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# Complementarity Enforced by Random Classical Phase Kicks

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Quantum phase difference is used to analyze which-path detectors in which the loss of interference predicted by complementarity cannot be attributed to classical momentum transfer between the interfering particle and the measuring apparatus. It is shown that the dynamics of the measurement disturbs the interference via random classical phase shifts. [S0031-9007(98)07560-7]

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Double-slit interferometers provide perhaps the simplest example in which the coherent addition of quantum probability amplitudes leads to interference, and supply an excellent illustration of complementarity: if a "which-path" detector is inserted in order to determine the path taken by the particle within the interferometer, the interference is necessarily destroyed.

From a dynamical point of view, it is natural to consider this destruction of interference as a consequence of the perturbation of the interfering beams induced by the interaction with the quantum apparatus. In all of the classic examples, the disturbance is somehow uncontrollable random momentum transfer [1,2]. In addition to a loss of coherence, the exchange of momentum also causes the broadening of the single-slit diffraction pattern.

However, a which-path measurement has been presented by Scully, Englert, and Walther [3], where the interference is lost seemingly without momentum transfer or any other alteration of the wave functions of the interfering paths. This would be supported by the absence of any broadening of the diffraction pattern. As further evidence of the lack of alteration, it has been shown that a quantum eraser can be devised which allows an interference pattern of high visibility to be extracted from within a featureless pattern if a suitable measurement is made on the path detector which erases the path information.

This intriguing novelty has been followed by a vivid and enlightening debate on the connections between complementarity, uncertainty relations, and momentum transfer [1,2,4,5].

Recently, this situation has been readdressed by considering that quantum mechanics allows different definitions of what constitutes a random momentum kick [2,5]. In particular, it seems possible to discriminate between classical and quantum momentum transfer: if the particle experiences random classical momentum kicks its output momentum probability distribution $P_{\mathrm{ob}}(p)$ results from the convolution of the initial one $P(p)$ with a probability distribution $\Omega(p)$ of momentum transfer:

$$P_{\mathrm{ob}}(p) = \int dp'\, P(p - p')\Omega(p')\,. \qquad (1)$$

This definition fits properly with our classical intuition and leads to the broadening of the single-slit diffraction pattern (unless the slits have zero width, in which case there cannot be further broadening).

On the other hand, a quantum transfer involves the convolution of momentum amplitudes instead of probabilities. Quantum kicks can destroy the interference without modifying the diffraction pattern. However, they lack the simple classical picture that Eq. (1) provides.

In this Letter we present a new approach that reconciles the loss of interference with the notion of classical randomization expressed in Eq. (1). To this end, the analysis of the phenomenon in terms of momentum is precluded and another dynamical variable must be used. It can be expected that the mechanisms which enforce complementarity may vary from one experimental situation to another. Thus detection schemes such as the one introduced in Ref. [3] suggest the investigation of other disturbing agents different from momentum transfer.

  

In this regard, phase difference is distinguished as the central tool in solving and understanding classical interference problems. A full quantum analysis of the problem in terms of the phase difference needs a quantum description of this variable. Although this issue may encounter some difficulties, currently there are adequate and useful solutions [6]. This will allow us to examine how this variable transforms due to the interaction and whether such a transformation is the result of random classical phase shifts.

Let us begin our analysis by considering a double-slit arrangement in which the two paths for the interfering particle are represented by the normalized vectors $|\psi_1\rangle$ and $|\psi_2\rangle$. For definiteness, we assume the problem to be one dimensional, and $\psi_1(x)$ and $\psi_2(x)$ will denote the position wave functions on the corresponding aperture. The two apertures are taken to be identical and spaced $d$ apart in the $x$ direction so that $\psi_2(x) = \psi_1(x + d)$, which implies that $\tilde{\psi}_2(p) = e^{ipd/\hbar}\tilde{\psi}_1(p)$ in the transverse-momentum representation. Since the two apertures do not overlap, we have $\langle\psi_1 \mid \psi_2\rangle = 0$.

