model is described by the action

$$S = -\beta \sum_{r,\mu < \nu} \Re U_{r,\mu\nu} - \kappa \sum_{r,\mu} \Re \bar{\Phi}_r U_{r,\mu} \Phi_{r+\mu} + \lambda \sum_r (|\Phi_r|^2 - 1)^2 + \sum_r |\Phi_r|^2 \quad (1)$$

In the $\lambda \rightarrow \infty$ limit, $|\Phi| \rightarrow 1$ and the action simplifies to

$$S = -\beta \sum_{r,\mu < \nu} \Re U_{r,\mu\nu} - \kappa \sum_{r,\mu} \Re \bar{\Phi}_r U_{r,\mu} \Phi_{r+\mu} \quad (2)$$

The phase diagram of that restricted version was considered first in [3] and has been discussed by us in [4], and [5]. We discuss here the global aspects of the phase diagram in the fixed modulus case (See Fig. 1). Point A is the pure compact U(1) phase transition, a well established first order point [6–8], even though this transition for small lattices seems to be second order. We point out that some authors [9]

have cast a new hue on its nature, pointing out the possibility of this transition to be second order.

As we showed in [4], particularly in the neighbourhood of the triple point C, the line joining A with the latter is also first order. The line CD was studied by us [10] and is a first order line ending in a second order point D. The precise location of D is $\kappa = 0.5260(9)$ and $\beta = 0.8485(8)$, with measured critical exponents compatible with the classical (Mean Field) values $\alpha = 0, \beta = 1/2, \gamma = 1, \nu = 1/2$. The line BC is more controversial. The difficulty to identify the order of the Coulomb-Higgs phase transition was made apparent because of the large finite size effects. The $\beta \rightarrow \infty$ end of that line is the X-Y model phase transition, a well established second order one with Mean Field exponents, which has induced researchers to think the line to be a prolongation of its end point, and as such, second order [11]. Yet, the relationship of the model to the $q > 1$ version and to the Z_N model reported in [12,4] and [5] points towards a possible first order transition. However that relationship is based on perturbative arguments (expansion around $\beta = \infty$) and might not be applicable.

The difficulty to study directly the $\lambda \rightarrow \infty$ limit has lead us to study the problem at finite, variable λ . This had been done earlier [2], and we intend to improve on the statistics and the lattice sizes.

3. Simulation and observables

We have fixed $\beta = 1.15$, which is on the Coulomb-Higgs side of the first order Confining-Coulomb phase transition.

If we consider larger values of β , the system has a larger correlation length, and then in order to approach the thermodynamical limit, $L \gg \xi$, we need very large volumes. Also, a tricritical point along the Coulomb-Higgs transition, where the transition order changes, is not expected.

Then, we select some value of λ and search for the “critical” κ , i.e. we are looking for the Coulomb-Higgs transition. As is known, for small enough values of λ the simulations on this transition show clear signals of metastability, giving place to two-peaked histograms, while for large values the

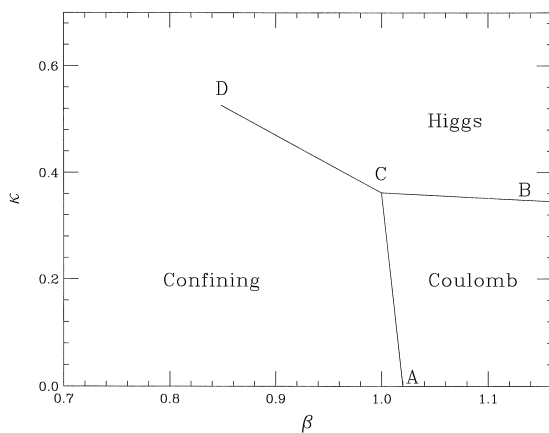


Fig. 1. Phase diagram of the model with fixed modulus.

