

Dark matter clues in the muon anomalous magnetic momentJ. A. R. Cembranos,¹ A. Dobado,² and A. L. Maroto²¹*Department of Physics and Astronomy, University of California, Irvine, California 92697 USA*²*Departamento de Física Teórica, Universidad Complutense de Madrid, 28040 Madrid, Spain*

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We study the possibility to explain the nonbaryonic dark matter abundance and improve the present fits on the muon anomalous magnetic moment through the same new physics. In this work we show that massive brane fluctuations (branons) in large extra-dimensions models can provide an economical way to deal with these two issues. This is so because the low-energy branon physics depends effectively on essentially only three parameters. Next collider experiments, such as LHC or ILC, will be sensitive to branon phenomenology in the natural parameter region where the theory is able to account for the two effects.

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The existence of dark matter (DM) is one of the long-standing problems in astrophysics and cosmology, dating back to the early thirties when F. Zwicky observed for the first time that the total and visible masses of rich galaxies disagree in a factor 10–100. Since then, additional evidence has been obtained from galaxy rotation curves, galaxy motions in clusters and, more recently, by precise measurements of the temperature fluctuations of the Cosmic Microwave Background (CMB) radiation [1], Type Ia supernovae, large scale distribution of galaxies or Ly α clouds.

The fact that all these data seems to strongly suggest that the total amount of DM cannot be made of known particles is one of the most pressing arguments for the existence of New Physics (NP), be it in the form of new particles or as a modification of gravity at large distances. The most favored particle candidate to account for the DM energy density is a Weakly Interacting Massive Particle (WIMP) which can provide us with the nonbaryonic DM abundance $\Omega_{\text{NBDM}}h^2 = 0.095\text{--}0.129$ measured by WMAP [1], in the form of a standard thermal relic. Decoupling from thermal equilibrium typically occurs at $T \sim M/20$, where M is the mass of the WIMP which we are equating to the scale of NP, $\Lambda_{\text{NP}} \sim M$. If we assume a typical annihilation cross section $\sigma_A \sim \alpha^2/\Lambda_{\text{NP}}^2$ (where α is the electromagnetic coupling constant), the present abundance can be roughly estimated to be $\Omega_{\text{Wimp}} \sim (\Lambda_{\text{NP}}/(100 \text{ GeV}))^2$. The interesting feature of this result is that the NP which is able to explain the missing matter problem ($\Omega_{\text{Wimp}}h^2 \sim 0.1$), could be related with the electroweak sector ($\Lambda_{\text{NP}} \sim 100 \text{ GeV}$) and be accessible in the next generation of collider experiments. The most popular WIMP candidate is the stable lightest supersymmetric particle which typically corresponds to a neutralino [2] but other candidates have been proposed recently in the context of universal extra dimension theories [3] and little Higgs models (see [4] for a recent review), where some discrete symmetry stabilizes the lightest neutral new particle.

On the other hand, the success of the Standard Model (SM) of particles and interactions has been tested in many

different experiments without finding very important discrepancies so far. A very remarkable example is the electron magnetic moment: $\vec{\mu}_e = g_e(e/(2m_e))\vec{s}$, whose gyromagnetic ratio deviates from the value $g_e = 2$, given by the Dirac equation, as predicted by quantum radiative corrections. This fact has been tested up to a relative precision of 0.03 parts per million (ppm) and confirms Quantum Electrodynamics (QED) as the most precise physical theory [5] (see [6] for an updated analysis).

Curiously one of the most interesting deviation from the SM prediction is provided by the muon magnetic moment. Indeed $a_\mu = (g_\mu - 2)/2$ is not only more sensitive to strong and weak interactions than the electron moment, but also to NP. The 821 Collaboration at the Brookhaven Alternating Gradient Synchrotron has reached a precision of 0.5 ppm in the measurement of such a parameter [7]. Taking into account e^+e^- collisions data in order to calculate the $\pi^+\pi^-$ spectral functions, the deviation with respect to the SM prediction is at 2.6σ [8]: $\delta a_\mu \equiv a_\mu(\text{exp}) - a_\mu(\text{SM}) = (23.4 \pm 9.1) \times 10^{-10}$. On the other hand, the contribution of NP to this parameter can be written generically as $\delta a_\mu = k \times (m_\mu/\Lambda_{\text{NP}})^2$ where the order of magnitude of the constant k depends on the particular model under consideration. Notice that in order to be able to explain the current discrepancy ($\delta a_\mu \sim 10^{-9}$) with the same NP as for dark matter, i.e. $\Lambda_{\text{NP}} \sim 100 \text{ GeV}$, we should have $k \sim 10^{-3}$. This is again the case for some particular supersymmetric, universal extra dimensions or little Higgs models. In this work, however, we point out that the brane-world scenario, originally proposed as a new setting for solving the gauge hierarchy problem [9], is also a viable alternative here.

