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Magnetic dipole moments for composite dark matter

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Abstract. We study neutral dark matter candidates with a nonzero magnetic dipole moment. We assume that they are composite states of new fermions related to the strong phase of a new gauge interaction. In particular, invoking a dark flavor symmetry, we analyze the composition structure of viable candidates depending on the assignments of hypercharge and the multiplets associated to the fundamental constituents of the extended sector. We determine the magnetic dipole moments for the neutral composite states in terms of their constituents masses.

Keywords: dark matter experiments, dark matter theory, particle physics - cosmology connection

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1 Introduction

Many different observational evidences prove the existence of Dark Matter (DM). Galaxy clusters and dynamics, structure formation, big-bang nucleosynthesis, and the cosmic microwave background show that baryons can only account for a small part of the total matter density of the Universe. Many extensions of the Standard Model (SM) provide viable DM candidates, however no clear evidence for any particular extension has been found. This fact motivates the analysis of DM properties from a broader approach.

DM is typically assumed to have negligible direct couplings to photons but there are other interesting possibilities [1–6] such as Magnetic DM (MDM), i.e. DM particles with a nonzero magnetic dipole moment (μ_{DM}). This has been explored in different scenarios with some interesting results. For example in ref. [7, 8] one finds a general study of its phenomenological signatures and constraints. Ref. [9] studies direct detection in experimental observations to constraint different types of MDM. In [10] the authors analyze the situation particularly for the CoGeNT data for a DM mass of several GeVs. Ref. [11] remarks similar results for the DAMA signature. The works in [12] and [13] also analyze direct detection results for MDM, but they compare their conclusions with other constraints from indirect searches and colliders. The indirect detection of MDM is studied in ref. [14] as a possible explanation of the 130 GeV line observed by Fermi-LAT. The constraining power of supernova SN 1987A data in order to restrict the viability of light MDM is shown in [15]. A similar analysis for the beam dump experiment E613 is done in ref. [16].

A particle with a permanent μ_{DM} must have a nonzero spin. In this work we only consider fundamental spin-1/2 Dirac fermions ψ_{DM} , since a Majorana fermion cannot have

this type of moments. In contrast with the electric dipole moment, the magnetic dipole moment is an axial vector and can couple to the spin without violating time-reversal and parity symmetries. In contrast to charged particles or neutral particles with electric dipole moments [17], particles provided with a μ_{DM} do not have the ability to form atom-like bound states with other charged particles or with each other. This fact changes completely the phenomenology of MDM.

The magnetic dipole moment is expected to be small enough to satisfy perturbative constraints: $\mu_{\text{DM}} \lesssim e m_{\text{DM}}^{-1}$, where m_{DM} is the mass of the DM particle. A more rigorous bound can be imposed by unitarity arguments. Indeed, the total s -wave annihilation cross section must be $\sigma \lesssim 4\pi/m_{\text{DM}}^2$ [18]. By using the expression for two photons annihilation [7, 8, 12], it is possible to find $\mu_{\text{DM}} m_{\text{DM}} \lesssim 20 (m_e/m_{\text{DM}})$.

The viability of MDM can be divided in three different regions depending on its mass: if $m_{\text{DM}} \lesssim 10$ MeV, the constraints on additional relativistic degrees of freedom from big-bang nucleosynthesis (BBN) introduce the important restrictions. However MDM can decouple before the QCD phase transition and evade these bounds [7, 8]. In any case, it is difficult to find a production mechanism associated to this light MDM in order to account for the total amount of DM. In addition, there are more constraining bounds for very light MDM even if it just constitutes part of the total non-baryonic matter. For example, the energy-loss analysis of stellar objects in globular clusters constraints dipole moments more strongly for masses $m_{\text{DM}} \lesssim 5$ keV [19]. Similar bounds can be found by taking into account the data from the supernova 1987A [15], but in this case, it can be extended up to masses of order $m_{\text{DM}} \lesssim 10$ MeV or even 100 MeV depending on different assumptions about the thermal properties of the supernova.

For the *middle* region, with $10 \text{ MeV} \lesssim m_{\text{DM}} \lesssim 1 \text{ GeV}$, the experimental and observational constraints may be satisfied for a larger value of μ_{DM} . In this case, the most robust constraints come from precision measurements and, in particular, from the contribution of the MDM to the running of the fine-structure constant, which modifies the mass of the W^\pm boson predicted in the SM [7, 8]. Similar constraints can be placed by MDM direct production in particle accelerators [20–28]. The cleaner environment makes the single photon channel data at LEP slightly more constraining than mono-jet signatures at the Tevatron or at the LHC [12].

For values close to the above bound, MDM can achieve the abundance by the classical thermal freeze-out mechanism in order to account for the total DM density [7, 8, 12]. However, this type of DM suffers the constraints associated to general light WIMPs, and it is difficult to think that DM with masses below 10 GeV can constitute the total missing matter (due to restrictions coming from observations of cosmic-ray positrons, cosmic-ray antiprotons and radio observations [29]).

Finally, for MDM heavier than ~ 1 GeV, the constraints on the value of μ_{DM} are even more important due to direct detection experiments. However, in this and the former case, the DM abundance can be produced by the thermal freeze-in mechanism. Within the standard inflationary framework, the preferred value for μ_{DM} depends on the reheat and maximum temperatures with respect to the MDM mass [30].

By the definition of MDM, the magnetic interaction with photons is its leading interaction with SM particles. However, the possible values for the magnetic moment commented above can be different if we assume a more involved cosmological setup, for example the relic abundance can be larger if exotic processes increase the expansion rate during freeze-out [31], or if there is a particle-antiparticle asymmetry for the MDM. In general, a particle with a non-

zero magnetic dipole, it is not its own anti-particle and this fact can play an important role in order to compute the relic abundance of this type of DM. Indeed, an asymmetry between the abundance of particles and anti-particles generally increases the amount of DM. This improves the viability of MDM since the constraints commented in the introduction typically restrict severely the abundance of this type of DM. There are different possibilities for the production of the initial asymmetry. In general, a linear combination of the SM baryon number and the new baryon number has to be broken. It means that one of them can be preserved, but not both. There are also different mechanisms for the creation of this asymmetric DM: the mentioned freeze-out and freeze-in processes are modified in this context, they can also be produced by the decays of heavy particles out of equilibrium, or the Affleck-Dine mechanism or even a spontaneous genesis can be the responsible of the relic density of asymmetric DM [32].

Other laboratory constraints, as the one coming from the Lamb shift [33, 34] or the targeted experiment at SLAC [35], are also subdominant. Astrophysical analyses related to the stability of the Galactic disk, annihilations in the solar neighborhood, or lifetimes of compact objects, are not competitive either [7, 8, 36–43]. The same situation was found by [7, 8] for the constraints derived from Large-Scale Structure or the Cosmic Microwave Background. In contrast, different conclusions can be found for the indirect signatures of MDM as we have already commented, although there are important uncertainties involved in these studies associated with different assumptions [7, 8, 44–47].