metastability disappears. Of course, the simulations are made on finite lattices and then a conclusion on the order of the transition cannot be extracted directly from that behaviour. We have selected an intermediate region of λ values, ranging from the point where the two peak signal practically disappears for our lattices sizes ($\lambda = 0.3$) to the point where the metastability is so strong that it makes the work with our computational disponibilities difficult ($\lambda = 0.003$). The total set of λ -values that we have used is 0.003, 0.005, 0.01, 0.03, 0.1, 0.3 on lattices $6^4, 8^4, 12^4, 16^4$ and 24^4 (depending on λ), with statistics ranging between 10^5 and 10^6 Monte Carlo iterations per measurement, on workstations, on parallel machines at the Edinburgh Parallel Computing Centre and on our custom, 64 T800 processor computer RTN [13]. We have used an over-relaxed Metropolis update method, with a multicanonical procedure to accelerate the flip-flop rate in some case. We use the spectral density technique [14] in order to improve the determination of the transition point, and the jack-knife method in the error estimation.

We have observed the different energies one can define from the action. In particular we have paid special attention to the link energy $E_{\text{link}} = \langle \bar{\Phi} U \Phi \rangle$ as we have fixed the value of β and the transition is almost parallel to the β axis. All the observables we will define hereafter will be referred to this energy.

In the range of λ and L studied, the histograms of E_{link} present the following features (examples can be seen in Fig. 2):

- Two-peak structure.
- Asymmetry of the peaks, with a narrow low-energy peak and a broad high-energy one.
- Strong size dependence, consisting on a narrowing of the gap and width of the peaks as the lattice size L increases, caused by a much faster displacement towards lower energy of the broad peak.

It is known that, due to finite size effects, two-peaked histograms on finite lattices can become single peaked in the thermodynamical limit and vice versa. For instance, the transition in the $q = 4$ Potts model is known analytically to be second order but the simulation on finite lattices gives a double peaked

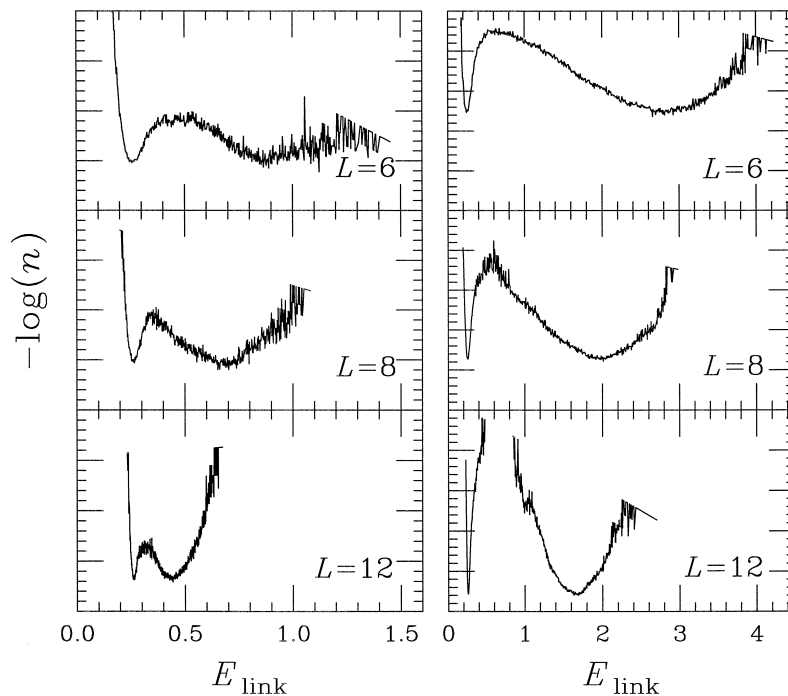


Fig. 2. Minus logarithm of the E_{link} histogram at $\lambda = 0.01$ (left) and $\lambda = 0.003$ (right).

structure [15]. On the contrary, single peaked histograms on small lattices can become double peaked when the lattice size increases, as in [16].

Then an accurate study of finite size effects has to be made. We will use several independent Finite Size Scaling methods, which allow to determine correctly the order of the transition whenever one uses large enough lattices (but not so large as if one only looked for metastability signals). Now we will introduce the different quantities which we have considered in order to extract our conclusions.

