

## Sinks, sources, and centers in some history of economic thought

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**EN Abstract.** We propose a unifying dynamical systems approach to states of rest and fluctuations grounded in four models offered by Jump & Stockhammer (2023). Our treatment permits reflection on the claim of heterodoxy made of the models. We conclude that only models 2 and 3 are heterodox. Models 1 and 4 are orthodox.

**Keywords.** equilibrium, adjustment, fluctuations.

**JEL Codes:** B41, B50, E32.

### ES Sumideros, fuentes y centros en alguna historia del pensamiento económico

**ES Resumen.** El artículo propone un enfoque unificador de sistemas dinámicos sobre los estados de reposo y las fluctuaciones, basado en cuatro modelos presentados por Jump y Stockhammer (2023). Nuestro tratamiento permite reflexionar sobre la afirmación de heterodoxia atribuida a los modelos. Concluimos que solo los modelos 2 y 3 son heterodoxos. Los modelos 1 y 4 son ortodoxos.

**Palabras clave.** equilibrio, ajuste, fluctuaciones.

**Códigos JEL:** B41, B50, E32.

### PT Sumidouros, fontes e centros em parte da história do pensamento econômico

**PT Resumo.** Propomos uma abordagem unificadora de sistemas dinâmicos sobre estados de repouso e flutuações, fundamentada em quatro modelos apresentados por Jump e Stockhammer (2023). Nossa análise permite refletir sobre a alegação de heterodoxia atribuída aos modelos. Concluímos que apenas os modelos 2 e 3 são heterodoxos. Os modelos 1 e 4 são ortodoxos.

**Palavras-chave.** equilíbrio, ajuste, flutuações.

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**Sumario:** 1. Introduction. 2. Cycles and fluctuations in economic thought. 3. The multiplier-accelerator model. 4. The Goodwin model. 5. The Minsky model. 6. Momentum trader models. 7. Conclusions. 8. References.

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## 1. Introduction

'Equilibrium is blither', J. M. Keynes famously proclaimed and stirred up a hornet's nest for future scholars who were to grapple with a short-run equilibrium where markets, notably the labor market, did not clear, and a long-run equilibrium where, as famously, everybody was dead. At a different remove is general equilibrium where markets clear. The conditions for stability of the equilibrium are specified. Fluctuations in time around a dynamic equilibrium path are permitted. Quite separately, the business cycle, traditionally, was a separate field of enquiry with pure theorists on one side, and great miners of masses of empirical and historical data on the other. Scholars of the long cycle are clear and separate from the purview of the subject under review. The word suggests regularity and rhythms, peaks and troughs, and turning points separating them. Instead, both orthodox and heterodox conflate cycles with fluctuations after the definition of the subject by Robert Lucas and the rational expectations school.

An economy is always in flux. The system is locally stable inside the walls of a corridor between which the elasticities of demand and supply functions are within known bounds. The demands of global stability go beyond calculus into heavier technology. A pioneer in this regard was Hyman Minsky, who was unorthodox in analyzing the capitalist economy as a real-monetary-financial coupled system. Using the metaphor of modern physics, the economy is a vibrating string. The string can be stretched to breaking point resulting in rollercoaster comebacks. (The snapping of the string either in economics or physics would mean the end of the model world!). In addition, Minsky superimposed upon the constantly-perturbed economy a cycle that "happened" repeatedly. Over a relatively tranquil period, debts are undertaken to produce goods and services and employ people. The monetary authorities are supportive. Over the stretch of a 'great moderation', conservative postures give way to undue risk taking, the central bank lowers defenses, and financial positions turn from hedge to speculative. Reactions of participants are endogenous. Propensities are not given. We will elaborate in section 5. To be sure, speculation found on fundamentals is the definition of the future. The literature distinguishes between chartists and fundamentalists behind the movement of stock prices. The former work with time series of asset prices, the latter with structural models, even if back-of-the-envelope. As with Minsky, optimism and pessimism with chartists is self-generating. However, with the usual shapes of demand and supply functions, fundamentalists ensure that the system reverts to equilibrium. The distinction is taken up in section 6. In contrast, Minsky's model is non-equilibrium, locally stable, but globally unstable. Another, and earlier, epoch-marking event in the history of heterodox cycle theory was the contribution of Richard Goodwin and the subject of section 4. The difference with Minsky was that Goodwin's models were innocent of money and finance. The prime mover was the accumulation of capital à la Karl Marx and the conflict between capitalists and workers that drove and derived from the capitalist dynamic. To start with, in section

3 we examine the multiplier-accelerator model that is staple fare in orthodox models of fluctuations.