One motivated way of having MDM arises for composite DM [48–50]. If that is the case, it is possible that the constituents that form these *dark hadrons* might have non zero electric charge and thus contribute to a non zero dipole magnetic moment for the bound or composite state. Motivated by the situation in QCD, where the use of an $SU(3)_F$ flavor symmetry at low energies facilitates a description of the different mesons and baryons, we consider a similar situation for DM, where new fundamental fermionic degrees of freedom are introduced and assumed to interact strongly through an unspecified new interaction present at a high energy scale. At lower energies, a flavor symmetry is assumed to exist that allows us to consider the different composite states to be analyzed in terms of their possible values for μ_{DM} . The new fundamental fermionic particles, denoted by q in analogy to quarks, can be electrically charged and thus contain $SU(2)_L \times U(1)_Y$ quantum numbers.

Following the example of QCD, we think of the new strong interaction as a scaled-up version of it and thus consider, at low energies, a situation where a $SU(3)_D$ *dark* flavor symmetry is present for three new dark quarks that transform in its fundamental representation **3**. With respect to the Standard Model (SM) gauge group, they are singlets of $SU(3)_C$ and might transform non-trivially under $SU(2)_L \times U(1)_Y$. We then have the following possibilities: they can form a triplet of $SU(2)_L$ with one hypercharge $Y = y_1$; two of them can form a doublet and the third one a singlet with hypercharges $Y = y_1$, and $Y = y_2$, respectively; or they can all be singlets with independent hypercharges. We analyze each case separately.

The paper is organized as follows: in section 2 we present how the three new fermions lead to composite states associated to the dark flavor symmetry $SU(3)_D$. After these are presented, in section 3 we explore the different possibilities for the $SU(2)_L \times U(1)_Y$ quantum numbers of the new states in order to determine if there are neutral composite states that can play the role of DM. Section 4 shows the expressions for μ_{DM} in all cases considered. In section 5, we discuss a generalized version of the Gell-Mann-Nishijima relation for our case, and finally, we conclude in section 6.

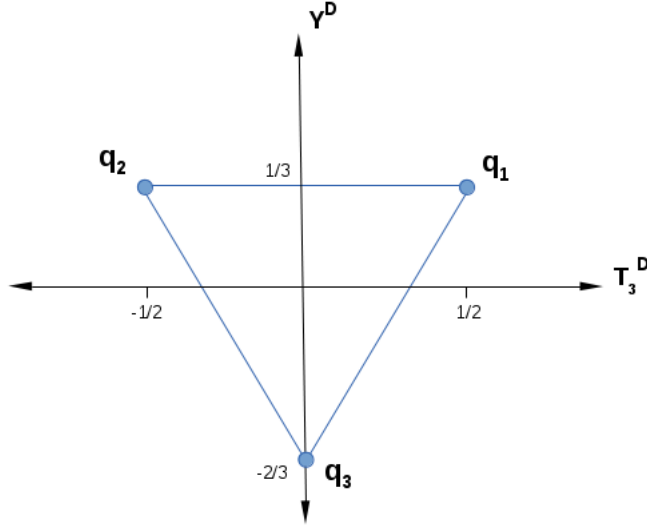


Figure 1. $SU(3)_D$ triplet of elementary particles.

2 Composite states from $SU(3)_D$

First, we describe how three fundamental fermions in the $\mathbf{3}$ of $SU(3)_D$ can form the composite states. Using the fact that they belong to the fundamental representation, the new elementary particles can be characterized by their T_3^D and Y_D “quantum numbers”, corresponding to the eigenvalues of the two diagonal generators of $SU(3)_D$ (see appendix A for the QCD case). If we denote each fundamental state by $q_i = q(Y_D, T_3^D)$, then we have (see figure 1)

$$q_1 = q\left(\frac{1}{3}, \frac{1}{2}\right), \quad q_2 = q\left(\frac{1}{3}, -\frac{1}{2}\right), \quad q_3 = q\left(-\frac{2}{3}, 0\right). \quad (2.1)$$

Composite states made up of three constituents are obtained in the triple product of the fundamental representation $\mathbf{3}$ (see appendix B for the description of the spin wavefunctions used in the analysis),

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}. \quad (2.2)$$

The octet states are denoted by $D_i(q_k, q_l, q_m) = D(Y^D, T_3^D)$, with $i = 1, 2, \dots, 8$ and $k, l, m = 1, 2, 3$. From this notation, we have (see figure 2)

$$\begin{aligned} D_1(q_1, q_1, q_2) &= D\left(1, \frac{1}{2}\right), & D_2(q_1, q_2, q_2) &= D\left(1, -\frac{1}{2}\right), & D_3(q_1, q_3, q_3) &= D\left(-1, \frac{1}{2}\right), \\ D_4(q_2, q_3, q_3) &= D\left(-1, -\frac{1}{2}\right), & D_5(q_1, q_2, q_3) &= D(0, 0), & D_6(q_1, q_2, q_3) &= D(0, 0), \\ D_7(q_2, q_2, q_3) &= D(0, -1), & D_8(q_1, q_1, q_3) &= D(0, 1). \end{aligned} \quad (2.3)$$

In the same way we can denote the decuplet states shown in figure 3 as $D_i^*(q_k, q_l, q_m) = D^*(Y^D, T_3^D)$:

$$D_1^*(q_1, q_1, q_2) = D^*\left(1, \frac{1}{2}\right), \quad D_2^*(q_1, q_2, q_2) = D^*\left(1, -\frac{1}{2}\right), \quad D_3^*(q_1, q_3, q_3) = D^*\left(-1, \frac{1}{2}\right),$$

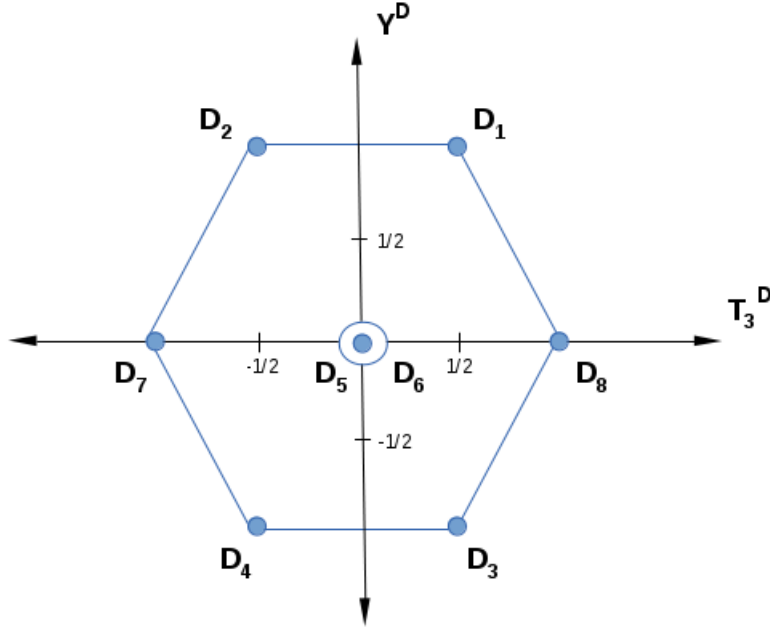


Figure 2. Octet states.

