Instituto Complutense de Análisis Económico

UNIVERSIDAD COMPLUTENSE

FACULTAD DE ECONOMICAS

Campus de Somosaguas

28223 MADRID

Teléfono 394 26 11 - FAX 294 26 13

W
49
(9719)

Documento de trabajo

Testing for Invertibility in a MA(1) Process

Rafael Flores de Frutos

Miguel Jerez Méndez

No.9719 Diciembre 1997

Instituto Complutense de Análisis Económico

UNIVERSIDAD COMPLUTENSE

TESTING FOR INVERTIBILITY IN A MA(1) PROCESS

Rafael Flores de Frutos
Miguel Jerez Méndez

Universidad Complutense de Madrid
Campus de Somosaguas
28223 Madrid

BIBLIOTECA UCM

ABSTRACT

In this paper we propose a test for detecting overdifferencing in a MA(1) process. Unlike the standard practice, we use invertibility as the null hypothesis to be tested. By so doing it is possible to use a standard likelihood ratio test with the standard χ^2 distribution. Simulation results indicate that its performance is comparable to that of the best tests available in this literature.

RESUMEN

En este trabajo proponemos un test para detectar sobrediferenciación en un proceso MA(1). A diferencia de la práctica habitual, nuestra hipótesis nula es la de invertibilidad. Esto permite plantear un contraste de razón de verosimilitud con una distribución χ^2 estándar bajo la hipótesis nula. Los resultados de simulación indican que su comportamiento es comparable al de los mejores contrastes disponibles en la literatura.

n.c.: X-53-297052-6

Key words: Nonstationarity; Overdifferencing; Time Series; Unit-root test.

JEL classification codes: C12, C22

Mailing address: Departamento de Economía Cuantitativa. Universidad Complutense. Somosaguas. 28223 Madrid. Spain. Fax: (34) 1 394 26 13. Rafael Flores de Frutos e-mail: eccua04@sis.ucm.es.

N.E.: 5310279611

1. Introduction

Testing for noninvertibility in univariate ARIMA models is an important issue. First, overdifferencing a time series results in a noninvertible model, see Plosser and Schwert (1977). Second, noninvertible processes may occur in rational expectations models, see Christiano (1987). Third, noninvertibility can be used to distinguish trend stationary from difference stationary, see Tsay (1993).

The direct approach of testing for a unit root in a MA polynomial involves, in general, the use of statistical tests with non-standard distributions. Even in the most simple case of a zero mean MA(1) process, using a direct estimate of θ to test for noninvertibility is problematic because: a) traditional likelihood asympthotics do not apply when the parameter space is constrained to the interval [-1, 1], b) the finite-sample distribution of the Maximum Likelihood estimator has a positive probability at unity and c) unlike the Least Squares (LS) estimates of autoregressive models, there are no close-form formulae for the MA estimates.

Given that the development of both Wald or likelihood ratio tests becomes intractable under $\theta=1$, Tanaka (1990) proposes a general score-type test (S_T) for the moving average unit root hypothesis that is a locally best invariant unbiased (LBIU) test in the special case of a MA(1). Later, Saikkonen and Luukkonen (1993) propose two additional tests, R1 and R2, the former identical to a one-side version of the Lagrange multiplier test, valid if the mean of the process is known, and the later a LBIU test closely related to Tanaka (1990) test.

As an alternative approach, Tsay (1993) propose to transform the noninvertible problem into a nonstationary problem. By so doing, all of the tests statistic available in the literature for nonstationarity can be applied to noninvertibility. Tsay (1993) shows the better performance of his test against Arellano and Pantula (1990) tests but fails to beat the performance of Tanaka (1990) test.

As Tsay (1993) does, we transform the problem to be solved, but unlike Tsay, we change the null hypothesis (H_0) to be tested. The main feature of our test is that H_0 is $-1<\theta<1$ instead of $\theta=1$. Thus a standard likelihood ratio test with a standard χ^2 distribution can be used. We show through simulation that the performance of this test is comparable to that of the alternatives mentioned above.