Recently, the concatenation of ideas has been brought under the rubric hysteresis (Palley, 2023). The claim is that in the history of thought the origin can be traced to Joan Robinson and Nicholas Kaldor and the Cambridge England School of the 1960s and 1970s. Robinson strongly advocated historical time in the study of societies instead of non-equilibrium. Non-equilibrium was different from disequilibrium in that the former was undefined while the latter was defined in relation to equilibrium. Non-equilibrium is the same as flux in the previous paragraph. The definition of hysteresis is that a phenomenon moving in time can be irreversibly changed when it crosses a threshold. One-way hysteresis and two-way hysteresis have been distinguished. The first is the case of a regime change, the second is a reversion to the original regime in the event of crossing a different threshold, as in cycles. The different points of return are equilibria.

We salute the admirable effort of Jump & Stockhammer (2023) (hereafter, J&S) in proposing unifying formal structure for the above concepts and then testing four well-known models against mathematics. Their numbering of equations is reproduced. Our response is a constructive critique. J&S confine themselves to heterodox theories. However, even at a first glance, their first (section 3) and the last (section 6) models are orthodox. Secondly, J&S resort to the commonplace that orthodox business cycles are generated by exogenous shocks to tastes and technology whereas heterodox business cycles are endogenous. Yet, even on orthodox turf, in a faceoff with Robert Lucas, Jean-Michel Grandmont (1985) had a counter, "On Endogenous Competitive Business Cycles". The fact that the former carried the day must do more with the geopolitics of the spread of ideas than anything else. More importantly and reverting to our earlier separating out of cycles as an independent field of enquiry, all the classical perturbations like those of F. A. Hayek and the Austrian school, were endogenous.

We scour some economic thought for support in the next section. In sections 3 to 6 we walk in the tracks of the four models of the J&S paper rewriting and extending. The final section is a conclusion.

## 2. Cycles and fluctuations in economic thought

The classic conflict on the use of the equilibrium method to analyze business cycles was the dispute between F. A. von Hayek and A. Lowe on the subject (Hagemann, 2024). In the 1920s, European research adopted NBER methods in the US in depicting cycles as a complex of fluctuations in prices and quantities with leading, coinciding, and lagging indicators. Both von Hayek and Lowe were joined in the conviction that theory must inform the movement of statistical aggregates. Lowe was a member of the Kiel group that held that cycles were a constituent of the long-term growth of economies and the two should, therefore, be examined jointly. For Hayek, an illustrious member of the Austrian school, the theory of money and the theory of capital must be integrated into the theory of the business cycle. Lowe stated

his issue with the equilibrium method neatly. A general equilibrium is closed, and exogenous shocks are required to perturb the system. Cycles, on the other hand, are out-of-equilibrium phenomena and must be addressed on different terms. von Hayek conceded the artificiality and arbitrariness of generating cycles by introducing extraneous factors. The task then was to develop the equilibrium method to endogenize fluctuations. Money and credit were the potential culprits, according to him, that displaced the time path of a dynamic equilibrium. Cycles were a disequilibrium phenomenon. Lowe, on the other hand, held that if theory was to illuminate real fluctuations, the notion of tendencies towards market clearing was a dead end. Cycles were generated by technical change and money and credit were, at best, superimpositions. Translated to our study below, the Goodwin model respects the Lowe agenda, the Minsky model falls within the ambit of the von Hayek scheme.

Amid the to and fro of theorems, a heretical idea originating with Schumpeter that was later to be embraced by the modern theory of the monetary circuit was the 'loans make deposits' proposition. Voiced by the banker L. A. Hahn and developed independently by H. D. Macleod, banks were theorized to be the manufactory of credit. Capital formation was the result of credit creation and not prior savings. A specification is offered in section 5.

