

ORTHOGONALLY ADDITIVE POLYNOMIALS ON SPACES OF CONTINUOUS FUNCTIONS

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ABSTRACT. We show that, for every orthogonally additive homogeneous polynomial P on a space of continuous functions $C(K)$ with values in a Banach space Y , there exists a linear operator $S : C(K) \rightarrow Y$ such that $P(f) = S(f^n)$. This is the $C(K)$ version of a related result of Sundaresam for polynomials on L_p spaces.

1. INTRODUCTION

Given a Banach lattice X and a Banach space Y (possibly the scalars), a function $\varphi : X \rightarrow Y$ is called *orthogonally additive* if, for every $f, g \in X$ with disjoint support, $\varphi(f + g) = \varphi(f) + \varphi(g)$. Orthogonally additive functions and their representations have been studied by several authors since the sixties (see for instance [6], [7], [8] and the references therein).

In [9], the author studies the orthogonally additive polynomials defined on L_p spaces and proves, among other things, that, for $1 \leq p < \infty$ and $1 \leq n < p$, for every scalar n -homogeneous polynomial $P : L_p \rightarrow \mathbb{K}$ there exists $g \in L_{\frac{p}{p-n}}$ such that, for every $f \in L_p$,

$$P(f) = \int f^n g d\mu.$$

In this paper we prove the analogue for $C(K)$ spaces, that is, if Y is a Banach space (in particular Y can be the scalars \mathbb{K}) and $P : C(K) \rightarrow Y$ is an orthogonally additive n -homogeneous polynomial, then there exists a linear operator $S : C(K) \rightarrow Y$ such that $P(f) = S(f^n)$ for every $f \in C(K)$.

First we introduce our notation and some known facts which we will use. Y will always be a Banach space and K a compact Hausdorff space. Σ will be the σ -algebra of the Borel sets of K . $S(K)$ is the space of Σ -simple scalar functions defined on K , and $B(K)$ is the completion of $S(K)$ under the supremum norm. Throughout the paper, ‘operator’ (linear, multilinear or polynomial) will mean ‘continuous operator’.

We suppose the reader well acquainted with the theory of representation of linear operators on $C(K)$ spaces by vector measures. An excellent exposition of this theory can be read in [4, Chapter VI].

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For multilinear operators, if we let $T : C(K) \times \cdots \times C(K) \longrightarrow Y$ be a multilinear operator, then we know that T can be extended to a multilinear operator $\bar{T} : B(K) \times \cdots \times B(K) \longrightarrow Y^{**}$, in a unique way if we ask \bar{T} to be separately weak*-weak* continuous, when the weak* topology we consider in $B(K)$ is given by the isometric inclusion $B(K) \hookrightarrow C(K)^{**}$ (see [3]). In fact, this extension is nothing but the restriction of the Aron-Berner extension $AB(T) : C(K)^{**} \times \cdots \times C(K)^{**} \longrightarrow Y^{**}$ to the product $B(K) \times \cdots \times B(K)$ (for information about the Aron-Berner extension, see [5] and the references therein). It follows from the uniqueness of the Aron-Berner extension in the particular case of $C(K)$ spaces that, if T is symmetric, then $AB(T)$, and hence \bar{T} , are also symmetric (for a proof of this fact see [1, page 83]).

Once we have defined \bar{T} , we can define the set function

$$\gamma : \Sigma \times \cdots \times \Sigma \longrightarrow Y^{**}$$

given by

$$\gamma(A_1, \dots, A_n) = \bar{T}(\chi_{A_1}, \dots, \chi_{A_n}).$$

Thus defined, γ is a weak* regular countably additive *polymeasure*, that is, a separately additive set function such that, for every $y^* \in Y^*$, $y^* \circ \gamma$ is separately countably additive and separately regular. For the theory of polymeasures and its applications to the study of multilinear operators on $C(K)$ spaces see [3], [10] and the references therein. In particular, given a polymeasure γ we later use the definition of its *semivariation* $\|\gamma\|$, which the reader can find in [3].