$$\begin{aligned}
D_4^*(q_2, q_3, q_3) &= D^*\left(-1, -\frac{1}{2}\right), & D_6^*(q_1, q_2, q_3) &= D^*(0, 0), & D_7^*(q_2, q_2, q_3) &= D^*(0, -1), \\
D_8^*(q_1, q_1, q_3) &= D^*(0, 1), & D_9^*(q_1, q_1, q_1) &= D^*\left(1, \frac{3}{2}\right), & D_{10}^*(q_2, q_2, q_2) &= D^*\left(1, -\frac{3}{2}\right), \\
D_{11}^*(q_3, q_3, q_3) &= D^*(-2, 0).
\end{aligned} \tag{2.4}$$

These are the composite states that we consider in this work. The next step is to explore the different possibilities emanating from the different $SU(2)_L \times U(1)_Y$ assignments of the new fermions q_i in order to determine the neutral composite states. Note that we label one of the states of the decuplet with D_{11}^* instead of D_5^* . In this way, we can relate the particles D_i of the octet with the D_i^* associated with the decuplet since they are formed by the same constituents, as we will discuss.

3 Charge assignments and neutral states

Since we are interested in the electrically neutral composite states, we want to know the conditions under which these states will be neutral for each of the three different charge assignments mentioned above, i.e. whether the three new fundamental fermions form a triplet, a doublet plus a singlet or three singlets of $SU(2)_L$. The electric charge of the fermion f_i is then determined using the relation $Q_i = T_3 + Y/2$, where T_3 and Y correspond to the third component of isospin and hypercharge of the fermion f_i , respectively.

Note that D_i and D_i^* are made up of the same particles for $i = 1, \dots, 8$ and so, when specifying the neutral states, we only do it for D_i , D_9^* , D_{10}^* and D_{11}^* .

It is important to note that at this level of discussion, we are assuming that there is a flavor symmetry $SU(3)_D$ that is present in the new sector at low energies. Although in

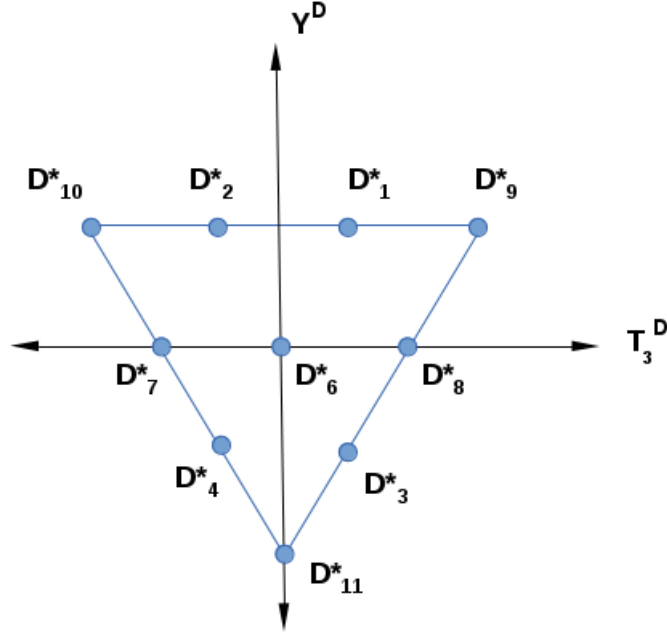


Figure 3. Decuplet states.

principle, and as a first and natural extension, we do have in mind a scaled-up version of QCD, we do not explicitly consider the strongly interacting gauge group that is assumed to be present at high energy.

3.1 Triplet

Let the three new fermions q_i form a triplet of $SU(2)_L$ with $Y = y_1$. A priori, we have the freedom of assigning any q_i in the $SU(2)_L$ triplet to any *position* in the $SU(3)_D$ triplet, however, as we will see later, the different combinations can be related and so, we use the simplest one in which they occupy the same position.

The electric charges of the fundamental particles q_i are then given by

$$Q_1 = 1 + \frac{y_1}{2}, \quad Q_2 = \frac{y_1}{2}, \quad Q_3 = -1 + \frac{y_1}{2}. \quad (3.1)$$

Using these charges we find that the composite states will be neutral in the following situations:

1. D_1 is neutral for $y_1 = -\frac{4}{3}$.
2. D_4 is neutral for $y_1 = \frac{4}{3}$.
3. D_2 and D_8 are neutral for $y_1 = -\frac{2}{3}$.
4. D_3 and D_7 are neutral for $y_1 = \frac{2}{3}$.
5. D_5 , D_6 and D_{10}^* are neutral for $y_1 = 0$.
6. D_9^* is neutral for $y_1 = -2$.

7. D_{11}^* is neutral. for $y_1 = 2$.

Note that D_9^* , D_{10}^* , and D_{11}^* are made up of three neutral fundamental particles and therefore they do not have magnetic moment (in a model where only the valence contribution is considered).

3.2 Doublet and singlet

Consider now the case where we have a $SU(2)_L$ doublet and a singlet. Let the hypercharge of the doublet be y_1 , and the one of the singlet be y_2 . The corresponding electric charges are given by

$$Q_1 = \frac{1}{2} + \frac{y_1}{2}, \quad Q_2 = -\frac{1}{2} + \frac{y_1}{2}, \quad Q_3 = \frac{y_2}{2}. \quad (3.2)$$

The composite states will be neutral in the following situations:

1. D_1 is neutral if $y_1 = -\frac{1}{3}$, independently of y_2 .
2. D_2 is neutral if $y_1 = \frac{1}{3}$, independently of y_2 .
3. D_3 is neutral if $y_1 = -(1 + 2y_2)$.
4. D_4 is neutral if $y_1 = 1 - 2y_2$.
5. D_5 and D_6 are neutral if $y_1 = -\frac{y_2}{2}$.
6. D_7 is neutral if $y_1 = 1 - \frac{y_2}{2}$.
7. D_8 is neutral if $y_1 = -(1 + \frac{y_2}{2})$.
8. D_9^* is neutral if $y_1 = -1$, independently of y_2 .
9. D_{10}^* is neutral if $y_1 = 1$, independently of y_2 .
10. D_{11}^* is neutral if $y_2 = 0$, independently of y_1 .

