New model of relativistic slowly rotating neutron stars with surface layer *crust:* application to

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Abstract.
Introducing a surface layer of matter on the edge of a neutron star in slow rigid rotation, we analyze, from an intrinsic point of view, the junction conditions that must be satisfied between the interior and exterior solutions of the Einstein equations. In our model the core-*crust* transition pressure arise as an essential parameter in the description of a configuration. As an application of this formalism, we describe giant *glitches* of the Vela pulsar as a result of variations in the transition pressure, finding that these small changes are compatible with the expected temperature variations of the inner crust during *glitch* time

1. Introduction

Many pulsars show sudden spin jumps, *glitches*, superimposed to the gradual spin down due to the continued loss of angular momentum suffered by the star. The study of the properties of the star during the *glitch* time is essential to understand the structure of the neutron star that models the pulsar [1, 2]. In Vela pulsar, giant *glitches* with relative period variation of the order of 10^{-6} have been observed. Many mechanisms have been proposed to explain *glitches*.

The models that deal with *glitches* are associated with the layer structure of neutron stars. Two regions of the star can be differentiated: the core and the *crust*. It is thought that the *crust* region has a solid crystalline structure similar to a metal [3]. Some theories propose that the *glitch* is triggered by the rupture of the *crust* as a consequence of the tensions on the *crust* that try to adequate the ellipticity of the *crust* to the changing angular velocity of the star.

Because of the large density gradient close to the transition between the core and the *crust*, most of the crustal matter resides in the shells in the inner part of the *crust* [4]. Furthermore, it is noteworthy that the dynamical properties of the neutron star depend strongly on the transition pressure between the star's core and the *crust* as have been pointed up by [5] and [6]. Because most of the matter of the *crust* is found in the shells near the transition region between the core of the star and the inner *crust*, this is the region where the properties of the *crust* are relevant [4]. Hence, we will treat the *crust* as a surface layer that envelops the star's core. In the next section we will describe how our model of slowly rotating neutron star with a surface layer *crust* is constructed.

2. Construction of models of slowly rotating neutron stars with surface *crust*.

We assume that the neutron star is in permanent rigid rotation. The star rotates with constant angular velocity Ω around an axis, so the resulting space-time is axisymmetric and stationary. We assume that the rotation of the star is sufficiently slow and, hence, the Hartle-Thorne perturbative solution can be used [7, 8]. To second order in the angular velocity of the star Ω, appropriate coordinates can be chosen so that the metric can be written as follows

$$ds^2 = -e^{2\psi(r)}[1+2h(r,\theta)]dt^2 + e^{2\lambda(r)}[1+2m(r,\theta)]dr^2 + r^2[1+2k(r,\theta)]\left[d\theta^2 - \sin^2\theta(d\phi - \omega(r)dt)^2\right].$$

We have to solve the Einstein equations in the interior of the star, where the matter is described by a perfect fluid tensor with equation of state $\rho = \rho(p)$ and in the outer region, where space is empty. Also, we have to impose appropriate boundary conditions in the surface of the star. It is very desirable to introduce a new radial coordinate R adapted to the surface of the star [7, 8].

We assume that the core of the star is surrounded by a thin surface layer (simplified *crust* model). Then we find that in the surface of the star $R = a$, the stress-energy tensor is non null and it can be written as $T_c^{\mu\nu}(R) = \rho_c(\theta)u_c^\mu u_c^\nu(a)$, i.e. the surface layer can be described as a perfect fluid with its own angular velocity Ω_c. The surface energy density can be written up to second order as $\rho_c(\theta) = \varepsilon + \delta\varepsilon(\theta)$. We must impose the usual junction conditions on the interior and exterior solutions, but taking into account the introduction of a surface layer of matter on the border of the star. We use the intrinsic formulation of these conditions [9] [10]: the first fundamental form must be continuous across the surface of the star, and the second fundamental form presents a discontinuity given by the stress-energy tensor in the surface of the star. In order to use these junction conditions in the slow rotation approximation, we expand them in terms of Ω, finding the following results for each one of the metric functions:

Order zero: we obtain important conditions for the mass function and for the pressure of transition between core and *crust*

$$M_{ext} = M_{int} + 4\pi a^2 \sqrt{1 - \frac{2M_{int}}{a}}\,\varepsilon - 8\pi^2 a^2 \varepsilon^2 \tag{1}$$

$$\frac{M_{ext}}{a^2\sqrt{1 - 2M_{ext}/a}} - \frac{M_{int} + 4\pi a^3 p_{int}}{a^2\sqrt{1 - 2M_{int}/a}} = 4\pi\varepsilon \tag{2}$$

The equation (1) gives the total mass of the star to zero order (M_{ext}) as the addition of three terms: the core mass ($M_{int} \equiv 4\pi \int_0^a \rho R^2 dR$), the *crust* mass, and a negative bounding energy term. We interpret (2) as an equation giving the surface density of energy ε in terms of the total mass of the star, the interior mass, and the core-*crust* transition pressure p_{int}. This condition is used to obtain the radius of the star.