Using the convention $[\cdot]$ to mean that the i -th coordinate is not involved, it follows also from [3] that, for every $(g_1, [\cdot], g_n) \in B(K) \times \binom{n-1}{\cdot} \times B(K)$ there is a unique Y^{**} -valued bounded weak*-Radon measure $\gamma_{g_1, [\cdot], g_n}$ on K (i.e., a Y^{**} -valued finitely additive bounded vector measure on the Borel subsets of K , such that for every $y^* \in Y^*$, $y^* \circ \gamma_{g_1, [\cdot], g_n}$ is a regular countably additive measure on K), satisfying

$$\int g_i d\gamma_{g_1, [\cdot], g_n} = \bar{T}(g_1, \dots, g_{i-1}, g_i, g_{i+1}, \dots, g_n), \text{ for every } g_i \in B(K).$$

Similarly, for every $g_i \in B(K)$ there is a unique Y^{**} -valued bounded weak*-regular countably additive $(n-1)$ -*polymeasure* γ_{g_i} , satisfying

$$\int (g_1, [\cdot], g_n) d\gamma_{g_i} = \bar{T}(g_1, \dots, g_{i-1}, g_i, g_{i+1}, \dots, g_n),$$

for every $(g_1, [\cdot], g_n) \in B(K) \times \binom{n-1}{\cdot} \times B(K)$.

$\mathcal{P}^n(C(K); Y)$ will denote the Banach space of all n -homogeneous polynomials from $C(K)$ to Y . When $Y = \mathbb{K}$ we will omit it. Further notation will be introduced when needed.

2. THE RESULT

Our main result is the following.

Theorem 2.1. *Let $P \in \mathcal{P}^n(C(K); Y)$ be an orthogonally additive n -homogeneous polynomial with associated symmetric multilinear operator T . Then, there exists a linear operator $S \in \mathcal{L}(C(K); Y)$, with associated (finitely additive) measure $\nu : \Sigma \longrightarrow Y^{**}$, such that $\|S\| = \|T\|$ and such that, for every $f \in C(K)$,*

$$P(f) = S(f^n) = \int_K f^n d\nu.$$

To prove our theorem, we need some previous results.

Proposition 2.2. *Let $P : C(K) \longrightarrow \mathbb{K}$ be in $\mathcal{P}^n(C(K))$ and let $T : C(K) \times \dots \times C(K) \longrightarrow \mathbb{K}$ be its associated symmetric n -linear operator. Then P is orthogonally additive if and only if for every $1 < s \leq n$ and $1 \leq n_1, \dots, n_s \leq n$ such that $n_1 + \dots + n_s = n$ and for every mutually orthogonal $f_1, \dots, f_s \in C(K)$, we have that*

$$(1) \quad T(f_1, \overset{n_1}{\cdot}, f_1, \dots, f_s, \overset{n_s}{\cdot}, f_s) = 0.$$

Proof. One of the implications is clear. For the other, we fix s, n_1, \dots, n_s and f_1, \dots, f_s as above and we take scalars $\lambda_1, \dots, \lambda_s$. The orthogonal additivity of the polynomial gives us that

$$P(\lambda_1 f_1 + \dots + \lambda_s f_s) = \lambda_1^n P(f_1) + \dots + \lambda_s^n P(f_s).$$

Moreover, we have that

$$P(\lambda_1 f_1 + \dots + \lambda_s f_s) = \sum_{k_1, \dots, k_n=1}^s \lambda_{k_1} \dots \lambda_{k_n} T(f_{k_1}, \dots, f_{k_n})$$

and, using the symmetry of T , we can rearrange to get

$$P(\lambda_1 f_1 + \dots + \lambda_s f_s) = \sum_{\substack{0 \leq \alpha_j \leq n \\ \alpha_1 + \dots + \alpha_s = n}} \frac{n!}{\alpha!} \lambda_1^{\alpha_1} \dots \lambda_s^{\alpha_s} T(f_1, \overset{\alpha_1}{\cdot}, f_1, \dots, f_s, \overset{\alpha_s}{\cdot}, f_s),$$

where $\alpha! = \alpha_1! \dots \alpha_s!$.