J&S propose a general algebraic setup to examine their theses but confine themselves to the predator-prey model. We are inspired, as a result, to not so restrict ourselves but encompass J&S's four models in the methodology of dynamical systems theory. Systems, expressed as difference or differential equations, can be linear or nonlinear. The constant solution of the dynamical equation is called the equilibrium point for the equation. The equilibrium solution is a source if nearby solutions tend away from it. It is a sink when nearby solutions tend towards it.

A special contributor to the literature on cycles in the context of pioneering systematic policy analysis was Jan Tinbergen (Dekker, 2024). In addition, Tinbergen's orientation towards policy was slanted by his membership of the Social-Democratic Labor Party and their Plan of Labor. His mathematics was, following John Neville Keynes, the art of economics in the service of policy. In the 1930s, he participated in debates in the Party about the coexistence of high wages, unemployment, public works and policy designed to lift the economy out of the depression. On cycles, Tinbergen was able to unearth an endogenous cycle in shipbuilding. He followed it up quickly with a study of cycles in agricultural commodities which knowledge could be used to stabilize prices. Thereafter, he addressed himself to the hard question whether the downturn was a fluctuation and an upturn to equilibrium could be expected. His answer was multiple equilibria, each of which could be stable. An economy could get stuck in an unemployment equilibrium he foresaw, anticipating Keynes. Among the policy measures he offered which might seem inconsistent, but which excite imaginations today were a) industrial policy targeted at cartel busting, b) state credits to support flagging production and demand, c) centralization of produc-

tion to reap economies of scale especially among small and medium enterprises. The overall stance was not dissimilar to President Roosevelt's New Deal which he cited enthusiastically. The upshot was the restoration of purchasing power for the unemployed. Tinbergen was skeptical of notions like the invariant structure of the economy but insisted that stabilization policies changed the structure of the economy. For instance, built-in public works altered investment relationships. Indeed, he held that policy models were less uncertain than "uninfluenced systems". The multiplier-accelerator model that follows is closest to Tinbergen-Keynes thinking and to stabilizing government intervention. We proceed to explore J&S using the numbering and titles of their sections.

### 3. "3. The multiplier-accelerator model"

Among the chief features of the model is a disequilibrium adjustment mechanism. We drop autonomous expenditure from the J&S multiplier-accelerator model of (1) and (2) only to write down a two-by-two homogenous equation system. We are aware of the importance of that dimension in post-Keynesian theory but matters arising from Sraffian supermultiplier theory, for instance, are not our concern here. In the notation of the authors, we have

$$\begin{bmatrix} \dot{Y} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} a(c + \beta k - 1) & -a\beta \\ \beta k & -\beta \end{bmatrix} \begin{bmatrix} Y \\ K \end{bmatrix}. \quad 1$$

$Y$  is total output and  $K$  is the capital stock;  $c$  is the marginal propensity to consume, and  $\beta$  is the marginal propensity to invest;  $k$  is the coefficient connecting the desired capital stock and income, and  $a$  is the disequilibrium adjustment parameter operating between aggregate demand and aggregate supply. We state the following:  $c, \beta, k, a \leq 1$ .

A compendium of possible behaviors of the system is along the Trace-Development Plane of Hirsch, Smale and Devaney (2004, pp. 61-64). The trace  $\text{tr } A$  of the coefficient matrix is  $T$  and the determinant  $\det A$  is  $D$ . We summarize the criteria. i. If  $D > 0$ , and  $T < 0$ , we have a sink. Nearby solutions tend to the equilibrium solution. ii. If  $D > 0$ ,  $T > 0$ , we have a source. Nearby solutions spiral away from the equilibrium solution. iii. If  $T = 0$  we have a center. Solutions lie on circles centered at the origin. iv. If  $D < 0$ , we have a saddle, an equilibrium.