3.3 Singlets

Let the hypercharges of the three $SU(2)_L$ singlets be y_1 , y_2 , y_3 , respectively. Their electric charges are given by

$$Q_1 = \frac{y_1}{2}, \quad Q_2 = \frac{y_2}{2}, \quad Q_3 = \frac{y_3}{2}. \quad (3.3)$$

Now we find that the composite states will be neutral if at least one of the following conditions is satisfied:

1. D_1 is neutral if $y_1 = -\frac{y_2}{2}$, independently of y_3 .
2. D_2 is neutral if $y_1 = -2y_2$, independently of y_3 .
3. D_3 is neutral if $y_1 = -2y_3$, independently of y_2 .
4. D_4 is neutral if $y_3 = -\frac{y_2}{2}$, independently of y_1 .
5. D_5 and D_6 are neutral if $y_1 + y_2 + y_3 = 0$.
6. D_7 is neutral if $y_2 = -\frac{y_3}{2}$, independently of y_1 .

7. D_8 is neutral if $y_1 = -\frac{y_3}{2}$, independently of y_2 .
8. D_9^* , D_{10}^* or D_{11}^* are neutral if $y_1 = 0$, $y_2 = 0$ or $y_3 = 0$, respectively. However we are not interested in these conditions, since the states are made up of three neutral particles and therefore their magnetic moments is zero.

Each one of the conditions in 1-4, 7 and 8 can be considered individually or combined with one of the others (1 with 4, 2 with 3, and 7 with 8) so that we get $y_i = y_j = -\frac{y_k}{2} \neq 0$, for $i \neq j \neq k$. Then, depending on the values of y_1 , y_2 and y_3 it is possible to obtain two, three or four neutral states.

4 Magnetic moments

The expressions for the magnetic moments of the octet and decuplet states, in terms of the magnetic moments of their constituents, are the same as those obtained in the Quark Model using the spin-flavor wavefunctions given in appendix B. For the octet states the results are

$$\begin{aligned}
 \mu_{D_1} &= \frac{1}{3}(4\mu_1 - \mu_2), & \mu_{D_2} &= \frac{1}{3}(4\mu_2 - \mu_1), \\
 \mu_{D_3} &= \frac{1}{3}(4\mu_3 - \mu_1), & \mu_{D_4} &= \frac{1}{3}(4\mu_3 - \mu_2), \\
 \mu_{D_5} &= \mu_3, & \mu_{D_6} &= \frac{2}{3}(\mu_2 + \mu_1) - \frac{1}{3}\mu_3, \\
 \mu_{D_7} &= \frac{1}{3}(4\mu_2 - \mu_3), & \mu_{D_8} &= \frac{1}{3}(4\mu_1 - \mu_3),
 \end{aligned} \tag{4.1}$$

and for the decuplet states we get

$$\begin{aligned}
 \mu_{D_1^*} &= 2\mu_1 + \mu_2, & \mu_{D_2^*} &= 2\mu_2 + \mu_1, & \mu_{D_3^*} &= 2\mu_3 + \mu_1, \\
 \mu_{D_4^*} &= 2\mu_3 + \mu_2, & \mu_{D_6^*} &= \mu_1 + \mu_2 + \mu_3, & \mu_{D_7^*} &= 2\mu_2 + \mu_3, \\
 \mu_{D_8^*} &= 2\mu_1 + \mu_3, & \mu_{D_9^*} &= 3\mu_1, & \mu_{D_{10}^*} &= 3\mu_2, \\
 \mu_{D_{11}^*} &= 3\mu_3.
 \end{aligned} \tag{4.2}$$

We see that, except for D_5 and D_6 , there are symmetries under the exchanges $i \leftrightarrow j$, $j \leftrightarrow k$, $k \leftrightarrow i$, and $i \rightarrow j \rightarrow k \rightarrow i$: the same magnetic moments are obtained among different states. Consider for example the case $1 \leftrightarrow 2$ and note that $\mu_{D_1} \leftrightarrow \mu_{D_2}$. The origin of these relations can be traced to the symmetries of the spin-wavefunctions and this, in fact, is the reason of why we obtain the same magnetic moments (though corresponding to different symmetry-related states), independently of how we assign the order between the components in the $SU(2)_L$ and $SU(3)_D$ multiplets.

Defining the mass ratios $r_{1j} \equiv m_1/m_j$, we can express all the magnetic moments in units of $\frac{e\hbar}{2m_1}$. We obtain the results shown in tables 1, 2 and 3 for the magnetic moments of the neutral dark hadrons when the constituents are in a $SU(2)_L$ triplet, doublet plus singlet and three singlets respectively. The first column presents the hypercharges leading to the neutral composite states shown in the second column. The third column displays the expressions for the magnetic moments in units of $\frac{e\hbar}{2m_1}$. The last three columns are added for curiosity: they show specific values for degeneracies in the constituents masses. Note that the zero magnetic moments in these three last columns are *accidental* since they exist only in the case of exact mass degeneracy.

Hypercharge (y_i) Charge (Q_1, Q_2, Q_3)	Dark hadron	$\mu_{D_i, D_i^*} \left(\frac{e\hbar}{2m_1} \right)$	$r_{12} = 1$	$r_{13} = 1$	$r_{12} = r_{13}$
$y_1 = -4/3$ $(1/3, -2/3, -5/3)$	D_1	$\frac{2}{9}(2 + r_{12})$	$2/3$	—	—
	D_1^*	$\frac{2}{3}(1 - r_{12})$	0^{**}	—	—
$y_1 = 4/3$ $(5/3, 2/3, -1/3)$	D_4	$-\frac{2}{9}(2r_{13} + r_{12})$	—	—	$-\frac{2}{3}r_{12}$
	D_4^*	$\frac{2}{3}(r_{12} - r_{13})$	—	—	0^{**}
$y_1 = -2/3$ $(2/3, -1/3, -4/3)$	D_2	$-\frac{2}{9}(1 + 2r_{12})$	$-2/3$	—	—
	D_8	$\frac{4}{9}(2 + r_{13})$	—	$4/3$	—
	D_2^*	$\frac{2}{3}(1 - r_{12})$	0^{**}	—	—
	D_8^*	$\frac{4}{3}(1 - r_{13})$	—	0^{**}	—
$y_1 = 2/3$ $(4/3, 1/3, -2/3)$	D_3	$-\frac{4}{9}(1 + 2r_{13})$	—	$-4/3$	—
	D_7	$\frac{2}{9}(r_{13} + 2r_{12})$	—	—	$\frac{2}{3}r_{12}$
	D_3^*	$\frac{4}{3}(1 - r_{13})$	—	0^{**}	—
	D_7^*	$\frac{2}{3}(r_{12} - r_{13})$	—	—	0^{**}
$y_1 = 0$ $(1, 0, -1)$	D_5	$-r_{13}$	—	-1	—
	D_6	$\frac{1}{3}(2 + r_{13})$	—	1	—
	D_6^*	$1 - r_{13}$	—	0^{**}	—
	D_{10}^*	0^*	—	—	—
$y_1 = -2$ $(0, -1, -2)$	D_9	0^*	—	—	—
$y_1 = 2$ $(2, 1, 0)$	D_{11}^*	0^*	—	—	—

Table 1. Magnetic moments of the neutral states if the three constituents are in a $SU(2)_L$ triplet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents ($r_{1j} \equiv m_1/m_j$). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0^*) and those where a degeneracy in constituents mass is present (denoted by 0^{**}).

