Using Squeeziness to test from Finite State Machines

Trabajo de fin de grado del Doble Grado en Ingeniería Informática - Matemáticas

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To my family and my supervisor, for their invaluable help.

Acknowledgements

If I have seen further it is by standing on the shoulders of Giants.

Isaac Newton

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Abstract

The art and science of asking questions
is the source of all knowledge.

Thomas Berger

Squeeziness is an information theoretic measure designed to quantify the likelihood of a form of fault masking called *failed error propagation*. It has been shown that Squeeziness correlates strongly with failed error propagation in white-box scenarios. In this thesis, we adapt Squeeziness to a black-box scenario and show how it can be used to estimate the likelihood of failed error propagation.

Key Words: Squeeziness, Failed Error Propagation (FEP), Fault Masking, Testing, Correction, Black-box, Finite State Machine (FSM), Information Theory.

Resumen

La ciencia y el arte de hacer preguntas
es la fuente de todo conocimiento.

Thomas Berger

Squeeziness es una medida de Teoría de la Información diseñada para cuantificar la probabilidad de una forma de enmascaramiento de errores llamada *fallo en la propagación de errores*. Se ha demostrado que Squeezinees correlaciona fuertemente con el fallo en la propagación de errores en escenarios de caja blanca. En este TFG, adaptamos Squeeziness a un escenario de caja negra y mostramos como puede usarse para estimar la probabilidad de un fallo en la propagación de errores.

Palabras Clave: Squeeziness, Fallo en la Propagación de Errores (FEP), Enmascaramiento de Errores, Testing, Corrección, Caja Negra, Máquina de Estados Finita (FSM), Teoría de la Información.

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Part I

Thesis

This first part of the document presents the work performed during the thesis, the methodology, the obtained results and our conclusions.

Chapter 1

Introduction

Every story has a beginning, a middle, and an end. Not necessarily in that order.

Tim Burton

This chapter presents an introduction to our work and briefly sketches our design decisions during the development of the thesis. The idea of this thesis comes from previous work (Clark and Hierons, 2012) showing the application of elements from Information Theory to the detection of FEP (*Failed Error Propagation*) in software testing. The authors proposed a measure of FEP called *Squeeziness*. They developed and tested the usefulness of the work in a white box scenario. In our work, we start with the theoretical development of the Squeeziness measure adapted to a black box scenario and test it over both simulated and real cases. In the theoretical development, in addition to define Squeeziness in a black box scenario, we prove several properties of this measure. In the practical part, we develop a simulation and four real case testing, which give us different insights about what this measure is capable of.

The rest of the chapter is structured as follows. Section 1.1 presents an explanation of the motivations that lead us to the development of this work. Section 1.2 enumerates our goals. Finally, in Section 1.3 we review the structure of the rest of the document.

1.1 Motivation

Software testing (Ammann and Offutt, 2017; Myers et al., 2011) is the main validation technique used to increase the reliability of complex software systems. Software testing has traditionally been considered to be an *informal* technique (Gaudel, 1995). However, it is now known that testing activities can have a formal basis. Formal testing is an active research area (Binder et al., 2015; Cavalli et al., 2015; Hierons et al., 2009) and the existence of several tools that support formal testing has led to the recognition that the combination of formal methods and testing facilitates test automation (Shafique and Labiche, 2015).

FEP is the situation in which a faulty statement in the SUT (*System Under Test*) is executed during testing, the fault *corrupts* the internal state of the SUT, but the expected output is observed. Naturally, in order for a statement to be a fault there must be at least one input under which FEP does not occur. FEP is a form of fault masking and can reduce the effectiveness of testing: we might fail to find a fault despite executing the faulty statement in testing. Empirical studies have shown that many systems suffer from FEP (Androutsopoulos et al., 2014; Masri et al., 2009). For example, Masri et al. (2009) found that in 13% of the programs that they examined, a total of 60% or more of tests suffered from FEP. Previous work introduced the notion of Squeeziness (Androutsopoulos et al., 2014; Clark and Hierons, 2012) to capture this FEP, with Squeeziness being a measure of the information (entropy) lost by a channel (the SUT) that takes input and returns output. In experiments, there was a rank correlation of close to 0.95 between measures of Squeeziness and the likelihood of FEP (Androutsopoulos et al., 2014). In addition, it has been found that the likelihood of FEP more strongly correlates with Squeeziness than with the DRR (*Domain to Range Ratio*) (Clark and Hierons, 2012). There are two practical reasons for the interest in measures associated with FEP. First, such measures might be used to estimate testability; we might expect it to be particularly difficult for testing to find a fault in an SUT with high Squeeziness. Second, there may be potential to generate test cases that achieve a given test purpose, such as covering part of the SUT or a model, and that have a low probability of FEP.

There is a significant body of work on FEP and fault masking for white-box testing (Apiwattanapong et al., 2006; Masri et al., 2009; Woodward and Al-Khanjari, 2000; Wang et al., 2009a) and black-box testing (Guo et al., 2006; Petrenko, 2001; Petrenko et al., 2004; Wang et al., 2009b). As mentioned, previous work has also defined Squeeziness in a white-box scenario (Androutsopoulos et al., 2014; Clark and Hierons, 2012). However, we are not aware of any work that uses an information theory foundation for addressing FEP in a black-box context, so we decided to explore this way.

1.2 Goals

The main goal of our work is to adapt the notion of Squeeziness to a black box testing scenario and using FSMs (*Finite State Machines*). Although the FSM (*Finite State Machine*) formalism is relatively simple, we establish the basis of a framework to test in more complex black-box contexts because the basis of testing is similar: we apply a sequence of inputs and decide whether the observed sequence of outputs is consistent with the specification of the system. In addition to extending the notion of Squeeziness to a black box scenario, we evaluated its usefulness through experiments. Importantly, we found that there was an extremely high rank correlation between our proposed measures and the probability of FEP. As a result, the proposed measures could act as testability measures for state-based testing and have the potential to help direct testing.

There are several differences between the original scenario (Clark and Hierons, 2012) and ours. First, we have to reshape the actual definition of Squeeziness because inputs and outputs have a different treatment in each scenario. In the previously considered white-box case, a program receives an input (a tuple of values) and returns an output (again, a tuple of values). In the scenario that we consider in our work, an *input* is a sequence of input actions while an output is also a sequence, in this case of output actions. Therefore, the first adaption is that a uniform distribution over the sets of inputs and outputs is not suitable because, for example, a prefix of a sequence should have a higher probability than the whole sequence. Second, in white-box testing we can *follow* the path that a specific execution of the program is traversing. In black box testing we do not know the internal structure of the SUT and, therefore, we cannot take advantage of it to guide the testing process.

Other side goals that we pursue in this work are:

- Show that Squeeziness still holds its characteristics in this new black box testing scenario (and look for potential new characteristics).
- Show that Squeeziness is still better than other measures in this new black box testing scenario.
- Look for a normalization of Squeeziness that can help us to use it as a measure.
- Determine if Squeeziness can be used over the FSM specification or it only works over the SUT.
- Suggest how we can use Squeeziness to test and how we cannot use it.

1.3 Workplan

Our workplan tries to mimic the steps that Professors David Clark and Robert M. Hierons followed in their work (Clark and Hierons, 2012). We can divide the work into two main parts: a theoretical one and a practical one.

1.3.1 Theoretical workplan

The theoretical part consists in developing the theory around FEP detection, using techniques from Information Theory in a black box testing scenario, starting with the tools we will use to model systems and ending with the proposed measure applied to this new scenario. All of this is addressed in Chapter 2, which is divided into the following sections:

- Section 2.1: a definition of the version of FSM that we use in this work.
- Section 2.2: a definition of our main measure, Squeeziness, and some of its properties. Here we also explain some useful cases.
- Section 2.3: a definition of our alternative measure, Probabilistic Squeeziness.

1.3.2 Practical workplan

The practical part of our work consists in testing the proposed measures. We can distinguish two parts. First, we use simulation to test if we have a good measure. Next, we perform experiments to test the use of the measures in real FSMs.

1.3.2.1 Simulation

In Chapter 3 we explain the simulation that we performed to test how well our measures work. The chapter is divided into the following sections:

- Section 3.1: a definition of DRR, a previous measure of the probability of FEP, and a comparison with our measures.
- Section 3.2: a definition of a formal measure of FEP and a comparison with our measures.
- Section 3.3: an explanation of the simulation, how it was done, our conclusions and a comparison between results.

1.3.2.2 Experiments

In Chapter 4 we explain the different experiments that we performed and what we conclude from their results. The chapter is divided into the following sections:

- Section 4.1: an explanation of our tool to generate random FSMs.
- Section 4.2: an explanation of the first experiment, what we wanted to prove, what results we got, and what we concluded.

- Section 4.3: an explanation of the second experiment, what we wanted to prove, what results we got, and what we concluded.
- Section 4.4: an explanation of the third experiment, what we wanted to prove, what results we got, and what we concluded.
- Section 4.5: an explanation of the fourth experiment, what we wanted to prove, what results we got, and what we concluded.
- Section 4.6: an overview of the results of the experiments.

The code used to perform the experiments in this work has been developed from scratch and can be found at `https://github.com/Colosu/Bachelor-Thesis`.

1.3.3 Conclusions

Finally, in Chapter 5 we discuss the conclusions from our work and the practical uses of our measures. The chapter is divided in the following sections:

- Section 5.1: a discussion about the results of the experiments and what they imply.
- Section 5.2: a discussion about the practical uses of our measures, once we saw the results from the experiments.
- Section 5.3: an overview of our work and possible lines of future work.

Chapter 2

Theoretical Framework

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it is wrong.

Richard P. Feynman

In this chapter we explain the theoretical framework underlying our work. In order to develop a framework to use Squeeziness in a black box scenario, we have chosen to follow the same order as the one used by Professors David Clark and Robert M. Hierons in the original work (Clark and Hierons, 2012). In addition to adapt the existing definitions, properties and results to the new setting, we also need to develop the proofs of all this results for the new scenario. Unfortunately, most of the proofs are not included in Clark and Hierons (2012) and, therefore, have to be produced from scratch.

The rest of the chapter is structured as follows. In Section 2.1 we introduce the FSMs formalism. In Section 2.2 we formally define Squeeziness, present some of its properties and explain some useful cases. Finally, Section 2.3 includes a definition of our other alternative measure: a probabilistic version of Squeeziness.

2.1 Finite State Machines

First of all, we need to define the formalism that we will use to model systems. As we will work in a black box scenario, it is common in the literature to refer to systems as FSMs. For our purposes, we take most of the concepts from the original sources (Lee and Yannakakis, 1996), while some notation is adapted to facilitate the formulation of subsequent definitions. Next, we introduce some auxiliary notation.

Given a set A, we let A^* denote the set of finite sequences of elements of A; $\epsilon \in A^*$ denotes the empty sequence. We let A^+ denote the set of non-empty sequences of elements of A. A^k denotes the set of sequences with length $k \geq 1$. We let $|A|$ denote the cardinal of set A. Given a sequence $\sigma \in A^*$, we have that $|\sigma|$ denotes its length. Given a sequence $\sigma \in A^*$ and $a \in A$, we have that σa denotes the sequence σ followed by a and $a\sigma$ denotes the sequence σ preceded by a.

We let I be the set of input actions and O be the set of output actions. It is important to differentiate between input actions and *inputs* of the system. In our context an input of a system will be a non-empty sequence of input actions, that is, an element of I^+ (similarly for outputs and output actions). An FSM is a (finite) labelled transition system in which transitions are labelled by an input/output pair. We use this formalism to define processes.

Definition 1 *We say that $M = (Q, q_{in}, I, O, T)$ is an* FSM*, where Q is a finite set of states, $q_{in} \in Q$ is the initial state, I is a finite set of inputs, O is a finite set of outputs, and $T \subseteq Q \times (I \times O) \times Q$ is the transition relation. A transition $(q, (i, o), q') \in T$, also denoted by $q \xrightarrow{i/o} q'$ or by $(q, i/o, q')$, means that from state q after receiving input i it is possible to move to state q' and produce output o.*

We say that M is deterministic *if for all $q \in Q$ and $i \in I$ there exists at most one pair $(q', o) \in Q \times O$ such that $(q, i/o, q') \in T$. In our work we consider deterministic* FSMs.

We say that M is input-enabled *if for all $q \in Q$ and $i \in I$ there exists $(q', o) \in Q \times O$ such that $(q, i/o, q') \in T$.*

We let $\mathtt{FSM}(I, O)$ denote the set of finite state machines with input set I and output set O.

A process can be identified with its initial state and we can define a process corresponding to a state q of M by making q the initial state. Thus, we use states and processes and their notation interchangeably. An FSM can be represented by a diagram in which nodes represent states of the FSM and transitions are represented by arcs between the nodes. We use a double circle to denote the initial state.

As usual, we assume that SUTs (*Systems Under Test*) are input-enabled: the SUT should be able to react, somehow, to any external stimulus. In particular, if the tester applies an input action at a certain stage, then the SUT should be able to provide a response (that is, an output action). Actually, if an input cannot be applied in some state of the SUT, then we can assume that there is a response to the input that reports that this input is blocked, so that this assumption of input-enableness is not a significant restriction. However, we do not force specifications to be input-enabled. In particular, all the definitions and results concerning Squeeziness will not assume input-enableness.

As stated in the previous definition, we consider the case where both specifications and SUTs are deterministic. This is similar to the previously explored white-box scenario that assumed that programs are deterministic.

Our main goal while testing is to decide whether the behaviour of an SUT conforms to the specification of the system that we would like to build.

In order to detect differences between specifications and SUTs, we need to compare the behaviours of specifications and SUTs and the main notion to define such behaviours is given by the concept of *trace*.

Definition 2 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM. *We use the following notation.*

1. *Let $\sigma = (i_1, o_1) \dots (i_k, o_k) \in (I \times O)^*$ be a sequence of input/output actions and q be a state. We say that M can perform σ from q if there exist states $q_1 \dots q_k \in Q$ such that for all $1 \leq j \leq k$ we have $(q_{j-1}, i_j/o_j, q_j) \in T$, where $q_0 = q$. We denote this by either $q \stackrel{\sigma}{\Longrightarrow} q_k$ or $q \stackrel{\sigma}{\Longrightarrow}$. If $q = q_{in}$ then we say that σ is a* trace *of M. We denote by $\mathtt{traces}(M)$ the set of traces of M. Note that for every state q we have that $q \stackrel{\epsilon}{\Longrightarrow} q$ holds. Therefore, $\epsilon \in \mathtt{traces}(M)$ for every* FSM *M.*

2. *Let $\alpha = i_1 \dots i_k \in I^*$ be a sequence of input actions and q be a state. We define $\mathtt{out}_M(q, \alpha)$ as the set*

$$\{o_1, \dots, o_k \in O^* | q \xrightarrow{(i_1,o_1)\dots(i_k,o_k)} \}$$

Note that if M is deterministic then this set is either empty or a singleton. In the last case we will sometimes write $\mathtt{out}_M(q, \alpha) = o_1, \dots, o_k$.

3. *Let $q \in Q$ be a state. We define $\mathtt{dom}_M(q)$ as the set*

$$\{\alpha \in I^* | \mathtt{out}_M(q, \alpha) \neq \emptyset\}$$

If $q = q_{in}$ then we simply write $\mathtt{dom}_M$. Similarly, we define $\mathtt{image}_M(q)$ as the set

$$\{o_1 \dots o_k \in O^* | \exists i_1 \dots i_k \in I^* : q \xrightarrow{(i_1,o_1)\dots(i_k,o_k)} \}$$

If $q = q_{in}$ then we simply write $\mathtt{image}_M$. We denote by $\mathtt{dom}_{M,k}$ the set $\mathtt{dom}_M \cap I^k$. Similarly, We denote by $\mathtt{image}_{M,k}$ the set $\mathtt{image}_M \cap O^k$.

Note that if M is input-enabled then for all $k > 0$ we have that $\mathtt{dom}_{M,k} = I^k$ and, therefore, for all $\alpha \in I^k$ we have that $\mathtt{out}_M(q, \alpha) \neq \emptyset$.

Now, an FSM M can be seen as a function transforming sequences of input actions belonging to $\mathtt{dom}_M$ into sequences of output actions belonging to $\mathtt{image}_M$. Therefore, we could say that M receives an *input* (an element of I^*) and returns an *output* (an element of O^*, with the same length as the input).

We define *projections* of this function: for a natural number k, we restrict the function to the set of sequences of input actions that are of length k. In particular, these projections will allow us to consider finite sets of inputs (all the sequences of inputs of a certain length). We also introduce the notion of *collision*: two inputs collide if they produce the same output.

Definition 3 *Let* $M = (Q, q_{in}, I, O, T)$ *be an* FSM. *We define* $f_M : \mathtt{dom}_M \longrightarrow \mathtt{image}_M$ *as the function such that for all* $\alpha \in \mathtt{dom}_M$ *we have* $f_M(\alpha) = \beta$ *for* β *such that* $\mathtt{out}_M(q_{in}, \alpha) = \{\beta\}$.

Let $k > 0$. *We define* $f_{M,k}$ *to be the function* $f_M \cap (I^k \times O^k)$, *where we use the function* f_M *to denote the associated set of pairs. Let* $\beta \in \mathtt{image}_M$. *We define* $f_M^{-1}(\beta)$ *to be the set* $\{\alpha \in I^* | f_M(\alpha) = \beta\}$.

Let $\alpha_1, \alpha_2 \in I^*$. *We say that* α_1 *and* α_2 *collide for* M *if* $\alpha_1 \neq \alpha_2$ *and* $f_M(\alpha_1) = f_M(\alpha_2)$.

Note that if two sequences of input actions collide then they must have the same length (otherwise, the returned sequences of output actions would have different length and, therefore, cannot be equal).

2.2 Squeeziness

Once we have defined the basic model to work with, we introduce some notation for random variables and recall the concept of *entropy* (Shannon, 1948) associated with a random variable and the concept of Squeeziness (Clark and Hierons, 2012) of a function.

Definition 4 *Let* A *be a set and* ξ_A *be a random variable over* A. *We denote by* σ_{ξ_A} *the probability distribution induced by* ξ_A. *The* entropy *of the random variable* ξ_A, *denoted by* $\mathcal{H}(\xi_A)$, *is defined as:*

$$\mathcal{H}(\xi_A) = -\sum_{a \in A} \sigma_{\xi_A}(a) \cdot \log_2(\sigma_{\xi_A}(a))$$

Let $f : A \longrightarrow B$ *be a total function. The* Squeeziness *of* f, *denoted by* $\mathtt{Sq}(f)$, *is defined as the loss of information after applying* f *to* A, *that is,* $\mathcal{H}(A) - \mathcal{H}(B)$.

As we said, Squeeziness represents the amount of information lost by a given function. Since we have shown that FSMs can be seen as functions from a set of sequences of input actions to a set of sequences of output actions, we can try and adapt Squeeziness to deal with FSMs.

First, we need to define how inputs are chosen and outputs are returned. We consider a probabilistic view where a random variable associated with each set of relevant inputs/outputs is taken into account. We studied two possible alternatives:

- We associate a random variable with the whole set of inputs/outputs (that is, a random variable induces a probability distribution over I^* and O^*, respectively).

- We associate a random variable with the set of inputs/outputs of a certain length (that is, there are different random variables associated with I^1, I^2, ..., O^1, O^2, ...).

In our work we consider the second approach for two main reasons. First, it gives us an incremental procedure to compute a sequence of consecutive values of Squeeziness so that we can analyse how the series is *evolving*. Second, but strongly related to the first one, it provides us with the basis for a *stopping rule*: we can compute consecutive values until the difference between them drops below a threshold. In other words, we reach a certain k such that we test inputs of length k and consider that the costs of further testing to locate faults will not be compensated by the likelihood of finding these faults.

