Dynamical control of cold bosons using oscillating potentials

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Abstract. The dynamics of cold bosonic atoms held in an optical lattice can be profoundly modified by using a periodic driving potential to induce the quantum interference effect termed "coherent destruction of tunneling". In this way the entanglement and localization of individual particles [1] can be very precisely controlled, pointing to the attractive possibility of using these systems for quantum information processing. This tunneling control also allows the effects of inter-particle interactions to be manipulated in a novel fashion, thereby giving precise control of the quantum phase transition [2] between the Mott insulator and the superfluid, and also providing a convenient means to manipulate the self-trapping effect [3].

1. Introduction

Systems of ultracold atoms trapped in optical lattice potentials are currently the focus of intense theoretical and experimental investigation. In contrast to the normal lattice systems encountered in solid state physics, the experimental parameters of these systems can be varied and controlled to an extremely high degree of precision, and their high degree of isolation from the environment allows their quantum dynamics to remain coherent over long timescales. These properties naturally make them an excellent starting point for engineering and manipulating entangled states, which are of great relevance to quantum information processing applications, and also present the exciting prospect of observing novel coherent effects in a well-controlled arena.

Here we will show how a quantum interference effect [4] termed "coherent destruction of tunneling" (CDT), which occurs when a system is subjected to a time-periodic driving force, can be used to control the dynamics of a lattice boson system. This effect has been observed experimentally in such a system very recently [5]. We will firstly show how a single particle can be manipulated, to produce directed motion and to realize quantum gates. We will then demonstrate how introducing interactions enriches the system's behaviour, and how balancing the CDT effect with photon-assisted tunneling allows the system's dynamics to be precisely controlled.

2. Model

A one-dimensional system of optically-confined bosons can be described very well [6] by the Bose-Hubbard model

$$H_{\mathrm{BH}} = -J \sum_{\langle j,k \rangle} \left[a_j^\dagger a_k + H.c. \right] + \frac{U}{2} \sum_j n_j \left(n_j - 1 \right), \tag{1}$$
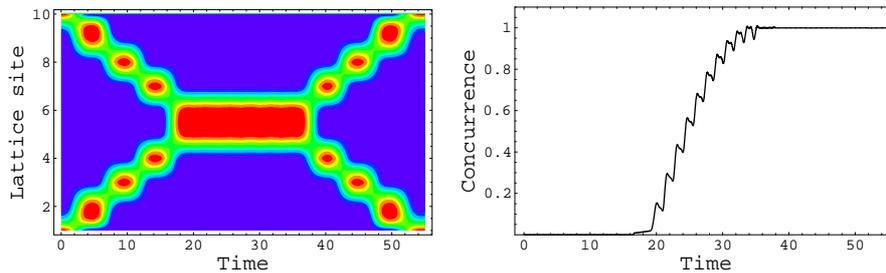
Figure 1. Left: A two-particle system is initialised with particle a in site 1, and particle b in site 10. The driving field is used to bring the particles to the centre of the lattice, where they interact, and then to separate them. Right: The concurrence measures the degree of entanglement between the particles. This rises during the interaction period, the duration of which is chosen to produce the maximally-entangled state. This value remains constant as the particles are then separated.

where $a_j/a_j^\dagger$ are the standard annihilation/creation operators for a boson on site j, $n_j = a_j^\dagger a_j$ is the number operator, J is the tunneling amplitude between neighboring sites, and U is the repulsion between a pair of bosons occupying the same site.

We now consider adding a periodic driving potential to the system, $H(t) = H_{\mathrm{BH}} + K \sin \omega t \sum_j ja \, n_j$. where a is the lattice spacing. As the Hamiltonian is periodically time-dependent, $H(t) = H(t+T)$, the Floquet theorem can now be invoked to define quasienergies and Floquet states which generalize the energies and eigenstates of the static case. When the driving frequency is the dominant energy scale of the problem, Floquet analysis reveals that the effect of the driving is to renormalize the intersite tunneling as $J_{\mathrm{eff}} = J\mathcal{J}_0(Ka/\omega)$, where J_0 is the zeroth Bessel function of the first kind. Accordingly, whenever the Bessel function vanishes, the tunneling is suppressed, giving rise to CDT.

3. Single particle control

Although CDT allows us to control the amplitude of J_{eff}, in order to produce directed motion it is necessary to distinguish between motion to the left, and motion to the right. This can be done by employing a bipartite lattice with two *different* lattice spacings x_1 and x_2. If this system were driven by a high-frequency field such that $\mathcal{J}_0(Kx_1/\omega) = 0$, then tunneling processes between sites separated by x_1 would be destroyed, and a particle could only tunnel to its other neighbour, separated by x_2. If we instead set $\mathcal{J}_0(Kx_2/\omega) = 0$, then the opposite effect would occur. Thus by periodically modulating the amplitude of the driving field between these values, a particle initialised in a single lattice site can be successively stepped through the lattice in a ratchet-like fashion [1].

This level of control over a particle's position allows us to induce entanglement between particles, and thereby realise quantum gates. Consider initialising the system with particle a in site 1, and particle b in site N, where a and b are distinguishable (Fig.1). The periodic driving field can then be used to move them towards the centre of the lattice, where they interact with each other. For sufficiently large values of U, this interaction takes the form of an effective Heisenberg interaction, where $J_{\mathrm{H}} = 4J_{\mathrm{eff}}^2/U$. Controlling the length of time for which the particles interact allows us to set the degree of entanglement between the particles; for $t = \pi/2J_{\mathrm{H}}$ the entanglement is maximised, and the particles undergo a $\sqrt{\mathrm{SWAP}}$ operation. By then applying the driving field to separate the particles, we are able to produce a mesoscopically-separated, maximally entangled two-particle state.