In the absence of any path detector, the most general pure state for the interfering particle in the Hilbert space of the system $\mathcal{H}_s$ can be written as

$$|\psi\rangle = \cos(\theta/2)\,|\psi_1\rangle + \sin(\theta/2)e^{i\phi}|\psi_2\rangle. \tag{2}$$

The parameter $\theta$ gives the probability of each path, and $\phi$ is the relative phase. The interference is recorded on a distant screen such that the position pattern on it is equivalent to the momentum distribution of $|\psi\rangle$ on the plane of the apertures,

$$P(p) = |\tilde{\psi}_1(p)|^2[1 + \sin\theta\cos(pd/\hbar + \phi)]. \tag{3}$$

In this expression $|\tilde{\psi}_1(p)|^2$ is the single-aperture diffraction pattern. If, as usual, $|\tilde{\psi}_1(p)|^2$ is a slowly varying function of $p$, the visibility of the fringes is $\mathcal{V} = \sin\theta$.

The observation of the trajectory requires the use of auxiliary degrees of freedom. The system-apparatus interaction is represented by a unitary transformation $U$ acting on $\mathcal{H}_s \otimes \mathcal{H}_a$, where $\mathcal{H}_a$ is the Hilbert space of the apparatus.

We are interested in detection schemes in which there is at least one initial state of the apparatus $|A\rangle$ such that the system-apparatus coupling does not modify the wave function associated with the particle passing through a single slit, which means that $U|\psi_j\rangle|A\rangle = |\psi_j\rangle|A_j\rangle$.

When the unobserved state of the particle is given by Eq. (2), the final state after the interaction is

$$U|\psi\rangle|A\rangle = \cos(\theta/2)\,|\psi_1\rangle|A_1\rangle + \sin(\theta/2)e^{i\phi}|\psi_2\rangle|A_2\rangle. \tag{4}$$

If the final state of the apparatus is not measured, we have the following after the interaction:

$$P_{\rm ob}(p) = |\tilde{\psi}_1(p)|^2[1 + \sin\theta|\langle A_1 \mid A_2\rangle| \times \cos(pd/\hbar + \phi + \delta)], \tag{5}$$

where $\delta = \arg\langle A_1 \mid A_2\rangle$. The interaction effectively does not modify the diffraction pattern in any case, which is still given by $|\tilde{\psi}_1(p)|^2$. However, the fringe visibility has changed to $\mathcal{V}_{\rm ob} = |\langle A_1 \mid A_2\rangle|\mathcal{V}$, which is less than or equal to the visibility $\mathcal{V}$.

The loss of visibility is related to the performance of the path detection, which depends on the inner product $\langle A_1 \mid A_2\rangle$. The lesser the value of $\langle A_1 \mid A_2\rangle$, the better the accuracy that is achievable. Optimum path detection occurs when $\langle A_1 \mid A_2\rangle = 0$, since in this case the states of the apparatus are perfectly distinguishable and the path followed can be discriminated unambiguously. Therefore, $\mathcal{V}_{\rm ob} = 0$ and the interference is completely lost.

Whenever the single-slit diffraction pattern is not modified, the degradation of the interference cannot be explained in terms of random classical momentum kicks. Since in Eq. (3) neither $|\tilde{\psi}_1(p)|^2$ nor the path probability $\theta$ are modified by the observation, we can conclude that the interaction with the apparatus will modify the phase difference $\phi$ between the interfering paths (we are also excluding the possibility of any modification of the separation $d$ of the apertures). In order to translate this reasoning into quantitative relations, a quantum description of this dynamical variable is necessary.

The effective system space $\mathcal{H}_s$ is always two dimensional because we focus only on meaningful runs, where the particle actually goes through the slits. Moreover, the observation arrangements we are considering do not modify the dependence on $x$ of the wave functions $\psi_1(x)$ and $\psi_2(x)$, so that no other vectors are necessary to describe the interfering particle, even after the interaction. In these conditions, it is possible to describe the phase-difference variable (we shall denote it by $\varphi$) by using the positive-operator measure

$$\Delta(\varphi) = \frac{1}{2\pi}[|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + e^{-i\varphi}|\psi_1\rangle\langle\psi_2| + e^{i\varphi}|\psi_2\rangle\langle\psi_1|], \tag{6}$$

where $\varphi$ can take any value in a $2\pi$ interval [7]. This defines a probability distribution for the phase difference as $P(\varphi) = {\rm tr}[\rho\Delta(\varphi)]$, where $\rho$ is the density matrix of the system.