Still, we think that the first approach is also interesting. In particular, it can be used to compare the two notions for a large sample of FSMs and we consider this to be a line of future work.

We have that $\texttt{dom}_{M,k}$ represents the possible inputs of length equal to k that M can perform (therefore, other elements of I^k have probability equal to zero) and $\texttt{image}_{M,k}$ represents the possible outputs of length equal to k that M can produce after receiving an element of $\texttt{dom}_{M,k}$. Therefore, the difference of entropy, that is $\mathcal{H}(\xi_{\texttt{dom}_{M,k}})-\mathcal{H}(\xi_{\texttt{image}_{M,k}})$, represents the amount of information destroyed by M. This is the notion of Squeeziness that we will use in our work.

Definition 5 *Let* $M = (Q, q_{in}, I, O, T)$ *be an* FSM *and* $k > 0$. *Let us consider two random variables* $\xi_{\texttt{dom}_{M,k}}$ *and* $\xi_{\texttt{image}_{M,k}}$ *ranging, respectively, over the domain and image of* $f_{M,k}$. *The Squeeziness of* M *at length* k *is defined as*

$$\texttt{Sq}_k(M) = \mathcal{H}(\xi_{\texttt{dom}_{M,k}}) - \mathcal{H}(\xi_{\texttt{image}_{M,k}})$$

Squeeziness for state-machines is an interesting notion that has some unexpected properties. For example, it is not monotonic with respect to k. That is, there exist finite state machines where using longer sequences can solve a loss of information produced by shorter sequences. That was something that didn't happen in the white box testing scenario, because in our case we have the deterministic property of the FSMs that the white box testing scenario didn't have.

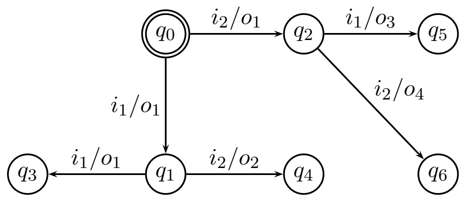

Figure 2.1: Machine M

Example 1 *Consider the machine M from Figure 2.1 where q_0 is the initial state. We have that Squeeziness for $k = 1$ is equal to $\log_2(2) = 1$ while for $k = 2$ is equal to 0.*

An important remark concerning random variables associated with inputs and outputs is that given an FSM M, $k > 0$ and a random variable $\xi_{\mathtt{dom}_{M,k}}$ we have that the probability distribution of the random variable $\xi_{\mathtt{image}_{M,k}}$ is completely determined. This is so because for each element $\beta \in \mathtt{image}_{M,k}$ we have that

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) = \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)$$

The following result is immediate from the definition of entropy and the previous explanation concerning how the random variable associated with outputs is determined by the one corresponding to inputs.

Lemma 1 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM *and $k > 0$. If $f_{M,k}$ is bijective then $\mathtt{Sq}_k(M) = 0$.*

Next, we present an alternative formulation of Squeeziness. The proof of the following result follows from the partition property of entropy (Cover and Thomas, 2006) and the definition of $\sigma_{\xi_{\mathtt{image}_{M,k}}}$ in terms of $\sigma_{\xi_{\mathtt{dom}_{M,k}}}$. First, we give an auxiliary result concerning conditional distributions of random variables. In the following, $\xi_1 | \xi_2$ denotes the conditional random variable ξ_1 given ξ_2.

Lemma 2 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM *and $k > 0$. Let us consider two random variables $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. We have that $\mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}}) = 0$.*

Proof

Consider the entropy of the conditional random variable $\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}}$, that is,

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}}) = \sum_{\alpha \in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}} = \alpha)$$

If we unfold the second term of the sum we have that the previous expression is equal to

$$\sum_{\alpha\in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)\cdot \left(\sum_{\beta\in \mathtt{image}_{M,k}} \sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}(\beta|\alpha)\cdot \log_2(\sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}(\beta|\alpha)) \right)$$

We will prove that all the summands of the previous expression are equal to zero. Considering that M is deterministic we have that $\sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}$ can be either 0 or 1. Using this fact in the previous expression, we have two cases:

- If $\sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}(\beta|\alpha) = 0$ then the result obviously holds.
- If $\sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}(\beta|\alpha) = 1$ then $\log_2(\sigma_{(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})}(\beta|\alpha)) = 0$ and, again, the result holds.

We finally conclude that $\mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}}) = 0$. □

Proposition 1 *Let* $M = (Q, q_{in}, I, O, T)$ *be an* FSM *and* $k > 0$. *Let us consider two random variables* $\xi_{\mathtt{dom}_{M,k}}$ *and* $\xi_{\mathtt{image}_{M,k}}$ *ranging, respectively, over the domain and image of* $f_{M,k}$. *We have that*

$$\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{image}_{M,k}}) - \sum_{\beta\in \mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)\cdot \left(\sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)\cdot \log_2(\sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)) \right)$$

Proof
By the definition of conditional entropy (Cover and Thomas, 2006) we have that

$$\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}}) = \sum_{\beta\in \mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)\cdot \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}} = \beta)$$

Next, we apply the notion of conditional probability and consider that $\xi_{\mathtt{dom}_{M,k}}$ restricted to $\xi_{\mathtt{image}_{M,k}} = \beta$ is the random variable $\xi_{f_M^{-1}(\beta)}$ ranging over $f_M^{-1}(\beta)$ and whose probabilities are equal to

$$\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}$$

Therefore, we have that

$$\begin{aligned}
\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}} = \beta) &= \mathcal{H}(\xi_{f_M^{-1}(\beta)}) \\
&= - \sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)\cdot \log_2(\sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)) \\
&= - \sum_{\alpha\in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\cdot \log_2\left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\right)
\end{aligned}$$

Therefore, the term $\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}})$ is equal to

$$-\sum_{\beta\in\mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)\cdot\left(\sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)\cdot\log_2(\sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha))\right) \tag{2.1}$$

If we apply the *Chain rule* then we have

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}},\xi_{\mathtt{dom}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{image}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}})$$

where $\mathcal{H}(\xi_{\mathtt{image}_{M,k}},\xi_{\mathtt{dom}_{M,k}})$ is the joint probability of the two random variables. Considering that, applying again the *Chain rule*, we also have

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}},\xi_{\mathtt{dom}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})$$

then we obtain

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})$$

Finally, given that by Lemma 2 we have $\mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}}) = 0$ and given the value of $\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}})$ from equation (2.1), we obtain the desired reformulation of $\mathcal{H}(\xi_{\mathtt{dom}_{M,k}})$. $\square$

A trivial corollary of the previous result provides an alternative definition of Squeeziness where the value is computed in terms of the inverse images partition of the input space considering, as previously explained, that we have

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) = \sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)$$

Therefore, we only use the probability distribution on inputs given by $\xi_{\mathtt{dom}_{M,k}}$.

Corollary 1 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM *and $k > 0$. Let us consider a random variable $\xi_{\mathtt{dom}_{M,k}}$ ranging over the domain of $f_{M,k}$. We have that*

$$\mathtt{Sq}_k(M) = -\sum_{\beta\in\mathtt{image}_{M,k}}\left(\sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)\right)\cdot \left(\sum_{\alpha\in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\cdot\log_2\left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\right)\right)$$

2.2.1 Useful Implementations

In general, it is not possible to know the probability distribution that ranges over the inputs. Therefore, if we want to have an estimation of the different values of Squeeziness for a given FSM we need to make an assumption about this distribution. There are different possibilities. For example, we can assume *maximum entropy*, that is, we choose a probability distribution that maximizes the entropy. Another strategy considers the worst case scenario, that is, we may suppose that the chosen probability distribution induces the maximum loss of information, i.e., we look for a probability distribution that maximises Squeeziness.

2.2.1.1 Maximum entropy principle

In order to consider maximum entropy, and assuming no further restrictions, we need to use a uniform distribution (Cover and Thomas, 2006). In this case, the weight of a single element of $\sigma_{\xi_{\mathtt{dom}_{M,k}}}$ is $\frac{1}{|\mathtt{dom}_{M,k}|}$. Thus, the weight of the inverse image of an output $\beta \in \mathtt{image}_{M,k}$ is equal to $\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}$. Finally, Squeeziness under the assumption of having a uniform distribution over inputs is equal to

$$
\begin{aligned}
\mathtt{Sq}_k(M) &= -\sum_{\beta\in\mathtt{image}_{M,k}} \left(\sum_{\alpha\in f_M^{-1}(\beta)} \frac{1}{|\mathtt{dom}_{M,k}|} \right) \cdot \left(\sum_{\alpha\in f_M^{-1}(\beta)} \frac{\frac{1}{|\mathtt{dom}_{M,k}|}}{\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}} \cdot \log_2 \left(\frac{\frac{1}{|\mathtt{dom}_{M,k}|}}{\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}} \right) \right) \\
&= -\sum_{\beta\in\mathtt{image}_{M,k}} \frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|} \cdot \left(\frac{|f_M^{-1}(\beta)|}{|f_M^{-1}(\beta)|} \cdot \log_2 \left(\frac{1}{|f_M^{-1}(\beta)|} \right) \right) \\
&= -\sum_{\beta\in\mathtt{image}_{M,k}} \frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|} \cdot \log_2 \left(\frac{1}{|f_M^{-1}(\beta)|} \right) \\
&= \frac{1}{|\mathtt{dom}_{M,k}|} \cdot \sum_{\beta\in\mathtt{image}_{M,k}} |f_M^{-1}(\beta)| \cdot \log_2(|f_M^{-1}(\beta)|)
\end{aligned}
$$

2.2.1.2 Maximum loss of information

If we want to consider maximum loss of information, then we need to consider a probability distribution such that it is uniformly distributed in the bigger inverse image of an element of the outputs and zero elsewhere (Clark and Hierons, 2012). Formally, consider $\beta' \in \mathtt{image}_{M,k}$ such that for all $\beta \in \mathtt{image}_{M,k}$ we have $|f_M^{-1}(\beta')| \geq |f_M^{-1}(\beta)|$. Then,

$$
\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) = \begin{cases} \frac{1}{|f_M^{-1}(\beta')|} & \text{if } \alpha \in f_M^{-1}(\beta') \\ 0 & \text{otherwise} \end{cases}
$$

Using this probability distribution, Squeeziness is defined as follows:

$$\begin{aligned}
\mathtt{Sq}_k(M) &= -\left(\sum_{\alpha\in f_M^{-1}(\beta')} \frac{1}{|f_M^{-1}(\beta')|}\right) \cdot \left(\sum_{\alpha\in f_M^{-1}(\beta')} \frac{1}{|f_M^{-1}(\beta')|} \cdot \log_2\left(\frac{1}{|f_M^{-1}(\beta')|}\right)\right) \\
&= -\frac{|f_M^{-1}(\beta')|}{|f_M^{-1}(\beta')|} \cdot \left(\frac{|f_M^{-1}(\beta')|}{|f_M^{-1}(\beta')|} \cdot \log_2\left(\frac{1}{|f_M^{-1}(\beta')|}\right)\right) \\
&= -\log_2\left(\frac{1}{|f_M^{-1}(\beta')|}\right) \\
&= \log_2(|f_M^{-1}(\beta')|)
\end{aligned}$$

Let us remark that this probability distribution maximises Squeeziness because for any other distribution $\xi_{\mathtt{dom}_{M,k}}$ we have $\mathtt{Sq}_k(M) \leq \log_2(|f_M^{-1}(\beta')|)$. This result is an immediate consequence of the following result (Clark and Hierons, 2012).

Lemma 3 *Let us consider non-negative real numbers $a_1, \ldots, a_n, p_1, \ldots, p_n \in \mathbb{R}^+$. If for all $1 \leq i \leq n$ we have that $a_1 \geq a_i$ and $\sum_i p_i \leq 1$, then $\sum_i (p_i \cdot a_i) \leq a_1$.*

2.3 Probabilistic Squeeziness

Now that we have an upper bound for Squeeziness, we can develop a probability measure based on this notion. The idea is that *Probabilistic Squeeziness* will provide a value between 0 and 1 (and therefore, similar to a probability) associated with the probability of having FEP for a certain input.

Definition 6 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM *and $k > 0$. Let us consider two random variables $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. Let us consider $\beta' \in \mathtt{image}_{M,k}$ such that for all $\beta \in \mathtt{image}_{M,k}$ we have that $|f_M^{-1}(\beta')| \geq |f_M^{-1}(\beta)|$. The Probabilistic Squeeziness of M at length k is defined as*

$$\mathtt{PSq}_k(M) = \frac{\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) - \mathcal{H}(\xi_{\mathtt{image}_{M,k}})}{\log_2(|f_M^{-1}(\beta')|)}$$

Although these values are more complicated to compute, they might be more useful at the time of comparing results and automate their use because they can be treated as probabilities. It is immediate to reformulate Probabilistic Squeeziness as follows: $\mathtt{PSq}_k(M) =$

$$-\frac{\sum_{\beta\in\mathtt{image}_{M,k}} \left(\sum_{\alpha\in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)\right) \cdot \left(\sum_{\alpha\in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))} \cdot \log_2\left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\right)\right)}{\log_2(|f_M^{-1}(\beta')|)}$$

Chapter 3

Simulation

In theory there is no difference between theory and practice. In practice there is.

Yogi Berra

In this chapter we explain why we decided to do a simulation, present the tools needed to make the simulation, and explain how we did the simulation and what were the results. In order to test the goodness of our proposed measures, we decided to make a first step *simulating* the results. By simulation we mean that instead of using real FSMs we generate input/output pairs (that is, a sequence of input actions and a corresponding sequence of output actions). These pairs are appropriately encoded as a pair of natural numbers. Using this *trick* we can work with very long, randomly generated sequences. We compare the results from Squeeziness and Probabilistic Squeeziness with the results from DRR, another measure that has been proposed before to address this problem. In order to do the comparison, we will start by adapting DRR to our scenario and compare its properties to our measures. Also, we need a measure that works as a formal reference, that is, a measure returning the *real* probability of having FEP. This measure is the probability of collisions: we will define it and we will compare its properties to our measures. Finally, once we have defined all our measures, we can proceed with the simulation and analyse the results.

The rest of the chapter is structured as follows. In Section 3.1 we define DRR and compare it with our measures. Section 3.2 presents the definition of a formal measure of FEP and a comparison with our measures. Section 3.3 provides an explanation of our simulation and an overview of the obtained results.

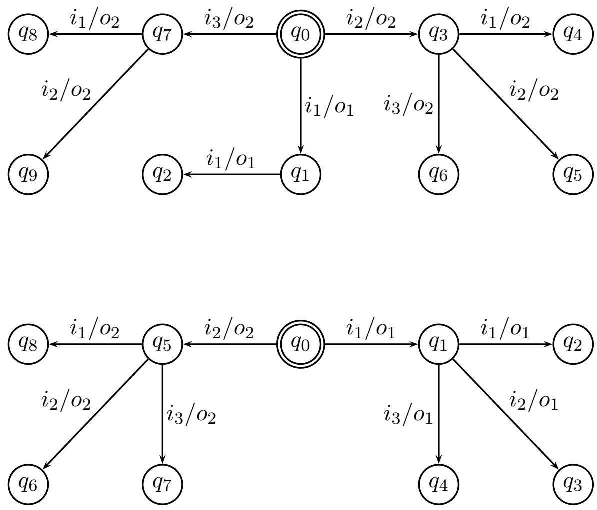

Figure 3.1: Machines M_1 (up) and M_2 (down)

3.1 Domain to Range Ratio

It is difficult to compare Squeeziness with other notions that can compute fault masking because the literature is very scarce. One of the few notions in this line is DRR (Woodward and Al-Khanjari, 2000). First we will give the original definition of DRR.

Definition 7 *Let $f : I \longrightarrow O$ be a total and surjective function. We define the Domain to Range Ratio of f, denoted by $\mathtt{DRR}(f)$, as $\frac{|I|}{|O|}$.*

Next, we adapt this notion to our framework. Actually, our *functions* are total and surjective because we restrict ourselves to domain and range.

Definition 8 *Let $M = (Q, q_{in}, I, O, T)$ be an* FSM *and $k > 0$. Let us consider $f_{M,k} : \mathtt{dom}_{M,k} \longrightarrow \mathtt{image}_{M,k}$. We define the Domain to Range Ratio for M and k, denoted by $\mathtt{DRR}(f_{M,k})$, as $\frac{|\mathtt{dom}_{M,k}|}{|\mathtt{image}_{M,k}|}$.*

The next result shows that this measure is inconsistent with Probabilistic Squeeziness (the proof that this measure is inconsistent with Squeeziness can be found in the original work on Squeeziness Clark and Hierons (2012)).

Lemma 4 *There exist* FSMs *M_1 and M_2 and $k > 0$ such that $\mathtt{DRR}(f_{M_1,k}) = \mathtt{DRR}(f_{M_2,k})$ but $\mathtt{PSq}_k(M_1) \neq \mathtt{PSq}_k(M_2)$.*

There exist FSMs *M_1 and M_2 and $k > 0$ such that $\mathtt{DRR}(f_{M_1,k}) < \mathtt{DRR}(f_{M_2,k})$ but $\mathtt{PSq}_k(M_1) > \mathtt{PSq}_k(M_2)$.*

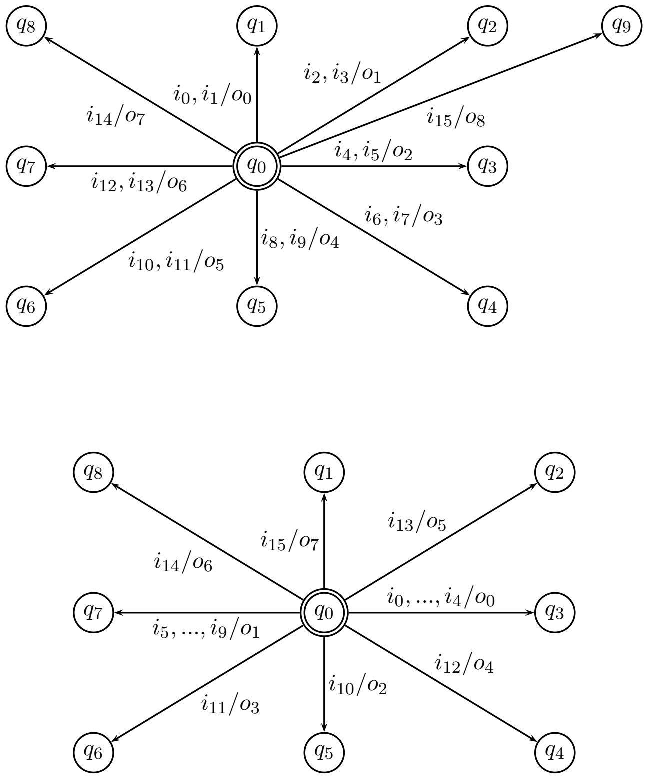

Figure 3.2: Machines M_3 (up) and M_4 (down)

Proof
First, let us note that in this proof we assume uniform distributions over inputs (and outputs) of the FSMs. However, the result holds for any probability distribution: we would only need to slightly modify the definition of the given machines.