Recall that there is an ordering symmetry among the components in the multiplets of $SU(2)_L$ and $SU(3)_D$ except for D_5 and D_6 . In this case the way we relate the components of the multiplets matters. We show this in tables 4, 5, and 6, where we use the following notation: call \tilde{q}_i the components of the $SU(2)_L$ multiplet and q_i those of the $SU(3)_D$ one (our previous analysis and results in tables 1, 2 and 3 corresponds to the case $\tilde{q}_i = q_i$ for the $SU(2)_L$). The first column of each of these tables contains the relation among the multiplets, followed by the expressions for the magnetic moments.

5 Gell-Mann-Nishijima formula

The electric charge of a particle is given by

$$Q = T_3 + \frac{1}{2}Y_W, \quad (5.1)$$

Hypercharge Charge (y_i) (Q_1, Q_2, Q_3)	Dark hadron	$\mu_{D_i, D_i^*} \left(\frac{e\hbar}{2m_1} \right)$	$r_{12} = 1$	$r_{13} = 1$	$r_{12} = r_{13}$
$y_1 = -1/3, y_2$	D_1	$\frac{2}{9} (2 + r_{12})$	$2/3$	—	—
$(1/3, -2/3, y_2/2)$	D_1^*	$\frac{2}{3} (1 - r_{12})$	0^{**}	—	—
$y_1 = 1/3, y_2$	D_2	$-\frac{2}{9} (1 + 2r_{12})$	$-2/3$	—	—
$(2/3, -1/3, y_2/2)$	D_2^*	$\frac{2}{3} (1 - r_{12})$	0^{**}	—	—
$y_1 = -(1 + 2y_2)$	D_3	$\frac{y_2}{3} (1 + 2r_{13})$	—	y_2	—
$(-y_2, -(1 + y_2), \frac{y_2}{2})$	D_3^*	$-y_2 (1 - r_{13})$	—	0^{**}	—
$y_1 = (1 - 2y_2)$	D_4	$\frac{y_2}{3} (2r_{13} + r_{12})$	—	—	$y_2 r_{13}$
$(1 - y_2, -y_2, y_2/2)$	D_4^*	$y_2 (r_{13} - r_{12})$	—	—	0^{**}
$y_1 = -y_2/2$ $(\frac{(2-y_2)}{4}, \frac{-(2+y_2)}{4}, \frac{y_2}{2})$	D_5	$\frac{y_2}{2} r_{13}$	—	$y_2/2$	—
	D_6	$\frac{1}{3} (1 - r_{12})$	—	—	—
	D_6^*	$-\frac{y_2}{6} (1 + r_{12} + r_{13})$	—	—	—
		$\frac{1}{2} (1 - r_{12})$ $-\frac{y_2}{4} (1 + r_{12} - 2r_{13})$	—	—	—
$y_1 = 1 - y_2/2$	D_7	$-\frac{y_2}{6} (r_{13} + 2r_{12})$	—	—	$-y_2 r_{13}/2$
$(1 - \frac{y_2}{4}, \frac{-y_2}{4}, \frac{y_2}{2})$	D_7^*	$\frac{y_2}{2} (r_{13} - r_{12})$	—	—	0^{**}
$y_1 = -(1 + y_2/2)$	D_8	$-\frac{y_2}{6} (2 + r_{13})$	—	$-y_2/2$	—
$(-\frac{y_2}{4}, -(1 + \frac{y_2}{4}), \frac{y_2}{2})$	D_8^*	$-\frac{y_2}{2} (1 - r_{13})$	—	0^{**}	—
$y_1 = -1, y_2$	D_9^*	0^*	—	—	—
$(0, -1, y_2/2)$					
$y_1 = 1, y_2$	D_{10}^*	0^*	—	—	—
$(1, 0, y_2/2)$					
$y_1, y_2 = 0$	D_{11}^*	0^*	—	—	—
$(\frac{(y_1+1)}{2}, \frac{(y_1-1)}{2}, 0)$					

Table 2. Magnetic moments of the neutral states when the new fermions form a $SU(2)_L$ doublet and a singlet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents ($r_{1j} \equiv m_1/m_j$). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0^*) and those where a degeneracy in constituents mass is present (denoted by 0^{**}).

where T_3 and Y_W are the third component of isospin and the hypercharge, associated to the generators of the gauge groups $SU(2)_L$ and $U(1)_Y$, respectively.

In the Quark Model, where it is considered a flavor $SU(3)_F$ symmetry, there are two group diagonal generators, I_3 and Y_F . As it turns out, because the diagonal generators in this case correspond to $SU(2)$ and $U(1)$ symmetries, the specific assignment of the light quarks under the $SU(3)_F$ as a fundamental allows us to express electric charge in terms of the $SU(3)_F$ generators through the well known Gell-Mann-Nishijima formula

$$Q = I_3 + \frac{1}{2}Y_F. \quad (5.2)$$

Hypercharge Charge (y_i) (Q_1, Q_2, Q_3)	Dark hadron	$\mu_{D_i, D_i^*} \left(\frac{e\hbar}{2m_1} \right)$	$r_{12} = 1$	$r_{13} = 1$	$r_{12} = r_{13}$
$y_1 = -y_2/2, y_3$ ($-y_2/4, y_2/2, y_3/2$)	D_1 D_1^*	$-\frac{y_2}{6} (2 + r_{12})$ $-\frac{y_2}{2} (1 - r_{12})$	$-y_2/2$ 0^{**}	$-$ $-$	$-$ $-$
$y_1 = -2y_2, y_3$ ($-y_2, y_2/2, y_3/2$)	D_2 D_2^*	$\frac{y_2}{3} (1 + 2r_{12})$ $-y_2 (1 - r_{12})$	y_2 0^{**}	$-$ $-$	$-$ $-$
$y_1 = -2y_3, y_2$ ($-y_3, y_2/2, y_3/2$)	D_3 D_3^*	$\frac{y_3}{3} (1 + 2r_{13})$ $-y_3 (1 - r_{13})$	$-$ $-$	y_3 0^{**}	$-$ $-$
$y_1, y_3 = -y_2/2$ ($y_1/2, y_2/2, -y_2/4$)	D_4 D_4^*	$-\frac{y_2}{6} (r_{12} + 2r_{13})$ $\frac{y_2}{2} (r_{12} - r_{13})$	$-$ $-$	$-$ $-$	$-y_2 r_{13}/2$ 0^{**}
$y_1 + y_2 + y_3 = 0$ ($-\frac{(y_2+y_3)}{2}, \frac{y_2}{2}, \frac{y_3}{2}$)	D_5	$y_3 r_{13}/2$	$-$	$y_3/2$	$-$
	D_6	$\frac{y_2(r_{12}-1)}{3} - \frac{y_3(2+r_{13})}{6}$	$-$	$-$	$-$
	D_6^*	$\frac{y_2(r_{12}-1)}{2} - \frac{y_3(1-r_{13})}{2}$	$-$	$-$	$-$
$y_1, y_3 = -2y_2$ ($y_1/2, y_2/2, -y_2$)	D_7 D_7^*	$\frac{y_2}{3} (2r_{12} + r_{13})$ $y_2 (r_{12} - r_{13})$	$-$ $-$	$-$ $-$	$y_2 r_{13}$ 0^{**}
$y_1 = -y_3/2, y_2$ ($-y_3/4, y_2/2, y_3/2$)	D_8 D_8^*	$-\frac{y_3}{6} (2 + r_{13})$ $-\frac{y_3}{2} (1 - r_{13})$	$-$ $-$	$-y_3/2$ 0^{**}	$-$ $-$
$y_1 = 0, y_2, y_3$ ($0, y_2/2, y_3/2$)	D_9	0^*	$-$	$-$	$-$
$y_1, y_2 = 0, y_3$ ($y_1/2, 0, y_3/2$)	D_{10}^*	0^*	$-$	$-$	$-$
$y_1, y_2, y_3 = 0$ ($y_1/2, y_2/2, 0$)	D_{11}^*	0^*	$-$	$-$	$-$