For the unobserved state (2), we have

$$P(\varphi) = \frac{1}{2\pi}[1 + \sin\theta\cos(\varphi - \phi)], \tag{7}$$

which is centered on $\phi$, while, for the observed state (4), we have

$$P_{\rm ob}(\varphi) = \frac{1}{2\pi}[1 + \sin\theta|\langle A_1 \mid A_2\rangle|\cos(\varphi - \phi - \delta)], \tag{8}$$

which is centered on $\phi + \delta$. The probability distribution after the interaction (8) is broader than (7). This can be shown using the dispersion

$$D^2 = 1 - |\langle e^{i\varphi}\rangle|^2 = 1 - \left| \int d\varphi\, e^{i\varphi} P(\varphi) \right|^2, \tag{9}$$

which provides a suitable measure of the phase uncertainty [8]. The quantity $|\langle e^{i\varphi}\rangle|$ is directly related to the visibility of the interference fringes: for the unobserved case we have $|\langle e^{i\varphi}\rangle| = \mathcal{V}/2$, while after the interaction $|\langle e^{i\varphi}\rangle_{\rm ob}| = \mathcal{V}_{\rm ob}/2$. In consequence, larger phase uncertainty means lesser visibility, as expected.

The explicit form of the probability distributions (7) and (8) allows us to examine whether the broadening of $P_{\rm ob}(\varphi)$ is the result of random classical phase kicks performed on the initial $P(\varphi)$ in a form analogous to (1). In fact, this is the case, since

$$P_{\rm ob}(\varphi) = \int d\varphi'\, P(\varphi - \varphi')\Omega(\varphi'), \tag{10}$$

provided that $\Omega(\varphi)$ is real and

$$\int d\varphi\, e^{i\varphi}\Omega(\varphi) = \langle A_1 \mid A_2\rangle. \tag{11}$$

This condition is compatible with normalization and the natural constraint $\Omega(\varphi) \geq 0$, as the following particular example demonstrates:

$$\Omega(\varphi) = \frac{1}{2\pi}(1 - |\langle A_1|A_2\rangle|^2) \left| \sum_{n=0}^{\infty} \langle A_1 \mid A_2\rangle^n e^{-in\varphi} \right|^2. \tag{12}$$

These relations show that increasing the accuracy of the observation implies the convolution of $P(\varphi)$ with a broader $\Omega(\varphi)$, leading to a $P_{\rm ob}(\varphi)$ with larger phase uncertainty. This in turn implies lesser visibility of the interference fringes. In other words, complementarity is dynamically enforced by the phase-difference uncertainty introduced by the interaction with the quantum apparatus. Moreover, the phase randomization has precisely the form of random classical phase kicks. Phase difference, unlike momentum, provides a classical and simple dynamical picture of the degradation of interference predicted by complementarity.

Why the phase difference, instead of momentum, is the variable that experiences a classical randomization can be understood from the peculiarities of this kind of observation. The detection schemes we are considering cannot be regarded as measurement of the position of the interfering particle (leaving aside the case of apertures with zero width). Position measurements are usually described by couplings of the form $U = \exp(ixB)$, where $B$ is an observable depending on variables of the apparatus. If this was the case, the momentum of the particle will experience a transformation of the form (1). Instead of position, what is actually measured is just the observable described by the projection measure $\Lambda_j = |\psi_j\rangle\langle\psi_j|$. Then, the observable we can expect to be directly and unavoidably disturbed by observation is not momentum but the observable complementary to $\Lambda_j$ that is the phase difference. Let us note that $\mathrm{tr}[\Delta(\varphi)\Lambda_j] = 1/(2\pi)$, as expected for complementary observables.

There is another way to arrive at the classical randomization expressed by the convolution (10) that provides an explicit form for $\Omega(\varphi)$. The states $|A_1\rangle$ and $|A_2\rangle$ can always be written as $V_1|A\rangle$ and $V_2|A\rangle$, respectively, where $V_1$ and $V_2$ are unitary operators acting on $\mathcal{H}_a$. Provided that the initial state of the apparatus is $|A\rangle$, we can consider the following form for $U$:

$$U = \Lambda_1 V_1 + \Lambda_2 V_2, \tag{13}$$

and compute how $\Delta(\varphi)$ transforms under the system-apparatus interaction,

$$U^\dagger\Delta(\varphi)U = \frac{1}{2\pi}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + e^{-i\varphi}V_1^\dagger V_2|\psi_1\rangle \times \langle\psi_2| + e^{i\varphi}V_2^\dagger V_1|\psi_2\rangle\langle\psi_1|). \tag{14}$$

