Optik 121 (2010) 2269–2272

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Evaluation of the meridional longitudinal spherical aberration from corneal topography measurements

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ARTICLE INFO

Article history:
Received 14 May 2009
Accepted 20 September 2009

Keywords:
Corneal profile
Videokeratometry
Spherical aberration

ABSTRACT

This paper shows how corneal topographic data can be used to determine the value of the longitudinal spherical aberration. We have obtained the corneal profiles and the values of the longitudinal spherical aberration for the rays propagating within the steepest and flattest meridional planes, by using a real raytracing algorithm. These corneal profiles have been also fitted to conicoids and the asphericity parameter has been calculated. We have found that the longitudinal spherical aberration follows a parabolic dependence for a circular region of 5 mm in diameter. This parabolic dependence has been fitted with a polynomial function. The data provided by commercial topographic systems can be used to obtain the longitudinal spherical aberration along the selected meridians.

1. Introduction

Longitudinal spherical aberration (LSA) has been used in the design of modern multifocal contact lenses for presbyopes [1]. The basic principle of these lenses is to control the LSA of the compound lens-eye system obtaining an LSA equivalent to a depth of focus which in turn is related to the required accommodation amplitude producing a permissible confusion disc. This depth of focus allows the presbyope to optimize his perception. The LSA of the total eye is a balance between the LSA of the cornea and the LSA from the lens of the eye [2,3]. Therefore, the knowledge of the corneal LSA can be used to improve the performance of the lens-eye system.

Modern videokeratometry produces an invaluable information that can be used not only for surgical and optometric evaluation but also to explore the optical characteristics of the cornea in depth [4–7]. There are many studies that have analyzed the shape of the cornea and this has been fitted with several different models [8–14]. Some other recent papers have evaluated the Zernike composition of the wavefront aberration produced by this refracting surface [15–19]. In this work we investigate the LSA of the cornea calculated for rays propagating within meaningful meridional planes. Our approach uses two-dimensional real raytracing algorithms. Therefore, it provides an exact evaluation of LSA directly related with its definition as the discrepancy between the paraxial and real location of the image along the optical axis. The intrinsic limitations of this approach are compensated by the simplicity of the model that can be rapidly understood by a wide audience. We have evaluated this aberration in terms of vergences. We consider this approach easier to relate with the compensation of ametropies and with the variation of the refractive power of the eye due to accommodation than the description of the spherical aberration as a polynomial coefficient or RMS value. However, the values obtained here can be connected to those expressed as wavefront aberration by using well established relations [20,21]. Previous studies have analyzed the power distribution of the cornea and its relation to the LSA of soft and rigid contact lenses fitted on the eye [22–25]. This study obtains the actual LSA of the cornea expressed in diopters by using the experimental data produced by a videokeratometer. We present the method of calculation of the LSA induced by the cornea from the topographic data obtained from the videokeratometer. We show not only the LSA results but the average profile of the corneal surface, and a basic population distribution of the corneal shape.

2. Data acquisition and modeling

The EyeSys videokeratometer model EyeSys Windows Workstation version 1.20 W was used. The system provides 16 readings of the radius of the cornea every 1° at positions determined by the rings of the keratometer's cone and spaced about $\frac{1}{4}$ mm. Further information could be extracted from this topographic data [4,5]. For example, it should be possible to use a Zernike fitting procedure to determine the surface of the cornea and relate it with the classical third order aberration of the whole surface [15,17].

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0030-4026/$ - see front matter © 2010 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2009.09.011

Due to the fact that we are mainly interested in the behavior of meridional rays we have chosen the flattest and steepest meridians of the cornea. The angular location of these privileged meridians are also provided by the software of the videokeratometer by means of an internal utility that simulates a keratometric reading (SIM-K). This simplification may compromise the rigorous definition of spherical aberration because of its intrinsic rotational symmetry. However, when analyzing these privileged meridional planes we still may define and calculate the LSA for those rays propagating within these meridional planes [1]. The topographic data along the selected meridians have been processed with an utility provided by EyeSys to obtain the sagita of the cornea profile. The calculated profile is represented in Fig. 1. We have used these sagitae data to calculate the spherical aberration [6,7,9,11,22,26–28]. Once the flattest or the steepest meridian has been selected, we have made a calculation of the intersection of the rays incident on the corneal surface at a given height. To obtain regularly spaced values we first made an interpolation of the sagita values obtained at the measured points to a equally spaced points (spatial distance along the meridian of 1/4 mm). The intersection of the rays were made by a real raytracing procedure assuming that the corneal profile along the meridian is composed of straight lines connecting the sagita points obtained by the measuring system. The scheme of the calculation is shown in Fig. 2. The intersection position z is obtained as

$$z = \frac{z_a + z_b}{2} + \frac{y_a + y_b}{2}\left\{\tan\left[\tan^{-1}\left(\frac{z_b - z_a}{y_b - y_a}\right) - \sin^{-1}\left(\frac{1}{n}\sin\left[\tan^{-1}\left(\frac{z_b - z_a}{y_b - y_a}\right)\right]\right)\right]\right\}^{-1}, \quad (1)$$

where $n = 1.376$ is the value of the index of refraction of the cornea, and (z_a, y_a), (z_b, y_b) are the coordinates of consecutive