Recently it has been found that massive brane fluctuations (branons) are natural candidates to dark matter in brane-world models with low tension [10,11]. Branon physics can be described at low energies by an effective action which depends essentially on only three parameters: the branon mass M , the brane tension scale f and the cut-off Λ which sets the range of validity of the effective theory. The number of branon fields N , is an additional

discrete free parameter which is just the number of extra dimensions.

From the point of view of the four dimensional effective phenomenology, branons are new massive pseudoscalar fields, which are stable due to parity invariance of the gravitational interaction on the brane [12–15]. The SM-branon low-energy effective Lagrangian [12,14,15] can be written as:

$$\mathcal{L}_{Br} = \frac{1}{2} g^{\mu\nu} \partial_\mu \pi^\alpha \partial_\nu \pi^\alpha - \frac{1}{2} M^2 \pi^\alpha \pi^\alpha + \frac{1}{8f^4} \times (4\partial_\mu \pi^\alpha \partial_\nu \pi^\alpha - M^2 \pi^\alpha \pi^\alpha g_{\mu\nu}) T^{\mu\nu}. \quad (1)$$

where $\alpha = 1 \dots N$, with N the number of branon species. The above low-energy effective action is completely general assuming Lorentz invariance on the brane and that all the branons have the same mass [15] and it is valid in principle only for energies and branon masses much smaller than brane tension scale. We see that branons interact by pairs with the SM energy-momentum tensor $T^{\mu\nu}$, and that the coupling is suppressed by the brane tension f^4 . Limits on the model parameter from tree-level processes in colliders are briefly summarized in Table I, where one can find not only the present restrictions coming from HERA, Tevatron and LEP-II, but also the prospects for future colliders such as ILC, LHC or CLIC [15,16]. Additional bounds from astrophysics and cosmology can be found in [17].

In order to obtain the first branon contribution to the μ anomalous magnetic moment, we compute the one-loop effective action for SM particles, by integrating out the branon fields with cut-off regularized integrals. At the level of two-point functions, branon loops result only in a renor-

malization of the SM particle masses, which is not observable. However new couplings appear at higher-point functions which can be described by an effective Lagrangian [16,18] whose more relevant terms are:

$$\mathcal{L}_{SM}^{(1)} \simeq \frac{N\Lambda^4}{192(4\pi)^2 f^8} \{2T_{\mu\nu} T^{\mu\nu} + T_\mu^\mu T_\nu^\nu\}. \quad (2)$$

As we have commented above, Λ is the cut off which limits the validity of the effective description of branon and SM dynamics. This new parameter appears when dealing with branon radiative corrections since the Lagrangian in (1) is not renormalizable. A one-loop calculation with the new effective four-fermion vertices coming from (2), whose Feynman digrams are given in Fig. 1, is equivalent to a two-loop computation with the Lagrangian in (1), and allows us to obtain the contribution of branons to the anomalous magnetic moment:

$$\delta a_\mu \simeq \frac{5m_\mu^2}{114(4\pi)^4} \frac{N\Lambda^6}{f^8}. \quad (3)$$

This result is qualitatively similar to other $g_\mu - 2$ contributions obtained in different analyses in the brane-world scenario [19,20]. We can observe that the correction has the right sign and that it is thus possible to improve the agreement with the experimental value. In fact, by using the commented difference between the experimental and the SM prediction [7,8], we can estimate the preferred parameter region for branon physics:

$$6.0 \text{ GeV} \gtrsim \frac{f^4}{N^{1/2}\Lambda^3} \gtrsim 2.2 \text{ GeV (95\% c.l.)} \quad (4)$$

TABLE I. Limits from direct branon searches in colliders. The results for HERA, LEP-II and Tevatron-I have been obtained from real data, whereas the rest are estimations. \sqrt{s} is the center of mass energy of the total process; \mathcal{L} is the total integrated luminosity; f_0 is the bound on the brane tension scale for one massless branon ($N = 1$) and M_0 is the limit on the branon mass for small tension $f \rightarrow 0$ (see [15] for details).

Experiment	\sqrt{s} (TeV)	\mathcal{L} (pb $^{-1}$)	f_0 (GeV)	M_0 (GeV)
HERA ^a	0.3	110	16	152
Tevatron-I ^a	1.8	78	157	822
Tevatron-I ^b	1.8	87	148	872
LEP-II ^b	0.2	600	180	103
Tevatron-II ^a	2.0	10 ³	256	902
Tevatron-II ^b	2.0	10 ³	240	952
ILC ^b	0.5	2 × 10 ⁵	400	250
ILC ^b	1.0	10 ⁶	760	500
LHC ^a	14	10 ⁵	1075	6481
LHC ^b	14	10 ⁵	797	6781
CLIC ^b	5	10 ⁶	2640	2500

^aUpper indices.

