Geometry and Physics of the Fermions. 2. (Interactions).
Towards a boson structure $\{ \gamma , Z^0 , W^{\pm} , g \}$.

Eugenio Olmedilla Moreno [a]
Departamento de Física Teórica, Facultad de Ciencias Físicas,
Universidad Complutense, 28040 Madrid, Spain.
(Dated: October 3, 2024)

We have developed a general formalism for defining distinct creation and annihilation operators for each one of the leptons and of the quarks (up to the families) in the previous $\mathcal{PART}$ *1*. The physical magnitudes: the electric charge, the spin (up and down, an scalar), the vector-spin and the chirality are intrinsic to them. We defined the electric charges and the vectors-spin in terms of specific values of a discrete angle parameter and also of a set of discrete values of the time and the space. In this way, we established the elementary fermions as geometrical structures of a local time and space. Arguments of symmetry are beneath this construction.

We devote this Part **2** (interactions) to the study of several of their mutual interactions and decays, from the geometrical point of view. We view the adjustments of the geometries as a starting point for the definitions of the bosons. The method is essentially algebraic, still with some deficiencies that require more research.

Keywords: Leptons, quarks, bosons. Creation annihilation operators. Algebra - Geometry. Interactions and decays.

$\mathcal{PART}$ *II.* INTERACTIONS AND DECAYS.

I) Guided by: ' *particle* $\wedge$ *antiparticle* $\longrightarrow$ *bosons* : photons / Z^0 '.

II) Guided by the leptons: ' *neutrinos* $\wedge$ *charged leptons* $\longrightarrow$ $W^{\pm}$ *bosons* '. *Muon decays* ([1]).

III) Guided by the Quarks. Appearance of the gluons: $\{ g \} \cup$ ✻ ?

- **i)** A β decay: the non conservation of the parity. Madame Wu C.S. et al. (1956).
- **ii)** Helicity of the neutrino. Goldhaber M.L. et al. (1957).
- **iii)** Proton – antiproton collision. A relation for the **W** bosons with leptons and with quarks. Geometry.
- **iv)** The decay of a charged pion (π^-).

([1]) behind a *lepton universality*.

Previous point: our model with a geometric structure of the leptons and quarks. [1] The contents of this part show the possibility for the first steps in a geometric construction for the operators of a boson structure. It is only a question of time and "brute force" to get more algebraic relationships. The difficulty is to discern their geometry and to understand their physical significance, if any, for some of them.

An important point with the algebraic treatment: the determinants of these operators are zero . We also have a large variety of algebraic equalities. We need to clarify their possible physical significance in a theory for a geometrical model for the bosons. Besides, the conservation theorems could forbid some of the possible algebraic (geometrical) treatments.

We define the interactions and the decays in terms of straight products of creation operators. We leave apart the intermediary products defined for some anticommutators, which we use for inverting the order in the corresponding products, when necessary.

Treatment of the three families or generations of quarks and leptons and of their masses are out of scope. Even though we study the μ decay (different generations) thanks to an imposed *lepton universality*.

This presentation is a starting point. This motivates the necessity for more research. Let us describe two specific directions:

- motivated by the algebra: the already mentioned determinants equal to zero and the many different physical possibilities. In subsection **I)** the annihilation of a particle with its antiparticle(-s) (the spin), as shown in equations (8) and (12), producing two photons, or three photon, or Z^0, or , ... A particular important algebraic equality in equation (11), with a proof in (19):

$$\varphi_R = 0 , \qquad \left(\{ \nu^{\dagger}_{1,\mathcal{M}} , \nu^{\dagger}_{5,\mathcal{M}} \} = \right) \; \nu^{\dagger}_{1,\mathcal{M}} \nu^{\dagger}_{5,\mathcal{M}} + \nu^{\dagger}_{5,\mathcal{M}} \nu^{\dagger}_{1,\mathcal{M}} = (- \check{\sigma})_m + (- \hat{\sigma})_m = \left(- \mathbb{1} \right)_m . \tag{1}$$

What does this mean? What does the "+" represent? Also, a distinction of a right and a left? Considering the inverse way, neutrino – antineutrino with antineutrino – neutrino production; out of what? This relation is at the core of the electron – anti-electron (or any particle – antiparticle) annihilation with equations (8), as already mentioned, and producing something,

- motivated by the geometry: $\{ \mathbb{1}, \sigma^z, \sigma^x, \sigma^y \}$ ($\{ \mathrm{i}\mathbb{1}, \mathrm{i}\,\sigma^z, \mathrm{i}\,\sigma^x, \mathrm{i}\,\sigma^y \}$) represent local geometrical coordinate axes. We use their matrix forms to work with them (the algebraic instrument). This has deep geometrical and physical implications.

We present some brief comments in the appendixes.

---

[a] email: eugenio.o.m@ucm.es

From *PART 1* [1]