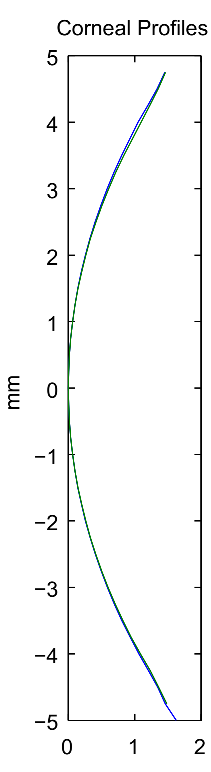

Fig. 1. Profile of the cornea obtained by averaging the sagitae obtained from the videokeratometric data. We have plotted both the mean flattest and steepest meridians. In this sagita representation the differences are very small. Moreover the presented data are averaged, and this averaging minimizes the differences.

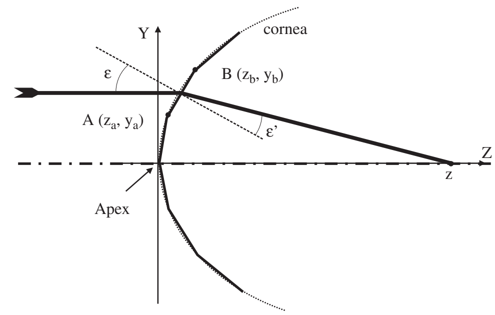

Fig. 2. Diagram of the raytracing used to obtain the longitudinal spherical aberration.

couples of the measured points of the corneal profile. We have assumed that the input rays are perpendicular to the reference plane defined by the tangent to apex of the cornea. They are incident at the midpoint point between the measured points of the cornea. From these intersection points we have made a calculation of the LSA by subtracting the impact point values from the intersection of the rays that comes close to the apex of the corneal surface, that is $x = 0$. These paraxial data have been evaluated to obtain the paraxial power of the cornea, and the astigmatism induced by the asymmetry of this surface. The results are expressed in vergences to relate them with the power values obtained from the videokeratographic maps. Several studies have considered an alternative approach evaluating the wavefront aberration from the height maps. The wavefront aberration can be related with transversal and longitudinal aberrations measured at the image plane [20]. This relation can be expressed as follows:

$$\mathrm{LSA}(\mathrm{m}^{-1}) = \frac{12}{h^2}\langle \mathrm{WA_{SSA}} \rangle, \quad (2)$$

where h is the maximum radius of the aperture, and $\langle \mathrm{WA_{SSA}} \rangle$ is the mean value of the Seidel spherical aberration measured as a wavefront aberration (these values need to be introduced in meters). To obtain this equation we have used the form of the Seidel spherical aberration (SSA subindex) as $\mathrm{WA} = \mathrm{WA_{SSA}}\rho^4$, being ρ a radial coordinate normalized to the aperture radius. Besides, the relation between the mean value and the aberration coefficient is given as $\langle \mathrm{WA_{SSA}} \rangle = 1/3\mathrm{WA_{SSA}}$ [20]. When the value of the aberration is given at some specific height the relation between the wavefront aberration and the corresponding LSA in diopters is

$$\mathrm{LSA}(\mathrm{m}^{-1}) = \frac{4}{h^2}\mathrm{WA_{SSA}}. \quad (3)$$

Thibos [21] presents a similar expression relating the root mean square error of the wavefront aberration, σ_{WA}, and the equivalent defocus, $\Delta F'$:

$$\Delta F' = \frac{16\sqrt{3}}{h^2}\sigma_{\mathrm{WA}}. \quad (4)$$

This equation can be also used to relate more complex compositions of optical aberrations. For example, the results obtained by Wang [19] can be treated by using this expression.

3. Experimental results and discussion

Corneal videokeratometry data have been obtained from 33 eyes of 17 young subjects aged 19–27 years old, (6 male and 11 female). All these corneas can be considered as healthy corneas.