First, coming back to the histograms, in Ref. [17] an interesting method is proposed. They explain how in a two-peaked structure the increment between the minima and the local maximum of the free energy is directly related to the analogous increment for the logarithm of the histogram of the energy. The idea is that if the transition is first order the gap has to be more pronounced for larger L , while if it is second order the depth of the gap does not grow with L . Our procedure will be the following. From the histogram of E_{link} obtained in the simulation, we calculate a new histogram where the height of the two peaks is the same by using basically the spectral density method [14]. Then we take the logarithm of it and change its sign. In this way we obtain a figure analogous to the free energy (see Fig. 2). Here we can measure directly the depth of the gap, let us call it ΔF . A ΔF growing with L indicates a first order transition, while if it is constant we have a second order one.

Second, we have measured the specific heat

$$c_V = \frac{\partial \langle E_{\text{link}} \rangle}{\partial \kappa}. \quad (3)$$

Its maximal value should scale with an exponent α/ν . It will be d for a first order transition and 0 for a mean field second order one (in this case c_V^{max} will be a constant plus logarithmic corrections).

Third, we use the maximum of c_V as definition of *critical* $\kappa_c(L, \lambda)$. The scaling of $\kappa_c(L)$ with L allows us to extract ν . It will be $\nu = 1/d$ if the transition is first order, and $\nu = 1/2$ if we have second order with mean field exponents.

Fourth, we consider the latent heat at $\kappa_c(L, \lambda)$, ΔE , as the difference between the positions of the maxima for a fit to each peak separately to a cubic spline, after the histogram has been shifted to the

apparent critical point by the spectral density method. A cubic spline has been preferred to other functional shapes because of its ability to reproduce a maximum and accommodate the mixed states, whose influence on the histogram is otherwise difficult to account for. If we have $\Delta E \neq 0$ in the $V \rightarrow \infty$ limit we have a first order phase transition.

Fifth, we have computed the Binder cumulant

$$B = 1 - \frac{\langle E_{\text{link}}^4 \rangle}{3 \langle E_{\text{link}}^2 \rangle^2}. \quad (4)$$

If the energy distribution is single peaked (and then second order) in the thermodynamical limit, this quantity can be computed exactly and becomes $2/3$.

4. Results

First of all, let us comment on which have been our limits on λ and L .

For the biggest value of λ studied, $\lambda = 0.3$, it was impossible to distinguish the two peaks on the histogram, and then we could not make the type of treatment described above. For $\lambda = 0.1$ two peaks are visible but the gap is so small that it is difficult to measure with precision observables such as ΔF and ΔE ; for this value of λ we have only simulated $L = 6, 8$.

On the other hand, for the smallest value $\lambda = 0.003$ the metastability is so strong that we have only been able to use $L = 6, 8, 12$. Besides, the simulation in the $L = 12$ have been made fixing a value $\kappa = 0.2703$ and making two different runs, one starting from a cold configuration and the other one starting from a hot one. In this case ($L = 12$) no flip was observed; then we can only say that the two peaks are rather separated, but a precise estimation of the latent heat and of the other observables is not possible. In particular, the latent heat is greatly affected by a displacement in κ (because the broad peak is much more sensitive than the narrow one, as the displacement in the peak will be proportional to the squared width) and a bad estimation of κ_c would give place to an erroneous measurement.

For the intermediate values of λ things are more feasible and we have used $L = 6, 8, 12, 16$ for $\lambda = 0.005, 0.01, 0.03$ and also $L = 24$ for $\lambda = 0.005$.

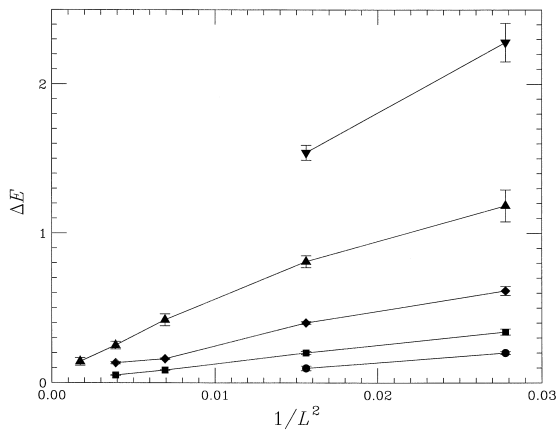


Fig. 3. Latent heat as a function of $1/L^2$ at different values of λ . From top to bottom, $\lambda = 0.003, 0.005, 0.01, 0.03, 0.1$.