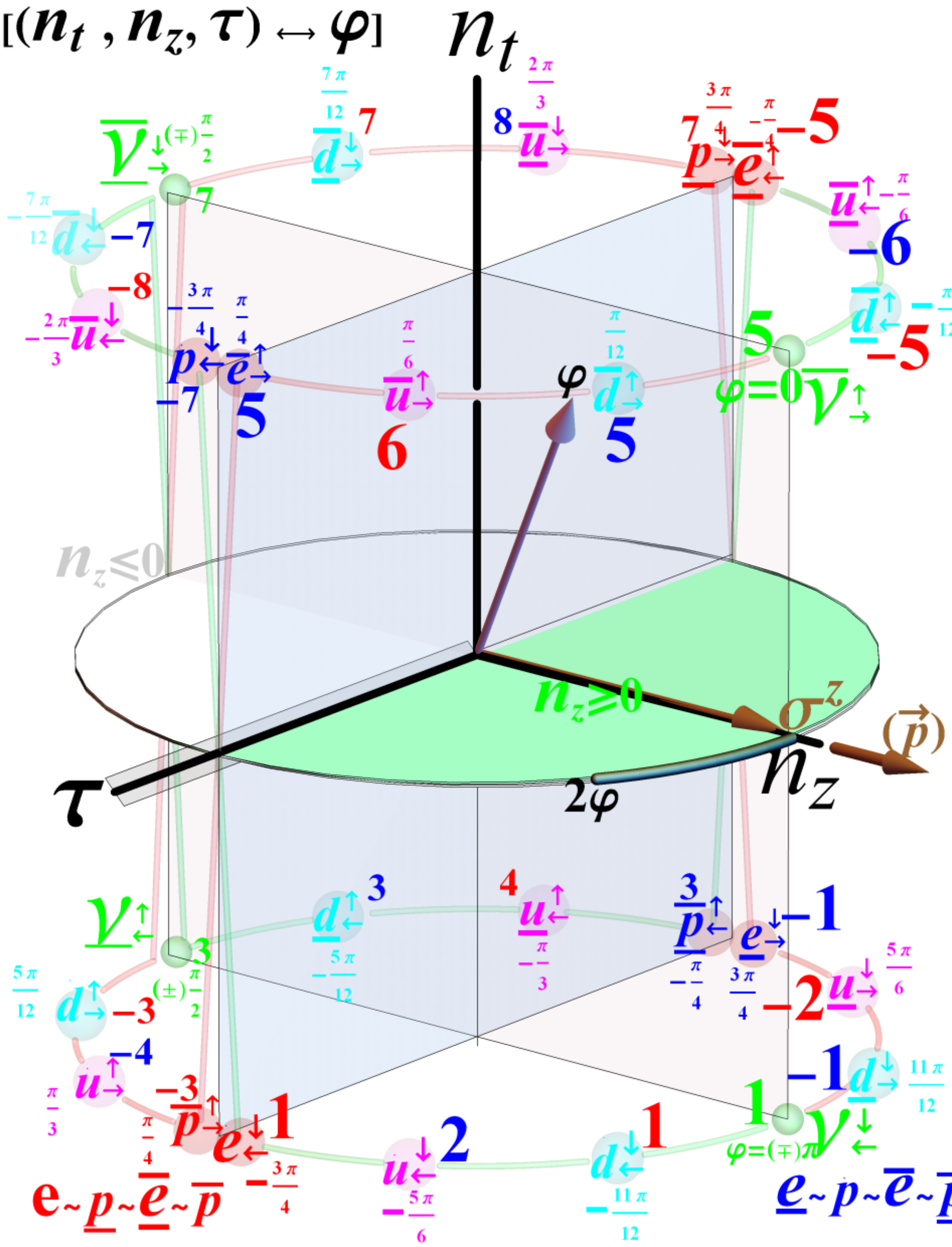

FIG. 1. The geometric of the elementary fermions (with n_t, n_z, τ and the specific angles φ).

Negative electric charge with red color $(e, \underline{p})$, $(\underline{\overline{e}}, \overline{p})$; d-quarks (1, 7), (−5, −3); u-antiquarks (6, 4), (−2, −8)
Positive electric charge with blue color $(\underline{e}, p)$, $(\overline{e}, \underline{\overline{p}})$; d-antiquarks (−1, −7), (5, 3); u-quarks (−6, −4), (2, 8) .
Without electric charge with green color: neutrinos $(\nu, \underline{\overline{\nu}})$ (1,7)) ; antineutrinos $(\overline{\nu}, \underline{\nu})$ (5,3))

Spin down with ↓
Spin up with ↑ .

$(\varphi < 0)$ Left chirality in the green quarters of the circles, with ←
$(\varphi \geq 0)$ Right chirality in the red quarters of the circles, with → .

Oppositions in n_t alone and in τ alone or in the 3d-space alone change the sign of the electric charge
Oppositions in n_t alone and in n_z alone or in the 3d-space alone change the spin (φ behind) .
Opposition in τ do not change the spin, and opposition in n_z do not change the sign of the electric charge

An "overline" or an "underline" indicate: oppositions in "n_t" or in "τ" (respectively), and also oppositions in the sign of the electric charge; with the particular notation for e (electron) and p (anti−): $e \leftrightarrow \overline{p}$, $\overline{e} \leftrightarrow p$.

$\vec{\mathbf{p}}$ is the momentum of the created particle as it is created, in a rest frame of the producing particles. With our construction we impose $\vec{\mathbf{p}} \Rightarrow \sigma^z$ and $\sigma^z \Rightarrow (\vec{\mathbf{p}})\big|_{\text{(own frame)}}$ locally, in the reference frame of the charged lepton, although $\|(\vec{\mathbf{p}})\| = 0$. This is 'its own frame'. This has implications for the helicity and its relation with the chirality. Thomson writes in his *Modern Particle Physics* in the page 105 [2]: "spin along its direction of flight". See figures 2 and 4. (Spin, the scalar, ↑, ↓).

We are going to use from *PART 1* the following creation operators ("daga"), with $\lambda = \pm 1$, $\epsilon = \pm 1$:

$$
\begin{array}{lll}
1\,(2) & \mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}}(\varphi_R) \equiv \mathbf{R}_{O\,M}^{\lambda\varphi_R} \left[\left(\sigma^z \right)_{M-1} \left(\mathbf{R}_O^{\epsilon\frac{\pi}{2}} (-\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\epsilon\frac{\pi}{2}} \right)_m \right] \mathbf{R}_{O\,M}^{-\lambda\varphi_R} & (\varphi = \varphi_R - \pi) \\
7\,(8) & \mathbf{U}^{\dagger}_{\lambda 7,\mathcal{M}}(\varphi_R) \equiv \mathbf{R}_{O\,M}^{\lambda\varphi_R} \left[\left(-\sigma^z \right)_{M-1} \left(\mathbf{R}_O^{\epsilon\frac{\pi}{2}} (\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\epsilon\frac{\pi}{2}} \right)_m \right] \mathbf{R}_{O\,M}^{-\lambda\varphi_R} & (\varphi = \varphi_R + \frac{\pi}{2}) \\
\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\
5\,(6) & \mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}}(\varphi_R) \equiv \mathbf{R}_{O\,M}^{\lambda\varphi_R} \left[\left(\sigma^z \right)_{M-1} \left((\mathrm{m}^- \sigma^+) \right)_m \right] \mathbf{R}_{O\,M}^{-\lambda\varphi_R} & (\varphi = \varphi_R) \\
3\,(4) & \mathbf{U}^{\dagger}_{\lambda 3,\mathcal{M}}(\varphi_R) \equiv \mathbf{R}_{O\,M}^{\lambda\varphi_R} \left[\left(-\sigma^z \right)_{M-1} \left((-\mathrm{m}^- \sigma^+) \right)_m \right] \mathbf{R}_{O\,M}^{-\lambda\varphi_R} & (\varphi = \varphi_R - \frac{\pi}{2})
\end{array}
\qquad (2)
$$

with; $\quad \mathrm{m}^+ \equiv e^{\,i(\phi - \frac{\pi}{2})} = -i\, e^{\,i\phi}, \qquad \mathrm{m}^- \equiv e^{-i(\phi - \frac{\pi}{2})} = i\, e^{-i\phi} = \overline{\mathrm{m}^+} = {\mathrm{m}^+}^{-1},$

and: $\quad \mathbf{A}_{M-1} = \underset{1}{\mathbf{A}} \otimes \cdots \otimes \underset{k}{\mathbf{A}} \otimes \cdots \otimes \underset{m-1}{\mathbf{A}} \otimes \underset{m}{\mathbb{1}} \otimes \ldots \otimes \underset{N}{\mathbb{1}}$ [3] , $\quad \mathbf{A} \in \left\{ \underset{\le m}{\mathbf{R}_O^{(\pm)\lambda\varphi_R}},\ \underset{\le m-1}{\left((\pm)\sigma^z \right)} \right\}, \qquad \left(\left(*((\pm)\, \mathrm{m}^- \sigma^+)\, ** \right)_m = \Sigma_m \right).$

An algebraically equivalent form for (2) is:

$$
\begin{array}{lll}
\mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}}(\varphi_R) & \equiv \left(\mathbf{R}_e^{2\lambda\varphi_R} \right)_{M-1} & \left(\mathbf{R}_O^{\lambda\varphi_R + \epsilon\frac{\pi}{2}} (-\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\lambda\varphi_R - \epsilon\frac{\pi}{2}} \right)_m \\
\mathbf{U}^{\dagger}_{\lambda 7,\mathcal{M}}(\varphi_R) & \equiv \left(-\mathbf{R}_e^{2\lambda\varphi_R} \right)_{M-1} & \left(\mathbf{R}_O^{\lambda\varphi_R + \epsilon\frac{\pi}{2}} (\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\lambda\varphi_R - \epsilon\frac{\pi}{2}} \right)_m \\
\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}}(\varphi_R) & \equiv \left(\mathbf{R}_e^{2\lambda\varphi_R} \right)_{M-1} & \left(\mathbf{R}_O^{\lambda\varphi_R} (\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\lambda\varphi_R} \right)_m \\
\mathbf{U}^{\dagger}_{\lambda 3,\mathcal{M}}(\varphi_R) & \equiv \left(-\mathbf{R}_e^{2\lambda\varphi_R} \right)_{M-1} & \left(\mathbf{R}_O^{\lambda\varphi_R} (-\mathrm{m}^- \sigma^+)\, \mathbf{R}_O^{-\lambda\varphi_R} \right)_m
\end{array}
\qquad (3)
$$

with: $\quad \mathbf{U}_{\mathcal{M}} \equiv \mathbf{B}_{M-1}\Sigma_m = \underset{1}{\mathbf{B}} \otimes \cdots \otimes \underset{m-1}{\mathbf{B}} \otimes \underset{m}{\Sigma} \otimes \mathbb{1} \otimes \cdots \otimes \underset{N}{\mathbb{1}}\ . \quad \left(\mathbf{B} = \underset{\le m-1}{\left((\pm)\mathbf{R}_e^{2\lambda\varphi_R} \right)}, \quad \Sigma_m = \left(\mathbf{R}_O^{\lambda\varphi_R} ((\pm)\, \mathrm{m}^{\mp}\sigma^{\pm})\, \mathbf{R}_O^{-\lambda\varphi_R} \right)_m \right).$