The article is organized as follows. Section 2 describes the proposed test statistic. Section 3 illustrates the performance of this test in finite samples. Finally, Section 4 presents the most important conclusions.

2. A test for the null of invertibility in a MA(1) process

Consider the simple zero mean MA(1) model:

$$z_t = a_t - \theta a_{t-1} \tag{1}$$

where $-1<\theta<1$ and a_t follows a white noise process with variance σ^2.

As an approximation to (1), consider the AR(L) model:

$$z_t = \pi_1 z_{t-1} + \pi_2 z_{t-2} + \dots + \pi_L z_{t-L} + r_t \tag{2}$$

where the order L is a function of T, say n_T, such that:

$$\begin{aligned} &n_T \to \infty \\ &n_T^3 \to 0 \\ &\sqrt{T}\sum_{i=n_T+1}^{\infty} \|\pi_i\| \to 0 \end{aligned} \tag{3}$$

as $T \to \infty$.

It is well known (see Lütkepohl, 1993, pages 305-309) that LS to model (2) yields consistent estimates of π_j ($j = 1, 2, \dots, L$), and that :

$$\hat{\sigma}_r^2 = \frac{\sum_{t=L+1}^{T} \hat{r}^2_t}{T-2L} \tag{4}$$

is a consistent estimator of σ^2.

Now, consider the integrated variable y_t ($\nabla y_t = z_t$) as defined in Saikkonen and Luukkonen (1993):

$$\begin{aligned} y_1 &= y_0 + a_1 \\ \nabla y_t &= a_t - \theta a_{t-1} \quad t=2,\dots,T \end{aligned} \tag{5}$$

If z_t follows an invertible MA(1) process with (2) being a good approximation, then y_t will behave according to the nonstationary AR($L+1$) process:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_{L+1} z_{t-L-1} + u_t \tag{6}$$

with coefficients satisfying (as $T \to \infty$) :

$$\begin{aligned} \phi_1 &= 1 + \pi_1 \\ \phi_2 &= \pi_2 - \pi_1 \\ \phi_3 &= \pi_3 - \pi_2 \\ &\vdots \\ \phi_L &= \pi_L - \pi_{L-1} \\ \phi_{L+1} &= -\pi_L \end{aligned} \tag{7}$$

that is: $$\sum_1^{L+1} \phi_i = 1 \tag{8}$$

Under invertibility and for a large enough L, (8) will hold and (2) will become a constrained version of (6).

Therefore, under (8) the likelihood ratio test (LR) below which compares the residual variances obtained from the constrained model ($\hat{\sigma}_r^2$) versus the unconstrained model ($\hat{\sigma}_u^2$), follows a standard χ_1^2 distribution:

$$LR = (T-2L)\ \ln\left(\frac{\hat{\sigma}^2{}_r}{\hat{\sigma}^2{}_u}\right) \tag{9}$$

where

$$\sigma_{\hat{u}}^2 = \frac{\sum_{t=L+2}^{T} \hat{u}_t^2}{T-(2L+2)} \tag{10}$$

is a consistent estimator of σ^2 and $\hat{u}_t$ are the residuals of the unconstrained model (6). Note that this test is similar to the proposed in Flores and Novales (1997) for testing univariate seasonality.

It is important to note that:

2. Excluding the case of θ = -1.0, which is too far from the alternative of interest θ = 1.0, restriction (8) holds if and only if $-1<\theta<1$. Thus, using (8) as the null hypothesis it is equivalent to using $-1<\theta<1$ as the null. The alternative is $\theta=1$, i.e. the sum of coefficients in (6) is not 1. Our test as that of Tsay (1993) is a test for nonstationarity, but note that in this paper both the model and the integrated variable y_t are different. While in Tsay's (1993) paper nonstationarity implies noninvertibility, in ours nonstationarity implies invertibility.