In order to prove the first part of the result, we define two machines M_1 and M_2, both with initial state q_0, fulfilling the conditions. Let M_1 be the machine from Figure 3.1 (up). We have

$$\texttt{dom}_{M_1,2} = \{(i_1, i_1), (i_2, i_1), (i_2, i_2), (i_2, i_3), (i_3, i_1), (i_3, i_2)\}$$

and

$$\texttt{image}_{M_1,2} = \{(o_1, o_1), (o_2, o_2)\}$$

On the one hand we have $\texttt{DRR}(f_{M_1,2}) = 6/2 = 3$ while, on the other hand, we have $\texttt{PSq}_2(M_1) = \frac{(5 \cdot log_2(5) + 1 \cdot log_2(1))/6}{log_2(5)} \approx 0.833$. Now, let M_2 be the machine from Figure 3.1 (down). We have

$$\texttt{dom}_{M_2,2} = \{(i_1, i_1), (i_1, i_2), (i_1, i_3), (i_2, i_1), (i_2, i_2), (i_2, i_3)\}$$

and

$$\texttt{image}_{M_2,2} = \{(o_1, o_1), (o_2, o_2)\}$$

We have, on the one hand, that $\texttt{DRR}(f_{M_2,2}) = 6/2 = 3$ while, on the other hand, $\texttt{PSq}_2(M_2) = \frac{(2 \cdot 3 \cdot log_2(3))/6}{log_2(3)} = 1$.

In order to prove the second part of the result, let us consider again two machines M_3 and M_4, with initial state q_0, and we will show that they fulfil the required conditions. In these machines, we consider that $x_1, \ldots, x_n/y$ is a shorthand for n different transitions labelled, respectively, by $x_1/y, x_2/y, \ldots, x_n/y$. Let M_3 be the machine from Figure 3.2 (up). We have $\texttt{dom}_{M_3,1} = \{i_0, ..., i_{15}\}$ and $\texttt{image}_{M_3,1} = \{o_0, ..., o_8\}$. Therefore, $\texttt{DRR}(f_{M_3,1}) = 16/9 \approx 1.778$ while $\texttt{PSq}_1(M_3) = \frac{(7 \cdot 2 \cdot log_2(2) + 2 \cdot 1 \cdot log_2(1))/16}{log_2(2)} = 0.875$. Finally, let M_4 be the machine from Figure 3.2 (down). We have $\texttt{dom}_{M_4,1} = \{i_0, ..., i_{15}\}$ and $\texttt{image}_{M_4,1} = \{o_0, ..., o_7\}$, so $|\texttt{image}_{M_4,1}| = 8$ and $\texttt{DRR}(f_{M_4,1}) = 16/8 = 2, \texttt{PSq}_1(M_4) = \frac{(2 \cdot 5 \cdot log_2(5) + 6 \cdot 1 \cdot log_2(1))/16}{log_2(5)} = 0.625$. $\square$

Finally, note that the results in this section are independent of the actual value of k. We have used functions over inputs of different lengths to show that the length of the sequences do not influence the computations. The idea is that each sequence of input actions works as a single input and the computations consider only the number of inputs of the same length.

3.2 Probability of Collisions

In order to have a reference measure for our experiments, we need to define the probability of collisions. In our context, fault masking (FEP) happens when the expected and faulty input sequences produce the same sequence β of output actions. Besides, if given an FSM M and $k > 0$ we have that there exist $\beta \in \texttt{image}_{M,k}$ such that $\alpha, \alpha' \in f^{-1}_{M,k}(\beta)$, with $\alpha \neq \alpha'$, then there is a collision and this might hide a fault. Next we provide a notion to compute the probability of having a collision.

Definition 9 *Let M be an* FSM *and $k > 0$. Let $\texttt{image}_{M,k} = \{\beta_1, ..., \beta_n\}$ and for all $1 \leq i \leq n$ let $I_i = f^{-1}_{M,k}(\beta_i)$ and $m_i = |f^{-1}_{M,k}(\beta_i)|$. We have that $d = \sum_{i=1}^{n} m_i$ is the size of the input space.*

Given a uniform distribution over the inputs, the probability of α and α' belonging to the same set I_i is equal to $p_i = \frac{m_i \cdot (m_i - 1)}{d \cdot (d-1)}$. We have that the probability of having a collision in M for sequences of length k, denoted by $\texttt{PColl}_k(M)$, is given by

$$\texttt{PColl}_k(M) = \sum_{i=1}^{n} \frac{m_i \cdot (m_i - 1)}{d \cdot (d-1)}$$

The original work (Clark and Hierons, 2012) states that this can be seen as a probability of collisions when the probability distribution over the inputs

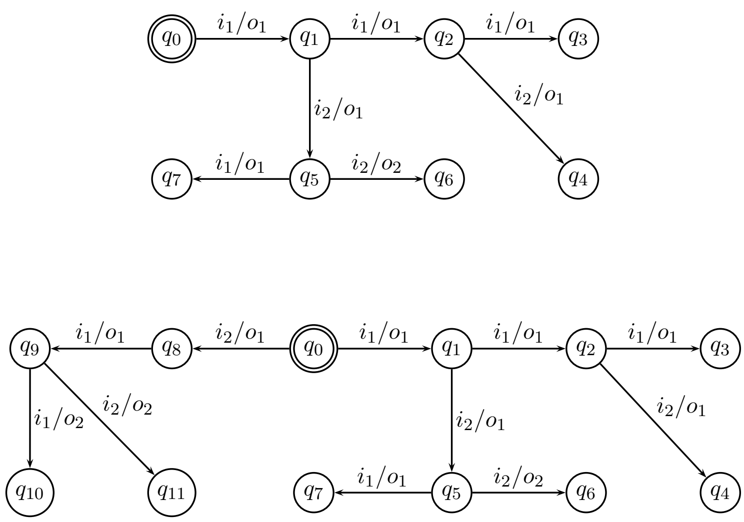

Figure 3.3: Machines M_5 (up) and M_6 (down)

is uniform, but the relationship between $\texttt{PColl}_k(M)$ and $\texttt{PSq}_k(M)$ is not, in general, monotonic, and thus it is neither between $\texttt{PColl}_k(M)$ and $\texttt{Sq}_k(M)$ (the proof that the relationship between $\texttt{PColl}_k(M)$ and $\texttt{Sq}_k(M)$ is not, in general, monotonic is in the original work Clark and Hierons (2012)).

Lemma 5 *There exist* FSMs M_1 *and* M_2 *and* $k > 0$ *such that* $\texttt{PSq}_k(M_1) < \texttt{PSq}_k(M_2)$ *but* $\texttt{PColl}_k(M_1) > \texttt{PColl}_k(M_2)$*.*

Proof
First, let us note again that, similar to the proof of Lemma 4, in this proof we assume uniform distributions over inputs (and outputs) of the FSMs. Again, if we have a different probability distribution then we only need to adapt the definition of the machines so that the result still holds.

First, we consider M_5 with initial state q_0, the machine from the Figure 3.3 (up). Second, let M_6, again with initial state q_0, be the machine from Figure 3.3 (down). On one hand we have $\texttt{PColl}_3(M_5) = 0.5$ and $\texttt{PColl}_3(M_6) = 0.4$ while, on the other hand, we have $\texttt{PSq}_3(M_5) = 0.75$ and $\texttt{PSq}_3(M_6) = 1$. □

3.3 The Simulation

In order to compare PColl, PSq, Sq and DRR we made a simulation. We defined the four measures assuming uniform distributions over the inputs, in terms of the sizes of the subdomains ($f^{-1}_{M,k}(\beta)$). Our methodology to perform simulations followed the approach used in the original work on

Squeeziness (Clark and Hierons, 2012). This methodology consist on simulate an FSM by setting some parameters about the I/O correspondence of the FSM. The firs parameter to be settled is the size of the input space (that we will denote by d), that is, the number of inputs for the simulated FSM. The second parameter we have to set is the maximum inverse domain size (that we will denote by m), that is, the maximum number of inputs that can lead to a same output. Then, we can set the fundamental parameters, the size of the inverse domains of the outputs of the simulated FSM. In order to do so we generate random integers between 1 and m until the values summed to d, that is, we generated outputs with an inverse domain of a random size (between 1 and m) until each input is on the inverse domain of an output. Once we have those inverse domains, we can compute the four measures over the simulated FSM.

This process was repeated 200 times for each pair (d, m), with 149 pairs being used (d ranging between 10 and $2 \cdot 10^9$ and m ranging between 5 and 10^4). Then we computed the Pearson correlation coefficient between `PColl` and the other three measures. Actually, we also computed the Spearman Rank correlation coefficient, but the results where almost identical, so we will not comment about these correlation coefficients.

For each pair (d, m) we performed this process twice. The complete results can be seen in Appendix A.1. Our main conclusion is that there is a relatively strong correlation between `PColl` and `PSq`, with all of the values being greater than 0.89 for *big* sets, but getting lower correlations (with a minimum of 0.37) for the smallest sets. Actually, the values are bigger than 0.96 for input sets with $5 \cdot 10^6$ or more elements. Moveover, we also obtained a correlation bigger than 0.96 between `PColl` and `Sq`, similar to the one obtained in Clark and Hierons (2012).

On the contrary, we obtained a not so strong correlation between `PColl` and `DRR`, with all correlations being between 0.91 and 0.60. Interestingly enough, in contrast to the case of `PSq` and `Sq`, the correlations deteriorate when the size of the input space increases. This shows that this measure is not so good at detecting fault masking, although it is certainly easy to compute.

As a final comment, it is worth noting that standard Squeeziness has a better correlation than Probabilistic Squeeziness. This situation is created by the normalization that transforms Squeeziness into a probability measure. However, Probabilistic Squeeziness can be more useful than standard Squeeziness because it gives a fixed and bounded set of values that can be easily compared because we know that all of them belong to the interval $[0, 1]$. This advantage is achieved with a small additional computational cost because only few computations are needed to transform Squeeziness into a probability measure.

Chapter 4

Experiments

No amount of experimentation can ever prove me right; a single experiment can prove me wrong.

Albert Einstein

In this chapter we explain all the experiments we did in order to assess the usefulness of our measures and discuss the results. In the previous chapter we concluded that both Squeeziness and its probabilistic version are related to the probability of fault masking. However, our study has an obvious limitation: we considered correlations in the context of *big* sets of inputs, with their respective maximum partition values, under a uniform distribution. Therefore, the question remains as to whether the results are similar if we consider finite states machines, not having such a *symmetric* behaviour. In a first step we evaluated our measures on 50 randomly generated FSMs, having between 25 and 50 states, and we used different scenarios. Then, we realized some problems of this approach and therefore evaluated our measures on 500 randomly generated *input-enabled* FSMs with 25 states each one. We have not considered bigger FSMs because of resources limitation, in terms of computational power and memory limits, but these relatively small FSMs allowed us to extract relevant conclusions about our measures.

The rest of the chapter is structured as follows. In Section 4.1 we explain our tool to generate random FSMs. In Sections 4.2, 4.3, 4.4 and 4.5 we explain our four experiments, what we wanted to prove, the results that we got, and our conclusions. Finally, Section 4.6 presents an overview of the results of the experiments.

4.1 FSM Generator

In order to perform our experiments we need to generate FSMs. In order to do so, we developed an FSM generator that generates random FSMs given some parameters.

The first issue we solved was to fix the internal representation of FSMs. Since our work is not the first dealing with FSMs we decided to review the literature and found the OpenFST library (Allauzen et al., 2007). This library is intended to work with FSTs (*Finite State Transducers*) (as its name indicates). These are a kind of FSMs with an input/output par in each transition and a weight. Therefore, we simply ignore the weight. This library also provides some shell commands that we can use, in particular, to generate the associated binary files and to generate the topological representation of each FSM as an image.

Once we have a proper representation for our FSMs, we developed the tool for generating those FSMs. An important part of this tool has as goal to generate a huge range of different FSMs fulfilling some specific properties. In order to do so, we defined some basic parameters:

- $NREP$: the number of FSMs we want to generate.
- MAX_STATES: the maximum number of states an FSM can have.
- MIN_STATES: the minimum number of states an FSM must have.
- $MAX\_TRANSITIONS$: the maximum number of transitions each state of an FSM can have.
- $MIN\_TRANSITIONS$: the minimum number of transitions each state of an FSM must have.
- $NINPUTS$: the number of inputs.
- $NOUTPUTS$: the number of inputs.

After setting these basic parameters, the program can be executed. The execution flow for each one of the $NREP$ FSMs is:

- Create a folder to save the FSM files.
- Set a random number of states between MIN_STATES and MAX_STATES for the FSM.
- Choose one of this states as initial state.
- For each state of the machine:
  - Set a random number of transitions between $MIN\_TRANSITIONS$ and $MAX\_TRANSITIONS$ for the state.

  - For each transition of the machine:
    * Set a random state as an end of the transition.
    * Set a random input label for the transition not previously used for another transition of this state (so the FSMs are deterministic).
    * Set a random output label for the transition.
    * Save this transition to the FSM file.

- Create the binary file that the OpenFST library uses to interpret FSTs using the FSM file we created.

- Create a pdf image with the FSM topology.

In order to create input-enabled FSMs, in our tool is as simple as setting $MIN\_TRANSITIONS = MAX\_TRANSITIONS = NINPUTS$.

4.2 First Experiment: Squeeziness vs. location of FEP

Our first conjecture was that the Squeeziness of sequences might tell us something about where a fault masked by FEP is likely to be. In order to test it, we develop the following experiment. For each FSM M we computed `Sq` and `PSq` using all sequences of input actions of length k for $1 \leq k \leq 25$. Then, we mutated M by modifying the ending state of a randomly chosen transition, that is, we induced a *transfer error*[1]. Using a *mutation test*[2] approach as in Clark and Hierons (2012), we checked whether the mutant exhibited FEP. We iterated the process until we had a total of 100 valid mutants of M presenting FEP. Given a mutant M', we executed the input sequences of length 25 on M' until we used an input sequence that executed the faulty transition; the position ℓ of the faulty transition within the sequence was said to be the *position* of the fault. We then computed the rank correlation between the FEP for sequences of length k and the number of mutants that had score $k-1$ (i.e. whose faulty transition was first executed in position $k-1$).

We ran the previous procedure twice and obtained similar results: if we consider non-trivial FSMs then there is no correlation between where the fault is produced and the Squeeziness and Probabilistic Squeeziness obtained for the length of the input sequence reaching the mutated transition. This

[1] We did not consider mutations that change the input or the output of a transition, or the initial state of a transition because these mutations are easier to find in testing and generate less FEP.

[2] The interested reader is referred to previous work (Hierons et al., 2010) on mutation testing for additional details.

(negative) result suggests that we cannot use the Squeeziness of input sequences of different lengths to determine the likely *position* of a fault. This also shows that computing Squeeziness for sequences of length k when there are sequences of length $l > k$ could lead us to probabilities of FEP in the FSM that are useless as the FEP produced for length k can be *solved* with a sequence of length l by the detection of an invalid output[3]. In this way, whenever we use Squeeziness to compute the probability of having FEP during testing, we have to compute Squeeziness for sequences of the maximum length we will test in order to get a proper measure of having FEP in the test. In Table 4.1 we present the results for two FSMs showing the announced lack of correlation. The data shows both runs of the experiment. Similar values are obtained for the 50 FSMs.

FSM	Number of states	Pearson PSq	Spearman PSq	Pearson Sq	Spearman Sq
$\mathbf{M_{15}}$	35	−0.0421031	−0.0601407	−0.0421031	−0.0601407
$\mathbf{M_{15}}$	35	−0.0429902	−0.0601407	−0.0429902	−0.0601407
$\mathbf{M_{19}}$	25	−0.0434405	−0.0601407	−0.0434405	−0.0601407
$\mathbf{M_{19}}$	25	−0.0421031	−0.0601407	−0.0421031	−0.0601407

Table 4.1: Table with some results from the first experiment.

4.3 Second Experiment: Squeeziness vs. probability of FEP in mutants

Our second experiment studied whether Squeeziness appropriately *predicts* the probability of having FEP in a mutant version of the FSM. We started with the same 50 FSMs. In this case, we computed Squeeziness and Probabilistic Squeeziness for sequences of length 15 for 10 machines. Then, we mutated the machines by modifying, again, the state reached by one of the transitions. We produced 1000 valid mutants for each machine (we considered both mutants with and without FEP) and computed the number of mutants having FEP. Finally, we computed the Pearson and Spearman correlations between the number of mutants having FEP produced for sequences of inputs of length equal to 15 and both Squeeziness and Probabilistic Squeeziness for the 10 FSMs. We also performed all the experiments twice.

Interestingly, we obtained again almost no correlation. This fact tell us that both Squeeziness and Probabilistic Squeeziness have no correlation with the probability of an FEP being produced in their mutants, even given that

[3]This is due to the fact that we are using deterministic FSMs, as we have previously discussed.

they are similar FSMs, but not the same. This is an important result, as it shows us that we cannot rely on the Squeeziness values for the specification FSM when testing, as it will be a value that has no correlation with the SUT. This is due to that in the moment the SUT has a modification in their input/output behaviour from the specification FSM, the Squeeziness of this SUT has changed and so the probability of FEP. That will make harder to use Squeeziness for testing, as we cannot rely on the specification. In Table 4.2 we present the results for four FSMs showing the announced lack of correlation. The data shows both runs of the experiment. Similar values are obtained for the 50 FSMs.

Test Number	**Pearson** PSq	**Spearman** PSq	**Pearson** Sq	**Spearman** Sq
7	−0.189714	0.329743	−0.189714	0.329743
7	−0.186373	0.344158	−0.186373	0.344158
12	−0.62552	−0.974084	−0.62552	−0.974084
12	−0.616673	−0.971429	−0.616673	−0.971429
25	−0.162697	−0.287494	−0.162697	−0.287494
25	−0.162697	−0.287494	−0.162697	−0.287494
37	−0.369106	−0.567857	−0.369106	−0.567857
37	−0.362629	−0.589813	−0.362629	−0.589813

Table 4.2: Table with some results from the second experiment.

4.4 Third Experiment: Squeeziness vs. probability of FEP in the original FSM

Our first conjectures were too ambitious. Once we obtained both negative results (given by a lack of correlation between the studied events) we decided to check whether Squeeziness appropriately *predicts* the probability of having FEP on the same machine. We started with the same 50 FSMs. In this case, we computed Squeeziness and Probabilistic Squeeziness over the mutants. First, we mutated the FSMs by modifying, again, the state reached by one of the transitions. We produced 10 valid mutants (we considered both mutants with and without FEP). Then, we computed the probability of an FEP to be produced and the Squeeziness and Probabilistic Squeeziness of input sequences of length 20. Finally, we computed the Pearson and Spearman correlations between the probability of the mutants of having FEP produced for sequences of inputs of length equal to 20 and both Squeeziness and Probabilistic Squeeziness for each mutant. We perform all this experiment twice for each FSM. In order to compute the probability of producing an FEP for an input and a mutant, we computed all the possible inputs (and outputs) and counted how many did/did not detect the mutation. We used the following formula (Androutsopoulos et al., 2014):

$$p(FEP) = \frac{\text{\# tests reaching the wrong state but generating the correct output}}{\text{\# tests reaching the wrong state}}$$

Again, we got almost no correlation between both measures and the probability of FEP, what leads us to think about the measure we are comparing to. In Table 4.3 we present the results for four FSMs showing the announced lack of correlation. The data shows both runs of the experiment. Similar values are obtained for the 50 FSMs.

FSM	**Numberof States**	**Pearson** PSq	**Spearman** PSq	**Pearson** Sq	**Spearman** Sq
M_1	43	0.327585	0.223298	0.256473	0.223298
M_1	43	−0.278164	−0.190777	−0.227735	−0.276935
M_{17}	40	0.593344	0.69765	0.332521	0.555651
M_{17}	40	−0.415348	−0.0747667	−0.400539	−0.0747667
M_{21}	28	−0.121012	0.205656	−0.286948	−0.0311601
M_{21}	28	−0.512128	−0.596973	−0.417908	−0.596973
M_{47}	25	0.340424	0.237809	0.411806	0.310981
M_{47}	25	−0.525826	−0.647114	−0.0576659	−0.241851

Table 4.3: Table with some results from the third experiment.