Now,  $T = a(c + \beta k - 1) - \beta$ . Since the three parameters in the bracket are less than or equal to one, their product and sum are less than or equal to one. The term in brackets is negative. The condition for negativity of  $T$  is trivially met. At the same time, the adjustment parameter  $a$  is worthy of independent attention. In the case of a sluggish adjustment to the disequilibrium between demand and supply,  $a$  tending to zero for instance, the criterion of negativity of  $T$  is additionally met.  $D > 0$ . Thus, one solution is a real sink. What if adjustments are instantaneous, so that  $a = 1$ . In addition, suppose the actual capital stock is equal to the desired capital stock with  $k = 1$ . In that case, capital does not accumulate and  $\beta = 0$ . All income would be consumed,  $c = 1$ . Plugging these values into equation 1, the right-hand side is the null vector. We have an equilibrium point of the system. An equilibrium point corresponds to the con-

stant solution  $(Y, K) = (Y_0, K_0)$ . Economics suggests a stationary state. The case for state intervention to pull economies out of the trough is made. Government expenditure must crowd in private investment as well as increase private consumption. The case for “built-in” or “automatic” stabilizers arose here. In support, Roncaglia (2023) does not classify the model as heterodox, albeit in the context of crises, but claims that the algebra fits with the “marginalist theories of value” as fluctuations around a neoclassical equilibrium.

#### 4. “4. The Goodwin model”

The building blocks of the model are a real wage resistance Phillips curve, equation (8) with the real wage,  $w$ , and employment rate,  $v$ , and positive parameters  $\rho$  and  $\gamma$ . A positive  $\rho$  indicates successful proletarian militancy as increasing employment correlates with real wage increases. On the other hand, without the depletion of the reserve army, real wage increases are as successfully resisted and captured by the negative value of the coefficient  $\gamma$ . Output  $Y$  is determined by the existing capital stock  $K$  given at a fixed capital output ratio  $\sigma$ . Labor is  $L$ , and grows at the rate,  $\beta$ , with labor productivity,  $a$ . Equation (9) is the assumption that all profits are reinvested in the accumulation of capital. The productivity of labor is assumed to grow at the rate  $\alpha$  and the rate of growth in employment as a linear function of the rate of profit, equation (10), is derived. The share of labor is denoted by  $u = w/a$ . If  $v$  is the employment rate, we contribute an  $N$  as the total available workforce for  $v = L/N$ . Corresponding with  $a$ , we will have a level of output given by  $a^* = Y/N$ . For the same reason,  $u^* = wN/Y$ . The starred status of variables must not be confused with the production possibility curve, or the potential level of output. They are the full-employment levels of output in the national product accounting sense of absorbing all willing and able hands. J&S put the above equations together to derive two two-dimensional differential equations in  $v$  and  $u$ , (11) and (12), reproduced below.

$$\dot{v} = \left[ \frac{1-u}{\sigma} - (\alpha + \beta) \right] v \quad 2$$

$$\dot{u} = [-(\alpha + \gamma) + \rho v] u \quad 3$$

A straight substitution of our starred modifications in the two equations above gives a state-space representation.

$$\begin{bmatrix} \dot{v} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} \left[ \frac{1}{\sigma} - (\alpha + \beta) \right] & -\frac{a^*}{\sigma} \\ \frac{\rho u^* a^*}{a} & -(\alpha + \gamma) \end{bmatrix} \begin{bmatrix} v \\ u \end{bmatrix}. \quad 4$$

If  $1/\sigma < (2\alpha + \beta + \gamma)$ ,  $T < 0$ ,  $D > 0$ . The solution is a sink. When  $1/\sigma > (2\alpha + \beta + \gamma)$ ,  $T > 0$ ,  $D > 0$ . We have a source. A sink is the outcome when output per unit of capital does not keep up with the growth of output per unit of labor (multiplicatively), plus the growth of the labor force, plus the state of class conflict. When, on the other hand, ‘productivity of capital’ is more than the growth of productivity of labor (multiplicatively), plus the growth in the labor force, plus the level of class conflict, the solution is a source. The center is the transition between the two regimes. We have  $1/\sigma -$