2. Under θ = 1, LR has not a standard distribution. Further, the statistic LR diverges due to the lack of consistence of LS estimators in (2). Thus, LR is expected to show a high power against θ = 1.

In practice, the test is very easy to implement. The null hypothesis (8) can be tested as follows: (i) apply LS to models (2) and (6), then (ii) compute (9) and compare its value with that of a χ_1^2.

3. Simulation exercise

In this Section we study the performance of our test in finite samples . We compare its performance with that of Saikkonen and Luukkonen's (1993) tests, R1 and R2, and with that of Tanaka's (1990) test, S_T.

We simulate 1,000 realizations of model (1), for different values of θ (θ= .6, .8, .9, .95 and 1.0), different sample sizes (T=100, 200 and 300) and different lag lengths ($L=3\,T^{1/4}$, $4\,T^{1/4}$, $5\,T^{1/4}$ and $6\,T^{1/4}$). The integrated series y_t is generated according to (5), starting from $y_0=0$.

Table 1 illustrates the performance of our test for different choices of L. Figures in this Table represent empirical LR sizes (theoretical size = 5%) when the parameter on the left hand side is used in the generating process. Note that when θ= 1.0 ,figures represent empirical powers.

[Insert Table 1]

Table 2 illustrates the performance of R1, R2 and S_T respectively, it replicates the results in Saikkonen and Luukkonen (1993) and Tanaka (1990). Figures in this Table represent the empirical powers associated with each value of θ. Given the H_0 used by these tests, when θ = 1 figures represent empirical sizes.

[Insert Table 2]

The most important results are:

1. If θ < .8 there are not important distortions in the size of LR. In these cases neither the choice of L nor T seems to affect the size (see Table 1).

2. For θ > .8 and a given sample size, the choice of L becomes more important (see Table 1). In general, the larger L the smaller the distortion in size. However the largest distortion in size occurs for the combination: T=100, $L=3\,T^{1/4}$ and θ = .95. In this case a 43.6% of times the null ($-1<\theta<1$) is wrongly rejected against the alternative θ = 1. In other words, our test detects θ = .95 a 56.4% of times. Our competitors, R1, R2 and S_T detect this parameter in a 59.1%, a 30.7% and a 31.9% of times (see Table 2). The good performance of our test is also evidenced when θ =.9. In this case invertibility is detected a 80% of times, while R1, R2 and S_T only do it a 74.4%, a 57.8% and a 59.3% of times, respectively. The power of our test under T=100 and $L=3\,T^{1/4}$ is 92%, i.e. comparable to the empirical probability level attained by R1, R2 and S_T : 93.7%, 95% and 93.6% respectively.

3. In general, the larger L the lower the power of our test. This is a problem for T=*100*. In this case the power falls up to 29.1% when L is large (L=19). However, if T=200 or 300 the power is larger than 74% for any choice of L. Then, we recommend to estimate θ by any consistent method and make the choice of L taking into account both the estimate of θ and the sample size. Alternatively, one could compute LR using different values of L.

4. If the most favorable L is chosen for each T, i.e. $3\,T^{1/4}$ for T=100, $5\,T^{1/4}$ for T=200 and $6\,T^{1/4}$ for T=300, then LR shows the best performance among all tests considered. For θ = .90 and T=100, 200 and 300, our test detect invertibility in a 80.2%, a 91.9% and a 93.6% of times respectively. A value of θ = .95 is detected in a 56.4%, a 81.1% and a 84.7% of times. R1 and LR perform very similar and both outperform R2 and S_T. In the case of R2, when θ = .9, the percentages are: 57.8%, 84.2% and 94.7%. When θ = .95 these percentages decrease up to: 30.7%, 62.5%, 77.3%. In the case of S_T , when θ = .9, the percentages are: 59.3%, 84.4% and 94.6%. When θ = .95 these percentages decrease up to: 31.9%, 63.2% and 77.4%, respectively.