4.5 Fourth Experiment: Squeeziness vs. Probability of Collisions

For our last experiment, we analysed what we were comparing in our previous experiments. In those experiments we compared two measures. The first one was related to Squeeziness and it was computed on one FSM (in some experiments it was the SUT while in others was the specification). The other measure was the probability of FEP. This measure computes a certain relation between the specification and the SUT, using the following formula Androutsopoulos et al. (2014):

$$p(FEP) = \frac{\text{\# tests reaching the wrong state but generating the correct output}}{\text{\# tests reaching the wrong state}}$$

The problem is that we were comparing a measure obtained from one FSM with another measure obtained from two. Consider, for example, that we have one specification and n mutants. Our previous experiments were trying to correlate one value with different values and the results were bad, showing a low correlation. Therefore, we thought that we should compare Squeeziness and a value that depends only on the FSM that we are using to compute Squeeziness. The other option would be to compare to probability of FEP, but computing it over all the possible SUTs, for a given specification, what is only feasible for small specifications, even assuming that each faulty SUT has at most one error. We concluded that we might consider the same measure that we used in Chapter 3:

$$\texttt{PColl}_k(M) = \sum_{j=1}^{n} \frac{m_j \cdot (m_j - 1)}{d \cdot (d-1)}$$

where m_j is the cardinal of the inverse image of the j-th output (i.e. the number of inputs that lead to this output) and d is the cardinal of the inputs (i.e. the total number of inputs). The only drawback of this measure is that it can be applied only to input-enabled FSMs. Therefore, the first step is to generate a new set of (input-enabled) FSMs.

We generated 500 machines with 25 states and 5 outgoing transitions from each state (making them input-enabled). Then, we computed Squeeziness, Probabilistic Squeeziness and PColl for each FSM. The next step was to compute the correlation between the results for 10 different machines for Squeeziness and PColl, and Probabilistic Squeeziness and PColl. Due to memory limits, we computed these measures for input sequences of length 8. This number is certainly low for a proper experiment, but it is the highest that we could achieve with our current computation setting. We repeat this experiment twice for each block of 10 FSMs.

This time we obtain positive results concerning correlation but there are some *downsides*. The results show a correlation between 0.7 and 1 for most of the cases of Squeeziness vs. PColl, with similar values for Pearson and Spearman correlations (again, in most of the cases). This shows that our simulation was not only useful as a theory reinforcement, but also that it is close to the results for real FSMs. Actually, we observe a similar pattern between the results of this experiment and the experiments reported in Chapter 3. So, it is safe to assume that the correlations will increase for bigger FSMs and that the bad correlation results are due to the limited size of the input sequences length (what limits the total number of considered inputs). Unfortunately, the results for Probabilistic Squeeziness are really bad, showing a lack of correlation. However, due to the relative correspondence between the results of this experiment and the ones in Chapter 3, we can also assume that these bad results are due to the small size of the experiment. We expect that the correlation will increase for bigger FSMs. After all, for this small experiment we obtained good correlation for a reduced number of cases.

The results of our fourth experiment can be found in three tables given in Appendix A.2.

4.5.1 Application Scope

The previous measures were computed on the same FSM and this fact raised another research question. Specifically, what does happen if we compute these values on the specification and on a *slightly different* SUT? In order to reach a conclusion, we developed the following additional experiment:

1. Consider an FSM (the specification).
2. Generate 10 valid mutants and compute Squeeziness, Probabilistic Squeeziness and PColl for each of them. We obtain three vectors $\overrightarrow{Sq}$, $\overrightarrow{PSq}$ and $\overrightarrow{PColl}$.
3. Compute the means of the values in each vector: $\overline{Sq}$, $\overline{PSq}$ and $\overline{PColl}$.
4. Subtract from each value of the vectors their corresponding mean and obtain new vectors $\overrightarrow{Sq_1}$, $\overrightarrow{PSq_1}$ and $\overrightarrow{PColl_1}$.
5. Compute the norm of the new vectors and divide by the mean of the measure.
$$\frac{\left\|\overrightarrow{Sq_1}\right\|}{\overline{Sq}}, \quad \frac{\left\|\overrightarrow{PSq_1}\right\|}{\overline{PSq}} \text{ and } \frac{\left\|\overrightarrow{PColl_1}\right\|}{\overline{PColl}}$$
This value comprises the *deviation* among the different values of the measures of each set of mutants.

We did this twice with all the FSMs that we used in the fourth experiment and although most of the results were around 10% or less, some of them were up to 60% for PColl and up to 20% for Squeeziness and Probabilistic Squeeziness. These results lead us to reinforce the idea, already deduced from the first experiments, that a small deviation from the specification in the SUT can lead to totally different values for Squeeziness and the other measures.

In Table 4.4 we present the most interesting results (the highest and lowest ones). The notation M_x^y denotes the y-th experiment on the x-th machine of the set. The complete 1000 results (2 for each FSM) of this experiment can be found in Appendix A.3.

4.6 Concluding remarks

Our experiments show that Squeezines and Probabilistic Squeeziness correlate with the probability of having a collision (and therefore of having a case of FEP) when testing from an FSM. However, and this is an interesting (unfortunately negative) result, these measures are not useful when computed over the FSM specification. In this case they are useless because the potential differences between the specification and the SUT can lead to

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{19}	0.330245	0.0437058	0.143475
M^2_{19}	0.092328	0.0567495	0.0497149
M^1_{54}	0.154943	0.0929899	0.0425949
M^2_{54}	0.512515	0.0997374	0.204219
M^1_{178}	0.502121	0.133631	0.206008
M^2_{178}	0.0501833	0.0805334	0.0581454
M^1_{193}	0.11518	0.050467	0.0379082
M^2_{193}	0.319289	0.210992	0.0788463
M^1_{195}	0.615883	0.0610384	0.191272
M^2_{195}	0.171396	0.0678891	0.0736381
M^1_{217}	0.0730744	0.12794	0.0280366
M^2_{217}	0.0332924	0.00887374	0.00887374
M^1_{307}	0.369638	0.240243	0.155753
M^2_{307}	0.322205	0.154143	0.130664
M^1_{351}	0.0836284	0.0349514	0.0481058
M^2_{351}	0.550097	0.048125	0.212203
M^1_{373}	0.331589	0.103725	0.106278
M^2_{373}	0.531864	0.212388	0.186662
M^1_{411}	0.113319	0.00985007	0.00705992
M^2_{411}	0.0943745	0.0301996	0.0301996
M^1_{435}	0.391234	0.106289	0.0983797
M^2_{435}	0.619814	0.0821379	0.144467
M^1_{447}	0.129423	0.0995404	0.0684297
M^2_{447}	0.179715	0.202542	0.0483879

Table 4.4: Table with the most interesting results from the scope experiment.

different values for our two measures. Therefore, our experiments show that our measures are valid only if computed over the SUT that we want to test. Actually, this is a good result because we only need to apply sequences of inputs to the SUT, observe the produced sequences of outputs, and compute our measures. Note that despite the fact the SUT is a black box, and therefore we do not have access to its internal structure, we can always apply inputs and observe outputs. The specification will be used, during this process, to provide the input domain that we will use to compute our measures. In addition, the specification will be used, as usual, as an *oracle* to decide whether the observed outputs are the expected ones.

Chapter 5

Conclusions

Every story has an end, but in life every end is just a new beginning.

Annonymous

In this chapter we summarize the obtained results and discuss how we can use our measures to test from systems whose underlying structure can be given by an FSM.

The rest of the chapter is structured as follows. In Section 5.1 we discuss the results of the experiments and what they imply. In Section 5.2 we discuss the practical uses of our measures after considering the results of the experiments. Finally, in Section 5.3 we overview our contribution and suggest some lines for future work.

5.1 Results

Our experiments have validated only one of our hypothesis: Squeeziness and Probabilistic Squeeziness give us an estimation of the probability of having a collision in a given SUT, indicating a probability of having a case of FEP. Unfortunately, these measures cannot give us direct information about the location of an FEP (what we wanted to check with our first experiment). This negative result makes sense because the measures are not monotonous with respect to k (the length of the sequence of inputs that we exercise). Therefore, due to this property, one can find really high values for one fixed length k (this is supposed to tell us that FEP is very likely) but lower values can appear when testing with sequences of length $k + 1$. Another conclusion that we obtain from the results is that the specification of the SUT is not useful when talking about Squeeziness and Probabilistic Squeeziness. This is so because any change in the input/output behaviour of the specification changes the correlation between the measures and the real probability of having FEP in the SUT. This implies that these measures have to be computed over the SUT itself. Actually, this makes sense given that the measures basically rely on the input/output behaviour of an FSM. Therefore, changes over the FSM, even the smallest ones, modify the values of the measures. This will *decorrelate* the values measured over the specification FSM and the real probability of having an FEP in the SUT. Finally, we get a positive result when we consider the correlation between the probability of having an FEP in a SUT and the Squeeziness and Probabilistic Squeeziness over this SUT. This implies that, although the three initial hypothesis do not hold, our measures do not only work in the theoretical plane (as the simulation performed in Chapter 3 suggests) but also work in the practical plane, when we use them to determine the likelihood of having FEP in an SUT whose underlying internal structure is given by an FSM.

Our experiments also show that our measures are useful to decide how testable the SUT is. This fact reinforces the idea that it makes sense to compute these measures over the SUT itself. Note that after the use of our measures to obtain an estimation of the likelihood of FEP, the black-box testing process should make a proper testing that involves the specification, in particular, to use it as an oracle (as we have already said, the specification also plays a role in the computation of our measures because it provides the input domain).

5.2 Practical Uses

Next we briefly discuss the practical use of our measures. In order to avoid redundancy, we will focus on Probabilistic Squeeziness (a similar discussion applies to Squeeziness, although it has the drawback of not being a probability measure).

One possible use of Probabilistic Squeeziness is to guide the process of finding good input sequence lengths. Specifically, before running tests one might calculate the Probabilistic Squeeziness for input sequences of length k, varying k in a certain range. The resultant values could then be used to choose a length that has a relatively low Probabilistic Squeeziness value over the range of acceptable lengths. This makes it less likely that FEP will affect testing. A special case appears when we find a length for which Probabilistic Squeeziness is equal to zero: we can use this length as a *checkpoint*. The idea is that we should use all the possible input sequences of this length to test the SUT in order to know whether there are faults in this part of the program. Note, however, that Probabilistic Squeeziness is not monotonic (and, therefore, we might consider multiple checkpoints).

Another possible use, but this needs further work from the theoretical point of view and experimentation to validate the hypothesis, is to use our measures when defining the specification of the system. Intuitively, if we compute Probabilistic Squeeziness on the specification, then we can (re)define it in a way that we get the lowest Probabilistic Squeeziness possible, ideally 0, without drastically modifying its expected behaviour. Therefore, any correct SUT should have almost no FEP. Although for simpler cases it is easier to just make a 1 to 1 correspondence between inputs and outputs, for complex cases this might not be easily achieved. Thus, producing specifications with low Probabilistic Squeeziness values can be a good way to assure a low number of FEP in the implementation.

Finally,note that Probabilistic Squeeziness is a probability measure that aims to estimate the probability of FEP when testing over the FSM with input sequences of a certain length k. Since this is a probability measure, so is its inverse: $1 - PSq$. This derived measure is interesting because it gives us the *reliability* of a test in the sense that it represents the probability that a correct output denotes that no fault has being executed.

5.3 Final Considerations

It is known that failed error propagation (FEP) can have a significant effect on testing and recent work has shown that an Information Theoretic measure (called Squeeziness) strongly correlates with the likelihood of FEP. This work considered a white-box scenario in which the SUT simply receives input and returns output; there is no persistent state. In our work we have adapted Squeeziness to work with black-box scenarios in which we are interested in fault masking. Having devised new notions of Squeeziness, for black-box state-based systems, we carried out experiments in order to evaluate these measures. We found that there is a strong correlation between the likelihood of collisions (and therefore the likelihood of having a case of FEP) and our two measures (Squeeziness and Probabilistic Squeeziness).

The results in this thesis have two potential uses. First, our measures might be used as measures of testability, allowing one to assess how easy it is to test a system or part of a system. This might be used as part of the process of deciding how much testing is required. In addition, there is potential to use these measures to direct testing. For example, we might want to execute a part of the system with a test case where the probability of FEP (following this component) is relatively low. Future work will have to explore these potential uses, develop tools, and evaluate these on case studies. Also it will have to generalise the framework and measures to introduce data into the models.

Part II

Appendices

Appendix A

Results

A.1 Simulation Results

Here are the raw results from the simulation explained in Section 3.3.

Input set lenght	**Maximum size**	**Correlation of** `PSq`	**Correlation of** `Sq`	**Correlation of** `DRR`
10	5	0.722419	0.982492	0.912124
10	5	0.725592	0.980483	0.90151
10	10	0.383566	0.962152	0.760265
10	10	0.472533	0.956568	0.743107
20	5	0.773518	0.974509	0.870086
20	5	0.858893	0.979125	0.897943
20	10	0.605163	0.967108	0.819643
20	10	0.592223	0.964445	0.81604
50	5	0.983333	0.983333	0.925018
50	5	0.984073	0.984073	0.922599
50	10	0.761072	0.961765	0.786001
50	10	0.816325	0.972962	0.840203
100	10	0.92453	0.97571	0.868372
100	10	0.919776	0.970633	0.857886
100	20	0.729808	0.973755	0.827209
100	20	0.772328	0.965189	0.797126
100	50	0.465251	0.962363	0.741697
100	50	0.549541	0.967853	0.776547
100	100	0.489419	0.968688	0.701156
100	100	0.469358	0.965432	0.628285
200	10	0.973502	0.973502	0.858282
200	10	0.978305	0.978305	0.883113
200	20	0.850969	0.965564	0.791423
200	20	0.831395	0.966462	0.787118
200	50	0.661863	0.971746	0.806116
200	50	0.697791	0.971603	0.795561
200	100	0.468793	0.958421	0.721154
200	100	0.503831	0.968021	0.771041
500	10	0.967251	0.967251	0.818355
500	10	0.96825	0.96825	0.828994
500	20	0.96093	0.98084	0.860986
500	20	0.949305	0.969495	0.795557
500	50	0.860276	0.965889	0.764926
500	50	0.841532	0.970989	0.785379
500	100	0.776246	0.972638	0.802795
500	100	0.778458	0.97467	0.80358

Table A.1: Table with the first part of the results from the simulation.

Input set lenght	**Maximum size**	**Correlation of** PSq	**Correlation of** Sq	**Correlation of** DRR
1000	100	0.820004	0.968629	0.770524
1000	100	0.875756	0.974068	0.809386
1000	200	0.716649	0.970774	0.771857
1000	200	0.746114	0.972044	0.819073
1000	500	0.448179	0.968221	0.762177
1000	500	0.476338	0.965733	0.792384
1000	1000	0.374508	0.965005	0.611607
1000	1000	0.373547	0.964423	0.60093
2000	100	0.890084	0.970319	0.765819
2000	100	0.890027	0.969404	0.783833
2000	200	0.846704	0.974528	0.808135
2000	200	0.837518	0.971235	0.784891
2000	500	0.615086	0.975174	0.790542
2000	500	0.633549	0.965611	0.778344
2000	1000	0.506566	0.968637	0.727308
2000	1000	0.457672	0.96406	0.730815
5000	100	0.952364	0.973766	0.808192
5000	100	0.943738	0.973478	0.808059
5000	200	0.912783	0.968631	0.761656
5000	200	0.905308	0.97061	0.777158
5000	500	0.834572	0.979689	0.842386
5000	500	0.744812	0.967755	0.763411
5000	1000	0.7261	0.96982	0.782368
5000	1000	0.71995	0.966707	0.747713

Table A.2: Table with the second part of the results from the simulation.

Input set lenght	**Maximum size**	**Correlation of** `PSq`	**Correlation of** `Sq`	**Correlation of** `DRR`
10000	100	0.958346	0.968366	0.763623
10000	100	0.961652	0.973918	0.783759
10000	200	0.950883	0.973016	0.823959
10000	200	0.926334	0.967349	0.77492
10000	500	0.898466	0.973849	0.828911
10000	500	0.898056	0.973267	0.803445
10000	1000	0.835717	0.963235	0.744021
10000	1000	0.832637	0.973658	0.804282
10000	2000	0.729078	0.969764	0.787121
10000	2000	0.696597	0.966409	0.753929
10000	5000	0.469301	0.968639	0.778538
10000	5000	0.471505	0.968937	0.768205
10000	10000	0.427412	0.967281	0.71496
10000	10000	0.470961	0.966184	0.670497
20000	100	0.967166	0.969669	0.780648
20000	100	0.97366	0.975364	0.824959
20000	200	0.95245	0.969587	0.778449
20000	200	0.956955	0.971707	0.771526
20000	500	0.942709	0.971942	0.798243
20000	500	0.936519	0.974174	0.792043
20000	1000	0.880857	0.971248	0.786153
20000	1000	0.910427	0.967574	0.769014
20000	2000	0.861626	0.967758	0.770978
20000	2000	0.858335	0.975119	0.819613
20000	5000	0.709371	0.972733	0.823052
20000	5000	0.674096	0.970411	0.780216
20000	10000	0.415143	0.960576	0.728561
20000	10000	0.515837	0.961688	0.724022
50000	100	0.963278	0.963278	0.74568
50000	100	0.975731	0.975731	0.817716
50000	200	0.968476	0.974623	0.795574
50000	200	0.967934	0.969418	0.746002
50000	500	0.954561	0.966153	0.777624
50000	500	0.961281	0.975947	0.84295
50000	1000	0.945005	0.967855	0.76079
50000	1000	0.923576	0.967894	0.789061
50000	2000	0.880547	0.96735	0.764992
50000	2000	0.91738	0.969433	0.804356
50000	5000	0.850672	0.97278	0.797072
50000	5000	0.849898	0.971647	0.792316
50000	10000	0.721962	0.970928	0.779042
50000	10000	0.654239	0.963673	0.723346

Table A.3: Table with the third part of the results from the simulation.

Input set lenght	**Maximum size**	**Correlation of** PSq	**Correlation of** Sq	**Correlation of** DRR
100000	100	0.97475	0.97475	0.797906
100000	100	0.972203	0.972203	0.799384
100000	200	0.972457	0.972457	0.788938
100000	200	0.96889	0.969988	0.78341
100000	500	0.972534	0.980028	0.836659
100000	500	0.96534	0.972878	0.769055
100000	1000	0.955834	0.976104	0.817482
100000	1000	0.963025	0.974571	0.820023
100000	2000	0.929032	0.971424	0.779667
100000	2000	0.949671	0.975182	0.787567
100000	5000	0.8751	0.96594	0.762143
100000	5000	0.860949	0.96303	0.73042
100000	10000	0.878154	0.970134	0.757703
100000	10000	0.818777	0.96836	0.778925
200000	100	0.970841	0.970841	0.801076
200000	100	0.974049	0.974049	0.798232
200000	200	0.971829	0.971829	0.776558
200000	200	0.973847	0.973847	0.79645
200000	500	0.976077	0.978293	0.822944
200000	500	0.962437	0.96523	0.748004
200000	1000	0.962282	0.968757	0.768184
200000	1000	0.961139	0.972733	0.808345
200000	2000	0.951086	0.971834	0.798966
200000	2000	0.948132	0.969003	0.749107
200000	5000	0.930642	0.970825	0.760313
200000	5000	0.913568	0.969484	0.76873
200000	10000	0.893991	0.970044	0.792676
200000	10000	0.91019	0.972554	0.788373
500000	100	0.97668	0.97668	0.836037
500000	100	0.977493	0.977493	0.809851
500000	200	0.963671	0.963671	0.743951
500000	200	0.974121	0.974121	0.807426
500000	500	0.971054	0.971647	0.774395
500000	500	0.973503	0.973467	0.800447
500000	1000	0.972197	0.976121	0.820915
500000	1000	0.967381	0.97081	0.769445
500000	2000	0.967251	0.976695	0.803875
500000	2000	0.967249	0.973124	0.787502
500000	5000	0.926782	0.95885	0.743651
500000	5000	0.949692	0.969437	0.765643
500000	10000	0.937438	0.971292	0.786862
500000	10000	0.957172	0.975993	0.819747

Table A.4: Table with the fourth part of the results from the simulation.