Then, we have that the polynomial Q in $\lambda_1, \dots, \lambda_s$ given by

$$Q(\lambda_1, \dots, \lambda_s) = \sum_{\substack{0 \leq \alpha_j \leq n-1 \\ \alpha_1 + \dots + \alpha_s = n}} \frac{n!}{\alpha!} T(f_1, \overset{\alpha_1}{\cdot}, f_1, \dots, f_s, \overset{\alpha_s}{\cdot}, f_s) \lambda_1^{\alpha_1} \dots \lambda_s^{\alpha_s}$$

is equal to zero. Identifying coefficients we get that

$$T(f_1, \overset{\alpha_1}{\cdot}, f_1, \dots, f_s, \overset{\alpha_s}{\cdot}, f_s) = 0$$

for $0 \leq \alpha_j \leq n-1$ with $\alpha_1 + \dots + \alpha_s = n$. In particular, we obtain (1). \square

We need a stronger version of Proposition 2.2. To obtain it, we prove first a simple auxiliar lemma, direct consequence of the polarization formula.

Lemma 2.3. *If $T : C(K) \times \cdots \times C(K) \longrightarrow \mathbb{K}$ is a symmetric n -linear operator, $n_1, \dots, n_s \geq 1$ are natural numbers such that $n_1 + \cdots + n_s = n$ and $(f_j^i)_{i=1}^{n_j} \subset C(K)$ for $1 \leq j \leq s$, we have that there exist a natural number N , real numbers $(\beta_r)_{r=1}^N$ and elements $(g_j^r)_{j,r=1}^{s,N} \subset C(K)$ such that*

$$T(f_1^1, \dots, f_1^{n_1}, \dots, f_s^1, \dots, f_s^{n_s}) = \sum_{r=1}^N \beta_r T(g_1^r, \dots, g_1^{n_1}, g_1^r, \dots, g_s^r, \dots, g_s^{n_s}, g_s^r)$$

and such that $g_j^r \in \text{span}\{f_j^1, \dots, f_j^{n_j}\}$ for every $1 \leq j \leq s$ and every $1 \leq r \leq N$.

Proof. We reason by induction in s . The case $s = 1$ is just the polarization formula [5, Corollary 1.6]. To obtain the case s from the case $s - 1$, we apply the induction hypothesis to $T_{f_1^1, \dots, f_1^{n_1}}$ to obtain that

$$T(f_1^1, \dots, f_1^{n_1}, \dots, f_s^1, \dots, f_s^{n_s}) = \sum_{r=1}^N \beta_r T_{f_1^1, \dots, f_1^{n_1}}(g_2^r, \dots, g_2^{n_2}, g_2^r, \dots, g_s^r, \dots, g_s^{n_s}, g_s^r).$$

Now we can apply the case $s = 1$ to the operator $T_{g_2^r, \dots, g_2^{n_2}, g_2^r, \dots, g_s^r, \dots, g_s^{n_s}, g_s^r}$ to conclude the result. \square

Now we can improve Proposition 2.2.

Proposition 2.4. *Let $P \in \mathcal{P}^n(C(K))$ and let $T : C(K) \times \cdots \times C(K) \longrightarrow \mathbb{K}$ be its associated symmetric n -linear operator.*

Then, P is orthogonally additive if and only if for every $1 < s \leq n$, for every A_1, \dots, A_s , mutually disjoint subsets of K , for every $1 \leq n_1, \dots, n_s \leq n$ such that $n_1 + \dots + n_s = n$ and for every $f_j^1, \dots, f_j^{n_j} \in C(K)$ such that f_j^i is supported in A_j for $1 \leq j \leq s$, $1 \leq i \leq n_j$, we have that

$$T(f_1^1, \dots, f_1^{n_1}, \dots, f_s^1, \dots, f_s^{n_s}) = 0.$$

Proof. One of the implications is clear. For the other, Lemma 2.3 allows us to write

$$T(f_1^1, \dots, f_1^{n_1}, \dots, f_s^1, \dots, f_s^{n_s}) = \sum_{r=1}^N \beta_r T(g_1^r, \dots, g_1^{n_1}, g_1^r, \dots, g_s^r, \dots, g_s^{n_s}, g_s^r)$$

where $N \in \mathbb{N}$, $\beta_r \in \mathbb{R}$ and $g_j^r \in \text{span}\{f_j^1, \dots, f_j^{n_j}\}$. But then g_1^r, \dots, g_s^r are mutually orthogonal for every $1 \leq r \leq N$. Therefore, Proposition 2.2 assures that

$$T(g_1^r, \dots, g_1^{n_1}, g_1^r, \dots, g_s^r, \dots, g_s^{n_s}, g_s^r) = 0$$

for every $1 \leq r \leq N$. \square

The following result is an easy consequence of the fact that the polymmeasure representing a multilinear form is separately regular [3].