First order: we obtain continuity of the rate of rotation of the inertial frames $\omega(a)_{ext} = \omega(a)_{int} \equiv \omega(a)$, and an expression for the total angular momentum:

$$J = a^6 e^{\lambda(a)_{ext}}(e^{-\lambda(a)_{int}}\left[\partial_R \overline{\omega}(a)\right]_{int} + 16\pi\varepsilon\overline{\omega_c})/6 \tag{3}$$

where $\overline{\omega_c} = \Omega_c - \omega(a)$. It can be seen [7, 8] that the first term of expression (3) has the same sign as Ω (in our case always positive). However, the second term depends on the sign of Ω_c. Hence, contra-rotating configuration are possible (the core and the *crust* contra-rotate), and even, we could find certain critical configuration in which the total angular momentum is null.

Second order: we obtain $\Delta[r^*(a,\theta)] = 0$ where r^* is defined as in [7, 8], so both the mean radius and the eccentricity are continuous. Also, conditions for the second order perturbation of the mass function and surface density ($\delta\varepsilon$) are obtained.

Finally the last of the second order conditions fixes the value of the angular velocity of the *crust* $\Omega_c = \omega(a) \pm (\Omega - \omega(a))$. We obtain two possible configurations for the same star core and transition pressure: one with the *crust* co-rotating with the interior fluid and with identical velocity, and a special configuration with contra-rotating *crust*.

3. Some properties of the model using realistic equations of state for the core
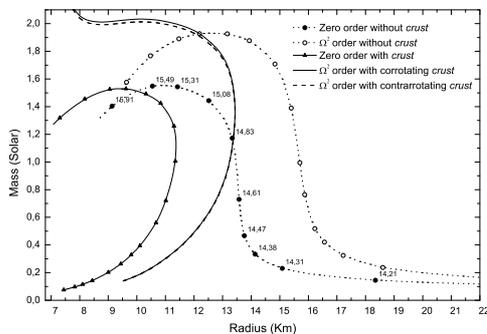
Figure 1. Mass vs radius for static/rotating configurations with a 10% *crust* (BPS)
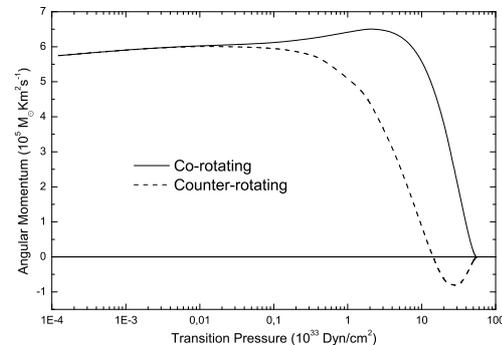
Figure 2. Angular momentum vs transition pressure for fixed ρ_c (BPS).

To obtain a model of a neutron star with a surface layer *crust* in permanent slow rotation, for a given equation of state of the core, we have to fix three parameters. We choose the total mass of the star, the central density, and the core-*crust* transition pressure. Then, we integrate the Einstein equations inside the star (using a realistic equation of state for the core) and we match this solution with the exterior solution found in [7, 8] using the junction conditions explained above. We have performed our simulations using an equation of state for high pressures from [11, K=240 MeV, $\frac{m^*}{m} = 0.78$ and $x_\sigma = 0.6$] and the BPS equation of state for the lower pressures. Similar results have been obtained for the analytical fit of the Sly equation presented in [3].

We present some of our results in figures 1 and 2. In fig. 1, we show multiple configurations for both static stars and rotating stars, with a *crust* of approximately 10% of the total mass and mass shedding angular velocity. On the static configurations without *crust* we write the value $\log 10(\rho_c)$. Note that the mass-radius behavior of the configurations with surface *crust* resembles those of the quark stars [11]. In fig. 2 we show the angular momentum for mass shedding angular velocity as a function of the core-*crust* transition pressure. In the counter rotating configuration we find that if the core-*crust* transition pressure is large enough, the angular momentum of the inner *crust* region could be equal or even higher than the angular momentum of the star core, resulting in a null or negative total angular momentum.

4. Core-*crust* transition pressure evolution in giant *glitches* of the Vela pulsar

On a typical pulsar, the rupture of the solid *crust* produces a release of energy 10^{41}-10^{43} ergs to the inner part of the *crust*. Several works study the thermal response of a neutron star after a *glitch* [12, 13], obtaining that increases in the temperature of the transition region of the order of $10^6 K$. This small rise of the temperature will produce an small but sensitive increase in the transition pressure [14]. We propose that the *glitch* can be explained by changes in the equation of the state of the star in the transition region between the *crust* and the core. In our model, these changes are represented by variations of the transition pressure due to the increase of the temperature produced by the deposition of the energy, coming from the rupture of the *crust*.