Input set lenght	Maximum size	Correlation of PSq	Correlation of Sq	Correlation of DRR
1000000	100	0.976936	0.976936	0.811867
1000000	100	0.971048	0.971048	0.775681
1000000	200	0.970973	0.970973	0.782711
1000000	200	0.977552	0.977552	0.839242
1000000	500	0.972066	0.972066	0.783899
1000000	500	0.974367	0.974367	0.770392
1000000	1000	0.973512	0.973926	0.79526
1000000	1000	0.97265	0.974027	0.830407
1000000	2000	0.967017	0.969736	0.780849
1000000	2000	0.972468	0.97408	0.805192
1000000	5000	0.964892	0.970854	0.809975
1000000	5000	0.959633	0.970388	0.787131
1000000	10000	0.948137	0.967924	0.778203
1000000	10000	0.953897	0.970411	0.769844
2000000	100	0.975097	0.975097	0.814434
2000000	100	0.968371	0.968371	0.768775
2000000	200	0.974395	0.974395	0.809679
2000000	200	0.97463	0.97463	0.800698
2000000	500	0.97177	0.97177	0.790358
2000000	500	0.970945	0.970945	0.809109
2000000	1000	0.978243	0.978102	0.826712
2000000	1000	0.971267	0.971722	0.810432
2000000	2000	0.967518	0.969418	0.755382
2000000	2000	0.968874	0.970523	0.779241
2000000	5000	0.971917	0.978818	0.810967
2000000	5000	0.958095	0.964455	0.698505
2000000	10000	0.962677	0.96991	0.776906
2000000	10000	0.9506	0.963563	0.781282
5000000	100	0.971105	0.971105	0.801428
5000000	100	0.975811	0.975811	0.806359
5000000	200	0.965705	0.965705	0.734183
5000000	200	0.975194	0.975194	0.787636
5000000	500	0.965762	0.965762	0.78538
5000000	500	0.977868	0.977868	0.816896
5000000	1000	0.970797	0.970797	0.782857
5000000	1000	0.974245	0.974245	0.807752
5000000	2000	0.973331	0.973636	0.783586
5000000	2000	0.973119	0.972639	0.782383
5000000	5000	0.976515	0.977712	0.793327
5000000	5000	0.961413	0.963994	0.708333
5000000	10000	0.97076	0.972559	0.773815
5000000	10000	0.972016	0.975021	0.788634

Table A.5: Table with the fifth part of the results from the simulation.

Input set lenght	**Maximum size**	**Correlation of** PSq	**Correlation of** Sq	**Correlation of** DRR
10000000	100	0.972085	0.972085	0.801643
10000000	100	0.96267	0.96267	0.74051
10000000	200	0.973476	0.973476	0.814127
10000000	200	0.978724	0.978724	0.817254
10000000	500	0.968369	0.968369	0.755809
10000000	500	0.976646	0.976646	0.784194
10000000	1000	0.97411	0.97411	0.792697
10000000	1000	0.970658	0.970658	0.782375
10000000	2000	0.973721	0.973856	0.793005
10000000	2000	0.974843	0.974945	0.782697
10000000	5000	0.975044	0.975649	0.814614
10000000	5000	0.965039	0.9663	0.780145
10000000	10000	0.97379	0.974921	0.808942
10000000	10000	0.973747	0.974783	0.821714
20000000	100	0.976361	0.976361	0.816832
20000000	100	0.969996	0.969996	0.785402
20000000	200	0.966911	0.966911	0.773231
20000000	200	0.975891	0.975891	0.830111
20000000	500	0.975834	0.975834	0.80509
20000000	500	0.971753	0.971753	0.761665
20000000	1000	0.970692	0.970692	0.800126
20000000	1000	0.972765	0.972765	0.780929
20000000	2000	0.975548	0.975548	0.79739
20000000	2000	0.97661	0.97661	0.790627
20000000	5000	0.975629	0.975512	0.81321
20000000	5000	0.969195	0.969801	0.778989
20000000	10000	0.969188	0.97061	0.79285
20000000	10000	0.974001	0.974807	0.823849
50000000	100	0.972157	0.972157	0.775908
50000000	100	0.97394	0.97394	0.744055
50000000	200	0.977712	0.977712	0.825954
50000000	200	0.964124	0.964124	0.754767
50000000	500	0.976058	0.976058	0.824369
50000000	500	0.971696	0.971696	0.792425
50000000	1000	0.968602	0.968602	0.773925
50000000	1000	0.975643	0.975643	0.813831
50000000	2000	0.972101	0.972101	0.80533
50000000	2000	0.96896	0.96896	0.763188
50000000	5000	0.967291	0.967312	0.733459
50000000	5000	0.971166	0.970914	0.792814
50000000	10000	0.973592	0.974186	0.831489
50000000	10000	0.970302	0.97075	0.794533

Table A.6: Table with the sixth part of the results from the simulation.

Input set lenght	Maximum size	Correlation of PSq	Correlation of Sq	Correlation of DRR
100000000	100	0.967785	0.967785	0.791843
100000000	100	0.973939	0.973939	0.79906
100000000	200	0.970936	0.970936	0.797435
100000000	200	0.971179	0.971179	0.792618
100000000	500	0.965457	0.965457	0.764338
100000000	500	0.967388	0.967388	0.749111
100000000	1000	0.967278	0.967278	0.762974
100000000	1000	0.975128	0.975128	0.816993
100000000	2000	0.976852	0.976852	0.809661
100000000	2000	0.973916	0.973916	0.811798
100000000	5000	0.964856	0.964856	0.752126
100000000	5000	0.975177	0.975177	0.804654
100000000	10000	0.973117	0.97333	0.797859
100000000	10000	0.979059	0.979012	0.839706
200000000	100	0.974298	0.974298	0.793441
200000000	100	0.974201	0.974201	0.817327
200000000	200	0.973198	0.973198	0.79773
200000000	200	0.969628	0.969628	0.752662
200000000	500	0.979169	0.979169	0.843415
200000000	500	0.975039	0.975039	0.830218
200000000	1000	0.975452	0.975452	0.842656
200000000	1000	0.973656	0.973656	0.81612
200000000	2000	0.974498	0.974498	0.799512
200000000	2000	0.980097	0.980097	0.843219
200000000	5000	0.97596	0.97596	0.81765
200000000	5000	0.973072	0.973072	0.794025
200000000	10000	0.972613	0.972525	0.790124
200000000	10000	0.975172	0.975228	0.812101
500000000	100	0.97099	0.97099	0.788888
500000000	100	0.971083	0.971083	0.798639
500000000	200	0.967438	0.967438	0.779869
500000000	200	0.977179	0.977179	0.832308
500000000	500	0.965965	0.965965	0.778361
500000000	500	0.968144	0.968144	0.764191
500000000	1000	0.974112	0.974112	0.800833
500000000	1000	0.973997	0.973997	0.779971
500000000	2000	0.971501	0.971501	0.782711
500000000	2000	0.970228	0.970228	0.743784
500000000	5000	0.976165	0.976165	0.825479
500000000	5000	0.973031	0.973031	0.779755
500000000	10000	0.969547	0.969547	0.772517
500000000	10000	0.966348	0.966348	0.773234

Table A.7: Table with the seventh part of the results from the simulation.

Input set lenght	**Maximum size**	**Correlation of** `PSq`	**Correlation of** `Sq`	**Correlation of** `DRR`
1000000000	100	0.96974	0.96974	0.779927
1000000000	100	0.974667	0.974667	0.824957
1000000000	200	0.978771	0.978771	0.859822
1000000000	200	0.968844	0.968844	0.759952
1000000000	500	0.975528	0.975528	0.799788
1000000000	500	0.972865	0.972865	0.806221
1000000000	1000	0.966998	0.966998	0.742382
1000000000	1000	0.970395	0.970395	0.795114
1000000000	2000	0.96474	0.96474	0.784384
1000000000	2000	0.966843	0.966843	0.768588
1000000000	5000	0.966975	0.966975	0.753142
1000000000	5000	0.969392	0.969392	0.777797
1000000000	10000	0.970387	0.970387	0.78255
1000000000	10000	0.966483	0.966483	0.741448
2000000000	100	0.968286	0.968286	0.797514
2000000000	100	0.974423	0.974423	0.78976
2000000000	200	0.97463	0.97463	0.779878
2000000000	200	0.969308	0.969308	0.776731
2000000000	500	0.97068	0.97068	0.77233
2000000000	500	0.964814	0.964814	0.741365
2000000000	1000	0.977148	0.977148	0.802956
2000000000	1000	0.972999	0.972999	0.824011
2000000000	2000	0.966897	0.966897	0.756296
2000000000	2000	0.967144	0.967144	0.731439
2000000000	5000	0.970575	0.970575	0.807333
2000000000	5000	0.965495	0.965495	0.781112
2000000000	10000	0.969172	0.969172	0.79843
2000000000	10000	0.972477	0.972477	0.783512

Table A.8: Table with the last part of the results from the simulation.

A.2 Fourth experiment Results

Here are the raw results from the experiment explained in Section 4.5.

Test Number	Pearson PSq	Spearman PSq	Pearson Sq	Spearman Sq
1_1	0.410929	0.187879	0.776577	0.236364
1_2	0.410929	0.187879	0.776577	0.236364
2_1	-0.463929	-0.490909	0.828186	0.69697
2_2	-0.463929	-0.490909	0.828186	0.69697
3_1	0.205106	0.0909091	0.904021	0.939394
3_2	0.205106	0.0909091	0.904021	0.939394
4_1	-0.00317279	-0.0909091	0.824818	0.806061
4_2	-0.00317279	-0.0909091	0.824818	0.806061
5_1	0.169992	0.139394	0.758513	0.721212
5_2	0.169992	0.139394	0.758513	0.721212
6_1	0.172371	0.0545455	0.806167	0.818182
6_2	0.172371	0.0545455	0.806167	0.818182
7_1	0.601034	0.733333	0.81336	0.769697
7_2	0.601034	0.733333	0.81336	0.769697
8_1	0.563977	0.50303	0.850133	0.878788
8_2	0.563977	0.50303	0.850133	0.878788
9_1	-0.33581	0.139394	0.805305	0.854545
9_2	-0.33581	0.139394	0.805305	0.854545
10_1	0.351802	0.357576	0.645551	0.515152
10_2	0.351802	0.357576	0.645551	0.515152
11_1	-0.434417	-0.175758	0.863196	0.806061
11_2	-0.434417	-0.175758	0.863196	0.806061
12_1	-0.0330206	0.260606	0.935489	0.709091
12_2	-0.0330206	0.260606	0.935489	0.709091
13_1	0.195111	0.115152	0.658232	0.612121
13_2	0.195111	0.115152	0.658232	0.612121
14_1	-0.230077	-0.187879	0.487063	0.29697
14_2	-0.230077	-0.187879	0.487063	0.29697
15_1	0.553214	0.381818	0.935108	0.806061
15_2	0.553214	0.381818	0.935108	0.806061
16_1	0.17371	0.127273	0.875405	0.842424
16_2	0.17371	0.127273	0.875405	0.842424
17_1	-0.066248	0.030303	0.788795	0.806061
17_2	-0.066248	0.030303	0.788795	0.806061
18_1	0.110038	0.115152	0.778921	0.733333
18_2	0.110038	0.115152	0.778921	0.733333
19_1	0.433426	0.0181818	0.940802	0.915152
19_2	0.433426	0.0181818	0.940802	0.915152
20_1	0.614432	0.551515	0.715185	0.672727
20_2	0.614432	0.551515	0.715185	0.672727

Table A.9: Table with the first part of the results from the fourth experiment.

Test Number	Pearson PSq	Spearman PSq	Pearson Sq	Spearman Sq
21_1	-0.00559347	0.0181818	0.931591	0.975758
21_2	-0.00559347	0.0181818	0.931591	0.975758
22_1	0.616079	0.393939	0.853011	0.90303
22_2	0.616079	0.393939	0.853011	0.90303
23_1	0.0015574	0.163636	0.837655	0.830303
23_2	0.0015574	0.163636	0.837655	0.830303
24_1	0.478062	0.345455	0.801985	0.684848
24_2	0.478062	0.345455	0.801985	0.684848
25_1	0.45871	0.29697	0.952795	0.939394
25_2	0.45871	0.29697	0.952795	0.939394
26_1	0.289878	0.430303	0.802949	0.830303
26_2	0.289878	0.430303	0.802949	0.830303
27_1	0.641054	0.490909	0.885834	0.915152
27_2	0.641054	0.490909	0.885834	0.915152
28_1	0.686382	0.660606	0.801872	0.854545
28_2	0.686382	0.660606	0.801872	0.854545
29_1	0.644446	0.624242	0.969037	0.951515
29_2	0.644446	0.624242	0.969037	0.951515
30_1	-0.119512	0.139394	0.899693	0.951515
30_2	-0.119512	0.139394	0.899693	0.951515
31_1	0.174788	0.357576	0.842033	0.806061
31_2	0.174788	0.357576	0.842033	0.806061
32_1	0.462198	0.551515	0.706882	0.624242
32_2	0.462198	0.551515	0.706882	0.624242
33_1	0.184176	0.175758	0.799839	0.757576
33_2	0.184176	0.175758	0.799839	0.757576
34_1	0.341318	0.393939	0.812711	0.757576
34_2	0.341318	0.393939	0.812711	0.757576
35_1	0.137284	0.127273	0.219772	-0.0181818
35_2	0.137284	0.127273	0.219772	-0.0181818

Table A.10: Table with the second part of the results from the fourth experiment.

Test Number	**Pearson PSq**	**Spearman PSq**	**Pearson Sq**	**Spearman Sq**
36_1	0.205909	0.175758	0.911315	0.878788
36_2	0.205909	0.175758	0.911315	0.878788
37_1	0.343904	0.381818	0.861319	0.890909
37_2	0.343904	0.381818	0.861319	0.890909
38_1	-0.447221	-0.0181818	0.329793	0.490909
38_2	-0.447221	-0.0181818	0.329793	0.490909
39_1	0.146021	0.175758	0.696415	0.612121
39_2	0.146021	0.175758	0.696415	0.612121
40_1	0.179521	0.309091	0.815152	0.842424
40_2	0.179521	0.309091	0.815152	0.842424
41_1	0.609291	0.284848	0.951844	0.733333
41_2	0.609291	0.284848	0.951844	0.733333
42_1	0.489005	0.406061	0.8339	0.854545
42_2	0.489005	0.406061	0.8339	0.854545
43_1	0.0843827	0.0666667	0.715362	0.830303
43_2	0.0843827	0.0666667	0.715362	0.830303
44_1	-0.289384	0.0787879	0.919747	0.842424
44_2	-0.289384	0.0787879	0.919747	0.842424
45_1	0.772454	0.490909	0.920541	0.939394
45_2	0.772454	0.490909	0.920541	0.939394
46_1	0.148455	0.272727	0.768264	0.878788
46_2	0.148455	0.272727	0.768264	0.878788
47_1	0.583788	0.684848	0.867355	0.806061
47_2	0.583788	0.684848	0.867355	0.806061
48_1	0.490787	0.345455	0.894378	0.878788
48_2	0.490787	0.345455	0.894378	0.878788
49_1	0.496066	0.139394	0.950662	0.842424
49_2	0.496066	0.139394	0.950662	0.842424
50_1	0.529419	0.539394	0.927733	0.90303
50_2	0.529419	0.539394	0.927733	0.90303

Table A.11: Table with the last part of the results from the fourth experiment.

A.3 Scope experiment Results

Here are the raw results from the experiment explained in Section 4.5.1.

FSM	PColl deviation	PSq deviation	Sq deviation
M_1^1	0.0950789	0.0703332	0.0387082
M_1^2	0.0749958	0.0952356	0.039985
M_2^1	0.182275	0.147906	0.0812145
M_2^2	0.0859465	0.102554	0.0257775
M_3^1	0.200358	0.031253	0.031253
M_3^2	0.122406	0.0956112	0.0576035
M_4^1	0.1108	0.0619166	0.0407203
M_4^2	0.116017	0.0879653	0.0538548
M_5^1	0.167367	0.0566661	0.0556639
M_5^2	0.121117	0.0341017	0.0435675
M_6^1	0.247408	0.104176	0.110493
M_6^2	0.116842	0.0200062	0.0295791
M_7^1	0.0909035	0.0391091	0.0313132
M_7^2	0.320194	0.0391855	0.0913402
M_8^1	0.279118	0.0849884	0.0981586
M_8^2	0.146865	0.0463582	0.0632131
M_9^1	0.0948726	0.0842807	0.0665105
M_9^2	0.307571	0.0900623	0.0402301
M_{10}^1	0.138399	0.0548657	0.0618528
M_{10}^2	0.32039	0.0737068	0.12743
M_{11}^1	0.0534865	0.0773609	0.0305317
M_{11}^2	0.0715425	0.116987	0.032441
M_{12}^1	0.221791	0.0771733	0.069286
M_{12}^2	0.113286	0.0429015	0.0493155
M_{13}^1	0.194512	0.149577	0.0487627
M_{13}^2	0.271059	0.130015	0.0529391
M_{14}^1	0.10515	0.0678593	0.0265715
M_{14}^2	0.101342	0.0833349	0.0443392
M_{15}^1	0.109012	0.0491771	0.0491771
M_{15}^2	0.107724	0.0571589	0.0371638
M_{16}^1	0.100109	0.0876177	0.0380637
M_{16}^2	0.366246	0.0476176	0.0638289
M_{17}^1	0.334802	0.0590221	0.0972634
M_{17}^2	0.189753	0.102213	0.085375
M_{18}^1	0.160188	0.0865485	0.0726993
M_{18}^2	0.0981993	0.133009	0.0632623
M_{19}^1	0.330245	0.0437058	0.143475
M_{19}^2	0.092328	0.0567495	0.0497149
M_{20}^1	0.228133	0.138484	0.0942126
M_{20}^2	0.211442	0.137835	0.0886251
M_{21}^1	0.113524	0.0617106	0.0404871
M_{21}^2	0.0971766	0.0813695	0.0573164
M_{22}^1	0.0818868	0.0663257	0.0321921
M_{22}^2	0.163391	0.0926745	0.0733553
M_{23}^1	0.0733843	0.0799325	0.0464522
M_{23}^2	0.144326	0.102849	0.0984305
M_{24}^1	0.09066	0.0222337	0.0277946
M_{24}^2	0.0836259	0.094271	0.0343725
M_{25}^1	0.200682	0.0600333	0.0832204
M_{25}^2	0.222674	0.0990115	0.0557218