With the paraxial data we have obtained the distribution of corneal astigmatism versus the power of the cornea. This is presented in Fig. 3. The corneal power is 43.15 ± 1.80 D (mean value $\pm$ standard deviation) with a corneal astigmatism of 0.87 ± 0.5 *D*. This distribution is in good agreement with the typical figures obtained in practice and suggests that we are studying a representative population of young individuals [10,11,29]. The fitting of the corneal profiles with an aspheric surface gives seven oblate ellipsoids, 53 prolate ellipsoids and six hyperboloids. These hyperbolic shapes are obtained for those profiles that fit the worst with a conicoid curve of the form: $y^2 = 2r_0x - px^2$. The mean value of the asphericity parameter is $p = 0.58$. When analyzing separately the flattest and steepest meridians, the fitting with a conicoid surface produces a mean value of the asphericity parameter of $p = 0.62$ for the flattest meridian, and $p = 0.53$ for the steepest one. The central radius of curvature obtained from the fitting, r_0, has a mean value of 7.85 ± 0.55 mm for the flattest meridian, and 7.6 ± 0.48 mm for the steepest one. The mean value of the 66 analyzed meridians is 7.7 ± 0.5 mm. The radii of curvature obtained in this study fit with those obtained by Kiely et al. [8]. However, the asphericity parameter founded by us is slightly below the one obtained by Kiely et al. The discrepancy is probably due to the appearance of hyperbolic shapes in some cases where the fitting is worse than in the rest of data.

The flattest and steepest meridians are treated separately to obtain the LSA. These meridians were selected from the results of the SIM-K function given by the videokeratometer. In Fig. 4a and b we show an histogram that represents the distribution of values of LSA for the flattest and steepest meridians, respectively. If we combine both sets of data the total spherical aberration distribution shown in Fig. 4c is obtained. As we can see the majority of the subjects is clustered around a parabolic shaped distribution. All the values are positive. The steepest and the flattest meridians behave very similarly. A more detailed analysis shows that the values for the flattest meridian are more variable. The spherical aberration of the corneas is < 1.00 *D* for apertures below 5 mm in diameter. This is one of the most important results of this study, and agree very well with those data obtained experimentally by other authors and presented in Table 1 [8,16,17,30,19]. In those cases where the aberration is given as a wavefront aberration, we have used the relations between the value, the mean value, or the RMS value of the spherical aberration (Eqs. (2)–(4)). The results given by Kiely et al. [8] are calculated by modeling the corneal surface as a conicoid.

In Fig. 5 we have plotted the mean value of the spherical aberration and its standard deviation. From this data it is also possible to obtain a polynomial fit to the data. We have made this fit within a region of 5 mm in diameter. The obtained polynomial is: $P_2(x) = 0.1121\,x^2 - 0.0002x + 0.0123$ D. If we extend the region to be fitted it is necessary to use a fourth degree polynomial that is given by $P_4 = -0.0100\,x^4 - 0.0015\,x^3 + 0.1742\,x^2 + 0.0122\,x - 0.0305$ *D* (with *x* in mm). Both curves lie inside the range limited by the standard deviation of the spherical aberration represented in the

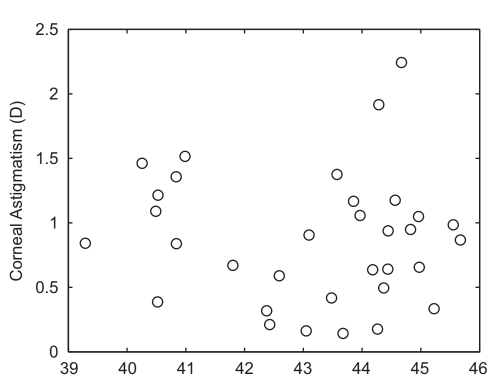

Fig. 3. Distribution of the data of individual subjects showing mean power of each cornea and its astigmatism.