Using the observational data from the ATNF Pulsar Catalogue [15] we can study the giant glitches of Vela pulsar using our model. We assume that during the *glitch* neither the total mass nor the central density of the star are changed. Hence, in our description, the changes in the pulsar during this epoch are due to readjustment of the core-*crust* transition pressure, i.e., a variation of the equation of state in transition region between the core and the *crust*. Assuming this, we can give a description of the effects of these *glitches* over the neutron star using our model of slowly rotating neutron star with superficial *crust*. We assume that the star has a total

Date (MJD)	$\delta\Omega(10^{-6})$	$\delta p_{int}(10^{-10})$	$\delta ecc(10^{-6})$	$\delta Q(10^{-6})$
40289	2.34	0.70	2.36	4.73
41192	2.05	0.58	1.97	3.94
43693	3.06	0.93	3.15	6.31
45192	2.05	0.58	1.96	3.93
48457	2.72	0.81	2.76	5.51
51559	3.09	0.93	3.14	6.28
53959	2.62	0.82	2.74	5.48

Table 1. Date of each giant *glitch* of the Vela pulsar in modified Julian days and relative increases of angular velocity, transition pressure, eccentricity and quadrupolar moment

mass of $1.44 M_\odot$, a central density of $1.279 \cdot 10^{15} gcm^{-3}$, and a transition pressure before the *glitch* of $3.751 \cdot 10^{33} dyn/cm^2$. We calculate the resulting configuration after the *glitch* varying the core-*crust* transition pressure until we reach an equivalent configuration (same mass and central density) with the resulting angular velocity just after the *glitch*. As it can be seen in table 1, our model predicts a relative transition pressure increase of the order of 10^{-10} for the considered *glitches*. This small variation in the equation of state on the surface of the star causes a redistribution of other properties of the star. The eccentricity of the star increases order 10^{-6}. Also, a movement of matter on the surface of the star, from the poles to the equator, produce an increase of the quadrupolar momentum (10^{-6}) which may cause gravitational radiation.

Using a estimation for the thermal pressure variations it can be seen that the corresponding increase of the inner *crust* temperature is order $10^6 K$ [14], which agree with the results obtained in the works previously commented. So the relative core-*crust* transition pressure changes we obtain in our model can be understood as variations in the equation of state of the transition region between the core and the *crust* of the neutron star, resulting from the deposition of energy of 10^{41}-$10^{43} erg$ in the outer layers of the neutron star during the *glitch* time.

In conclusion, we propose a new model for the giant *glitches* of the Vela pulsar: The tensions on the *crust* trying to adequate the ellipticity of the *crust* to the changed angular velocity produce the rupture of the *crust*. Then, there is a sudden energy deposition in the inner *crust* layers, which causes a rise of its temperature, resulting in an increase of the core-*crust* transition pressure of the neutron star (a change in the equation of the state in the core-*crust* transition region is produced). Hence, the properties of the neutron star (angular velocity, eccentricity, surface matter distribution and quadrupolar moment) change and a *glitch* is generated/observed.

References
[1] Lorenz C. P., Ravenhall D. G., Pethick C. J., 1993, *Phys. Rev. Lett.* **70**, 379
[2] Link B., Epstein R. I., Lattimer J. M., 1999, *Phys. Rev. Lett.* **83** 3362
[3] Haensel P., Potekhin A. Y., 2004, *Astron. Astrophys.* **428** 191 [arXiv:astro-ph/0408324]
[4] Pethick C. J., Lorenz C. P., Ravenhall D. G., 1995, *Nucl. Phys. A* **584** 675
[5] Lattimer J. M., Prakash M., 2001, *Astrophys. Journal* **550** 426
[6] Cheng K. S., Yuan Y. F., Zhang J. L., 2002, *Astrophys. Journal* **564** 909
[7] Hartle J. B., 1967, *Astrophys. Journal* **150** 1005
[8] Hartle J. B., Thorne K. S., 1968, *Astrophys. Journal* **153**, 807
[9] Misner C. W., Thorne K. S., Wheeler J. A., 1973, *Gravitation*, Freeman, San Francisco
[10] González-Romero L. M., 2003, *Phys. Rev. D* **67**, 064011
[11] Glendening N. K., 2000, *Compact stars* Astronomy & Astrophysics library, Springer, New York
[12] Hirano S. *et al* , 1997, *Astrophys. Journal* **491**, 286
[13] Van Riper K. A., Epstein I., Miller, G. S., 1991, *Astrophys. Journal* **381** L47
[14] Prakash M. *et al* , 1997, *Phys. Rep.* **280** 1
[15] Manchester, R. N. *et al* , 2005, *Astrophys. Journal* **129**, 1993