Since $V_1^\dagger V_2$ is unitary, it can be regarded as the complex exponential of a Hermitian operator, so that the observation shifts $\varphi$ by a quantity depending on variables of the apparatus. To be more specific we can introduce the eigenstates of $V_1^\dagger V_2$ as

$$V_1^\dagger V_2|\xi\rangle = e^{i\xi}|\xi\rangle. \tag{15}$$

With the help of these states, we have

$$P_{\rm ob}(\varphi) = \int d\xi\, P(\varphi - \xi)\mathcal{P}(\xi), \tag{16}$$

where $\mathcal{P}(\xi) = |\langle\xi \mid A\rangle|^2$. We have assumed, without loss of generality, a continuous range of variation for $\xi$. The transformation (16) has the same form (10) and $\mathcal{P}(\xi)$ satisfies condition (11).

This function $\mathcal{P}(\xi)$ was introduced previously as a probability distribution for the phase difference [9]. Here, we have shown that this is in fact a probability distribution of phase shifts entering in the classical randomization expressed by Eq. (10).

To illustrate these results we can describe in more detail the practical two-slit arrangement introduced by Scully, Englert, and Walther [3], where the interfering particle is an excited Rydberg atom (Fig. 1). Two identical microwave cavities $C_1$ and $C_2$, initially empty, are placed in front of the slits. The path followed can be detected if the atom deexcites, depositing a photon in one of them. In this example the apparatus involves two internal states of the atom (excited $|e\rangle$ and ground $|g\rangle$) and the field state in the cavities. The initial state of the apparatus is $|A\rangle = |0,0\rangle|e\rangle$, where $|n_1, n_2\rangle$ are the corresponding photon number states. We assume that the width of the slits is small enough in comparison with the wavelength of the

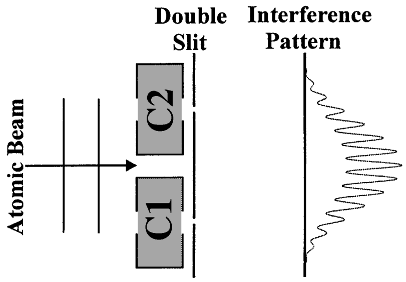


FIG. 1. A double-slit interferometer for atoms with path detection. The microwave cavities are initially empty and act as which-path detectors if the atom deexcites depositing a photon in one of them.

field modes, so that the deflection of the atom is negligible [i.e., the wave functions $\psi_j(x)$ are not modified]. In such a case, the atom-field interaction within each cavity can be described by the Hamiltonians $H_j = \lambda\hbar(|g\rangle\langle e|a_j^\dagger + |e\rangle\langle g|a_j)$ $(j = 1, 2)$, where $a_j$ and $a_j^\dagger$ are the annihilation and creation operators for the corresponding field modes, and $\lambda$ is a coupling constant. The final state after the interaction is of the form (4) with

$$\begin{aligned} |A_1\rangle &= \cos(\lambda t)\,|0,0\rangle\,|e\rangle - i\sin(\lambda t)\,|1,0\rangle\,|g\rangle\,, \\ |A_2\rangle &= \cos(\lambda t)\,|0,0\rangle\,|e\rangle - i\sin(\lambda t)\,|0,1\rangle\,|g\rangle\,, \end{aligned} \tag{17}$$

where $t$ is the time of passage through the cavities. In this example we have $\langle A_1 \mid A_2\rangle = \cos^2(\lambda t)$. Then both the fringe visibility and the efficiency of the path detection depend on the interaction time. The efficiency varies because there is a probability $\cos^2(\lambda t)$ that the atom crosses the cavities without depositing the photon. When this happens the path followed cannot be inferred. Optimum path detection occurs provided that $\cos(\lambda t) = 0$.

In Fig. 2 we have represented $P_{\mathrm{ob}}(\varphi)$ as a function of $\varphi$ and $\lambda t$, for $\theta = \pi/2$ and $\phi = 0$. For the initial state of the apparatus $|0,0\rangle|e\rangle$ there are only three possible values for the phase shift: $\xi = 0, \pm\xi_0$, with $\cos(\xi_0) = (c^2 + 2c - 1)/2$, and $c = \cos(\lambda t)$. The corresponding probabilities are $\mathcal{P}(0) = (1 - c)/(3 + c)$ and $\mathcal{P}(\pm\xi_0) = (1 + c)/(3 + c)$. In the optimum case

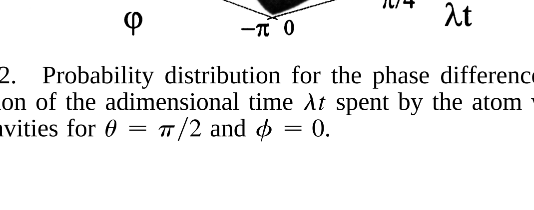


FIG. 2. Probability distribution for the phase difference as a function of the adimensional time $\lambda t$ spent by the atom within the cavities for $\theta = \pi/2$ and $\phi = 0$.