$(\alpha + \beta) = (\alpha + \gamma)$ . On the left-hand side, output after the growth of labor productivity and the growth of the labor force must be output per unit of capital or ‘capital productivity’. On the right-hand side, to the growth of labor productivity is added the increase in real profits which is the difference between employment and the increase in real wages. Capitalists and workers have called a truce. The two sides of the equation are connected by the assumption that all profits are invested. Serious scholars of the long run have reduced the growth and decline in the wealth of nations largely to a single measure, the productivity of labor. Its importance in the Goodwin model is, consequently, welcome. So-called neo-Goodwin models have been written along the tension between characterizing capitalist economies as so-called wage-led or profit-led regimes. VAR models are used for this purpose. In a state-of-the-art appraisal, it was found that the matter is inconclusive using the technique and other approaches will have to be found (Wheaton, 2025).

Can history enlighten? One application is the ongoing debate over the dating and composition of the Great Divergence between the growth trajectories between Europe and Asia (Broadberry, 2021). Apart from the capital-output ratio  $\sigma$ , the consensus is that the separation of paths dates from the eighteenth rather than the nineteenth century. The proximate causes are believed to lie in the growth of the labor force,  $\beta$ , increased labor effort, investment in human and physical capital. While the growth in total factor productivity (TFP) is common to all accounts, we highlight the growth in the ‘productivity’ of capital,  $\sigma$ . Northwestern Europe surged ahead of the rest of Europe not by growing faster during periods of positive growth, ‘spiraling’, but by reducing the frequency and rate of ‘sinking’ during periods of negative growth. Thus, while originating the process of growth is paramount, no less critical is ensuring that episodes of positive growth are not followed by episodes of negative growth. The capital-labor ratio,  $\sigma a$ , or the organic composition of capital grew initially in the aftermath of the Black Death but little thereafter until after the 1830s. A primary thesis is that TFP growth drove growth in labor productivity,  $\alpha$ . Physical and human capital deepening had a relatively limited impact on the productivity of labor.

The dynamics of growth was underpinned by the centralization of state power and the extension of state capacity. The movement of labor from agriculture, for instance, (an increase of  $u$  in the town driven by a fall in  $a$  in the countryside) could only be facilitated by the expectation of a higher wage,  $w$ , in industry; by the confidence that the rate of return on capital would match the capital-output ratio,  $\sigma$ . It was the job of the state to nurture a unified market and intervene on occasions of crises. The state was fiscally centralized and politically constrained. The former was required to ensure the provision of public goods like education and transportation, the latter to ensure that the budget was prudent and did not crowd out private investment. Some of these policy lessons were separately advocated in the development of the multiplier-accelerator model.

A related thesis for the US and OECD countries is that TFP growth has been indifferent from the 1970s (Block, 2022). The reference supports the inclusion of

both levels and change in productivity as well as the growth rate in employment,  $v$ , by considering production on the shop floor so to speak along with job-centered modifications to processes. The parameter  $\rho$  stands for class conflict at the workplace. The example provided is the ‘lean production’ model of Japan which was the counter to the engine of the American auto industry in the 1970s and 1980s. Just-in-time production enabled Japanese producers to employ fewer person hours per unit because of a far smaller scale of operation. A corollary of this empirical regularity is that the productivity of labor has been falling.

### 5. “5. The Minsky model”

The Minsky model, as presented by J&S, is nonlinear and is novel after a spate of linear Minsky models over the years (delli Gatti et al, 2025). We rewrite their notations. The output,  $Y$ , to capital,  $K$ , ratio is denoted by  $y$ ; the consumption,  $C$ , to capital ratio by  $\hat{c}$ ; the investment,  $I$ , to capital ratio by  $h$ . The rate of interest is  $i$ , the rate of profit is  $r$ , and the debt to capital ratio is  $d$ . The investment function is  $h = h(y, i, d)$ . The profit share is assumed to be fixed and the notation used is  $r = \zeta y$ . Consumption is equal to income, a standard assumption in models of this genre. That is,  $\hat{c} = (1 - \zeta)y$ . The equation for the motion of output is equation (18) reproduced below.

$$\dot{y} = a(h(y, i, d) - \zeta y) \quad 5$$

The budget equation for firms is equation (19).