Table 3. Magnetic moments of the neutral states if each of the three constituents is a $SU(2)_L$ singlet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents ($r_{1j} \equiv m_1/m_j$). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0^*) and those where a degeneracy in constituents mass is present (denoted by 0^{**}).

Since in our model we consider three fundamental particles with arbitrary electric charge, it is of interest to determine those cases where it is possible to define the electric charge in terms of the flavor generators T_3^D, Y^D of the group $SU(3)_D$, as in the Quark Model, with a generalization of the Gell-Mann-Nishijima formula $Q = c_T T_3^D + c_Y Y^D$. This of course depends crucially on the relation between \tilde{q}_i and q_i and the different $SU(3)_D$ representations used above. We now discuss each case separately.

5.1 Triplet

When the generalization can be used, the charges are given by

$$Q_i = \frac{c_T}{2} + \frac{c_Y}{3}, \quad Q_j = -\frac{c_T}{2} + \frac{c_Y}{3}, \quad Q_k = -\frac{2}{3}c_Y, \quad i \neq j \neq k. \quad (5.3)$$

Relation among multiplets	Magnetic moments $\mu_{D_{5,6}} \left(\frac{e\hbar}{2m_1} \right)$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_1, \tilde{q}_3 = q_3$	$\mu_{D_5} = -r_{13}$ $\mu_{D_6} = \frac{1}{3} (2r_{12} + r_{13})$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_2, \tilde{q}_3 = q_1$	$\mu_{D_5} = r_{13}$ $\mu_{D_6} = -\frac{1}{3} (2 + r_{13})$
$\tilde{q}_1 = q_1, \tilde{q}_2 = q_3, \tilde{q}_3 = q_2$	$\mu_{D_5} = 0^*$ $\mu_{D_6} = \frac{2}{3} (1 - r_{12})$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_3, \tilde{q}_3 = q_1$	$\mu_{D_5} = 0^*$ $\mu_{D_6} = -\frac{2}{3} (1 - r_{12})$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_1, \tilde{q}_3 = q_2$	$\mu_{D_5} = r_{13}$ $\mu_{D_6} = -\frac{1}{3} (2r_{12} + r_{13})$

Table 4. Magnetic moments of the states D_5 and D_6 if the three constituents are in a $SU(2)_L$ triplet for different relations among the multiplet components (see text for definition of \tilde{q}_i). The case $\tilde{q}_i = q_i$ is included in table 1. $r_{1j} \equiv m_1/m_j$.

Relation among multiplets	Magnetic moments $\mu_{D_{5,6}} \left(\frac{e\hbar}{2m_1} \right)$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_1, \tilde{q}_3 = q_3$	$\mu_{D_5} = y_2 r_{13}/2$ $\mu_{D_6} = -\frac{(1-r_{12})}{3} - \frac{y_2(1+r_{12}+r_{13})}{6}$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_2, \tilde{q}_3 = q_1$	$\mu_{D_5} = (2 - y_2) r_{13}/4$ $\mu_{D_6} = -\frac{(r_{13}+2r_{12})}{6} + \frac{y_2(4+r_{13}-2r_{12})}{12}$
$\tilde{q}_1 = q_1, \tilde{q}_2 = q_3, \tilde{q}_3 = q_2$	$\mu_{D_5} = -(2 + y_2) r_{13}/4$ $\mu_{D_6} = \frac{(2+r_{13})}{6} + \frac{y_2(r_{13}+4r_{12}-2)}{12}$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_3, \tilde{q}_3 = q_1$	$\mu_{D_5} = -(2 + y_2) r_{13}/4$ $\mu_{D_6} = \frac{(r_{13}+2r_{12})}{6} + \frac{y_2(4+r_{13}-2r_{12})}{12}$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_1, \tilde{q}_3 = q_2$	$\mu_{D_5} = (2 - y_2) r_{13}/4$ $\mu_{D_6} = -\frac{(2+r_{13})}{6} + \frac{y_2(r_{13}+4r_{12}-2)}{12}$

Table 5. Magnetic moments of the states D_5 and D_6 if the three constituents are in a $SU(2)_L$ doublet plus singlet for different relations among the multiplet components (see text for definition of \tilde{q}_i). The case $\tilde{q}_i = q_i$ is included in table 2. $r_{1j} \equiv m_1/m_j$.

Note that here there are only two independent relations while in (3.1) the three relations are dependent. If we want to express the charge in terms of the flavor generators, the system of equations is only consistent for $y_1 = 0$. For this particular value of the hypercharge we obtain four neutral states, D_5 , D_6 , D_6^* and D_{10}^* .

- For $\tilde{q}_i = q_i$, $c_Y = \frac{3}{2}$ and $c_T = 1$.
- For $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_1$, and $\tilde{q}_3 = q_3$, $c_Y = \frac{3}{2}$ and $c_T = -1$.
- For $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_2$, and $\tilde{q}_3 = q_1$, $c_Y = -\frac{3}{2}$ and $c_T = -1$.
- For $\tilde{q}_1 = q_1$, $\tilde{q}_2 = q_3$, and $\tilde{q}_3 = q_2$, $c_Y = 0$ and $c_T = 2$.