^bUpper indices.

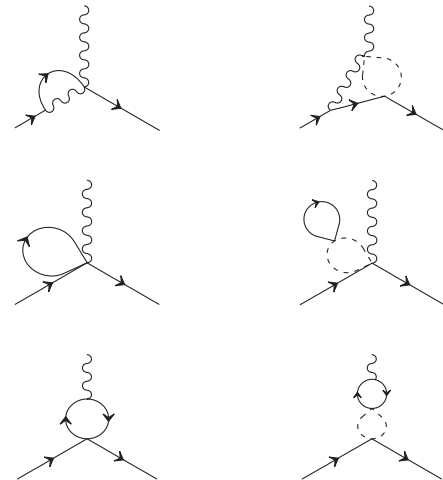


FIG. 1. The diagrams on the left are the three types of contributions from Lagrangian (2) to the muon anomalous magnetic moment at one loop. The diagrams on the right are the equivalent two-loop contributions from the branon theory in (1). The continuous, dashed and wavy lines represent the muon, branon and photon fields, respectively. Notice that in the first type of contribution, the fermion loop can also be attached to the outgoing muon.

However branon loops can have additional effects which should also be compatible with SM phenomenology. The most relevant ones could be the four-fermion interactions or the fermion pair annihilation into two gauge bosons. Following [21,22], we have used the data coming from HERA [23], Tevatron [24] and LEP [25] on this kind of processes in order to set bounds on the parameter combination $f^2/(\Lambda N^{1/4})$. The results are shown in Table II, where it is also possible to find the prospects for the future colliders mentioned above. These limits show that the first branon signals at colliders would be associated to radiative corrections [18] and not to the direct production studied in previous works [15].

Indeed, if there is NP in the muon anomalous magnetic moment and it is due to branon radiative corrections, the phenomenology of these particles should be observed at the LHC and in a possible future ILC, which have larger sensitivities for virtual effects working at a center of mass energy of 1 TeV (in contrast with the direct branon production, where the LHC presents a larger sensitivity in any case, see Tables I and II, and Figs. 2 and 3). In particular, the LHC should observe an important difference with respect to the SM prediction in channels like $pp \rightarrow e^+e^-$. The ILC should observe the most important effect in the Bhabha scattering.

Another limitation to the branon parameters could be obtained from electroweak precision measurements, which use to be very useful to constrain models of NP. The so called oblique corrections (the ones corresponding to the W , Z and γ two-point functions) use to be described in terms of the S , T , U [26] or the ϵ_1 , ϵ_2 and ϵ_3 parameters [27]. The experimental values obtained by LEP [25,28] are consistent with the SM prediction for a light Higgs $m_H \leq 237$ GeV at 95% c.l. In principle, it is necessary to know this parameter in order to put constraints on NP, but one

TABLE II. Limits from virtual branon searches at colliders (results at the 95% c.l.). The first four analysis have been performed with real data, whereas the final four are estimations. The first two columns are the same as in Table I, and the third one corresponds to the lower bound on $f^2/(N^{1/4}\Lambda)$.

Experiment	\sqrt{s} (TeV)	\mathcal{L} (pb $^{-1}$)	$f^2/(N^{1/4}\Lambda)$ (GeV)
HERA ^c	0.3	117	52
Tevatron-I ^{a,b}	1.8	127	69
LEP-II ^a	0.2	700	59
LEP-II ^b	0.2	700	75
Tevatron-II ^{a,b}	2.0	2×10^3	83
ILC ^b	0.5	5×10^5	261
ILC ^b	1.0	2×10^5	421
LHC ^b	14	10^5	383

^aDenotes the two-photon, e^+e^- and $e^+p(e^-p)$ channels, respectively.

^bDenotes the two-photon, e^+e^- and $e^+p(e^-p)$ channels, respectively.

^cDenotes the two-photon, e^+e^- and $e^+p(e^-p)$ channels, respectively.

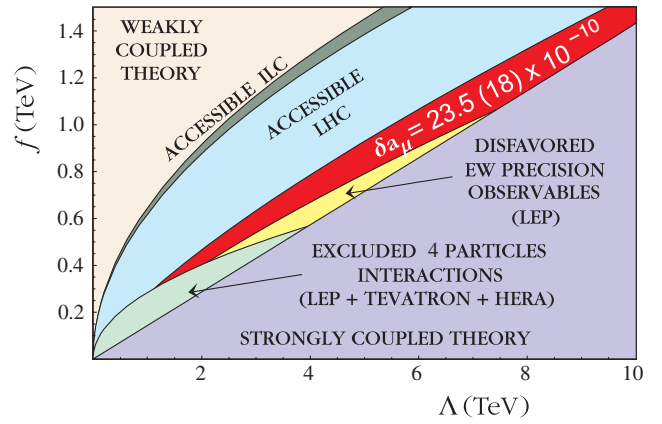


FIG. 2 (color online). Main limits from branon radiative corrections in the f - Λ plane for a model with $N = 1$. The (red) central area shows the region in which the branons account for the muon magnetic moment deficit observed by the E821 Collaboration [7,8], and at the same time, are consistent with present collider experiments (whose main constraint comes from the Bhabha scattering at LEP) and electroweak precision observables. Prospects for future colliders are also plotted.