Lemma 2.5. *Let γ be the polymeasure representing a multilinear form $T : C(K) \times \cdots \times C(K) \longrightarrow \mathbb{K}$. Given $(A_1, \dots, A_n) \in \Sigma^n$ and $\epsilon > 0$ there exist compact sets $(K_1, \dots, K_n) \in \Sigma^n$ and open sets $(G_1, \dots, G_n) \in \Sigma^n$ with $K_i \subset A_i \subset G_i$ ($1 \leq i \leq n$) such that*

$$|\gamma(A_1, \dots, A_n) - \gamma(K_1, \dots, K_n)| < \epsilon$$

and

$$|\gamma(A_1, \dots, A_n) - \gamma(G_1, \dots, G_n)| < \epsilon.$$

Moreover, if A_1, \dots, A_n are compact and mutually disjoint, we can choose the open sets G_1, \dots, G_n to be also mutually disjoint.

Proof. We prove the existence of the open sets in the statement, the existence of the compact sets being similar. We reason by induction on n . For $n = 1$, it is known that the regularity of the measures representing forms on $C(K)$ implies that for every $\mu \in C(K)^*$, for every $A \in \Sigma$ and for every $\epsilon > 0$ there exists an open set $G \supset A$ such that the variation $v(\mu)(G \setminus A) < \epsilon$, and this proves the result.

We suppose now the result true for $n - 1$ and consider sets $(A_1, \dots, A_n) \in \Sigma^n$. Then $\gamma(A_1, \dots, A_n) = \gamma_{A_n}(A_1, \dots, A_{n-1})$ (with the notation as in the introduction). The induction hypothesis assures the existence of open sets $(G_1, \dots, G_{n-1}) \in \Sigma^n$ such that

$$|\gamma_{A_n}(A_1, \dots, A_{n-1}) - \gamma_{A_n}(G_1, \dots, G_{n-1})| \leq \frac{\epsilon}{2},$$

and the case $n = 1$ provides an open set $G_n \in \Sigma$ such that

$$|\gamma_{G_1, \dots, G_{n-1}}(A_n) - \gamma_{G_1, \dots, G_{n-1}}(G_n)| < \frac{\epsilon}{2}$$

and this finishes the proof.

For the last statement, if A_1, \dots, A_n are compact and mutually disjoint, we start choosing mutually disjoint open sets $A_i \subset H_i$ ($1 \leq i \leq n$) and then we reason as above with the sets $G'_i := G_i \cap H_i$. \square

We also need the following technical lemma.

Lemma 2.6. *Let $P \in \mathcal{P}^n(C(K))$ be orthogonally additive and let $\gamma : \Sigma^n \longrightarrow \mathbb{K}$ be the representing polymeasure of the associated symmetric n -linear operator $T : C(K) \times \cdots \times C(K) \longrightarrow \mathbb{K}$. Then, for $1 < s \leq n$ and open sets $(G_1^1, \dots, G_1^{n_1}, \dots, G_s^1, \dots, G_s^{n_s}) \in \Sigma^n$, such that $G_i^{m_i} \cap G_j^{m_j} = \emptyset$ for every $i \neq j$, $i, j \in \{1, \dots, s\}$, $m_i \in \{1, \dots, n_i\}$, $m_j \in \{1, \dots, n_j\}$ we have that*

$$\gamma(G_1^1, \dots, G_1^{n_1}, \dots, G_s^1, \dots, G_s^{n_s}) = 0.$$

Proof. For simplicity in the notation we write the proof for the case of two open sets G_1, G_2 with $G_1 \cap G_2 = \emptyset$. The reasonings extend easily to the general case. Given an open set $G \in \Sigma$, we can consider the directed set of the Borel compact sets $C \subset G$ with the order given by the inclusion. Applying Urysohn's lemma, for every such C we can choose $f_C \in C(K)$, with $\chi_C \leq f_C \leq \chi_G$. It follows from the regularity of the measures representing

$C(K)^*$ that the net f_C converges weak* to χ_G . Hence, choosing two such nets (f_C) and (g_D) such that (f_C) weak* converges to χ_{G_1} and (g_D) weak* converges to χ_{G_2} , we have, applying Proposition 2.4 and the separate weak* continuity of the Aron-Berner extension, that

$$\gamma(G_1, G_2) = \overline{T}(\chi_{G_1}, \chi_{G_2}) = \lim_C \lim_D T(f_C, g_D) = 0.$$

□

The main ingredient in our proof is the following result.