In (2) and (3), we are writing the algebraic forms of the creation operators for the leptons and the quarks, which we consider as geometrical chains with m locations ($m > 1$). Each location in a complex space $\mathbb{C}^4$. It could be $m = 2$ or $m = 3$; in these cases, therefore, with dimensions over the real numbers 16 or 24, respectively.

For the expressions in (2): the vectors−matrices $\mathbf{R}_O^{(\pm)\lambda\varphi_R}$ outside of the squared bracket for each and all the locations, acting in a similar way as in the diagonalization of the real three dimensional vectors or as in the rotations. If we take the value $\varphi_R = 0$, then the four expressions get reduced to the inside of the squared brackets; we suggest that they represent **the neutrinos**. The four of them in terms of $((\pm)\mathrm{m}^-\sigma^+)$.

For the expressions in (3): by using $\mathbf{R}_O^{\lambda\varphi}\, ((\pm)\sigma^z)\, \mathbf{R}_O^{-\lambda\varphi} = (\pm)\mathbf{R}_e^{2\lambda\varphi}$, we separate the $m-1$ locations of the chain from the m location. Physically, we relate the $m-1$ locations with the **vectors-spin** and the m location with the **spin** (the scalar, up or down).

These operators for the leptons and quarks considered as $'$discrete flipping structures$'$ as they evolve ($\Downarrow\!\Uparrow$ and $\Uparrow\!\Downarrow$) in a time-space ($\mathbb{C}^{4M}$):

$$
\begin{array}{ll}
\langle\, \mathbf{U}^{\dagger}_{1,\mathcal{M}}\ \Uparrow\!\Downarrow,\ \Downarrow\!\Uparrow\ \mathbf{U}^{\dagger}_{7,\mathcal{M}}\, \rangle(\varphi_R), & \text{a particle with spin down and both chiralities, left - right} \\
\langle\, \mathbf{U}^{\dagger}_{-5,\mathcal{M}}\ \Downarrow\!\Uparrow,\ \Uparrow\!\Downarrow\ \mathbf{U}^{\dagger}_{-3,\mathcal{M}}\, \rangle(\varphi_R), & \text{a particle with spin up and both chiralities, left - right} \\
\langle\, \mathbf{U}^{\dagger}_{5,\mathcal{M}}\ \Downarrow\!\Uparrow,\ \Uparrow\!\Downarrow\ \mathbf{U}^{\dagger}_{3,\mathcal{M}}\, \rangle(\varphi_R), & \text{an antiparticle with spin up and both chiralities, right - left} \\
\langle\, \mathbf{U}^{\dagger}_{-1,\mathcal{M}}\ \Uparrow\!\Downarrow,\ \Downarrow\!\Uparrow\ \mathbf{U}^{\dagger}_{-7,\mathcal{M}}\, \rangle(\varphi_R). & \text{an antiparticle with spin down and both chiralities, right - left}
\end{array}
,
$$

and also both vectors-spin ($\mathbf{R}_e^{2\lambda\varphi_R}$ and $-\mathbf{R}_e^{2\lambda\varphi_R}$).

For every chosen value of $\lambda\varphi_R$ (and $\lambda(\varphi_R-\frac{\pi}{2})$, $\lambda(\varphi_R+\frac{\pi}{2})$, $\lambda(\varphi_R-\pi)$) in (3) or (4) we have a different family of fermions. With:

$\{\varphi_R = 0\}$, $q = 0$, the neutrinos , $\{\varphi_R = \frac{\pi}{4}\}$, $q = \mp 1$, the electrons-positrons ,
$\{\varphi_R = \frac{\pi}{12}\}$, $q = \mp\frac{1}{3}$, the d-type quarks , $\{\varphi_R = \frac{\pi}{6}\}$, $q = \pm\frac{2}{3}$, the u-type quarks .

Initially, the weak interaction sector with $0 \le |2\varphi_R| < \frac{\pi}{2}$, and the special treatment for $|2\varphi_R| = \frac{\pi}{2}$ (the charged leptons).

We have used the notation:

the **d-quark** family: after an angle parameter $\varphi_R = \frac{\pi}{12}$ (reference, 5) and the other odd sub-indexes; and for

the **u-quark** family with $\tilde{\varphi}_R = \frac{\pi}{6}$ and the even sub-indexes;

with the following correspondences and references to $\varphi_R = \frac{\pi}{12}$:

as "$-\lambda 6$" which correspond to $\lambda(\varphi_R - \frac{\pi}{4})$, as "$-\lambda 8$" which correspond to $\lambda(\varphi_R - \frac{3\pi}{4})$,
as "$-\lambda 4$" which correspond to $\lambda(\varphi_R + \frac{\pi}{4})$, as "$-\lambda 2$" which correspond to $\lambda(\varphi_R + \frac{3\pi}{4})$.

We also use the labeling with odd numbers $(\pm1, \pm7, \pm5, \pm3)$ for the leptons.

For the neutrinos without the unnecessary minus sign in the sub-index; the symmetry in $\tau\,(\tau = 0)$ cancels the λ dependence.

We denote $\begin{cases} e \text{ electron (particle) and } e \text{ anti-electron (antiparticle) to the parts acting with the weak interaction} \\ p \text{ anti-positron (particle) and } p \text{ positron (antiparticle) to the parts without that action.} \end{cases}$

An important algebraic equality with implications for the geometry – physics: $\mathbf{R}_O^{\epsilon\frac{\pi}{2}}\,(\mathrm{m}^{\mp}\sigma^{\pm})\,\mathbf{R}_O^{-\epsilon\frac{\pi}{2}} = \mathrm{m}^{\pm}\sigma^{\mp}$. (4)

We use this algebraic equality with the leptons, still subjected to discussion (see below). Its implementation with the quarks requires more research.