Table A.12: Table with the first part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{26}	0.146452	0.0425276	0.0500004
M^2_{26}	0.111927	0.0750084	0.0618829
M^1_{27}	0.180839	0.0634458	0.0663702
M^2_{27}	0.344779	0.0745289	0.105688
M^1_{28}	0.205471	0.0576941	0.0266888
M^2_{28}	0.141235	0.0500264	0.0560682
M^1_{29}	0.161485	0.027813	0.044236
M^2_{29}	0.150032	0.0456817	0.0467421
M^1_{30}	0.170951	0.0788736	0.0894238
M^2_{30}	0.416569	0.15985	0.15939
M^1_{31}	0.130102	0.024334	0.0300059
M^2_{31}	0.233224	0.0240878	0.0602772
M^1_{32}	0.173543	0.0501073	0.0699508
M^2_{32}	0.0519373	0.0340297	0.0250512
M^1_{33}	0.0993003	0.0632115	0.057719
M^2_{33}	0.0845636	0.0853346	0.0600748
M^1_{34}	0.0757057	0.0785033	0.0369472
M^2_{34}	0.0540204	0.0504756	0.0282128
M^1_{35}	0.228355	0.041907	0.0584282
M^2_{35}	0.051329	0.0289019	0.0277161
M^1_{36}	0.405388	0.116162	0.114474
M^2_{36}	0.154361	0.140656	0.045513
M^1_{37}	0.226829	0.103107	0.0635126
M^2_{37}	0.202945	0.0579514	0.064849
M^1_{38}	0.247621	0.0548181	0.0952067
M^2_{38}	0.141167	0.0740156	0.0395652
M^1_{39}	0.118233	0.0780228	0.0217101
M^2_{39}	0.153248	0.120387	0.048356
M^1_{40}	0.193821	0.124717	0.0723519
M^2_{40}	0.10875	0.0486825	0.070881
M^1_{41}	0.0734413	0.0729073	0.0226361
M^2_{41}	0.160393	0.0600171	0.0533946
M^1_{42}	0.410768	0.1285	0.137999
M^2_{42}	0.0894848	0.0711071	0.0711071
M^1_{43}	0.188865	0.0608428	0.0623437
M^2_{43}	0.16824	0.0837024	0.0493667
M^1_{44}	0.0628875	0.0321204	0.0285181
M^2_{44}	0.0590902	0.031853	0.0359773
M^1_{45}	0.343608	0.0751338	0.137585
M^2_{45}	0.228277	0.0752557	0.0736208
M^1_{46}	0.100331	0.0406228	0.0348343
M^2_{46}	0.303122	0.125679	0.0458033
M^1_{47}	0.125738	0.0758148	0.0592544
M^2_{47}	0.0990768	0.0305855	0.0335061
M^1_{48}	0.0844205	0.0469336	0.0367617
M^2_{48}	0.0747392	0.050212	0.0429968
M^1_{49}	0.273759	0.098691	0.0332833
M^2_{49}	0.179445	0.108899	0.06261
M^1_{50}	0.208328	0.0600992	0.0543456
M^2_{50}	0.211534	0.105022	0.0686775

Table A.13: Table with the second part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M_{51}^1	0.135969	0.072216	0.03921
M_{51}^2	0.14765	0.0610611	0.0485349
M_{52}^1	0.179811	0.13086	0.0425712
M_{52}^2	0.109345	0.0700381	0.0496059
M_{53}^1	0.287159	0.0401924	0.101824
M_{53}^2	0.171377	0.0642315	0.0832322
M_{54}^1	0.154943	0.0929899	0.0425949
M_{54}^2	0.512515	0.0997374	0.204219
M_{55}^1	0.164112	0.100697	0.089606
M_{55}^2	0.0805172	0.0389442	0.0521739
M_{56}^1	0.258327	0.0746952	0.0486011
M_{56}^2	0.221498	0.0604876	0.0830078
M_{57}^1	0.102095	0.0678515	0.0485712
M_{57}^2	0.116065	0.0571252	0.0493827
M_{58}^1	0.222202	0.0445676	0.100461
M_{58}^2	0.160713	0.0648392	0.0339851
M_{59}^1	0.0922697	0.114325	0.0392836
M_{59}^2	0.152339	0.0779662	0.0534211
M_{60}^1	0.0964332	0.0316583	0.0338506
M_{60}^2	0.197632	0.0312328	0.0845141
M_{61}^1	0.120822	0.0672468	0.041812
M_{61}^2	0.235578	0.079051	0.040635
M_{62}^1	0.136279	0.0518537	0.0426184
M_{62}^2	0.14009	0.0544163	0.0563986
M_{63}^1	0.20426	0.048189	0.0368091
M_{63}^2	0.0618376	0.0261753	0.0134116
M_{64}^1	0.239645	0.100136	0.0698116
M_{64}^2	0.264125	0.0387124	0.0603139
M_{65}^1	0.143398	0.0849325	0.0648596
M_{65}^2	0.18349	0.0702123	0.140543
M_{66}^1	0.088535	0.0583747	0.0299715
M_{66}^2	0.0953912	0.0685813	0.0462366
M_{67}^1	0.140132	0.0344606	0.0452552
M_{67}^2	0.0923699	0.0363215	0.026999
M_{68}^1	0.154591	0.110447	0.0561131
M_{68}^2	0.082975	0.0850383	0.0387928
M_{69}^1	0.167507	0.0829327	0.0894642
M_{69}^2	0.137595	0.10085	0.0336023
M_{70}^1	0.122308	0.0712289	0.0601406
M_{70}^2	0.139632	0.0472891	0.0366607
M_{71}^1	0.216472	0.0658322	0.0619258
M_{71}^2	0.225876	0.0672275	0.0609915
M_{72}^1	0.110128	0.0422674	0.0513186
M_{72}^2	0.101456	0.0417108	0.0280167
M_{73}^1	0.0754773	0.0665834	0.0452809
M_{73}^2	0.09956	0.0736397	0.0391385
M_{74}^1	0.187389	0.061739	0.0962819
M_{74}^2	0.154081	0.0633079	0.0759063
M_{75}^1	0.0767824	0.0282065	0.0340653
M_{75}^2	0.148949	0.0817115	0.0460042

Table A.14: Table with the third part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M_{76}^{1}	0.0974253	0.0649437	0.0383996
M_{76}^{2}	0.13086	0.0803395	0.0371369
M_{77}^{1}	0.126834	0.046984	0.047322
M_{77}^{2}	0.0666589	0.0567481	0.0306047
M_{78}^{1}	0.142421	0.0728291	0.0657716
M_{78}^{2}	0.135484	0.0413574	0.059526
M_{79}^{1}	0.0787972	0.0698102	0.0319994
M_{79}^{2}	0.132023	0.0707939	0.0686297
M_{80}^{1}	0.129335	0.0563609	0.0864538
M_{80}^{2}	0.124183	0.121535	0.0773668
M_{81}^{1}	0.0858395	0.0854437	0.0263411
M_{81}^{2}	0.248318	0.0772831	0.0894451
M_{82}^{1}	0.145843	0.0499052	0.0844324
M_{82}^{2}	0.128471	0.0425901	0.0449232
M_{83}^{1}	0.144199	0.0817007	0.0611257
M_{83}^{2}	0.144874	0.0604773	0.0573113
M_{84}^{1}	0.0509486	0.0628944	0.0350917
M_{84}^{2}	0.0503543	0.0539001	0.0536369
M_{85}^{1}	0.28953	0.102085	0.0505757
M_{85}^{2}	0.269303	0.0245646	0.0754339
M_{86}^{1}	0.0943483	0.0929761	0.0504982
M_{86}^{2}	0.10225	0.0519837	0.0480473
M_{87}^{1}	0.115419	0.0696148	0.0155263
M_{87}^{2}	0.0841211	0.0493197	0.0371748
M_{88}^{1}	0.313459	0.0956848	0.10059
M_{88}^{2}	0.143479	0.105883	0.0374557
M_{89}^{1}	0.199046	0.0587595	0.0702325
M_{89}^{2}	0.0328137	0.0417776	0.0185168
M_{90}^{1}	0.08223	0.0494104	0.0631411
M_{90}^{2}	0.0805961	0.0307027	0.0375822
M_{91}^{1}	0.203895	0.0611912	0.0996128
M_{91}^{2}	0.130726	0.0272775	0.0517429
M_{92}^{1}	0.0686052	0.0576632	0.0359208
M_{92}^{2}	0.0758512	0.0517596	0.0387012
M_{93}^{1}	0.165626	0.0565454	0.0492374
M_{93}^{2}	0.132438	0.0471261	0.0461808
M_{94}^{1}	0.0772225	0.0787881	0.0546716
M_{94}^{2}	0.072251	0.103199	0.0240037
M_{95}^{1}	0.147509	0.0431311	0.0455822
M_{95}^{2}	0.158812	0.036126	0.0596151
M_{96}^{1}	0.110288	0.0431687	0.0531826
M_{96}^{2}	0.103719	0.0442854	0.0395455
M_{97}^{1}	0.108832	0.0567255	0.0398672
M_{97}^{2}	0.121799	0.0575606	0.0331824
M_{98}^{1}	0.0673777	0.0806132	0.0384923
M_{98}^{2}	0.0872107	0.138243	0.0409666
M_{99}^{1}	0.156639	0.0402293	0.0565971
M_{99}^{2}	0.117663	0.08772	0.0782389
M_{100}^{1}	0.108151	0.056286	0.0588551
M_{100}^{2}	0.305576	0.112265	0.0611045

Table A.15: Table with the fourth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{101}	0.0844766	0.0599709	0.0378957
M^2_{101}	0.283562	0.0629163	0.0649796
M^1_{102}	0.146074	0.0372933	0.0447525
M^2_{102}	0.0814218	0.0681172	0.0473272
M^1_{103}	0.220781	0.0678471	0.0828049
M^2_{103}	0.185511	0.11293	0.0728425
M^1_{104}	0.20145	0.0431116	0.0960818
M^2_{104}	0.135999	0.0738722	0.0417661
M^1_{105}	0.0483363	0.0309682	0.0309682
M^2_{105}	0.104292	0.0693117	0.0466864
M^1_{106}	0.143024	0.0305789	0.0316344
M^2_{106}	0.141118	0.105002	0.0308516
M^1_{107}	0.216426	0.0867799	0.0548562
M^2_{107}	0.0702437	0.0723491	0.0603433
M^1_{108}	0.130258	0.0501329	0.0618744
M^2_{108}	0.13563	0.0739941	0.0739941
M^1_{109}	0.164022	0.0710539	0.0723892
M^2_{109}	0.12791	0.0645337	0.0378852
M^1_{110}	0.0689555	0.0524927	0.0406613
M^2_{110}	0.0428473	0.0280002	0.0280002
M^1_{111}	0.191615	0.11031	0.0376163
M^2_{111}	0.117217	0.0357249	0.0287088
M^1_{112}	0.0840409	0.0997353	0.0309793
M^2_{112}	0.0940412	0.107968	0.0433
M^1_{113}	0.153921	0.0584838	0.0588077
M^2_{113}	0.145272	0.062362	0.0623388
M^1_{114}	0.121859	0.0394848	0.0428094
M^2_{114}	0.21876	0.0926631	0.0893885
M^1_{115}	0.0912794	0.0444315	0.0450249
M^2_{115}	0.149166	0.0785472	0.0745509
M^1_{116}	0.0790619	0.0523054	0.0251431
M^2_{116}	0.108893	0.0601722	0.0507019
M^1_{117}	0.0855776	0.0425228	0.0365365
M^2_{117}	0.117766	0.173615	0.0413291
M^1_{118}	0.172183	0.0317841	0.0481532
M^2_{118}	0.0839136	0.0486966	0.0433486
M^1_{119}	0.107837	0.113698	0.0320349
M^2_{119}	0.16574	0.102346	0.0632889
M^1_{120}	0.047391	0.0153894	0.0178296
M^2_{120}	0.13007	0.0426523	0.0450756
M^1_{121}	0.105565	0.027227	0.027227
M^2_{121}	0.188941	0.0492386	0.0763803
M^1_{122}	0.198102	0.0327492	0.095699
M^2_{122}	0.219453	0.0640321	0.0684748
M^1_{123}	0.209157	0.0740377	0.0865089
M^2_{123}	0.352057	0.0765253	0.0967276
M^1_{124}	0.117708	0.0276473	0.0387273
M^2_{124}	0.0704975	0.119532	0.0257229
M^1_{125}	0.104222	0.0653418	0.041436
M^2_{125}	0.148375	0.0554637	0.0607576

Table A.16: Table with the fifth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{126}	0.0914777	0.0486279	0.0556591
M^2_{126}	0.205364	0.0414176	0.0820369
M^1_{127}	0.153284	0.0419661	0.0693078
M^2_{127}	0.191255	0.0551535	0.0578123
M^1_{128}	0.221614	0.0888669	0.0356835
M^2_{128}	0.145147	0.0774874	0.0757351
M^1_{129}	0.124546	0.105355	0.0558905
M^2_{129}	0.108438	0.0602756	0.026914
M^1_{130}	0.15936	0.0828102	0.0891357
M^2_{130}	0.0917881	0.0353718	0.0387383
M^1_{131}	0.17467	0.0694708	0.0597348
M^2_{131}	0.21275	0.035971	0.0676661
M^1_{132}	0.248657	0.0779216	0.0913048
M^2_{132}	0.153046	0.0550511	0.036208
M^1_{133}	0.120863	0.0777568	0.0519288
M^2_{133}	0.329576	0.110945	0.0622509
M^1_{134}	0.155251	0.10644	0.0671676
M^2_{134}	0.123067	0.0449835	0.0366785
M^1_{135}	0.171237	0.0921647	0.0611379
M^2_{135}	0.166176	0.0862205	0.059389
M^1_{136}	0.130262	0.0327552	0.0406223
M^2_{136}	0.183263	0.0660345	0.0619106
M^1_{137}	0.221797	0.0512838	0.0392418
M^2_{137}	0.128104	0.0916466	0.0629577
M^1_{138}	0.191807	0.0887032	0.0741373
M^2_{138}	0.0906809	0.0416269	0.0318906
M^1_{139}	0.0919258	0.0510027	0.0479386
M^2_{139}	0.137183	0.0682062	0.0443962
M^1_{140}	0.0814515	0.0628999	0.0308023
M^2_{140}	0.212336	0.0357221	0.034821
M^1_{141}	0.307923	0.0361928	0.0161623
M^2_{141}	0.202717	0.0293458	0.0567415
M^1_{142}	0.151375	0.0620119	0.043868
M^2_{142}	0.14693	0.0375828	0.0574405
M^1_{143}	0.268605	0.0849222	0.0632926
M^2_{143}	0.457164	0.0553189	0.0731294
M^1_{144}	0.119884	0.0503527	0.0291856
M^2_{144}	0.122672	0.0575767	0.0403385
M^1_{145}	0.147728	0.0465539	0.0619068
M^2_{145}	0.165448	0.0856024	0.0605216
M^1_{146}	0.117272	0.0569028	0.0666924
M^2_{146}	0.263706	0.0698919	0.0681954
M^1_{147}	0.132541	0.0580681	0.0531744
M^2_{147}	0.151678	0.0799734	0.0661626
M^1_{148}	0.199553	0.130119	0.0846456
M^2_{148}	0.10609	0.0489163	0.0552963
M^1_{149}	0.286599	0.0529339	0.0486027
M^2_{149}	0.164176	0.0336856	0.0606288
M^1_{150}	0.124621	0.0562229	0.0602435
M^2_{150}	0.0557401	0.0320024	0.0300159

Table A.17: Table with the sixth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{151}	0.214628	0.0641455	0.0576588
M^2_{151}	0.130729	0.0668157	0.0526234
M^1_{152}	0.164004	0.0432369	0.021959
M^2_{152}	0.120287	0.0266032	0.0452402
M^1_{153}	0.184779	0.092577	0.062778
M^2_{153}	0.185171	0.0820205	0.0533398
M^1_{154}	0.119267	0.0575744	0.0610703
M^2_{154}	0.0624143	0.0299985	0.0299985
M^1_{155}	0.160098	0.027385	0.0466827
M^2_{155}	0.127626	0.0496865	0.0481561
M^1_{156}	0.189844	0.105244	0.0723777
M^2_{156}	0.115825	0.0446958	0.0461897
M^1_{157}	0.155123	0.095274	0.0601376
M^2_{157}	0.0630051	0.0326579	0.0257701
M^1_{158}	0.277997	0.0681601	0.0677518
M^2_{158}	0.338781	0.0504093	0.0693117
M^1_{159}	0.0716252	0.0294645	0.0294645
M^2_{159}	0.101058	0.0367766	0.0365479
M^1_{160}	0.052383	0.0526103	0.0356691
M^2_{160}	0.131927	0.0727986	0.0516599
M^1_{161}	0.147549	0.0432905	0.0376181
M^2_{161}	0.126864	0.0392231	0.0172616
M^1_{162}	0.293867	0.057867	0.12441
M^2_{162}	0.10452	0.0597394	0.0408131
M^1_{163}	0.244765	0.128352	0.0918792
M^2_{163}	0.0807147	0.0353725	0.0225102
M^1_{164}	0.0727915	0.0531881	0.0255815
M^2_{164}	0.187796	0.0724204	0.0290426
M^1_{165}	0.128891	0.0815746	0.0323515
M^2_{165}	0.207986	0.0876257	0.0660454
M^1_{166}	0.124718	0.0777503	0.0614897
M^2_{166}	0.106747	0.030559	0.0638959
M^1_{167}	0.0645015	0.0677384	0.0488024
M^2_{167}	0.145528	0.0771552	0.0687874
M^1_{168}	0.239777	0.0684283	0.0918159
M^2_{168}	0.530166	0.127631	0.163047
M^1_{169}	0.126251	0.128943	0.0369049
M^2_{169}	0.0808908	0.13323	0.0361135
M^1_{170}	0.255709	0.0772762	0.080373
M^2_{170}	0.14729	0.0826296	0.0327279
M^1_{171}	0.110855	0.0459751	0.0414488
M^2_{171}	0.10475	0.0823686	0.0378294
M^1_{172}	0.0704059	0.0979399	0.0368073
M^2_{172}	0.13277	0.0572174	0.0455056
M^1_{173}	0.107222	0.0353752	0.0498633
M^2_{173}	0.146257	0.0727018	0.0693078
M^1_{174}	0.0854459	0.0544993	0.0475668
M^2_{174}	0.0877968	0.109689	0.0438812
M^1_{175}	0.0777855	0.0671701	0.0630091
M^2_{175}	0.102247	0.0524518	0.0368952