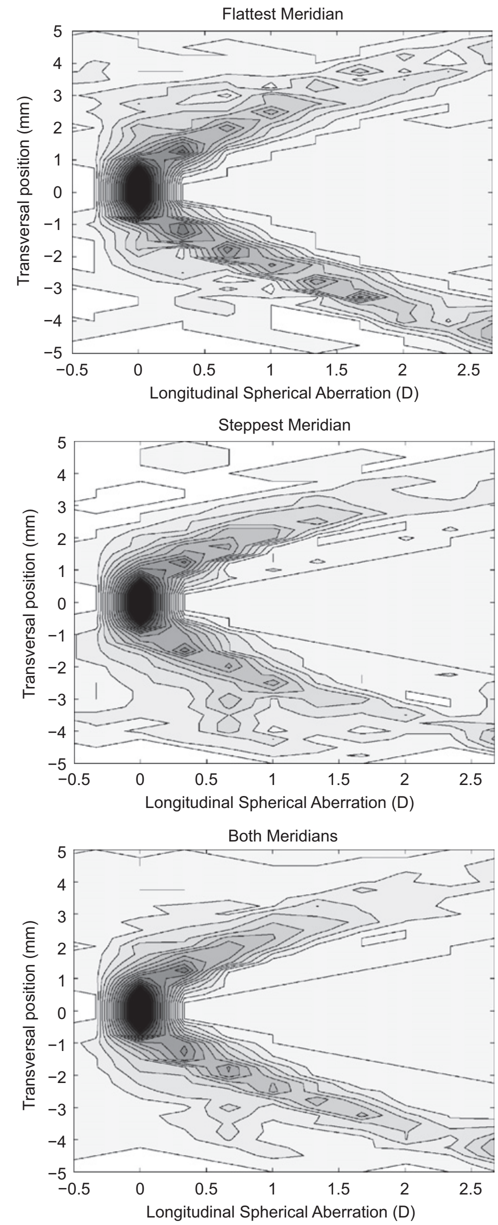

Fig. 4. Distribution of the population of the measured eyes with respect to their values of longitudinal spherical aberration and transversal position. Top: flattest meridian; center: steepest meridian; bottom: both meridians. The contour plots represents the number of people having a given value of longitudinal spherical aberration at a given transverse position with respect to the apex of the cornea. Due to the fact that we set a null value of spherical aberration at the corneal apex the maximum is obtained at this point. The contour lines are equally separated and represent a 5% of variation.

Table 1
Values of the LSA in diopters presented in the references. The values in parenthesis are the pupil diameters.

Reference	LSA (m^{-1}) (@ pupil diameter (mm))
Kiely et al. [8]	1 (@ 3)
Artal and Guirao [16]	0.35–0.89 (@ 4)
Guirao et al. [17]	1.51 (@ 6), 0.48 (@ 4)
Barbero et al. [30]	0.7 (@ 6.5), 0.2 (@ 5)
Wang et al. [19]	0.86 ± 0.26 (@ 6)

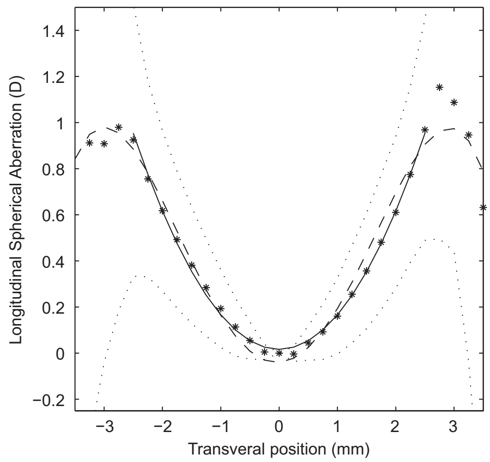

Fig. 5. Plot of the longitudinal spherical aberration. The solid circles correspond with the mean values obtained from the measured corneas. The dotted lines are the limits given by the standard deviation of the distribution. The solid line represents the parabolic fitting of the spherical aberration inside a region of 5 mm in diameter and centered at the corneal apex. The dashed line is the fitting with a polynomial of fourth degree.

Fig. 5 by two dotted lines. The standard deviation also increases when a large aperture is considered.

4. Conclusion

We have calculated values of the longitudinal spherical aberration by using a simple geometrical procedure from the data obtained with a commercial videokeratometer. This approach may help the professional working with videokeratometers to obtain added-value data from typical topographic data. This calculation shows the capability of these systems to provide sufficient amount of data to perform physiological optics research on the eye. We have performed a polynomial fitting of the LSA results that can be used to analytically obtain the LSA of the cornea at a given height. The LSA has been calculated for the two privileged meridians along the steepest and flattest meridians. The results show <1.00 D of longitudinal spheric aberration for apertures with diameter smaller than 5 mm, and are in good accordance with the values obtained by other researchers using a different approach. These results can be of interest when fitting aplanatic contact lenses, and expands the use of topographic data for analyzing the corneal optical aberrations by using real raytracing procedures.

Acknowledgments

We are deeply grateful to Dr. Eduard A. Phillipe, of EyeSys Laboratories Inc. for allowing us to use the "cornsag" utility. We also appreciate very much the collaboration of the Ophthalmology Department of the University Complutense of Madrid for facilitating the use of the videokeratometer.

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