Specifically, we can define the creation operators for the leptons in another algebraically equivalent form:

Particle (neutrino, electron – anti-positron) Antiparticle (antineutrino, anti-electron – positron)

$$\begin{array}{ll}
\varphi_R = 0 & \varphi_R = 0 \\
\nu\downarrow\ \ \boldsymbol{\nu}^{\dagger}_{1,\mathcal{M}} \equiv (\ \sigma^z)_{M-1}\,(\ (-\mathrm{m}^+\sigma^-)\ )_m & \overline{\nu}\uparrow\ \ \boldsymbol{\nu}^{\dagger}_{5,\mathcal{M}} \equiv (\ \sigma^z)_{M-1}\,(\ (\ \mathrm{m}^-\sigma^+)\ )_m \\
\underline{\nu}\downarrow\ \ \boldsymbol{\nu}^{\dagger}_{7,\mathcal{M}} \equiv (-\sigma^z)_{M-1}\,(\ (\ \mathrm{m}^+\sigma^-)\ )_m & \underline{\nu}\uparrow\ \ \boldsymbol{\nu}^{\dagger}_{3,\mathcal{M}} \equiv (-\sigma^z)_{M-1}\,(\ (-\mathrm{m}^-\sigma^+)\ )_m \\
\hline
\varphi_R = \frac{\pi}{4} & \varphi_R = \frac{\pi}{4} \\
e\downarrow\ \ l^{\dagger}_{1,\mathcal{M}} \equiv (\ \mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(-\mathrm{m}^+\sigma^-)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m & \overline{e}\uparrow\ \ l^{\dagger}_{5,\mathcal{M}} \equiv (\ \mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(\ \mathrm{m}^-\sigma^+)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m \\
\underline{p}\downarrow\ \ l^{\dagger}_{7,\mathcal{M}} \equiv (-\mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(\ \mathrm{m}^+\sigma^-)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m & \overline{p}\uparrow\ \ l^{\dagger}_{3,\mathcal{M}} \equiv (-\mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(-\mathrm{m}^-\sigma^+)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m \\
\cdots\cdots\cdots & \cdots\cdots\cdots \\
\overline{e}\uparrow\ \ l^{\dagger}_{-5,\mathcal{M}} \equiv (-\mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(\ \mathrm{m}^+\sigma^-)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m & \underline{e}\downarrow\ \ l^{\dagger}_{-1,\mathcal{M}} \equiv (-\mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(-\mathrm{m}^-\sigma^+)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m \\
\overline{p}\uparrow\ \ l^{\dagger}_{-3,\mathcal{M}} \equiv (\ \mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(-\mathrm{m}^+\sigma^-)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m & p\downarrow\ \ l^{\dagger}_{-7,\mathcal{M}} \equiv (\ \mathbf{R}_e^{\frac{\pi}{2}})_{M-1}\,(\mathbf{R}_O^{\frac{\pi}{4}}(\ \mathrm{m}^-\sigma^+)\,\mathbf{R}_O^{-\frac{\pi}{4}})_m
\end{array} \qquad (5)$$

In order to obtain these expressions, we have applied (4) in (3), in such a way that they remind a paragraph from Pauli´s *Selected Topics in Field Quantization* (page 4): "Note that here, in contrast with Bose-Einstein statistics, complete symmetry exists between a and a^*, and N and $1-N$, respectively." [4] Observe that in the book $'a'$ has the matrix form σ^+ and $'a^*'$ with σ^-. In our model, the spin is incorporated, and we distinguish a creation operator and a corresponding annihilation operator for the particle on one side and the operators for the antiparticles on another side, i.e. (with $e\ \downarrow$)

$$e:\ \left[l^{\dagger}_{1,\mathcal{M}}\Big|^{\downarrow}\right] \overset{\text{algebra}}{=} \begin{cases} -\ \left[l_{5,\mathcal{M}}\Big|^{\uparrow}\right] : \overline{e} \\ (-1)^{m-1}\left[l_{-1,\mathcal{M}}\Big|^{\downarrow}\right] : \underline{e} \end{cases},\ \text{but we have:}\ \begin{cases} \mathbf{N}_e + (\mathbf{1}-\mathbf{N}_e) \overset{\text{algebra}}{=} \mathbf{N}_e + \mathbf{N}_{\overline{e}} = l^{\dagger}_{1,\mathcal{M}}l_{1,\mathcal{M}} + l^{\dagger}_{5,\mathcal{M}}l_{5,\mathcal{M}} = \mathbf{1} \\ \mathbf{N}_e + (\mathbf{1}-\mathbf{N}_e) \overset{\text{algebra}}{=} \mathbf{N}_e + \mathbf{N}_{\underline{e}} = l^{\dagger}_{1,\mathcal{M}}l_{1,\mathcal{M}} + l^{\dagger}_{-1,\mathcal{M}}l_{-1,\mathcal{M}} = \mathbf{1} \end{cases}.$$

Besides, these charged leptons satisfy: $l^{\dagger}_{\lambda 5,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{-\lambda 7,\mathcal{M}}$ and $l^{\dagger}_{\lambda 1,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{-\lambda 3,\mathcal{M}}$,

but, beneath there are opposite spins and chiralities (geometry – physics); see Appendix II and [1].

I. INTERACTIONS. TOWARDS A BOSON STRUCTURE $\{\,\gamma,\ Z^0,\ W^{\pm},\ g\,\}$.

In particular (from (3)), we use explicitly:

$$
\text{Leptons}\ \begin{cases}
\overset{\downarrow}{\nu}_{(e,\mu)} & \longleftrightarrow & \nu^{\dagger}_{1,M} & = & \mathbf{R}^{0}_{e_{M-1}} & \Big(\quad (-\mathrm{m}^{+}\sigma^{-}) \quad \Big)_m & \\
\overset{\uparrow}{\bar{\nu}}_{(e,\mu)} & \longleftrightarrow & \nu^{\dagger}_{5,M} & = & \mathbf{R}^{0}_{e_{M-1}} & \Big(\quad (\ \mathrm{m}^{-}\sigma^{+}) \quad \Big)_m & \text{[anti-]} \\
\overset{\downarrow}{(e,\mu)^{-}} & \longleftrightarrow & l^{\dagger}_{1,M} & = & \mathbf{R}^{\frac{\pi}{2}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(-\mathrm{m}^{+}\sigma^{-})\ \mathbf{R}_O^{-\frac{\pi}{4}}\Big)_m & \\
\overset{\uparrow}{(\underline{\bar{e}},\underline{\bar{\mu}})^{-}} & \longleftrightarrow & l^{\dagger}_{-5,M} & = & \mathbf{R}^{-\frac{\pi}{2}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(\ \mathrm{m}^{+}\sigma^{-})\ \mathbf{R}_O^{-\frac{\pi}{4}}\Big)_m & \\
\overset{\uparrow}{(\bar{e},\bar{\mu})^{+}} & \longleftrightarrow & l^{\dagger}_{5,M} & = & \mathbf{R}^{\frac{\pi}{2}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(\ \mathrm{m}^{-}\sigma^{+})\ \mathbf{R}_O^{-\frac{\pi}{4}}\Big)_m & \text{[anti-]} \\
\overset{\downarrow}{(\underline{e},\underline{\mu})^{+}} & \longleftrightarrow & l^{\dagger}_{-1,M} & = & \mathbf{R}^{-\frac{\pi}{2}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(-\mathrm{m}^{-}\sigma^{+})\ \mathbf{R}_O^{-\frac{\pi}{4}}\Big)_m & \text{[anti-]}
\end{cases} \quad . \tag{6}
$$

$$
\text{Quarks}\ \begin{cases}
\overset{\downarrow}{d}{}^{-} & \longleftrightarrow & q^{\dagger}_{1,M} & = & \mathbf{R}^{\frac{\pi}{6}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{12}}\,(-\mathrm{m}^{+}\sigma^{-})\,\mathbf{R}_O^{-\frac{\pi}{12}}\Big)_m & \\
\overset{\downarrow}{u}{}^{+} & \longleftrightarrow & q^{\dagger}_{2,M} & = & \mathbf{R}^{\frac{\pi}{3}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{6}}\,(-\mathrm{m}^{+}\sigma^{-})\,\mathbf{R}_O^{-\frac{\pi}{6}}\Big)_m & \\
\overset{\uparrow}{\bar{u}}{}^{-} & \longleftrightarrow & q^{\dagger}_{6,M} & = & \mathbf{R}^{\frac{\pi}{3}}_{e_{M-1}} & \Big(\mathbf{R}_O^{\frac{\pi}{6}}\,(\ \mathrm{m}^{-}\sigma^{+})\,\mathbf{R}_O^{-\frac{\pi}{6}}\Big)_m & \text{[anti-]} \\
\overset{\uparrow}{\underline{\bar{d}}}{}^{-} & \longleftrightarrow & q^{\dagger}_{-5,M} & = & \mathbf{R}^{-\frac{\pi}{6}}_{e_{M-1}} & \Big(\mathbf{R}_O^{-\frac{\pi}{12}}(\ \mathrm{m}^{-}\sigma^{+})\,\mathbf{R}_O^{\frac{\pi}{12}}\Big)_m &
\end{cases} \quad . \tag{7}
$$

With an extensively use of (4) whenever is appropriate.