Let us now present the results for the different observables.

The study of the latent heat suggests an extrapolation to zero in the thermodynamical limit for $\lambda \geq 0.005$ as shown in Fig. 3.

The specific heat c_V is constant with L for the largest λ -values while it grows for the smallest ones, as shown in Table 1. In the presence of a stable double peaked structure (i.e. such that the peaks do not approach when changing L), c_V scales with the maximum possible speed, i.e. like V , indicating a first order transition. However in our case the peaks are not stable and then the scaling will be different. For a trivial second order transition in $d = 4$, it scales only logarithmically ($\alpha = 0$), then it must be almost constant. The results indicate a crossover between this two behaviors at $\lambda \approx 0.005$.

For the values of λ where we have more measurements, i.e. $\lambda = 0.005, 0.01, 0.03$ we have fitted $\kappa_c(L)$ as a function of L :

$$\kappa_c(L) = \kappa_c(\infty) + AL^{-1/\nu}. \quad (5)$$

The three parameter fit gives $\nu = 0.40(1), 0.43(1), 0.44(1)$, respectively, at those values of λ . These values of ν are closer to the mean field one, $\nu = 1/2$, than to the expected in a first order transition, $\nu = 1/4$. However, those apparent ν -exponents would probably derive to one of the two values ($1/2$ or $1/4$) in the thermodynamical limit.

The behaviour of the Binder cumulant has not

Table 1
Specific heat for the different values of L and λ .

λ	$L=6$	$L=8$	$L=12$	$L=16$	$L=24$
0.003	7710(490)	9810(1150)			
0.005	1820(80)	2400(70)	3850(170)	4260(890)	7760(810)
0.01	577(57)	652(53)	660(28)	897(105)	
0.03	185(41)	189(17)	172(15)	186(12)	
0.1	63(2)	58(3)			

given any conclusive result. In principle, it should approach the value $2/3$ with corrections of the order of $L^{2(1/\nu-d)}$ for a second order transition or a value different from $2/3$ with corrections as L^{-d} for a first order one. In practice, we obtain some value close to $2/3$ but neither the precision is enough nor the scaling is as cited. In Fig. 4 we show the results for $\lambda = 0.005$. We have a good scaling but with an exponent $-2.5(2)$ instead of -4 , which would be expected both in the first order case and in the gaussian second order one. The fit gives a value $0.64(1)$ in the infinite volume limit. However this difference from $2/3$ is not significant as the error is large and the scaling is still not as expected in the thermodynamical limit.

Finally, in Table 2 are shown the increments of the free energy ΔF . Again we can see that the behavior changes at $\lambda \approx 0.005$; for larger values ΔF is constant with L , while for smaller ones it grows. A look at Fig. 2 shows the different kind of behavior

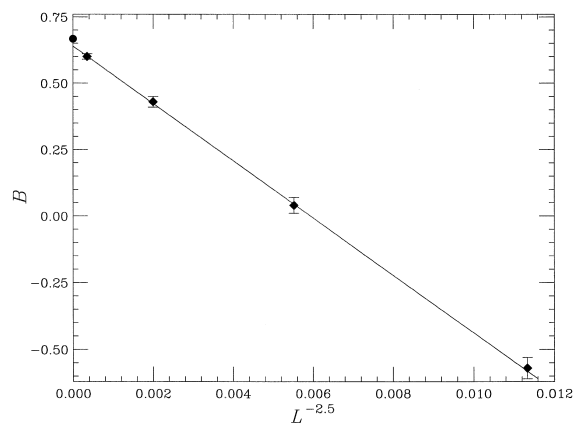


Fig. 4. Binder cumulant at $\lambda = 0.005$ as a function of $L^{-2.5}$. The circle corresponds to the value $2/3$.

Table 2
 ΔF for the different values of L and λ .

λ	$L=6$	$L=8$	$L=12$	$L=16$	$L=24$
0.003	4.0(4)	5.0(7)	large		
0.005	2.3(5)	2.1(4)	3.1(4)	3.4(4)	3.5(5)
0.01	1.7(4)	1.9(4)	1.3(4)	1.9(4)	
0.03	0.6(2)	0.8(3)	0.6(2)	0.7(2)	

for $\lambda = 0.01$ and $\lambda = 0.003$; while in the first case the gap tends to disappear when increasing the lattice size, in the second one it tends to become deeper.