$$\dot{D} = hK - s(rK - iD) \quad 6$$

Here,  $D$  is the stock of debt, and  $s$  the retention rate of profits by firms. The second differential equation which can be derived from the definitions is the motion for debt appearing as equation (20).

$$\dot{d} = h(y, i, d) - s(\zeta y - id) - h(y, i, d)d \quad 7$$

For reflections on the nonlinear system (18) and (20), we resort to Chapter 7 of Hirsch et al, 2004 (pp. 150-157). Let  $(y_0, d_0)$  be a particular solution of the system (18), (20). For each such point define the Jacobian matrix of the dynamical system (18), (20), at the point. Now consider the linear system

$$\begin{bmatrix} \dot{q} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} a(h_y - \zeta) & ah_d \\ h_y(1 - d) - s\zeta & h_d(1 - d) + si \end{bmatrix} \begin{bmatrix} q \\ e \end{bmatrix} \quad 8$$

where the square matrix is the Jacobian matrix of the J&S system (18), (20). The only difference with equation (21) of J&S is that  $d$  is not starred to stand for the steady-state value of that variable. This equation is called the variational equation along with the solution  $(y, d)$ . This equation has a solution along all points in a given interval for every  $(q_0, e_0)$ . The interesting feature of the equation is that if  $(q, e)$  is the solution of equation 8 that satisfies  $(q_0, e_0)$ , then the solution  $(y, d) + (q, e)$  is a good approximation of the solution of the equation with initial value  $(y_0, d_0) + (q_0, e_0)$ , provided  $(q_0, e_0)$  is sufficiently small.

We have  $T \leq 0$ , according as  $si \leq a(\zeta - h_y) + h_d(d - \eta)$ , and  $D \leq 0$ , according as  $(h_y - \zeta) \leq 0$ , and  $[s - (1 - d)] \leq 0$ . A sink would be the case  $T < 0$  and  $D > 0$ . While both

the level of debt with respect to capital as well as the ‘marginal propensity to invest with respect to debt’ are high, they are ‘dampened’ by the relatively low retention ratio and low interest rates. Another upper bound to  $si$ , is the negative value of  $(\zeta - h_y)$ . The ‘marginal propensity to invest with respect to income’ is constrained below by the given share of profits in income. In the event of a source,  $T > 0$  and  $D > 0$ . Firms both retain profits and borrow at arbitrarily high levels even though interest rates are high. The expression  $h_d(d - \eta)$  must swamp  $a(\zeta - h_y)$ . Debt rules the roost. Finally, we consider a saddle with  $D < 0$ . Now, the ‘marginal propensity to invest with respect to income’ is confronted by a ceiling, the given share of profits in income. The sum of the retention rate of profits and the debt to capital ratio of firms is less than unity.

Minsky’s famous cycle running from hedge to speculative to Ponzi financing was driven by different proclivities of the same agents coming to the fore under different phases of the cycle. Preferences are not given. The demand for investment driven positively by changes in income is captured by the partial derivative,  $h_y$ . Investment driven by debt is represented by the partial derivative,  $h_d$ . Debt-driven business cycles connect Minsky and Stock-Flow-Consistent (SFC) macroeconomics (Gross, 2022). The national income accounting matrix consists of households/workers, firms, banks, as column vectors. Of relevance here is the wide chasm between unremittingly rising financial wealth and the rate of growth in the creation of new assets in the USA,  $h$  in the model, taking the net investment ratio to GDP as a proxy for the latter (Canuto, 2021). One index of this divide is the fall in operating rates of return. Current capital income, which is the sum of corporate profits, proxied by  $\zeta$  above, noncorporate business income, rent, and interest have fallen relative to assets. Rates of operating returns on non-interest-bearing assets also fell as the falling interest rate,  $i$  above, increased valuations. Investors also faced declining rates of operating returns on profits on corporate equity. Besides, the growing component of intangible assets in investment implies low multiplier-accelerator effects. The disjuncture between the upward valuation of existing assets and the rising ratio of household debt to GDP, a portion of which is  $d$ , has occurred in conjunction with declining short-term and long-term interest rates. One explanation for low interest rates is the weakened position of the working class leading to limited upward pressure on wages during phases of low unemployment. Explanations for the indifferent growth of new assets run the gamut of variations on the theme of secular stagnation, from technical change to the concentration of income. The implication of the latter is clear. A fall in the retention ratio,  $s$ , means an increase in the surplus accruing to finance which, in turn, means a fall in the consumption to capital ratio,  $\hat{c}$ . The dispensation is sustained by public debt. Increasing public debts widen the space to accumulate private financial wealth. Keynes and Minsky distinguished between fluctuations in investment as an earlier prime generator of fluctuations in output, and the emerging propensity to increase leverage as the instigator of erratic swings in activity. Increasing leverage may come up against a bifurcation. Stringent financial conditions will interact with deleveraging in a spi-