We tentatively consider:

<u>g gluons</u> in **III)** : $\quad g_{m,\pm 12} = \Big(\mathbf{R}_O^{\pm\frac{\pi}{12}}\Big)_m \quad ; \quad g_{m,\pm 6} = \Big(\mathbf{R}_O^{\pm\frac{\pi}{6}}\Big)_m$.

We have not studied the spin of the possible gluon structures (the g´s).

<u>W bosons</u> in **II)** , **III)** iii) and **III)** iv) . $\quad \Big(\big(\mathbf{R}_O^{\pm\frac{\pi}{2}}\big)_{M-1}\ \big(\,*\,\big)_m \Big)$.

<u>$\{\gamma, Z^0\}$ bosons</u> $\ (\varphi_R \in \{0,\ \frac{\pi}{12},\ \frac{\pi}{6},\ \frac{\pi}{4}\})$ in **I)** :

$$
\{1,\ldots,m-1\}\ :\quad \Big(\mathbf{R}_O^{4\lambda\varphi_R}\Big) \qquad
\begin{cases}
\downarrow\uparrow\ ,\ \uparrow\downarrow\ : & \big(\ \mathbb{1}\big) & \text{(leptons and quarks)} \\
\downarrow\downarrow\ ,\ \uparrow\uparrow & \begin{cases} \Big(\mathbf{R}_O^{4\lambda\frac{\pi}{12}}\Big),\ \Big(\mathbf{R}_O^{4\lambda\frac{\pi}{6}}\Big) & \text{(quarks)} \\ \big(-\mathbb{1}\big) & \text{(charged leptons)} \end{cases}
\end{cases}
$$

$$
m\ :\quad
\begin{cases}
\downarrow\uparrow\ ,\ \uparrow\downarrow\ :\ \Big(\tfrac{1}{2}(\pm\mathbf{R}_e^{\lambda 2\varphi_R} - \mathbb{1})\Big) \quad \text{in particular} &
\begin{cases} (-\overset{\wedge}{\sigma})\ \ ,\ \ (-\overset{\vee}{\sigma}) & \text{(uncharged leptons)} \\ \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(-\overset{\wedge}{\sigma})\,\mathbf{R}_O^{-\frac{\pi}{4}}\Big),\ \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(-\overset{\vee}{\sigma})\,\mathbf{R}_O^{-\frac{\pi}{4}}\Big) & \text{(charged leptons)} \end{cases} \\
\downarrow\downarrow\ ,\ \uparrow\uparrow\ :\ \Big(\mathbf{R}_O^{\lambda\varphi_R}\sin(\pm 2\lambda\varphi_R)\,\mathrm{e}^{\mp\mathrm{i}\,\phi}\,\sigma^{\pm}\ \mathbf{R}_O^{\lambda\varphi_R}\Big) : \quad \text{in particular} & \Big(\mathbf{R}_O^{\frac{\pi}{4}}\,(\pm\,\mathrm{e}^{\mp\mathrm{i}\phi}\,\sigma^{\pm})\ \mathbf{R}_O^{\frac{\pi}{4}}\Big) \quad \text{(charged leptons)}
\end{cases}
$$

We have two different algebraic structures, $[\,(\,*\,)_m\,]$ and $[\,\big(\,*\,*\big)_{M-1}\ (\,*\,*\,*\,)_m\,]$, which we relate with the massless and the massive bosons respectively.

We do not write the parts with ± 3, ± 4, ± 7, ± 8 (they are in (2), (3) and (5)), as they are not under the weak interaction. They can be treated algebraically in the same way.

I) Guided by: ' *particle* $\wedge$ *antiparticle* $\longrightarrow$ *bosons* : photons / Z^0 '.

The starting point in the annihilation of an electron and a positron producing two photons (spin 0) or three photons (spin ±1).

We establish the following formal products for the generic creation operators related by a **time type opposition**, which implies a spin zero (↑↓ , ↓↑):

$$\overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} \equiv \mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}}\mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}} = \left(\mathbf{R}_o^{\lambda\varphi_R}(-\overset{\vee}{\sigma})\,\mathbf{R}_o^{-\lambda\varphi_R}\right)_m = \left(\tfrac{1}{2}(\ \mathbf{R}_e^{\lambda 2\varphi_R} - \mathbb{1})\right)_m , \qquad \left\langle \equiv \mathbf{U}^{\dagger}_{\lambda 7,\mathcal{M}}\mathbf{U}^{\dagger}_{\lambda 3,\mathcal{M}} \right\rangle \tag{8}$$
$$\overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} \equiv \mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}}\mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}} = \left(\mathbf{R}_o^{\lambda\varphi_R}(-\overset{\wedge}{\sigma})\,\mathbf{R}_o^{-\lambda\varphi_R}\right)_m = \left(\tfrac{1}{2}(-\mathbf{R}_e^{\lambda 2\varphi_R} - \mathbb{1})\right)_m , \qquad \left\langle \equiv \mathbf{U}^{\dagger}_{\lambda 3,\mathcal{M}}\mathbf{U}^{\dagger}_{\lambda 7,\mathcal{M}} \right\rangle .$$

The brackets: $\langle\ *\ \rangle$ indicate that there is not weak interaction (this is related to the z–axis).

In particular, for **neutrino** – **antineutrino** ($\varphi_R = 0$) in the forms:

$$\overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\nu\bar{\nu},m} \equiv \left(\overset{\downarrow}{\mathbf{v}}_e\,\overset{\uparrow}{\bar{\mathbf{v}}}_e\right) \equiv \mathbf{v}^{\dagger}_{1,\mathcal{M}}\mathbf{v}^{\dagger}_{5,\mathcal{M}} = \left(-\overset{\vee}{\sigma}\right)_m , \qquad \left\langle \equiv \overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\underline{\bar{\nu}}\underline{\nu},m} \equiv \left(\overset{\downarrow}{\underline{\bar{\mathbf{v}}}}_e\,\overset{\uparrow}{\underline{\mathbf{v}}}_e\right) \equiv \mathbf{v}^{\dagger}_{7,\mathcal{M}}\mathbf{v}^{\dagger}_{3,\mathcal{M}} \right\rangle \tag{9}$$
$$\overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{\bar{\nu}\nu,m} \equiv \left(\overset{\uparrow}{\mathbf{v}}_e\,\overset{\downarrow}{\bar{\mathbf{v}}}_e\right) \equiv \mathbf{v}^{\dagger}_{5,\mathcal{M}}\mathbf{v}^{\dagger}_{1,\mathcal{M}} = \left(-\overset{\wedge}{\sigma}\right)_m , \qquad \left\langle \equiv \overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{\underline{\nu}\underline{\bar{\nu}},m} \equiv \left(\overset{\uparrow}{\underline{\mathbf{v}}}_e\,\overset{\downarrow}{\underline{\bar{\mathbf{v}}}}_e\right) \equiv \mathbf{v}^{\dagger}_{3,\mathcal{M}}\mathbf{v}^{\dagger}_{7,\mathcal{M}} \right\rangle .$$

We have seen this in equation (1).