Table A.18: Table with the seventh part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{176}	0.0493413	0.0240178	0.0240178
M^2_{176}	0.236384	0.0273177	0.0273177
M^1_{177}	0.105683	0.0545788	0.051499
M^2_{177}	0.126939	0.0366448	0.0821855
M^1_{178}	0.502121	0.133631	0.206008
M^2_{178}	0.0501833	0.0805334	0.0581454
M^1_{179}	0.177361	0.0470784	0.0422568
M^2_{179}	0.160599	0.106426	0.0914263
M^1_{180}	0.09533	0.0747299	0.0344199
M^2_{180}	0.12197	0.114166	0.0673058
M^1_{181}	0.161714	0.0283374	0.0600085
M^2_{181}	0.161786	0.087904	0.0724184
M^1_{182}	0.164747	0.0908708	0.0856854
M^2_{182}	0.176832	0.0721107	0.0730407
M^1_{183}	0.0721706	0.0385128	0.0339832
M^2_{183}	0.119977	0.0342654	0.044556
M^1_{184}	0.0961821	0.0326587	0.0208317
M^2_{184}	0.22682	0.0581899	0.0649652
M^1_{185}	0.0745854	0.0256854	0.0308005
M^2_{185}	0.0573968	0.073575	0.0314416
M^1_{186}	0.0732334	0.0621747	0.0292452
M^2_{186}	0.261301	0.0125969	0.114724
M^1_{187}	0.0900539	0.0366144	0.0382513
M^2_{187}	0.249333	0.0397538	0.116899
M^1_{188}	0.18575	0.0791632	0.0403864
M^2_{188}	0.0897354	0.0595534	0.0355252
M^1_{189}	0.22324	0.102414	0.0481084
M^2_{189}	0.16201	0.0588293	0.0420487
M^1_{190}	0.139711	0.162801	0.0434869
M^2_{190}	0.161427	0.171295	0.0593966
M^1_{191}	0.0784647	0.0456441	0.0456441
M^2_{191}	0.142741	0.0433893	0.0880112
M^1_{192}	0.156988	0.0709328	0.0546674
M^2_{192}	0.106701	0.119741	0.0522467
M^1_{193}	0.11518	0.050467	0.0379082
M^2_{193}	0.319289	0.210992	0.0788463
M^1_{194}	0.054047	0.0442058	0.0158442
M^2_{194}	0.162021	0.0570734	0.0223556
M^1_{195}	0.615883	0.0610384	0.191272
M^2_{195}	0.171396	0.0678891	0.0736381
M^1_{196}	0.147129	0.0666865	0.0645558
M^2_{196}	0.176868	0.0694785	0.0738782
M^1_{197}	0.174025	0.0769187	0.0844488
M^2_{197}	0.145666	0.0769659	0.101004
M^1_{198}	0.0533901	0.0756466	0.0271886
M^2_{198}	0.0472839	0.0884422	0.0227125
M^1_{199}	0.122917	0.0490675	0.0518688
M^2_{199}	0.469743	0.0419683	0.132432
M^1_{200}	0.232111	0.0584861	0.114169
M^2_{200}	0.160918	0.0758837	0.0585696

Table A.19: Table with the eight part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M_{201}^{1}	0.320929	0.0915174	0.135178
M_{201}^{2}	0.194944	0.0668596	0.0623401
M_{202}^{1}	0.10212	0.048118	0.0280015
M_{202}^{2}	0.123118	0.121849	0.0403971
M_{203}^{1}	0.172405	0.10761	0.0683857
M_{203}^{2}	0.109712	0.045899	0.036106
M_{204}^{1}	0.109296	0.063936	0.0495415
M_{204}^{2}	0.0517274	0.0781413	0.0200832
M_{205}^{1}	0.14869	0.0520621	0.0542438
M_{205}^{2}	0.092611	0.0460969	0.0359107
M_{206}^{1}	0.25462	0.0645227	0.111061
M_{206}^{2}	0.119139	0.102157	0.0476853
M_{207}^{1}	0.0826792	0.132486	0.0451183
M_{207}^{2}	0.0610272	0.0845023	0.0297433
M_{208}^{1}	0.248716	0.180631	0.07764
M_{208}^{2}	0.23923	0.16745	0.0921881
M_{209}^{1}	0.106599	0.112782	0.0340541
M_{209}^{2}	0.0956643	0.0675092	0.0235404
M_{210}^{1}	0.0922222	0.0323893	0.0319545
M_{210}^{2}	0.158093	0.0501029	0.0653496
M_{211}^{1}	0.183116	0.0902302	0.0692288
M_{211}^{2}	0.0654867	0.0478722	0.0421683
M_{212}^{1}	0.107668	0.0595185	0.0423966
M_{212}^{2}	0.0459012	0.107833	0.0260634
M_{213}^{1}	0.27448	0.0544305	0.194911
M_{213}^{2}	0.181899	0.0995511	0.0770211
M_{214}^{1}	0.0655072	0.054067	0.0175885
M_{214}^{2}	0.131063	0.0803329	0.0454991
M_{215}^{1}	0.0823683	0.0311878	0.0386226
M_{215}^{2}	0.123834	0.0997431	0.058712
M_{216}^{1}	0.132688	0.0391275	0.0418114
M_{216}^{2}	0.0835506	0.109981	0.0384653
M_{217}^{1}	0.0730744	0.12794	0.0280366
M_{217}^{2}	0.0332924	0.00887374	0.00887374
M_{218}^{1}	0.127101	0.0812361	0.07276
M_{218}^{2}	0.148913	0.0706796	0.0752447
M_{219}^{1}	0.210453	0.0662656	0.073793
M_{219}^{2}	0.161557	0.035145	0.0584149
M_{220}^{1}	0.12434	0.0837517	0.0796094
M_{220}^{2}	0.103494	0.0283263	0.0355821
M_{221}^{1}	0.083641	0.03849	0.0426046
M_{221}^{2}	0.103591	0.0560226	0.0546263
M_{222}^{1}	0.115356	0.122027	0.0691752
M_{222}^{2}	0.0724865	0.127237	0.0384954
M_{223}^{1}	0.147823	0.0523881	0.0545882
M_{223}^{2}	0.208935	0.0703411	0.0894422
M_{224}^{1}	0.0845362	0.101135	0.0297763
M_{224}^{2}	0.1436	0.0907659	0.0458298
M_{225}^{1}	0.0729032	0.0537017	0.0489775
M_{225}^{2}	0.13208	0.0583676	0.0637316

Table A.20: Table with the ninth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{226}	0.066675	0.0627817	0.0378335
M^2_{226}	0.0728061	0.0661461	0.0235176
M^1_{227}	0.0949713	0.0505226	0.0308543
M^2_{227}	0.191036	0.0450683	0.0649042
M^1_{228}	0.0474001	0.0371107	0.0165736
M^2_{228}	0.133401	0.0617278	0.0439212
M^1_{229}	0.122374	0.0665126	0.0697959
M^2_{229}	0.146647	0.0384612	0.0479638
M^1_{230}	0.250148	0.0873959	0.104082
M^2_{230}	0.194104	0.0594211	0.0717124
M^1_{231}	0.0699153	0.0401544	0.0378323
M^2_{231}	0.11857	0.0517308	0.0333014
M^1_{232}	0.171835	0.0620839	0.111005
M^2_{232}	0.057153	0.0409218	0.042117
M^1_{233}	0.0783016	0.0351236	0.0351236
M^2_{233}	0.183598	0.0521553	0.0733566
M^1_{234}	0.157199	0.0637758	0.0700709
M^2_{234}	0.273699	0.0915263	0.0441587
M^1_{235}	0.0877706	0.11889	0.0322514
M^2_{235}	0.0792891	0.0676074	0.0382468
M^1_{236}	0.105288	0.0404291	0.0504072
M^2_{236}	0.194898	0.0800489	0.0930063
M^1_{237}	0.202047	0.0617091	0.0601799
M^2_{237}	0.124543	0.0590402	0.0494177
M^1_{238}	0.180059	0.132252	0.0627432
M^2_{238}	0.153479	0.0515442	0.0515442
M^1_{239}	0.0622773	0.0413479	0.0332475
M^2_{239}	0.126171	0.0245162	0.0297151
M^1_{240}	0.285131	0.0426491	0.0252248
M^2_{240}	0.0439781	0.0479556	0.0257062
M^1_{241}	0.275297	0.072067	0.121074
M^2_{241}	0.0684916	0.0571739	0.0272044
M^1_{242}	0.123954	0.115977	0.0433023
M^2_{242}	0.243066	0.107125	0.145532
M^1_{243}	0.120974	0.0508181	0.0530273
M^2_{243}	0.273139	0.0707985	0.042744
M^1_{244}	0.113722	0.101112	0.0348508
M^2_{244}	0.107245	0.0968577	0.0458462
M^1_{245}	0.329358	0.0893511	0.127548
M^2_{245}	0.0585442	0.0717083	0.0262594
M^1_{246}	0.212828	0.0686336	0.053447
M^2_{246}	0.210154	0.102289	0.080989
M^1_{247}	0.131076	0.0431643	0.0368589
M^2_{247}	0.106718	0.0635396	0.0421269
M^1_{248}	0.125279	0.0296497	0.0462179
M^2_{248}	0.116739	0.0479715	0.0420898
M^1_{249}	0.175949	0.105557	0.0686419
M^2_{249}	0.210539	0.0801715	0.0427538
M^1_{250}	0.197716	0.0589633	0.0778028
M^2_{250}	0.287449	0.0900582	0.051427

Table A.21: Table with the tenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{251}	0.141094	0.0390183	0.0530638
M^2_{251}	0.200839	0.0619043	0.071959
M^1_{252}	0.193962	0.094042	0.0593573
M^2_{252}	0.250122	0.0944683	0.0910919
M^1_{253}	0.17476	0.0725549	0.0608294
M^2_{253}	0.102149	0.0548417	0.0326269
M^1_{254}	0.120707	0.0506427	0.04314
M^2_{254}	0.079319	0.0220406	0.0338622
M^1_{255}	0.102217	0.100945	0.0300436
M^2_{255}	0.0561279	0.10956	0.0283264
M^1_{256}	0.153464	0.0828614	0.0513759
M^2_{256}	0.0864712	0.0904432	0.0301399
M^1_{257}	0.104343	0.0836763	0.0578935
M^2_{257}	0.0773851	0.0593843	0.0491363
M^1_{258}	0.12008	0.0384007	0.0528518
M^2_{258}	0.0877601	0.073502	0.0389848
M^1_{259}	0.0865356	0.0574114	0.0442272
M^2_{259}	0.0941158	0.0679937	0.0460864
M^1_{260}	0.0755704	0.0561131	0.029871
M^2_{260}	0.154404	0.0661945	0.0670694
M^1_{261}	0.177111	0.130211	0.0558384
M^2_{261}	0.14893	0.123687	0.0466886
M^1_{262}	0.0717966	0.0287465	0.0366246
M^2_{262}	0.142686	0.108111	0.0735273
M^1_{263}	0.155672	0.0561821	0.0587316
M^2_{263}	0.326915	0.0834482	0.126096
M^1_{264}	0.256933	0.06243	0.0680333
M^2_{264}	0.242616	0.121227	0.0635987
M^1_{265}	0.125309	0.0748321	0.0999525
M^2_{265}	0.134289	0.0899432	0.0418623
M^1_{266}	0.101069	0.0723155	0.0412336
M^2_{266}	0.0945022	0.0447822	0.0420619
M^1_{267}	0.123137	0.0982478	0.0493803
M^2_{267}	0.0654703	0.0400264	0.0400264
M^1_{268}	0.0646455	0.0519324	0.0266544
M^2_{268}	0.194833	0.0759964	0.0822875
M^1_{269}	0.0823866	0.0725328	0.0734576
M^2_{269}	0.0779122	0.0815766	0.0796027
M^1_{270}	0.107371	0.0925702	0.0368406
M^2_{270}	0.111394	0.0437777	0.0524355
M^1_{271}	0.11469	0.0718492	0.0262581
M^2_{271}	0.161362	0.0792488	0.0500438
M^1_{272}	0.0866802	0.0883648	0.0315621
M^2_{272}	0.126543	0.0619064	0.0331612
M^1_{273}	0.102331	0.151158	0.0452948
M^2_{273}	0.0877833	0.139118	0.0455388
M^1_{274}	0.28912	0.0580955	0.0396037
M^2_{274}	0.0676413	0.0599489	0.0607064
M^1_{275}	0.15168	0.0714352	0.0502067
M^2_{275}	0.153309	0.0502463	0.0520937

Table A.22: Table with the eleventh part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{276}	0.114089	0.0468405	0.0509689
M^2_{276}	0.126059	0.0640334	0.0640334
M^1_{277}	0.170807	0.0911448	0.0553544
M^2_{277}	0.133738	0.0467839	0.0596847
M^1_{278}	0.460824	0.0721443	0.140621
M^2_{278}	0.104701	0.0484774	0.0321138
M^1_{279}	0.0715977	0.0976689	0.0531812
M^2_{279}	0.0877401	0.087235	0.0343681
M^1_{280}	0.223377	0.136102	0.100698
M^2_{280}	0.169629	0.12194	0.0553103
M^1_{281}	0.160667	0.0571535	0.0551783
M^2_{281}	0.128525	0.0438124	0.0468212
M^1_{282}	0.0756296	0.0554665	0.0301116
M^2_{282}	0.118087	0.058236	0.0690887
M^1_{283}	0.163026	0.0759389	0.0453445
M^2_{283}	0.20086	0.0740739	0.0841777
M^1_{284}	0.203357	0.114661	0.0632884
M^2_{284}	0.147056	0.0818511	0.0774341
M^1_{285}	0.13755	0.0890331	0.0520915
M^2_{285}	0.133881	0.0498661	0.0317864
M^1_{286}	0.241841	0.0735911	0.0766967
M^2_{286}	0.464048	0.0919057	0.162669
M^1_{287}	0.0794803	0.0951128	0.0472314
M^2_{287}	0.0912918	0.0801973	0.054496
M^1_{288}	0.239192	0.036179	0.140227
M^2_{288}	0.110172	0.0855756	0.0367742
M^1_{289}	0.146085	0.058392	0.0619317
M^2_{289}	0.196843	0.0771836	0.079418
M^1_{290}	0.101416	0.0365206	0.0487504
M^2_{290}	0.119859	0.0456012	0.0571958
M^1_{291}	0.116864	0.0512095	0.0379064
M^2_{291}	0.132937	0.0573584	0.0543407
M^1_{292}	0.0696089	0.0792931	0.0330109
M^2_{292}	0.278049	0.103569	0.0797214
M^1_{293}	0.0817073	0.107813	0.042053
M^2_{293}	0.0828152	0.120854	0.0332964
M^1_{294}	0.204715	0.0971958	0.0264607
M^2_{294}	0.225482	0.0922726	0.0659926
M^1_{295}	0.12685	0.0400457	0.0400457
M^2_{295}	0.197612	0.134747	0.0740774
M^1_{296}	0.0741113	0.0486154	0.0369114
M^2_{296}	0.217722	0.0664447	0.0848397
M^1_{297}	0.0831666	0.126505	0.0365954
M^2_{297}	0.0963664	0.126616	0.0452842
M^1_{298}	0.118118	0.0841297	0.103975
M^2_{298}	0.142703	0.0773154	0.084301
M^1_{299}	0.0567002	0.129978	0.0238967
M^2_{299}	0.0751478	0.110387	0.0230378
M^1_{300}	0.109327	0.0475481	0.0377559
M^2_{300}	0.0637104	0.024211	0.024211

Table A.23: Table with the twelfth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{301}	0.0751681	0.0524621	0.0962008
M^2_{301}	0.133811	0.135539	0.0460798
M^1_{302}	0.0668363	0.0459721	0.0356969
M^2_{302}	0.0831301	0.119463	0.0649339
M^1_{303}	0.172878	0.0697953	0.0443404
M^2_{303}	0.133333	0.0406836	0.0636144
M^1_{304}	0.117497	0.0974513	0.0855335
M^2_{304}	0.224033	0.0723179	0.0746368
M^1_{305}	0.119178	0.0600124	0.0741318
M^2_{305}	0.105727	0.0744419	0.0369787
M^1_{306}	0.0766761	0.0631881	0.043166
M^2_{306}	0.0928634	0.0616329	0.0304961
M^1_{307}	0.369638	0.240243	0.155753
M^2_{307}	0.322205	0.154143	0.130664
M^1_{308}	0.0822149	0.0786954	0.0377644
M^2_{308}	0.21697	0.0472925	0.0468029
M^1_{309}	0.220095	0.0762443	0.0930257
M^2_{309}	0.118008	0.0222871	0.0347949
M^1_{310}	0.162176	0.104122	0.110252
M^2_{310}	0.131121	0.0721127	0.0438206
M^1_{311}	0.265844	0.0289179	0.099689
M^2_{311}	0.120044	0.0550444	0.0456639
M^1_{312}	0.127871	0.0473808	0.0534978
M^2_{312}	0.162177	0.0673043	0.0583468
M^1_{313}	0.125053	0.0610038	0.0538287
M^2_{313}	0.130294	0.104705	0.0740913
M^1_{314}	0.192956	0.0413812	0.065064
M^2_{314}	0.153958	0.0721573	0.0669193
M^1_{315}	0.119924	0.0593274	0.0523888
M^2_{315}	0.112644	0.0476461	0.0461444
M^1_{316}	0.127743	0.0577492	0.0700978
M^2_{316}	0.108994	0.10072	0.0762818
M^1_{317}	0.16051	0.0438569	0.0592306
M^2_{317}	0.537285	0.0888464	0.161864
M^1_{318}	0.100738	0.0596298	0.051539
M^2_{318}	0.112013	0.0669545	0.0468428
M^1_{319}	0.402804	0.0569587	0.101492
M^2_{319}	0.186568	0.0801016	0.0850162
M^1_{320}	0.309091	0.100051	0.11344
M^2_{320}	0.0922557	0.0206979	0.034221
M^1_{321}	0.166701	0.0692682	0.0497086
M^2_{321}	0.112431	0.0756044	0.0756044
M^1_{322}	0.0765915	0.0585519	0.0422726
M^2_{322}	0.154693	0.0864722	0.0773978
M^1_{323}	0.0776518	0.0700687	0.0467002
M^2_{323}	0.0571436	0.0475374	0.0307321
M^1_{324}	0.0538892	0.0327226	0.0187711
M^2_{324}	0.0862029	0.0398414	0.0398414
M^1_{325}	0.213891	0.0909445	0.0607545
M^2_{325}	0.0707111	0.047537	0.0432804

Table A.24: Table with the thirteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{326}	0.0761087	0.0622327	0.0328955
M^2_{326}	0.312854	0.0409126	0.0514986
M^1_{327}	0.102209	0.0504517	0.0551573
M^2_{327}	0.0759197	0.0463023	0.0341025
M^1_{328}	0.29064	0.0944766	0.0876885
M^2_{328}	0.102291	0.101437	0.0417158
M^1_{329}	0.0701764	0.0309814	0.0211304
M^2_{329}	0.223266	0.0859622	0.0866693
M^1_{330}	0.33104	0.0346499	0.0360339
M^2_{330}	0.18361	0.0530497	0.0700259
M^1_{331}	0.0542238	0.0238366	0.0192802
M^2_{331}	0.125369	0.0524815	0.0460164
M^1_{332}	0.107805	0.0812237	0.0431844
M^2_{332}	0.0998742	0.0680644	0.0665239
M^1_{333}	0.182451	0.0663325	0.0566946
M^2_{333}	0.305706	0.0969381	0.0979802
M^1_{334}	0.108633	0.0382457	0.0361673
M^2_{334}	0.119153	0.0618699	0.0243992
M^1_{335}	0.131964	0.0466529	0.0265864
M^2_{335}	0.0669222	0.0227871	0.0227871
M^1_{336}	0.103491	0.0975119	0.0402406
M^2_{336}	0.122469	0.0752941	0.0343733
M^1_{337}	0.157223	0.0888566	0.0485888
M^2_{337}	0.106672	0.0343578	0.0373323
M^1_{338}	0.120631	0.0645693	0.059336
M^2_{338}	0.272746	0.0911368	0.086843
M^1_{339}	0.0673564	0.0316809	0.0240207
M^2_{339}	0.171634	0.100511	0.0716773
M^1_{340}	0.173846	0.0955557	0.102227
M^2_{340}	0.156648	0.0875052	0.066874
M^1_{341}	0.114633	0.0544761	0.0402432
M^2_{341}	0.119136	0.0707378	0.0608722
M^1_{342}	0.0824055	0.0433543	0.0671879
M^2_{342}	0.145991	0.0793014	0.0456787
M^1_{343}	0.083107	0.0702971	0.0318762
M^2_{343}	0.100442	0.075337	0.0518129
M^1_{344}	0.141198	0.0693642	0.0550409
M^2_{344}	0.108963	0.128529	0.0361771
M^1_{345}	0.564628	0.0781958	0.125438
M^2_{345}	0.405552	0.0834236	0.102949
M^1_{346}	0.175331	0.0618822	0.0605833
M^2_{346}	0.261561	0.0894602	0.127099
M^1_{347}	0.135733	0.0547477	0.0612163
M^2_{347}	0.10666	0.0655543	0.0523571
M^1_{348}	0.130657	0.0425918	0.039
M^2_{348}	0.323741	0.0508387	0.0318503
M^1_{349}	0.122895	0.081154	0.0553859
M^2_{349}	0.178349	0.0730933	0.0537257
M^1_{350}	0.329279	0.0677303	0.139809
M^2_{350}	0.081584	0.0627299	0.0218783