ral. It is no surprise, then, that the IMF cautions that leverage is a double-edged sword. The evidence across a range of developed economies is of a vertical movement in the value of household assets relative to income over the last few cycles.

We capture some of these elements in the format of a SFC matrix. The precept of quadruple-entry

bookkeeping is an infallible accounting check. With every entry, the eye must scan up and down and sideways to ensure the rows and column of zeroes. The book by Godley and Lavoie (2007) is the well-known manual. Keeping to their notations, noting the differences where necessary, we have the following Transactions Flow matrix.

Households		Production Firms		Banks		$\Sigma$
		Current	Capital	Current	Capital	
Consumption	-C	+C				0
Investment		+I	-I			0
(GDP memo)		Y				
Wages	+WB	-WB				0
Profits, firms	+FD <sub>f</sub>	-F <sub>f</sub>	+FU <sub>f</sub>			0
Profits, banks	+FD <sub>b</sub>			-F <sub>b</sub>	+FU <sub>b</sub>	0
Loan interest		-i <sub>-1</sub> L <sub>-1</sub>		+i <sub>-1</sub> L <sub>-1</sub>		0
Change in Loans			+ $\Delta L_f$		- $\Delta L_f$	0
$\Sigma$	0	0	0	0	0	0

Begin with a blank Table. In the language of the monetary circuit, bank money is emitted in the wage bill, WB, the product of the average wage and the number of workers employed, the first entry in the matrix. A current account is written up for the borrowing firm, which is simultaneously a deposit account for the worker. As a first pass, workers spend what they earn on Basic Goods, C, which expenditure returns as revenues to firms. Firms honor their commitments to their banks and money is extinguished. With the reflux of money to the issuer, the circuit is closed. The GDP memo is a Savings = Investment crosscheck.  $S = Y - C = I$ . One attractive feature of the box above, in contrast to standard accounts, is the division of the firm into Production (Current) and Finance (Capital) Departments. Firm profits,  $F_f$ , are distributed to rentier households,  $FD_f$ , and/or undistributed, to be ploughed back into investment plans,  $FU_f$ . The same demarcation holds true for bank profits. Households will hold bank equity, and/or some portion of bank profits will be retained to support new investment proposals. The rate of interest  $r$  in J&S is  $i$  here. The notation for loans is  $L$ . The stock of debt,  $D$ , of J&S is loan commitments carried over from the 'previous period',  $L_{-1}$ , at the rate of interest,  $i_{-1}$ , prevalent then. Accounting can display regime changes. With his finger on the pulse of American capitalism, Minsky noted the creep of 'money manager capitalism'. Otherwise put, scholars have lamented the evaporation of so-called *long-termism*

and the triumph of so-called *short-termism*. Non-Financial firms turn into Financial Firms. Banks become universal banks or Financial Institutions (FIs). Accordingly, we may delete the capital account of firms and banks in the box. Real activity represented by wages paid out to produce consumption and investment goods, loans taken and given for the purpose, are rubbed out. All profits are distributed. The household is a rentier. Shadow banking and shadow assets, moving from plain vanilla to complex financial instruments not capturable in the accounts by definition, will be rife. Positive and negative signs assigned to purchase and sale of assets will interchange. For instance, the stock of shares of companies will be bought back by the same companies at continuously increasing prices to secure valuations. Buyers and sellers are both better off. The row and column of zeroes are a check on system integrity.