can talk about disfavored regions of parameters in order to avoid fine tunings. We can estimate this area by performing a computation of the parameter $\bar{\epsilon} \equiv \delta M_W^2/M_W^2 - \delta M_Z^2/M_Z^2$, in a similar way as it was done for the first order correction coming from the Kaluza-Klein gravitons in the ADD models for rigid branes [20]. The experimental value of $\bar{\epsilon}$ obtained from LEP [25,28] is $\bar{\epsilon} = (1.27 \pm 0.16) \times 10^{-2}$. The theoretical uncertainties are 1 order of magnitude smaller [27] and therefore, we can estimate the constraints for the branon contribution at 95% c.l. as

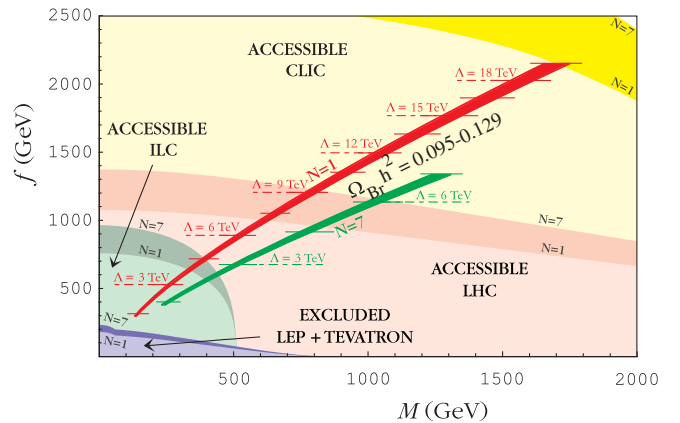


FIG. 3 (color online). Branon abundance in the range: $\Omega_{Br} h^2 = 0.095-0.129$, in the f - M plane (see [17] for details). The regions are only plotted for the preferred values of the brane tension scale f . The central values of Λ from Eq. (4) are also plotted. The lower area is excluded by single-photon processes at LEP-II and monojet signals at Tevatron-I [15]. The sensitivity of future collider searches for real branon production are also plotted (See [15] and Table I). The dependence on the number of branons in the range $N = 1-7$ can also be observed.

$|\delta\bar{\epsilon}| \lesssim 3.2 \times 10^{-3}$ with the result [18]:

$$\frac{f^4}{N^{1/2}\Lambda^3} \gtrsim 3.1 \text{ GeV (95\% c.l.)} \quad (5)$$

The constraints coming from this analysis are complementary to the previous ones since this bound has a different dependence on Λ . In Fig. 2, we have included all the limits in the $f - \Lambda$ plane from virtual branon effects. We have also plotted the region in which the effective theory can be considered as strongly coupled ($\Lambda \gtrsim 4\sqrt{\pi}fN^{-1/4}$), and for which the loop expansion is no longer valid [18]. We see that the region compatible with the Brookhaven results extends for $1100 \lesssim \Lambda \lesssim 15100N^{-1/2}$ (GeV) and $300N^{1/8} \lesssim f \lesssim 2130N^{-1/4}$ (GeV).

It is remarkable to note that the same parameter space which explains the magnetic moment deficit of the muon, is able to explain the DM content of the Universe and, in addition, the preferred scale is related with the electroweak sector. More precisely, if the branon mass is between $M \sim 100$ GeV and $M \sim 1.7$ TeV, branons could form the total nonbaryonic DM abundance observed by WMAP [1,10].

In Fig. 3, we have plotted the $f - M$ regions in which branons could explain the WMAP measurements. We include also the limits from colliders and the values of Λ corresponding to the central values of the muon anomalous magnetic moment observed at Brookhaven. In these regions branons decouple at $T < M < f < \Lambda$, i.e. they are nonrelativistic, behave as cold DM and the effective theory described by the Lagrangian (1) can be used to properly evaluate their thermal relic abundance [10].

To summarize, we have shown that massive branons could offer an alternative explanation for the observed dark matter abundance and the recent measurements of the muon anomalous magnetic moment. The preferred region compatible with current experiments will be tested by future colliders such as LHC or ILC.

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