5. Conclusions

We have investigated the order of the Coulomb-Higgs transition at fixed $\beta = 1.15$ and for different values of λ . The analysis of several observables indicates a crossover at $\lambda \approx 0.005$, the transition being first order for smaller and second order for larger values of λ . We remark that in order to extract this conclusion we have needed to consider different lattice sizes and, even though most of them present a two-peak structure, the latent heat goes to zero in the thermodynamical limit (at least for $\lambda \geq 0.005$). Previous works had estimated this crossover at $\lambda \approx 0.3$ [11] or $\lambda \approx 0.02$ [1] for $\beta = 1.5$. However for larger β a weaker character is expected (the correlation length grows and we approach the second order transition of the ϕ^4 theory). Then for $\beta = 1.5$ the crossover should occur at a smaller λ -value than for $\beta = 1.15$. Therefore we have lowered the estimation of the values of λ for which the Coulomb-Higgs transition changes order.

Acknowledgements

We are very grateful to the RTN collaboration for the use of their transputer machine where we have

run most of our simulations. D.I. acknowledges the EPCC and the Department of Physics of Edinburgh, as well as the Ministerio de Educación y Cultura (Spain) for a fellowship. This work has been partially supported by CICYT under contract numbers AEN96-1634 and AEN96-1670 and by the EU under contract number ERB-CHRX CT 92-0051.

References

- [1] D. Espriu, J.F. Wheeler, Nucl. Phys. B 258 (1985) 101.
- [2] H.G. Evertz, K. Jansen, J. Jersák, C.B. Lang, T. Neuhaus, Nucl. Phys. B 285 [FS19] (1987) 590.
- [3] E. Fradkin, S.H. Shenker, Phys. Rev. D 19 (1979) 3682.
- [4] A. Cruz, A. Tarancón, L.A. Fernández, A. Muñoz Sudupe, J.J. Ruiz-Lorenzo, Nucl. Phys. B (P.S.) 34 (1994) 638.
- [5] A. Cruz, A. Tarancón, L.A. Fernández, A. Muñoz Sudupe, J.J. Ruiz-Lorenzo, Int. Jour. of Mod. Phys. C, Vol. 5, No. 2 (1994) 343.
- [6] M. Creutz, L. Jacobs, C. Rebbi, Phys. Rev. D 20 (1979) 1915.
- [7] B. Lautrup, M. Nauenberg, Phys. Lett. B 95 (1980) 63.
- [8] L.A. Fernández, A. Muñoz Sudupe, R. Petronzio, A. Tarancón, Phys. Lett. B 267 (1991) 100.
- [9] J. Jersak, C.B. Lang, T. Neuhaus, Phys. Rev. Lett. 77 (1996) 1933.
- [10] The RTN Collaboration, J.L. Alonso et al. Nucl. Phys. B 405 (1993) 574.
- [11] K. Jansen et al., Nucl. Phys. B 265 [FS15] (1986) 187.
- [12] L.A. Fernández, A. Muñoz Sudupe, J.J. Ruiz-Lorenzo, A. Tarancón, Phys. Lett. B 312 (1993) 305.
- [13] RTN Collaboration, Computing in High Energy Physics'92 CERN 92-07, Geneva (1992) 353.
- [14] M. Falcioni, E. Marinari, M.L. Paciello, G. Parisi, B. Taglienti, Phys. Lett. B 108 (1982) 331; A.M. Ferrenberg and R. Swendsen, Phys. Rev. Lett. 61 (1988) 2635.
- [15] M. Fukugita, H. Mino, M. Okawa, A. Ukawa, J. Phys. A. Math. Gen. 23 (1990) L561.
- [16] J.L. Alonso, J.M. Carmona, J. Clemente Gallardo, L.A. Fernández, D. Iñiguez, A. Tarancón, C.L. Ullod, Phys. Lett. B 376 (1996) 148.
- [17] J. Lee, J.M. Kosterlitz, Phys. Rev. Lett. 65 (1990) 137.