Relation among multiplets	Magnetic moments $\mu_{D_{5,6}} \left(\frac{e\hbar}{2m_1} \right)$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_1, \tilde{q}_3 = q_3$	$\mu_{D_5} = y_3 r_{13}/2$ $\mu_{D_6} = \frac{y_2(1-r_{12})}{3} - \frac{y_3(2r_{12}+r_{13})}{6}$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_2, \tilde{q}_3 = q_1$	$\mu_{D_5} = -(y_2 + y_3) r_{13}/2$ $\mu_{D_6} = \frac{y_2(2r_{12}+r_{13})}{6} + \frac{y_3(2+r_{13})}{6}$
$\tilde{q}_1 = q_1, \tilde{q}_2 = q_3, \tilde{q}_3 = q_2$	$\mu_{D_5} = y_2 r_{13}/2$ $\mu_{D_6} = -\frac{y_2(2+r_{13})}{6} - \frac{y_3(1-r_{12})}{3}$
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_3, \tilde{q}_3 = q_1$	$\mu_{D_5} = y_2 r_{13}/2$ $\mu_{D_6} = -\frac{y_2(2r_{12}+r_{13})}{6} + \frac{y_3(1-r_{12})}{3}$
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_1, \tilde{q}_3 = q_2$	$\mu_{D_5} = -(y_2 + y_3) r_{13}/2$ $\mu_{D_6} = \frac{y_2(2+r_{13})}{6} + \frac{y_3(2r_{12}+r_{13})}{6}$

Table 6. Magnetic moments of the states D_5 and D_6 if the three constituents are singlets of $SU(2)_L$ for different relations among the multiplet components (see text for definition of \tilde{q}_i). The case $\tilde{q}_i = q_i$ is included in table 3. $r_{1j} \equiv m_1/m_j$.

- For $\tilde{q}_1 = q_2, \tilde{q}_2 = q_3$, and $\tilde{q}_3 = q_1$, $c_Y = 0$ and $c_T = -2$.
- For $\tilde{q}_1 = q_3, \tilde{q}_2 = q_1$, and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{2}$ and $c_T = 1$.

5.2 Doublet and singlet

From the charges given in equation (3.2), the generalization can be used for two relations between y_1 and y_2 :

1. $y_2 = -2y_1$ can be used with the following relations between \tilde{q}_i and q_i :

- $\tilde{q}_i = q_i$, $c_Y = \frac{3}{2}y_1$ and $c_T = 1$.
- $\tilde{q}_1 = q_2, \tilde{q}_2 = q_1$ and $\tilde{q}_3 = q_3$, $c_Y = \frac{3}{2}y_1$ and $c_T = 1$.
- $\tilde{q}_1 = q_3, \tilde{q}_2 = q_2$ and $\tilde{q}_3 = q_1$, $c_Y = -\frac{3}{4}(y_1 + 1)$ and $c_T = -\frac{1}{2}(3y_1 - 1)$.
- $\tilde{q}_1 = q_1, \tilde{q}_2 = q_3$ and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{4}(y_1 - 1)$ and $c_T = \frac{1}{2}(3y_1 + 1)$.
- $\tilde{q}_1 = q_3, \tilde{q}_2 = q_1$ and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{4}(y_1 + 1)$ and $c_T = \frac{1}{2}(3y_1 - 1)$.

Note that $y_2 = -2y_1$ is consistent with almost all the necessary conditions to obtain neutral states, except for D_7 and D_8 when $\tilde{q}_i = q_i$.

2. $y_2 = y_1 + 1$ works when $\tilde{q}_1 = q_2, \tilde{q}_2 = q_3$ and $\tilde{q}_3 = q_1$, $c_Y = -\frac{3}{4}(y_1 - 1)$ and $c_T = -\frac{1}{2}(3y_1 + 1)$. This relation between the hypercharges is consistent with all the conditions for neutral states.

5.3 Singlets

Here we have that each particle \tilde{q}_i has a charge given by $\frac{y_i}{2}$, so independently of the relation between \tilde{q}_i and q_i , we obtain the same relations between the three hypercharges, up to a change of the form $i \rightarrow j, j \rightarrow k, k \rightarrow i$.

For the particular relation $\tilde{q}_i = q_i$, it is found that the generalization can be used when

$$c_Y = \frac{3}{4}(y_1 + y_2), \quad c_T = \frac{1}{2}(y_1 - y_2) = \frac{1}{2}(y_2 - y_1), \quad (5.4)$$

this is

$$y_1 = y_2, \quad y_3 = -2y_1, \quad c_Y = \frac{3}{2}y_1, \quad c_T = 0. \quad (5.5)$$

These relations are satisfied by the conditions for the neutral states D_5 , D_6 , D_7 and D_8 . The conditions for the remaining states could also be satisfied, but this takes place only for $y_1 = y_2 = y_3 = 0$.

6 Conclusions

MDM posses an interesting and useful possibility that broadens up the spectrum of candidates and scenarios for the problem of DM. In this work, we consider DM candidates that can have a magnetic dipole moment due to the fact that they are composite states coming from a high energy strongly interacting sector.

Three additional elementary fermions have been introduced in addition to the SM particle content. These new fermions are singlets under $SU(3)_C$ but can have non trivial representations under the electroweak gauge group. Since there are three of them, then only three different possibilities for their $SU(2)_L$ transformation exist: triplet, doublet plus singlet or three singlets.

Assuming there is a low energy $SU(3)_D$ *dark flavor symmetry*, and in analogy with low energy QCD, we construct the composite states and determine those that can be neutral. We find that there are several possibilities for each of the three cases above. We have assumed that the lightest hidden state is neutral since it is needed for having a viable DM candidate. This is a strong assumption and it could be unnatural in some cases, but it is completely necessary. On the other hand, even within a specific model, it is not simple to compute the spectrum of the multiplets. Indeed, there may be additional constraints that we have not analyzed in our work. For instance, if the value of two hypercharges are the same in the singlet case, there is an unbroken $SU(2)_L \times SU(2)_R$ global chiral symmetry, which will be dynamically broken by the strong $SU(3)_D$ interaction. It implies the presence of electrically neutral Nambu-Goldstone bosons, which will be stable due to the unbroken diagonal $SU(2)$. This type of models suffer very important experimental restrictions.

The results for μ_{DM} are presented in terms of the constituents masses and hypercharges, and cases where they are zero are singled out for two different scenarios: i) when the constituents themselves are electrically neutral and ii) when there is a particular mass degeneracy among some of the elementary constituents masses.