## 6. "6. Momentum trader models"

Stock markets are the extension of known dates and states of the word to the future and unknown states of the world. With Minsky, hedge postures might overlap with speculative trading and Ponzi financing at points on the cycle till herd behavior overtakes all. The orthodox account includes bulls and bears, chartists and fundamentalists, as permanent fixtures. The model as framed by J&S is also nonlinear. Therefore, the formal steps taken with the Minsky model carry over.

Denoting by  $d^f$  and  $d^c$  the demand functions of fundamentalist and chartist traders respectively, we rewrite equations (23) and (24) in J&S as  $d^f = \varphi(v - p)$  and  $d^c = \chi\pi$ . The price in logs is  $p$ ,  $v$  is the fundamental value of the asset,  $\pi$  is the chartist perception of price trends. The coefficients  $\varphi$  and  $\chi$  are the reaction parameters of fundamentalists and chartists to their information sets. The rate of change of the two state variables is displayed over equations (25) and (26).

$$\dot{p} = \beta_p(d^f + d^c) \quad 9$$

$$\dot{\pi} = \beta_\pi(\dot{p} - \pi) \quad 10$$

Above,  $\beta_p \geq 0$  and  $\beta_\pi$  is the speed of adjustment of chartists to deviations of actual price changes from their expectations of price changes. In sum,  $\beta_p, \beta_\pi, \varphi, \chi > 0$ . Putting it all together, the coupled differential equations are summarized in (26) and (27).

$$\dot{p} = [\varphi(v - p) + \chi\pi] \quad 11$$

$$\dot{\pi} = \beta_\pi[\beta_p\varphi(v - p) - (\beta_p - 1)\pi] \quad 12$$

Following the steps of the previous section, we get

$$\begin{bmatrix} \dot{q} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} -\beta_p\varphi & \beta_p\chi \\ -\beta_\pi\beta_p\varphi & -\beta_\pi(\beta_p\chi - 1) \end{bmatrix} \begin{bmatrix} q \\ e \end{bmatrix}. \quad 13$$

Above,  $T < 0$ , and  $D > 0$ . The solution of the system is a sink.

All the variables are arrayed along  $T$ . For a negative  $T$ , it is sufficient that the reaction parameter of the fundamentalists,  $\varphi$ , to the deviation of actual prices from their fundamental values, along with  $\beta_p$ , their responsiveness to changes in the forces of demand and supply, are together stronger than the reaction parameter of the chartists,  $\chi$ , to their perception of statistical price trends, along with  $\beta_\pi$ , their responsiveness to deviations in actual price changes from their expectations of price changes,

in combination. The theorem is orthodox. The standard literature, which gave the distinction between the two types, holds that strong corrections to deviations from equilibrium is stabilizing overall. Only when elasticities of demand and supply responses are weak, and/or the extrapolation of chartists is dominant can deviations from equilibrium amplify.

Financial markets may be subjected to two-way hysteresis as defined in the Introduction. Traders may take less liquid positions in booms and more liquid positions in slumps. Asset prices will be high in a boom but may drop precipitously in a slump. The trigger would be the adoption of a high liquidity posture. Asset prices would stay subdued thereafter until signs of recovery are spotted, encouraging the adoption of less liquid portfolios. The duration of elevated and low prices can be protracted and dissociated from real activity.

Real markets and financial markets represented by the Goodwin model and the momentum trader model may interact and add to the stabilizing and destabilizing properties of the system (Sordi et al, 2023). The actions of the two classes in the two instances, capitalists and workers in the case of the Goodwin model, and chartists and fundamentalists in the momentum trader model with some interactions between them are sufficient.

## 7. Conclusions

We follow the intent of J&S's heterodox business cycle theory in offering a companion treatment of four models. We suggest that only two of the models are heterodox. The Goodwin model and that of Minsky models are non-equilibrium. They are models of fluctuations. Sinks and sources are equally likely solutions. The other two models are orthodox. The multiplier-accelerator model is an equilibrium model, and a sink is the only solution in the momentum trader model. We advocate a state space representation of the dynamics of economies. Further work in continuous-time models will elucidate different types of bifurcations and chaos.

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