And for the **electron** – **anti-electron** ($\varphi_R = \frac{\pi}{4}$) in the forms (algebraic equalities, with total equal spins):

$$\overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\bar{e}e,m} \equiv \left(\overset{\downarrow}{e}\overset{\pi/2}{\leftarrow}\right)\left(\overset{\uparrow}{\underline{e}}\overset{\pi/2}{\rightarrow}\right)\Big|_{(1,\ 5)} = \left(\mathbf{R}_o^{\frac{\pi}{4}}(-\overset{\vee}{\sigma})\mathbf{R}_o^{-\frac{\pi}{4}}\right)_m \overset{\text{algebra}}{=} \left(\mathbf{R}_o^{-\frac{\pi}{4}}(-\overset{\wedge}{\sigma})\mathbf{R}_o^{\frac{\pi}{4}}\right)_m = \left(\overset{\uparrow}{e}\overset{-\pi/2}{\leftarrow}\right)\left(\overset{\downarrow}{\underline{e}}\overset{-\pi/2}{\rightarrow}\right)\Big|_{(-5,-1)} \equiv \overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{\underline{e}\,\underline{e},m} \tag{10}$$
$$\overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\underline{e}\,\underline{e},m} \equiv \left(\overset{\downarrow}{\underline{e}}\overset{-\pi/2}{\rightarrow}\right)\left(\overset{\uparrow}{e}\overset{-\pi/2}{\leftarrow}\right)\Big|_{(-1,-5)} = \left(\mathbf{R}_o^{\frac{\pi}{4}}(-\overset{\wedge}{\sigma})\mathbf{R}_o^{-\frac{\pi}{4}}\right)_m \overset{\text{algebra}}{=} \left(\mathbf{R}_o^{-\frac{\pi}{4}}(-\overset{\vee}{\sigma})\mathbf{R}_o^{\frac{\pi}{4}}\right)_m = \left(\overset{\uparrow}{\underline{e}}\overset{\pi/2}{\rightarrow}\right)\left(\overset{\downarrow}{e}\overset{\pi/2}{\leftarrow}\right)\Big|_{(5,\ 1)} \equiv \overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{\bar{e}e,m} .$$

There are similar algebraic expressions for the **non weak** sector, the 'p' (positrons), and also mixing them ('e' and 'p').

It is verified:

$$\boxed{\overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} + \overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} = \left(-\mathbb{1}\right)_m ,} \qquad \overset{\downarrow\uparrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} - \overset{\uparrow\downarrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,m} = \left(\mathbf{R}_e^{\lambda 2\varphi_R}\right)_m . \tag{11}$$

(1) is a particular case of the first equation.

After a τ opposition, which implies a +1 (↑↑) or a −1 (↓↓) for the spins:

$$\overset{\downarrow\downarrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,\mathcal{M}} \equiv \mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}}\mathbf{U}^{\dagger}_{-\lambda 1,\mathcal{M}} = \left[\left(\mathbf{R}_o^{4\lambda\varphi_R}\right)_{M-1}\right]\left(\mathbf{R}_o^{\lambda\varphi_R}\sin(-2\lambda\varphi_R)e^{i\phi}\sigma^{-}\,\mathbf{R}_o^{\lambda\varphi_R}\right)_m \tag{12}$$
$$\overset{\uparrow\uparrow}{\mathbf{Y}}{}^{\varphi_R\,\dagger}_{\lambda,\mathcal{M}} \equiv \mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}}\mathbf{U}^{\dagger}_{-\lambda 5,\mathcal{M}} = \left[\left(\mathbf{R}_o^{4\lambda\varphi_R}\right)_{M-1}\right]\left(\mathbf{R}_o^{\lambda\varphi_R}\sin(\ 2\lambda\varphi_R)e^{-i\phi}\sigma^{+}\,\mathbf{R}_o^{\lambda\varphi_R}\right)_m .$$

Obviously these ones can not be produced with 'neutrinos' – 'antineutrinos' ($\varphi_R = 0 \Rightarrow \sin(2\varphi_R) = 0$).

And for the charged leptons ($\varphi_R = \frac{\pi}{4}$):

$$\overset{\downarrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{e\underline{e},\mathcal{M}} \equiv \left(\overset{\downarrow}{e}\overset{\pi/2}{\leftarrow}\ \overset{\downarrow}{\underline{e}}\overset{-\pi/2}{\rightarrow}\Big|_{(1,-1)}\right) = \left[\left(-\mathbb{1}\right)_{M-1}\right]\left(\mathbf{R}_o^{\frac{\pi}{4}}(-e^{i\phi}\sigma^{-})\,\mathbf{R}_o^{\frac{\pi}{4}}\right)_m \overset{\text{algebra}}{=}$$
$$\overset{!}{=}\left[\left(-\mathbb{1}\right)_{M-1}\right]\left(\mathbf{R}_o^{-\frac{\pi}{4}}(-e^{-i\phi}\sigma^{+})\mathbf{R}_o^{-\frac{\pi}{4}}\right)_m = \left(\overset{\uparrow}{e}\overset{-\pi/2}{\leftarrow}\ \overset{\uparrow}{\underline{e}}\overset{\pi/2}{\rightarrow}\Big|_{(-5,\ 5)}\right) \equiv \overset{\uparrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\underline{e}e,\mathcal{M}} , \tag{13}$$
$$\overset{\uparrow\uparrow}{\mathbf{Y}}{}^{\dagger}_{\bar{e}\underline{e},\mathcal{M}} \equiv \left(\overset{\uparrow}{\underline{e}}\overset{\pi/2}{\rightarrow}\ \overset{\uparrow}{e}\overset{-\pi/2}{\leftarrow}\Big|_{(5,-5)}\right) = \left[\left(-\mathbb{1}\right)_{M-1}\right]\left(\mathbf{R}_o^{\frac{\pi}{4}}(\ e^{-i\phi}\sigma^{+})\,\mathbf{R}_o^{\frac{\pi}{4}}\right)_m \overset{\text{algebra}}{=}$$
$$\overset{!}{=}\left[\left(-\mathbb{1}\right)_{M-1}\right]\left(\mathbf{R}_o^{-\frac{\pi}{4}}(\ e^{i\phi}\sigma^{-})\,\mathbf{R}_o^{-\frac{\pi}{4}}\right)_m = \left(\overset{\downarrow}{\underline{e}}\overset{-\pi/2}{\rightarrow}\ \overset{\downarrow}{e}\overset{\pi/2}{\leftarrow}\Big|_{(-1,\ 1)}\right) \equiv \overset{\downarrow\downarrow}{\mathbf{Y}}{}^{\dagger}_{\underline{e}e,\mathcal{M}}$$

although they are different geometrical and physical entities. Therefore these algebraic equalities would imply a misleading treatment of the spin. We need to impose the geometry – physics. The geometry – physics behind the algebraic significance of the spin in the boson structure has to be clarified. See in the appendixes for a geometrical interpretation.