Lemma 2.7. *Let $P \in \mathcal{P}^n(C(K); Y)$, T its associated symmetric multilinear operator and γ its representing polymeasure. If P is orthogonally additive then for every $1 < s \leq n$ and sets $(A_1, \dots, A_s) \in \Sigma^n$ such that $A_i \cap A_j = \emptyset$ for every $i \neq j$, we have that*

$$\gamma(A_1, \dots, A_s) = 0.$$

Proof. Let $T, \gamma, (A_1, \dots, A_s)$ be as in the hypothesis. We fix $y^* \in Y^*$. According to Lemma 2.5, for every $\epsilon > 0$ we have that there exist compact sets $(K_1^1, \dots, K_1^{n_1}, \dots, K_s^1, \dots, K_s^{n_s}) \in \Sigma^n$ with $K_j^i \subset A_j$ for every $j \in \{1, \dots, s\}, i \in \{1, \dots, n_j\}$ and such that

$$|(y^* \circ \gamma)(A_1, \dots, A_s) - (y^* \circ \gamma)(K_1^1, \dots, K_1^{n_1}, \dots, K_s^1, \dots, K_s^{n_s})| < \epsilon.$$

Using now the last part of Lemma 2.5 we can prove the existence of open sets $G_1^1, \dots, G_1^{n_1}, \dots, G_s^1, \dots, G_s^{n_s}$ such that $K_j^i \subset G_j^i$ for every $j \in \{1, \dots, s\}, i \in \{1, \dots, n_j\}$ with $G_j^i \cap G_{j'}^{i'} = \emptyset$ for every $j \neq j' \in \{1, \dots, s\}, i \in \{1, \dots, n_j\}, i' \in \{1, \dots, n_{j'}\}$ and such that

$$|(y^* \circ \gamma)(K_1^1, \dots, K_1^{n_1}, \dots, K_s^1, \dots, K_s^{n_s}) - (y^* \circ \gamma)(G_1^1, \dots, G_1^{n_1}, \dots, G_s^1, \dots, G_s^{n_s})|$$

is also less than ϵ .

Now, an application of Lemma 2.6 yields

$$|(y^* \circ \gamma)(A_1, \dots, A_s)| < 2\epsilon.$$

Since this is true for every $\epsilon > 0$, $y^* \in Y^*$, we get our result. □

Finally, we can prove Theorem 2.1

Proof of Theorem 2.1. We can define now the measure ν which will induce the operator S of the theorem.

Let $\nu : \Sigma \longrightarrow Y^{**}$ be the set function defined by

$$\nu(A) = \gamma(A, \dots, A).$$

We check that ν is a bounded finitely additive measure and, therefore, it defines a linear operator $U : B(K) \longrightarrow Y^{**}$:

Let us see that ν is additive. Let $(A_i)_{i=1}^m \subset \Sigma$ be a finite sequence of mutually disjoint sets. Then

$$\begin{aligned} \nu(\cup_{i=1}^m A_i) &= \gamma(\cup_{i=1}^m A_i, \dots, \cup_{i=1}^m A_i) = \sum_{i_1=1}^m \cdots \sum_{i_n=1}^m \gamma(A_{i_1}, \dots, A_{i_n}) = \\ &= \sum_{i=1}^m \gamma(A_i, \dots, A_i) = \sum_{i=1}^m \nu(A_i), \end{aligned}$$

where the second equality follows from the fact that γ is separately additive and the third equality follows from Lemma 2.7.