$\lambda t = \pi/2$, these phase shifts are $\xi = 0, \pm 2\pi/3$, with equal probabilities, which leads to a completely random phase difference $P_{\mathrm{ob}}(\varphi) = 1/(2\pi)$, and the interference is completely washed out.

Although the phase difference experiences a classical randomization, this variable also allows for a simple explanation of the erasure of the path information, which leads to the retrieval of the interference. A pattern of high visibility can be recovered if the interference records are classified according to the outcomes of a suitable measurement made on the apparatus. Such a measurement must provide no information about the path followed by the particle [3]. For example, this happens when the statistics of the measurement is governed by the vectors $|\bar{\xi}\rangle = V_2|\xi\rangle$. In such a case, we have $|\langle\bar{\xi} \mid A_1\rangle| = |\langle\bar{\xi} \mid A_2\rangle| = |\langle\xi \mid A\rangle|$.

To analyze the erasure in terms of the phase difference, we can consider a joint probability distribution for $\varphi$ and $\xi$ after the interaction $P_{\mathrm{ob}}(\varphi, \xi) = P(\varphi - \xi)\mathcal{P}(\xi)$. If the outcomes $\xi$ are discarded, we must integrate in $\xi$, which leads to Eq. (16) and the consequences already examined. On the other hand, if the outcomes are not discarded, we get a conditional probability distribution $P_{\mathrm{ob}}(\varphi \mid \xi)$, associated with each $\xi$, of the form $P_{\mathrm{ob}}(\varphi \mid \xi) = P(\varphi - \xi)$, which corresponds to an interference pattern shifted by $\xi$ and having the maximum visibility allowed by the initial state (2). Because of the quantum nature of the apparatus, the loss of interference need not be irreversible.

In summary, we have shown that the loss of interference may be explained in terms of a classical randomization of the phase difference. This is clearly compatible with the absence of broadening of the diffraction pattern, while the effect on the interference is indistinguishable from classical momentum kicks. The phase difference can be significantly disturbed even if there is neither energy nor momentum exchange between the interfering system and the apparatus.

While the phase difference has a clear classical counterpart when dealing with electromagnetic field modes, in situations involving matter it could be viewed as a variable of quantum origin, since it enters via the superposition principle underlying quantum phenomena. This means that the possibility of describing the loss of interference as a classical randomization of the phase difference does not contradict the fact that the detection arrangements studied in this work are in fact full of quantum features.

[1] S. M. Tan and D. F. Walls, Phys. Rev. A **47**, 4663 (1993).
[2] H. M. Wiseman, F. E. Harrison, M. J. Collet, S. M. Tan, D. F. Walls, and R. B. Killip, Phys. Rev. A **56**, 55 (1997).
[3] M. O. Scully, B.-G. Englert, and H. Walther, Nature (London) **351**, 111 (1991).
[4] P. Storey, S. Tan, M. Collet, and D. Walls, Nature (London) **367**, 626 (1994); B.-G. Englert, M. O. Scully,

and H. Walther, Nature (London) **375**, 367 (1995); E. P. Storey, S. M. Tan, M. J. Collet, and D. F. Walls, Nature (London) **375**, 368 (1995).
[5] H. Wiseman and F. Harrison, Nature (London) **377**, 584 (1995).
[6] A. Lukš and V. Peřinová, Quantum Opt. **6**, 125 (1994); R. Lynch, Phys. Rep. **256**, 367 (1995); R. Tanaś, A. Miranowicz, and T. Gantsog, Prog. Opt. **36**, 161 (1996).
[7] M. Grabowski, Int. J. Theor. Phys. **28**, 1215 (1989); A. Luis and L. L. Sánchez-Soto, Phys. Rev. A **56**, 994 (1997).
[8] J. M. Lévy-Leblond, Ann. Phys. (N.Y.) **101**, 319 (1976); T. Opatrný, J. Phys. A **27**, 7201 (1994).
[9] A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A **41**, 3436 (1990).