All of them have zero electric charge and different spins, +1 for ↑↑ , −1 for ↓↓ and 0 for ↑↓ or ↓↑. They would include the interactions: neutrino – antineutrino, electron – anti-electron, quark – antiquark. [5] Perhaps in a form similar to the $W^{(3)}$ and B mixtures for the bosons γ and Z. [2] [6]

The singlet and triplet states of the positronium, with the production of two (in (8)) and three (in (12)) photons respectively, should be studied in the present framework.

II) Guided by the leptons: ‘ *neutrinos* $\wedge$ *charged leptons* $\longrightarrow$ $W^{\pm}$ *bosons* ’.

$$\mathbf{R}_{O\mathcal{M}-1}^{\pm\frac{\pi}{2}} \longleftrightarrow \left(\mathbf{R}_O^{\pm\frac{\pi}{2}}\right)_{\mathcal{M}-1} = \left(\pm\,\mathbf{R}_O^{\frac{\pi}{2}}\right)_{\mathcal{M}-1} = \left(\pm\begin{pmatrix}0 & -e^{-i\phi}\\ e^{i\phi} & 0\end{pmatrix}\right)_{\mathcal{M}-1}.$$

We write with the **electrons and the antineutrinos**:

$$\overset{0}{W}{}^{-}\begin{cases}\overset{\downarrow\uparrow}{W}{}^{-\dagger}_{e\bar\nu,\mathcal{M}} \equiv \left(\overset{\downarrow}{e}{}^{-}\,\overset{\uparrow}{\bar\nu}_e\right) \equiv l^{\dagger}_{1,\mathcal{M}}\nu^{\dagger}_{5,\mathcal{M}} = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(-\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{-}\mathbf{R}_O^{-\frac{\pi}{4}}\sigma^{+}\right)_m = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{1}{2}\begin{pmatrix}0 & e^{-i\phi}\\ 0 & -1\end{pmatrix}\right)_m \\ \overset{\uparrow\downarrow}{W}{}^{-\dagger}_{\bar\nu e,\mathcal{M}} \equiv \left(\overset{\uparrow}{\bar\nu}_e\,\overset{\downarrow}{e}{}^{-}\right) \equiv \nu^{\dagger}_{5,\mathcal{M}}l^{\dagger}_{1,\mathcal{M}} = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(-\sigma^{+}\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{-}\mathbf{R}_O^{-\frac{\pi}{4}}\right)_m = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{-1}{2}\begin{pmatrix}1 & e^{-i\phi}\\ 0 & 0\end{pmatrix}\right)_m\end{cases}, \qquad (14a)$$

$$\overset{+1}{W}{}^{-}\begin{cases}\overset{\uparrow\uparrow}{W}{}^{-\dagger}_{\underline{e}\bar\nu,\mathcal{M}} \equiv \left(\overset{\uparrow}{\underline{e}}{}^{-}\,\overset{\uparrow}{\bar\nu}_e\right) \equiv l^{\dagger}_{-5,\mathcal{M}}\nu^{\dagger}_{5,\mathcal{M}} = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{-}\mathbf{R}_O^{-\frac{\pi}{4}}\sigma^{+}\right)_m = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{-1}{2}\begin{pmatrix}0 & e^{-i\phi}\\ 0 & -1\end{pmatrix}\right)_m \\ \overset{\uparrow\uparrow}{W}{}^{-\dagger}_{\bar\nu\underline{e},\mathcal{M}} \equiv \left(\overset{\uparrow}{\bar\nu}_e\,\overset{\uparrow}{\underline{e}}{}^{-}\right) \equiv \nu^{\dagger}_{5,\mathcal{M}}l^{\dagger}_{-5,\mathcal{M}} = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\sigma^{+}\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{-}\mathbf{R}_O^{-\frac{\pi}{4}}\right)_m = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{1}{2}\begin{pmatrix}1 & e^{-i\phi}\\ 0 & 0\end{pmatrix}\right)_m\end{cases}. \qquad (14b)$$

The $\overset{+1}{W}{}^{-}$ bosons for electrons with left chirallity but right helicity <u>in their own frames</u> (see Figure 2). Compare with the scheme depicted in Figure 4 for the pion decay and the $\overset{\downarrow\uparrow}{W}{}^{-}$.

And with the **anti-electrons and the neutrinos:**

$$\overset{0}{W}{}^{+}\begin{cases}\overset{\uparrow\downarrow}{W}{}^{+\dagger}_{\bar e\nu,\mathcal{M}} \equiv \left(\overset{\uparrow}{\bar e}{}^{+}\,\overset{\downarrow}{\nu}_e\right) \equiv l^{\dagger}_{5,\mathcal{M}}\nu^{\dagger}_{1,\mathcal{M}} = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(-\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{+}\mathbf{R}_O^{-\frac{\pi}{4}}\sigma^{-}\right)_m = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{-1}{2}\begin{pmatrix}1 & 0\\ e^{i\phi} & 0\end{pmatrix}\right)_m \\ \overset{\downarrow\uparrow}{W}{}^{+\dagger}_{\nu\bar e,\mathcal{M}} \equiv \left(\overset{\downarrow}{\nu}_e\,\overset{\uparrow}{\bar e}{}^{+}\right) \equiv \nu^{\dagger}_{1,\mathcal{M}}l^{\dagger}_{5,\mathcal{M}} = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(-\sigma^{-}\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{+}\mathbf{R}_O^{-\frac{\pi}{4}}\right)_m = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{1}{2}\begin{pmatrix}0 & 0\\ e^{i\phi} & -1\end{pmatrix}\right)_m\end{cases}, \qquad (15a)$$

$$\overset{-1}{W}{}^{+}\begin{cases}\overset{\downarrow\downarrow}{W}{}^{+\dagger}_{\underline{e}\nu,\mathcal{M}} \equiv \left(\overset{\downarrow}{\underline{e}}{}^{+}\,\overset{\downarrow}{\nu}_e\right) \equiv l^{\dagger}_{-1,\mathcal{M}}\nu^{\dagger}_{1,\mathcal{M}} = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{+}\mathbf{R}_O^{-\frac{\pi}{4}}\sigma^{-}\right)_m = \mathbf{R}^{-\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{1}{2}\begin{pmatrix}1 & 0\\ e^{i\phi} & 0\end{pmatrix}\right)_m \\ \overset{\downarrow\downarrow}{W}{}^{+\dagger}_{\nu\underline{e},\mathcal{M}} \equiv \left(\overset{\downarrow}{\nu}_e\,\overset{\downarrow}{\underline{e}}{}^{+}\right) \equiv \nu^{\dagger}_{1,\mathcal{M}}l^{\dagger}_{-1,\mathcal{M}} = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\sigma^{-}\mathbf{R}_O^{\frac{\pi}{4}}\sigma^{+}\mathbf{R}_O^{-\frac{\pi}{4}}\right)_m = \mathbf{R}^{\frac{\pi}{2}}_{O\mathcal{M}-1}\left(\frac{-1}{2}\begin{pmatrix}0 & 0\\ e^{i\phi} & -1\end{pmatrix}\right)_m.\end{cases} \qquad (15b)$$

The $\overset{-1}{W}{}^{+}$ bosons for anti-electrons with right chirallity but left helicity <u>in their own frames</u>.