Let us check that ν is bounded: $\|\nu\| =$

$$\begin{aligned} &= \sup \left\{ \left\| \sum_i a_i \nu(A_i) \right\| ; \text{ where } (A_i) \text{ is a finite partition of } K \text{ and } |a_i| \leq 1 \right\} \\ &= \sup \left\{ \left\| \sum_i a_i \gamma(A_i, \dots, A_i) \right\| \right\} \leq \|\gamma\| = \|T\|. \end{aligned}$$

Therefore ν defines an operator $U : B(K) \longrightarrow Y^{**}$. Let us see that, for every $g_1, \dots, g_n \in B(K)$,

$$\overline{T}(g_1, \dots, g_n) = U(g_1 \cdots g_n).$$

It clearly suffices to consider the case when the g_i 's are simple functions given by

$$g_i = \sum_{j=1}^m a_j^i \chi_{A_j}$$

where $(A_j)_{j=1}^m \subset \Sigma$ is a sequence of mutually disjoint sets. In that case, using again Lemma 2.7, we get

$$\begin{aligned} \overline{T}(g_1, \dots, g_n) &= \overline{T} \left(\sum_{j=1}^m a_j^1 \chi_{A_j}, \dots, \sum_{j=1}^m a_j^n \chi_{A_j} \right) = \sum_{j=1}^m a_j^1 \cdots a_j^n \gamma(A_j, \dots, A_j) \\ &= \sum_{j=1}^m a_j^1 \cdots a_j^n \nu(A_j, \dots, A_j) = U \left(\sum_{j=1}^m a_j^1 \cdots a_j^n \chi_{A_j} \right) = U(g_1 \cdots g_n). \end{aligned}$$

In particular, for every $f_1, \dots, f_n \in C(K)$,

$$T(f_1, \dots, f_n) = \overline{T}(f_1, \dots, f_n) = U(f_1 \cdots f_n).$$

Therefore, $S := U|_{C(K)} : C(K) \longrightarrow Y$ is the operator we were looking for.

Moreover, we have

$$\|\gamma\| = \|T\| \leq \|S\| = \|\nu\| \leq \|\gamma\|,$$

which yields the coincidence of all the norms involved. □

3. RELATION WITH THE INJECTIVE TENSOR NORM

We call a multilinear operator $T : X_1 \times \cdots \times X_n \longrightarrow Y$ ϵ -continuous if its linearization $\text{lin}(T) : X_1 \otimes \cdots \otimes X_n \longrightarrow Y$ is continuous when we consider the ϵ (injective) tensor norm. A n -homogeneous polynomial $P : X \longrightarrow Y$ is called ϵ -continuous if its associated symmetric n -linear form is so.

Note that, since the pointwise multiplication $M : C(K) \times \cdots \times C(K) \longrightarrow C(K)$ is ϵ -continuous, we obtain as a corollary to our main result that every orthogonally additive $P \in \mathcal{P}^n(C(K); Y)$ is ϵ -continuous. The converse is not true, as a simple example below shows, but it *is* true that every ϵ -continuous $P \in \mathcal{P}^n(C(K); Y)$ factors through an orthogonally additive $Q \in \mathcal{P}^n(C(B_{C(K)}^*); Y)$, see the comments below.

We start showing a simple example of a polynomial $P \in \mathcal{P}^2(C(K))$ ϵ -continuous and not orthogonally additive.

Example 3.1. Let $P : \mathbb{R}^2 = C(\{a, b\}) \longrightarrow \mathbb{R}$ the 2-homogeneous polynomial given by $P(x, y) = xy$. Clearly, $P(1, 0) + P(0, 1) = 0 \neq 1 = P(1, 1)$ and P is not orthogonally additive. However, being \mathbb{R}^2 finite dimensional, P is trivially ϵ -continuous.

Despite this, it follows as a simple consequence of [11, Corollary 2.2] that, given an ϵ -continuous polynomial $P \in \mathcal{P}^n(C(K); Y)$, there exists $U \in \mathcal{L}(C(K); Y)$ such that P factors as

$$P(f) = U(i(f)^n),$$

where $i : C(K) \hookrightarrow C(B_{C(K)}^*)$ is the canonical isometric injection given by $i(f)(x^*) = x^*(f)$. We remark that this injection does *not* preserve the lattice structure of $C(K)$.

Moreover, it follows from [11, Corollary 2.4] that, when K is metrizable and uncountable, there exists an injective isometry $j : C(K) \hookrightarrow C(K)$ such that, for every ϵ -continuous polynomial $P \in \mathcal{P}^n(C(K); Y)$, there exists $U \in \mathcal{L}(C(K); Y)$ such that P factors as

$$P(f) = U(j(f)^n).$$

Remark 3.2. Y. Benyamini, S. Lasalle and J.G. Llavona [2] have independently generalized Sundaresam's representation theorem to all Banach lattices.

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