Table A.25: Table with the fourteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{351}	0.0836284	0.0349514	0.0481058
M^2_{351}	0.550097	0.048125	0.212203
M^1_{352}	0.0471564	0.0408441	0.0227483
M^2_{352}	0.0761967	0.0661829	0.0387159
M^1_{353}	0.163491	0.0607554	0.0562085
M^2_{353}	0.104802	0.0379616	0.0463602
M^1_{354}	0.239421	0.150116	0.0876759
M^2_{354}	0.140764	0.0369418	0.0380205
M^1_{355}	0.23592	0.106742	0.0629564
M^2_{355}	0.210657	0.154168	0.120769
M^1_{356}	0.128	0.0827749	0.0531082
M^2_{356}	0.0850102	0.0616042	0.0353366
M^1_{357}	0.362391	0.0929328	0.117951
M^2_{357}	0.259907	0.128006	0.0699086
M^1_{358}	0.190927	0.069121	0.0523219
M^2_{358}	0.163931	0.0516221	0.0437683
M^1_{359}	0.106474	0.0453301	0.0451783
M^2_{359}	0.191318	0.0445829	0.0529826
M^1_{360}	0.110314	0.043088	0.0467028
M^2_{360}	0.223985	0.0435378	0.0819951
M^1_{361}	0.0642159	0.0412445	0.0308292
M^2_{361}	0.160141	0.0311946	0.0243725
M^1_{362}	0.0899054	0.0305344	0.0471437
M^2_{362}	0.115703	0.0710943	0.0363133
M^1_{363}	0.114217	0.105174	0.0454555
M^2_{363}	0.100206	0.0591558	0.0509891
M^1_{364}	0.137278	0.0451219	0.061306
M^2_{364}	0.0594534	0.074755	0.0269188
M^1_{365}	0.135136	0.075151	0.0622594
M^2_{365}	0.164412	0.071188	0.0574075
M^1_{366}	0.129829	0.071539	0.0624924
M^2_{366}	0.0571338	0.0415076	0.0298653
M^1_{367}	0.389936	0.0815426	0.131336
M^2_{367}	0.0736867	0.100805	0.0309125
M^1_{368}	0.161012	0.0342738	0.0342738
M^2_{368}	0.126231	0.161266	0.0453221
M^1_{369}	0.0712348	0.0262131	0.0253254
M^2_{369}	0.287443	0.107644	0.115839
M^1_{370}	0.195486	0.0519293	0.0699072
M^2_{370}	0.184579	0.0592937	0.0615913
M^1_{371}	0.149858	0.0601982	0.0776519
M^2_{371}	0.103234	0.0648942	0.0335496
M^1_{372}	0.233383	0.111732	0.199478
M^2_{372}	0.246185	0.0983152	0.114257
M^1_{373}	0.331589	0.103725	0.106278
M^2_{373}	0.531864	0.212388	0.186662
M^1_{374}	0.0721423	0.129319	0.0258241
M^2_{374}	0.157147	0.0503036	0.0546872
M^1_{375}	0.124403	0.0608135	0.0540222
M^2_{375}	0.0939925	0.0752171	0.0415776

Table A.26: Table with the fifteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{376}	0.444925	0.0968971	0.178013
M^2_{376}	0.123667	0.105442	0.0680788
M^1_{377}	0.154677	0.0398125	0.0407833
M^2_{377}	0.11623	0.0549689	0.057883
M^1_{378}	0.187577	0.120065	0.0871466
M^2_{378}	0.078331	0.0434595	0.0358646
M^1_{379}	0.165974	0.125887	0.0513576
M^2_{379}	0.256659	0.0315625	0.0300735
M^1_{380}	0.0936601	0.113404	0.0496139
M^2_{380}	0.345818	0.0592775	0.067112
M^1_{381}	0.121309	0.035956	0.0662846
M^2_{381}	0.139307	0.0437941	0.0438243
M^1_{382}	0.0815675	0.0677259	0.057582
M^2_{382}	0.224799	0.074479	0.100606
M^1_{383}	0.104524	0.0397682	0.0397682
M^2_{383}	0.134564	0.100412	0.038286
M^1_{384}	0.325631	0.0596833	0.116221
M^2_{384}	0.263611	0.0847591	0.0974182
M^1_{385}	0.142313	0.0727891	0.0646429
M^2_{385}	0.0904769	0.0739292	0.0318913
M^1_{386}	0.116867	0.068131	0.0391321
M^2_{386}	0.0926798	0.0948024	0.133598
M^1_{387}	0.143974	0.117427	0.0551444
M^2_{387}	0.131348	0.03251	0.0427484
M^1_{388}	0.0986484	0.0932067	0.0524849
M^2_{388}	0.0832164	0.0597179	0.0481906
M^1_{389}	0.107204	0.0485843	0.0622109
M^2_{389}	0.114703	0.0389334	0.0400975
M^1_{390}	0.172982	0.0815252	0.0490546
M^2_{390}	0.0909663	0.0423376	0.0189775
M^1_{391}	0.156726	0.0680384	0.0596362
M^2_{391}	0.153416	0.10335	0.051864
M^1_{392}	0.0503849	0.128545	0.0287593
M^2_{392}	0.0590931	0.0261697	0.0257582
M^1_{393}	0.174925	0.0708429	0.0768881
M^2_{393}	0.181906	0.036552	0.0610408
M^1_{394}	0.218717	0.154259	0.0911816
M^2_{394}	0.119315	0.056808	0.0466747
M^1_{395}	0.0454145	0.053361	0.0157583
M^2_{395}	0.254073	0.108671	0.139066
M^1_{396}	0.258569	0.102085	0.108523
M^2_{396}	0.175645	0.0268714	0.0754326
M^1_{397}	0.213471	0.0844783	0.0790643
M^2_{397}	0.125871	0.128276	0.0773134
M^1_{398}	0.312753	0.0370443	0.0952464
M^2_{398}	0.143189	0.103731	0.0246615
M^1_{399}	0.197057	0.0875514	0.0718046
M^2_{399}	0.15241	0.110996	0.0777289
M^1_{400}	0.206807	0.0770429	0.0846217
M^2_{400}	0.302098	0.0641582	0.0556008

Table A.27: Table with the sixteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{401}	0.183755	0.0955607	0.0542751
M^2_{401}	0.192876	0.0998242	0.0613003
M^1_{402}	0.0746597	0.0406848	0.0323447
M^2_{402}	0.0935185	0.0393443	0.0393443
M^1_{403}	0.419875	0.115111	0.126829
M^2_{403}	0.119129	0.0532776	0.0532776
M^1_{404}	0.114869	0.0449811	0.049377
M^2_{404}	0.108814	0.0664134	0.0414634
M^1_{405}	0.06945	0.0576064	0.0676418
M^2_{405}	0.238402	0.068933	0.0617063
M^1_{406}	0.197062	0.0977264	0.0632494
M^2_{406}	0.191327	0.0545338	0.0582374
M^1_{407}	0.117061	0.02056	0.02056
M^2_{407}	0.110962	0.0210372	0.0519275
M^1_{408}	0.114483	0.056631	0.056631
M^2_{408}	0.0864762	0.0382932	0.0382113
M^1_{409}	0.335769	0.0495142	0.0914708
M^2_{409}	0.0811428	0.0680475	0.0313734
M^1_{410}	0.291736	0.0455228	0.109149
M^2_{410}	0.132525	0.0336724	0.0399123
M^1_{411}	0.113319	0.00985007	0.00705992
M^2_{411}	0.0943745	0.0301996	0.0301996
M^1_{412}	0.0794742	0.0721892	0.0345698
M^2_{412}	0.0741632	0.0843467	0.0248976
M^1_{413}	0.289244	0.104897	0.0864094
M^2_{413}	0.109083	0.0898442	0.056836
M^1_{414}	0.101575	0.0699479	0.0472249
M^2_{414}	0.249285	0.0512601	0.0512601
M^1_{415}	0.153262	0.049352	0.0511312
M^2_{415}	0.180821	0.097483	0.0809946
M^1_{416}	0.162886	0.0834759	0.0609642
M^2_{416}	0.172094	0.149126	0.0547632
M^1_{417}	0.0681675	0.107257	0.0392274
M^2_{417}	0.0783026	0.0624943	0.059158
M^1_{418}	0.116726	0.0381089	0.0334993
M^2_{418}	0.116937	0.0524779	0.047273
M^1_{419}	0.29639	0.116795	0.139957
M^2_{419}	0.0877382	0.0317133	0.0317133
M^1_{420}	0.107736	0.0870905	0.0459494
M^2_{420}	0.112504	0.0506987	0.0470694
M^1_{421}	0.0798043	0.0407228	0.0455216
M^2_{421}	0.222901	0.0661685	0.081242
M^1_{422}	0.100643	0.0334949	0.0519131
M^2_{422}	0.149788	0.0794397	0.0760595
M^1_{423}	0.0763796	0.0533704	0.0450418
M^2_{423}	0.0990287	0.0501531	0.0674163
M^1_{424}	0.315723	0.0368914	0.046862
M^2_{424}	0.131669	0.0610594	0.0445548
M^1_{425}	0.13802	0.0835154	0.0344518
M^2_{425}	0.070792	0.0533706	0.0351079

Table A.28: Table with the seventeenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{426}	0.17418	0.0623736	0.0587066
M^2_{426}	0.117425	0.0732803	0.0492233
M^1_{427}	0.15393	0.146825	0.0692913
M^2_{427}	0.0892417	0.135324	0.0224043
M^1_{428}	0.222607	0.168566	0.0729746
M^2_{428}	0.265194	0.0744542	0.0961472
M^1_{429}	0.0853569	0.0576549	0.0409419
M^2_{429}	0.107962	0.122511	0.0518288
M^1_{430}	0.133292	0.0728919	0.0727617
M^2_{430}	0.106112	0.0783533	0.0849503
M^1_{431}	0.0820761	0.0911038	0.05445
M^2_{431}	0.111693	0.0755448	0.041594
M^1_{432}	0.202884	0.106852	0.05833
M^2_{432}	0.108716	0.0261364	0.045964
M^1_{433}	0.135571	0.0850128	0.0858141
M^2_{433}	0.315308	0.0702448	0.0595782
M^1_{434}	0.0991729	0.0468744	0.0485342
M^2_{434}	0.153356	0.0521722	0.0642107
M^1_{435}	0.391234	0.106289	0.0983797
M^2_{435}	0.619814	0.0821379	0.144467
M^1_{436}	0.278715	0.045039	0.045039
M^2_{436}	0.155101	0.0550049	0.0466262
M^1_{437}	0.120705	0.100806	0.148855
M^2_{437}	0.246964	0.0927485	0.0382826
M^1_{438}	0.128945	0.0646041	0.036106
M^2_{438}	0.117892	0.0991066	0.0654311
M^1_{439}	0.210998	0.0296153	0.0457495
M^2_{439}	0.113494	0.0482783	0.0558286
M^1_{440}	0.0957174	0.0372594	0.0372309
M^2_{440}	0.110521	0.0909291	0.0322345
M^1_{441}	0.0710094	0.0492488	0.0377337
M^2_{441}	0.284966	0.0382828	0.0431509
M^1_{442}	0.15132	0.0519909	0.0519905
M^2_{442}	0.213244	0.0646813	0.0709869
M^1_{443}	0.0935582	0.0595822	0.0425131
M^2_{443}	0.123917	0.0404854	0.0410097
M^1_{444}	0.162475	0.12577	0.0485455
M^2_{444}	0.125883	0.0851943	0.0365871
M^1_{445}	0.275712	0.175338	0.0818909
M^2_{445}	0.134816	0.0788528	0.036318
M^1_{446}	0.149046	0.0601671	0.0525369
M^2_{446}	0.116598	0.0792754	0.0845513
M^1_{447}	0.129423	0.0995404	0.0684297
M^2_{447}	0.179715	0.202542	0.0483879
M^1_{448}	0.110003	0.0335075	0.03667
M^2_{448}	0.30762	0.129075	0.118897
M^1_{449}	0.0783556	0.0458239	0.0299148
M^2_{449}	0.114831	0.0859966	0.0540543
M^1_{450}	0.0744275	0.0307327	0.0374451
M^2_{450}	0.1274	0.039806	0.0409177

Table A.29: Table with the eighteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M_{451}^1	0.172109	0.107416	0.0637079
M_{451}^2	0.229686	0.0872642	0.109568
M_{452}^1	0.194135	0.0546759	0.0907398
M_{452}^2	0.158362	0.0782163	0.0793955
M_{453}^1	0.0946186	0.035641	0.0192598
M_{453}^2	0.21465	0.128038	0.0558248
M_{454}^1	0.172598	0.0312893	0.06166
M_{454}^2	0.327203	0.0442363	0.0550292
M_{455}^1	0.100162	0.0269496	0.0413906
M_{455}^2	0.272833	0.0396243	0.0664567
M_{456}^1	0.124775	0.0476603	0.0693977
M_{456}^2	0.147411	0.0512621	0.0584165
M_{457}^1	0.044753	0.0272291	0.0272291
M_{457}^2	0.273165	0.0295533	0.135075
M_{458}^1	0.201697	0.0513578	0.0652211
M_{458}^2	0.211088	0.0868692	0.0650304
M_{459}^1	0.162094	0.0744186	0.0578904
M_{459}^2	0.305166	0.0981819	0.105881
M_{460}^1	0.092666	0.0643304	0.042764
M_{460}^2	0.402984	0.0628259	0.165882
M_{461}^1	0.156496	0.0741491	0.0649429
M_{461}^2	0.113452	0.070708	0.0321607
M_{462}^1	0.0756857	0.0537255	0.0528709
M_{462}^2	0.107118	0.0369478	0.0480374
M_{463}^1	0.0537231	0.061891	0.0268152
M_{463}^2	0.0974989	0.0787348	0.0384853
M_{464}^1	0.20478	0.0499588	0.0874904
M_{464}^2	0.0853001	0.0436542	0.0347427
M_{465}^1	0.0832011	0.0661142	0.0469648
M_{465}^2	0.0689071	0.132847	0.0318667
M_{466}^1	0.125299	0.0285004	0.0581205
M_{466}^2	0.140061	0.0689732	0.0568135
M_{467}^1	0.165538	0.0349194	0.0826046
M_{467}^2	0.0986379	0.0307522	0.0342973
M_{468}^1	0.111623	0.0964002	0.0250033
M_{468}^2	0.0739879	0.0479931	0.0192117
M_{469}^1	0.212829	0.0731508	0.060327
M_{469}^2	0.155654	0.072375	0.069009
M_{470}^1	0.102161	0.095448	0.0307732
M_{470}^2	0.23967	0.0627344	0.0385126
M_{471}^1	0.167334	0.0890129	0.0836267
M_{471}^2	0.200108	0.0624693	0.0932158
M_{472}^1	0.459428	0.0622839	0.165674
M_{472}^2	0.0773292	0.0526575	0.0482867
M_{473}^1	0.0838033	0.0387941	0.036914
M_{473}^2	0.204815	0.0295913	0.0713772
M_{474}^1	0.0579302	0.0405324	0.0343476
M_{474}^2	0.153993	0.0448958	0.0630911
M_{475}^1	0.298292	0.0474572	0.0857004
M_{475}^2	0.155505	0.0576917	0.0596003

Table A.30: Table with the nineteenth part of the results from the scope experiment.

FSM	PColl deviation	PSq deviation	Sq deviation
M^1_{476}	0.0820727	0.0424962	0.0411647
M^2_{476}	0.145468	0.0775201	0.0594205
M^1_{477}	0.100945	0.0497889	0.0476427
M^2_{477}	0.239191	0.0789397	0.0893663
M^1_{478}	0.277511	0.137918	0.139281
M^2_{478}	0.0475261	0.0396001	0.0187881
M^1_{479}	0.0693619	0.0880308	0.045945
M^2_{479}	0.239564	0.0645304	0.0845268
M^1_{480}	0.370271	0.127175	0.134167
M^2_{480}	0.351924	0.160024	0.125015
M^1_{481}	0.0745895	0.0899464	0.0323777
M^2_{481}	0.0910752	0.0886294	0.0478593
M^1_{482}	0.106205	0.0742103	0.0568929
M^2_{482}	0.059604	0.02957	0.0222431
M^1_{483}	0.083845	0.0971641	0.0235318
M^2_{483}	0.148673	0.124305	0.0446923
M^1_{484}	0.131703	0.0798524	0.0491
M^2_{484}	0.0960528	0.065247	0.0411868
M^1_{485}	0.0731827	0.0334442	0.0279392
M^2_{485}	0.219637	0.109743	0.0676613
M^1_{486}	0.0799472	0.0727899	0.0324964
M^2_{486}	0.10982	0.0730745	0.0475322
M^1_{487}	0.179289	0.0855079	0.0620036
M^2_{487}	0.215766	0.034421	0.0854916
M^1_{488}	0.17113	0.10345	0.0835216
M^2_{488}	0.181537	0.0373203	0.0483895
M^1_{489}	0.106505	0.0342126	0.0403732
M^2_{489}	0.0824679	0.0401266	0.0349669
M^1_{490}	0.119606	0.0670987	0.0458904
M^2_{490}	0.0932935	0.0740722	0.0393758
M^1_{491}	0.128965	0.0513455	0.0658652
M^2_{491}	0.0705518	0.0293303	0.0353849
M^1_{492}	0.0962019	0.0675422	0.0223416
M^2_{492}	0.123132	0.0473203	0.0473203
M^1_{493}	0.188326	0.0656698	0.0777106
M^2_{493}	0.265494	0.0483581	0.0391683
M^1_{494}	0.231715	0.128679	0.128679
M^2_{494}	0.256345	0.172227	0.0849967
M^1_{495}	0.164598	0.140646	0.119249
M^2_{495}	0.0998203	0.0880942	0.0945619
M^1_{496}	0.117457	0.0589744	0.0589744
M^2_{496}	0.314809	0.0562019	0.06904
M^1_{497}	0.287765	0.100214	0.155774
M^2_{497}	0.138707	0.0805146	0.073442
M^1_{498}	0.0988358	0.094019	0.0384221
M^2_{498}	0.166828	0.07395	0.0612405
M^1_{499}	0.16444	0.186858	0.0675427
M^2_{499}	0.215986	0.086802	0.0990668
M^1_{500}	0.237172	0.0663631	0.0749768
M^2_{500}	0.237097	0.0663329	0.0915869

Table A.31: Table with the twentieth part of the results from the scope experiment.

Bibliography

I find television very educating. The minute somebody turns it on, I go to the library and read a good book.

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List of Acronyms

DRR .... *Domain to Range Ratio*

FEP .... *Failed Error Propagation*

FSM .... *Finite State Machine*

FSMs .... *Finite State Machines*

FSTs .... *Finite State Transducers*

SUT .... *System Under Test*

SUTs .... *Systems Under Test*