Figure 2. The localization produced in a 5-site system with $U = 8J$ as a function of the frequency ω of the driving field and its amplitude K. When $n\omega = U$ the particles delocalise (the horizontal blue bands) due to photon assisted tunneling, except at sharply defined points (red) where J_{eff} vanishes. Between the bands localization is high (red), due to the relatively large size of the Hubbard-interaction.
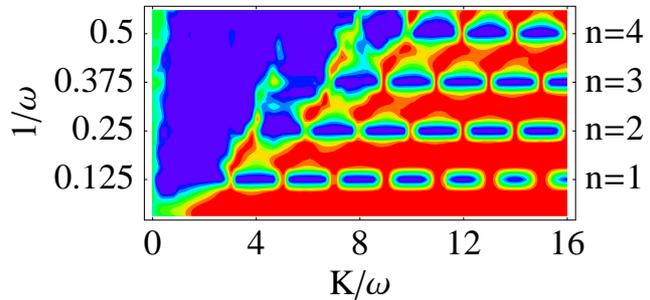

4. Tuning the Mott transition

The ground-state properties of the Bose-Hubbard model are governed by the ratio U/J which is set by the parameters of the optical lattice. When U/J is large, particles localize on the lattice sites and form a Mott insulator. In the opposite case when U/J is small, the bosons are able to delocalize across the lattice and form a superfluid; between these extremes the quantum phase transition known as the Mott transition occurs. This has been observed in experiment [7] by altering the depth of the optical lattice. By employing CDT to regulate the amplitude of J, however, it is possible to drive this transition *without* altering the lattice parameters [8].

When U is comparable in size to ω, the Floquet analysis of the system becomes more complicated [2], and U must be decomposed in the form $U = n\omega + u$. When the reduced interaction, u, is zero the driving field is resonant with the interaction energy, and it may be shown [2] that $J_{\text{eff}} = J\mathcal{J}_n(K/\omega)$. This clearly reduces to the previous result in the high-frequency limit for which $n = 0$. In Fig.2 we can see that as expected, when $J_{\text{eff}} = 0$, the tunneling is suppressed and the system enters the Mott state. However, away from these special points, the resonance between the driving frequency and U means that absorption of energy from the driving field can compensate for the Hubbard interaction, restoring a delocalised state (photon-assisted tunneling). The transition between the Mott and superfluid states can thus be regulated just by changing the value of K/ω by a very small amount.

5. Controlling self-trapping

In the Mott and superfluid states the bosons are distributed homogeneously across the lattice. If instead the bosons are concentrated in a small region of high density, the Hubbard interaction can produce a non-intuitive effect termed *self-trapping*. In this case, although the interaction is repulsive, the initial state does not spread with time. This can be understood through an energetic argument; the presence of the lattice potential means that the energy of free particles is confined to a Bloch band of width $4J$. If the initial configuration has an energy much higher than this, it cannot be converted into kinetic energy, and thus the initial state cannot decay.

As before, by controlling the ratio U/J we can regulate whether a given state is self-trapped [3]. The choice of frequency permits an additional level of control, as we show in Fig.3. Setting the frequency equal to the energy difference between N and $(N-1)$ localised bosons excites one particle out of the central site to its nearest neighbours. If, however, $\mathcal{J}_0(K/\omega) = 0$ it can propagate no further, and so executes a periodic oscillation between these sites. Tuning K/ω away from this value releases the particle, which is then free to travel across the lattice. This procedure may be used to successively depopulate the initial state, producing a sequence of well-defined, coherent atomic pulses.
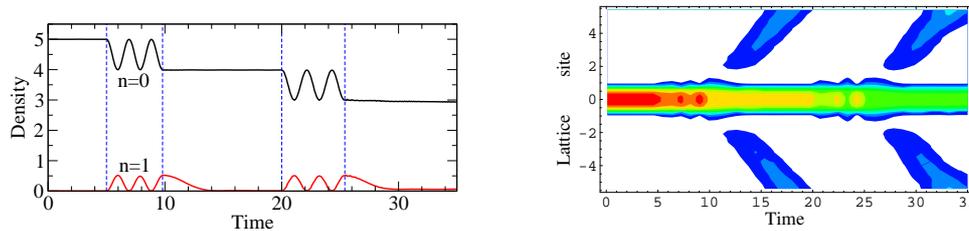
Figure 3. Left: Time evolution of the particle density of the trapping site ($n = 0$) and its neighbor ($n = 1$) for a 5-boson system. The driving parameters are chosen to first drive a Rabi oscillation of a single particle between the central site and its neighbours, and then to cause the emission of single bosons at $t \simeq 10$ and 25. Right: Particle density of the system. The emitted particles move away from the central site at a roughly constant speed, dispersing slightly as they propagate through the lattice.

6. Conclusions

By inducing CDT, a periodic driving field provides a new way to control and manipulate coherent particles held in optical lattices. It not only gives precise control over the dynamics of single particles, but also allows the manipulation of strongly-correlated states such as the Mott insulator and the self-trapped state. This opens up the prospect of studying non-equilibrium dynamics of strongly-correlated systems, and has many applications in quantum information processing.

References

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