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A QCD

QCD is a particular case for the charge assignment corresponding to one $SU(2)_L$ doublet and one singlet with $\tilde{q}_i = q_i$, $y_1 = \frac{1}{3}$, $y_2 = -2y_1 = -\frac{2}{3}$, $c_Y = \frac{3}{2}y_1 = \frac{1}{2}$ and $c_T = 1$. Here the three fundamental particles q_i correspond to the three light quarks

$$q_1 \rightarrow u, \quad q_2 \rightarrow d, \quad q_3 \rightarrow s, \quad (\text{A.1})$$

which carry a charge

$$Q_u = \frac{1}{2} + \frac{y_1}{2} = \frac{2}{3}, \quad Q_d = -\frac{1}{2} + \frac{y_1}{2} = -\frac{1}{3}, \quad Q_s = \frac{y_2}{2} = -\frac{1}{3}. \quad (\text{A.2})$$

For the octet states the correspondence is

$$\begin{aligned} D_1 &\rightarrow p, & D_2 &\rightarrow n, & D_3 &\rightarrow \Xi^0, & D_4 &\rightarrow \Xi^-, \\ D_5 &\rightarrow \Lambda, & D_6 &\rightarrow \Sigma^0, & D_7 &\rightarrow \Sigma^-, & D_8 &\rightarrow \Sigma^+, \end{aligned} \quad (\text{A.3})$$

while for the decuplet states we have

$$\begin{aligned} D_1^* &\rightarrow \Delta^+, & D_2^* &\rightarrow \Delta^0, & D_3^* &\rightarrow \Xi^{*0}, & D_4^* &\rightarrow \Xi^{*-}, & D_6^* &\rightarrow \Sigma^{*0}, \\ D_7^* &\rightarrow \Sigma^{*-}, & D_8^* &\rightarrow \Sigma^{*+}, & D_9^* &\rightarrow \Delta^{++}, & D_{10}^* &\rightarrow \Delta^-, & D_{11}^* &\rightarrow \Omega^-. \end{aligned} \quad (\text{A.4})$$

B Spin-flavor wavefunctions

If we consider the spin as an internal degree of freedom, then each fundamental particle q_i is in the spin-flavor group product $SU(3) \otimes SU(2)$. So the composite states are obtained in the product

$$(\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}) \otimes (\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2}). \quad (\text{B.1})$$

The decomposition of this product leads to singlets, octets and decuplets with spin 1/2 and 3/2. The spin-flavor wavefunctions of the octet states are [51]

$$\begin{aligned} |D_{1\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{1\uparrow}q_{1\uparrow}q_{2\downarrow}\rangle + 2|q_{1\uparrow}q_{2\downarrow}q_{1\uparrow}\rangle + 2|q_{2\downarrow}q_{1\uparrow}q_{1\uparrow}\rangle - |q_{1\uparrow}q_{1\downarrow}q_{2\uparrow}\rangle - |q_{1\uparrow}q_{2\uparrow}q_{1\downarrow}\rangle - |q_{2\uparrow}q_{1\uparrow}q_{1\downarrow}\rangle \right. \\ &\quad \left. - |q_{1\downarrow}q_{1\uparrow}q_{2\uparrow}\rangle - |q_{1\downarrow}q_{2\uparrow}q_{1\uparrow}\rangle - |q_{2\uparrow}q_{1\downarrow}q_{1\uparrow}\rangle \right) \\ &= \frac{1}{\sqrt{18}} \left(2|q_{1\uparrow}q_{1\uparrow}q_{2\downarrow}\rangle - |q_{1\uparrow}q_{1\downarrow}q_{2\uparrow}\rangle - |q_{1\downarrow}q_{1\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), \\ |D_{2\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{2\uparrow}q_{2\uparrow}q_{1\downarrow}\rangle - |q_{2\uparrow}q_{2\downarrow}q_{1\uparrow}\rangle - |q_{2\downarrow}q_{2\uparrow}q_{1\uparrow}\rangle + \text{permutations} \right), \\ |D_{3\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{3\uparrow}q_{3\uparrow}q_{1\downarrow}\rangle - |q_{3\uparrow}q_{3\downarrow}q_{1\uparrow}\rangle - |q_{3\downarrow}q_{3\uparrow}q_{1\uparrow}\rangle + \text{permutations} \right), \\ |D_{4\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{3\uparrow}q_{3\uparrow}q_{2\downarrow}\rangle - |q_{3\uparrow}q_{3\downarrow}q_{2\uparrow}\rangle - |q_{3\downarrow}q_{3\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), \\ |D_{5\uparrow}\rangle &= \frac{1}{\sqrt{12}} \left(|q_{2\uparrow}q_{3\uparrow}q_{1\downarrow}\rangle + |q_{3\uparrow}q_{2\uparrow}q_{1\downarrow}\rangle - |q_{3\uparrow}q_{1\uparrow}q_{2\downarrow}\rangle - |q_{1\uparrow}q_{3\uparrow}q_{2\downarrow}\rangle + \text{permutations} \right), \\ |D_{6\uparrow}\rangle &= \frac{1}{6} \left(2|q_{2\uparrow}q_{1\uparrow}q_{3\downarrow}\rangle + 2|q_{1\uparrow}q_{2\uparrow}q_{3\downarrow}\rangle - |q_{3\uparrow}q_{2\uparrow}q_{1\downarrow}\rangle - |q_{3\uparrow}q_{1\uparrow}q_{2\downarrow}\rangle - |q_{1\uparrow}q_{3\uparrow}q_{2\downarrow}\rangle - |q_{1\downarrow}q_{3\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), \\ |D_{7\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{2\uparrow}q_{2\uparrow}q_{3\downarrow}\rangle - |q_{2\uparrow}q_{2\downarrow}q_{3\uparrow}\rangle - |q_{2\downarrow}q_{2\uparrow}q_{3\uparrow}\rangle + \text{permutations} \right), \\ |D_{8\uparrow}\rangle &= \frac{1}{\sqrt{18}} \left(2|q_{1\uparrow}q_{1\uparrow}q_{3\downarrow}\rangle - |q_{1\uparrow}q_{1\downarrow}q_{3\uparrow}\rangle - |q_{1\downarrow}q_{1\uparrow}q_{3\uparrow}\rangle + \text{permutations} \right). \end{aligned} \quad (\text{B.2})$$

In a similar way, for the decuplet states the wavefunctions are given by

$$\begin{aligned}
D_{1\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{1\uparrow}q_{1\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), & D_{2\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{2\uparrow}q_{2\uparrow}q_{1\uparrow}\rangle + \text{permutations} \right), \\
D_{3\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{3\uparrow}q_{3\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), & D_{4\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{3\uparrow}q_{3\uparrow}q_{2\uparrow}\rangle + \text{permutations} \right), \\
D_{6\uparrow}^* &= \frac{1}{\sqrt{6}} \left(|q_{3\uparrow}q_{2\uparrow}q_{1\uparrow}\rangle + |q_{2\uparrow}q_{3\uparrow}q_{1\uparrow}\rangle + \text{permutations} \right), \\
D_{7\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{2\uparrow}q_{2\uparrow}q_{3\uparrow}\rangle + \text{permutations} \right), & D_{8\uparrow}^* &= \frac{1}{\sqrt{3}} \left(|q_{1\uparrow}q_{1\uparrow}q_{3\uparrow}\rangle + \text{permutations} \right), \\
D_{9\uparrow}^* &= |q_{1\uparrow}q_{1\uparrow}q_{1\uparrow}\rangle, & D_{10\uparrow}^* &= |q_{2\uparrow}q_{2\uparrow}q_{2\uparrow}\rangle, & D_{11\uparrow}^* &= |q_{3\uparrow}q_{3\uparrow}q_{3\uparrow}\rangle, \tag{B.3}
\end{aligned}$$

where permutations indicates the change $1 \rightarrow 3$ and $2 \rightarrow 3$ over each of the previous kets.

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