4. Conclusions

In this paper we propose a test for detecting overdifferencing in a MA(1) process. Our statistic is a standard likelihood ratio with a standard χ^2 distribution. It performs very well, shows a high power against the alternative (θ = 1.0) and is very easy to compute. The distortions in size when the parameter is close to 1.0 are important, however they are lower than the distortions in power experienced by either Tanaka's (1990) or Sukkonen and Luukkonen's (1993) tests when the parameter value approaches their null ($\theta = 1.0$). These features make our test very appealing.

References.

Arellano, C. and S.G. Pantula, 1990, Trend Stationarity Versus Difference Stationarity, in: *Proceedings of the Business and Economic Statistics Section (American Statictical Association)* 188-196.

Christiano, L.J. 1987, Cagan´s Model of Hyperinflation Under Rational Expectations, *International Economic Review* 28, 33-47.

Flores, R. and A. Novales, 1997, A General Test for Univariate Seasonality, *Journal of Time Series Analysis*, 18, 1, 29-48.

Lütkepohl, H. 1993, *Introduction to Multiple Time Series Analysis* , Springer-Verlag, Berlin.

Plosser, C.I. and G.W. Schwert, 1977, Estimation of a Non-invertible Moving Average Process: The Case of Overdifferencing, *Journal of Econometrics* 6, 199-224.

Saikkonen, P. and R. Luukkonen, 1993, Testing for a Moving Average Unit Root in Autoregressive Integrated Moving Average Models, *Journal of the American Statictical Association*, 88, 422, 596-601.

Tsay, R.S. 1993, Testing for Nonivertible Models With Applications, *Journal of Business and Economic Statistics* 11, 2, 225-233.

Tanaka, K. 1990, Testing for a Moving Average Unit Root, *Econometric Theory*, 6, 433-444.

Table 1: Empirical power and size of LR test (measured with 1,000 replications).

		$3T^{1/4}$	$4T^{1/4}$	$5T^{1/4}$	$6T^{1/4}$
$T = 100$		$L = 9$	$L = 13$	$L = 16$	$L = 19$
	.60	5.3	4.8	4.8	5.0
	.80	8.0	5.9	5.0	4.8
$\theta =$	.90	19.8	9.7	6.9	4.6
	.95	43.6	21.8	13.5	9.5
	1.00	92.0	61.8	44.2	29.1
$T = 200$		$L = 11$	$L = 15$	$L = 19$	$L = 23$
	.60	5.4	5.8	5.0	5.5
	.80	7.4	5.9	5.4	5.8
$\theta =$	.90	20.0	11.5	8.1	6.2
	.95	49.0	31.5	18.9	14.1
	1.00	100.0	98.0	89.3	74.3
$T = 300$		$L = 12$	$L = 17$	$L = 21$	$L = 25$
	.60	5.1	5.5	5.1	5.5
	.80	8.1	5.6	5.9	5.6
$\theta =$	.90	20.2	9.7	7.4	6.4
	.95	51.8	32.0	22.0	15.3
	1.00	100.0	100.0	99.0	95.6

Theoretical size = 5%.

Table 2: Empirical power and size of various tests (measured with 1,000 replications).

$T=$	$\theta=$	R1	R2	S_T
		$H_0: \theta=1$		
100	.60	89.1	97.4	97.7
	.80	85.4	83.9	84.1
	.90	74.4	57.8	59.3
	.95	59.1	30.7	31.9
	1.00	4.3	5.0	4.6
200	.60	93.0	99.7	99.8
	.80	93.6	97.2	97.2
	.90	85.7	84.2	84.4
	.95	75.6	62.5	63.2
	1.00	5.5	5.0	4.8
300	.60	94.8	100.0	100.0
	.80	93.4	99.4	99.3
	.90	91.2	94.7	94.6
	.95	84.0	77.3	77.4
	1.00	4.2	4.5	4.5

Theoretical size = 5%.