It is interesting to show the merely algebraic relationships that follows. They should not be identified as they represent different spins, different geometry and physics:

$$\begin{aligned}&\overset{0}{W}{}^{-\dagger}_{e\bar\nu,\mathcal{M}} = \overset{\downarrow\uparrow}{W}{}^{-\dagger}_{e\bar\nu,\mathcal{M}} \overset{(-1)^m}{\longleftrightarrow} \overset{\uparrow\uparrow}{W}{}^{-\dagger}_{\underline{e}\bar\nu,\mathcal{M}} = \overset{+1}{W}{}^{-\dagger}_{\underline{e}\bar\nu,\mathcal{M}}, \qquad &&\overset{0}{W}{}^{-\dagger}_{\bar\nu e,\mathcal{M}} = \overset{\uparrow\downarrow}{W}{}^{-\dagger}_{\bar\nu e,\mathcal{M}} \overset{(-1)^m}{\longleftrightarrow} \overset{\uparrow\uparrow}{W}{}^{-\dagger}_{\bar\nu\underline{e},\mathcal{M}} = \overset{+1}{W}{}^{-\dagger}_{\bar\nu\underline{e},\mathcal{M}}, \\ &\overset{0}{W}{}^{+\dagger}_{\bar e\nu,\mathcal{M}} = \overset{\uparrow\downarrow}{W}{}^{+\dagger}_{\bar e\nu,\mathcal{M}} \overset{(-1)^m}{\longleftrightarrow} \overset{\downarrow\downarrow}{W}{}^{+\dagger}_{\underline{e}\nu,\mathcal{M}} = \overset{-1}{W}{}^{+\dagger}_{\underline{e}\nu,\mathcal{M}}, \qquad &&\overset{0}{W}{}^{+\dagger}_{\nu\bar e,\mathcal{M}} = \overset{\downarrow\uparrow}{W}{}^{+\dagger}_{\nu\bar e,\mathcal{M}} \overset{(-1)^m}{\longleftrightarrow} \overset{\downarrow\downarrow}{W}{}^{+\dagger}_{\nu\underline{e},\mathcal{M}} = \overset{-1}{W}{}^{+\dagger}_{\nu\underline{e},\mathcal{M}}.\end{aligned} \qquad (16)$$

See the second appendix for a geometrical interpretation. Besides, with geometry and physics, should m be odd?

The lepton number conservation would ban with:

$$\begin{aligned}&\overset{\downarrow\downarrow}{W}{}^{-\dagger}_{e\nu,\mathcal{M}},\quad \overset{\downarrow\downarrow}{W}{}^{-\dagger}_{\nu e,\mathcal{M}},\quad \overset{\uparrow\downarrow}{W}{}^{-\dagger}_{\underline{e}\nu,\mathcal{M}},\quad \overset{\downarrow\uparrow}{W}{}^{-\dagger}_{\nu\underline{e},\mathcal{M}} \qquad &&(\text{ electrons with neutrinos }) \\ &\overset{\uparrow\uparrow}{W}{}^{+\dagger}_{\bar\nu\bar e,\mathcal{M}},\quad \overset{\uparrow\uparrow}{W}{}^{+\dagger}_{\bar e\bar\nu,\mathcal{M}},\quad \overset{\uparrow\downarrow}{W}{}^{+\dagger}_{\bar\nu\underline{e},\mathcal{M}},\quad \overset{\downarrow\uparrow}{W}{}^{+\dagger}_{\underline{e}\bar\nu,\mathcal{M}} \qquad &&(\text{anti-electrons with antineutrinos})\end{aligned}, \qquad (17)$$

(a rule, a law?)

Muon decays.

$$\mu^- \longrightarrow \nu_\mu \quad W^- \longrightarrow [\, \nu_\mu \quad \bar{\nu}_e \,]\; e^- ,$$
$$\mu^+ \longrightarrow \bar{\nu}_\mu \quad W^+ \longrightarrow [\, \bar{\nu}_\mu \quad \nu_e \,]\; e^+ .$$

We write a couple of muon decays with our geometrical formulation:

$$\overset{\downarrow}{\mu}{}^- \begin{cases} \nearrow \Big[\overset{\downarrow}{\nu}_\mu \overset{\uparrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}_e e,\, \mathcal{M}} + \overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{\nu_\mu e,\, \mathcal{M}} \Big] = \Big[\overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e + \overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu \Big] \overset{\downarrow}{e}{}^- \\ \searrow \Big[\overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{e \bar{\nu}_e,\, \mathcal{M}} \overset{\downarrow}{\nu}_\mu + \overset{\downarrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{e \nu_\mu,\, \mathcal{M}} \overset{\uparrow}{\bar{\nu}}_e \Big] = \overset{\downarrow}{e}{}^- \Big[\overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu + \overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e \Big] \end{cases}$$

$$. \tag{18}$$

$$\overset{\uparrow}{\underline{\mu}}{}^- \begin{cases} \nearrow \Big[\overset{\downarrow}{\nu}_\mu \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}_e \underline{e},\, \mathcal{M}} + \overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\nu_\mu \underline{e},\, \mathcal{M}} \Big] = \Big[\overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e + \overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu \Big] \overset{\uparrow}{\underline{e}}{}^- \\ \searrow \Big[\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\underline{e} \bar{\nu}_e,\, \mathcal{M}} \overset{\downarrow}{\nu}_\mu + \overset{\uparrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{\underline{e} \nu_\mu,\, \mathcal{M}} \overset{\uparrow}{\bar{\nu}}_e \Big] = \overset{\uparrow}{\underline{e}}{}^- \Big[\overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu + \overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e \Big] \end{cases}$$

Let us calculate the last square brackets (in each line):

$$\Big[\overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e + \overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu \Big] = \Big[\overset{\uparrow}{\bar{\nu}}_e \overset{\downarrow}{\nu}_\mu + \overset{\downarrow}{\nu}_\mu \overset{\uparrow}{\bar{\nu}}_e \Big].$$

The algebra with our geometrical - physical interpretation:

$$\begin{aligned} &\Big[\nu^{\dagger}_{\mu;\,1,\,\mathcal{M}} \; \nu^{\dagger}_{e;\,5,\,\mathcal{M}} + \nu^{\dagger}_{e;\,5,\,\mathcal{M}} \; \nu^{\dagger}_{\mu;\,1,\,\mathcal{M}} \Big] = \\ &= \Big[\mathbf{R}^0_{e_{M-1}} \big(-\mathrm{m}^+ \sigma^- \big)_m \; \mathbf{R}^0_{e_{M-1}} \big(\mathrm{m}^- \sigma^+ \big)_m + \mathbf{R}^0_{e_{M-1}} \big(\mathrm{m}^- \sigma^+ \big)_m \; \mathbf{R}^0_{e_{M-1}} \big(-\mathrm{m}^+ \sigma^- \big)_m \Big] = \\ &= \Big[\sigma^z_{M-1} \big(-\sigma^- \big)_m \; \sigma^z_{M-1} \big(\sigma^+ \big)_m + \sigma^z_{M-1} \big(\sigma^+ \big)_m \; \sigma^z_{M-1} \big(-\sigma^- \big)_m \Big] = \\ &= \Big[\big(-\overset{\vee}{\sigma} \big)_m + \big(-\overset{\wedge}{\sigma} \big)_m \Big] = \big(-\mathbb{1} \big)_m = -\mathbf{1} \; . \end{aligned} \tag{19}$$

The sum in this last squared bracket already in the equations in (1) and un (11).

The muon and the electron are not the same particle ! Although this, we have used a lepton universality, the same algebraic expressions for the electrons and muons and also for their corresponding antineutrinos (see (6)), for the weak couplings with the $W^\pm$ *bosons*:

$$l^{\dagger}_{1,\mathcal{M}} \quad \text{for} \quad \overset{\downarrow}{e}{}^- \text{ and } \overset{\downarrow}{\mu}{}^- \; ; \qquad l^{\dagger}_{-5,\mathcal{M}} \quad \text{for} \quad \overset{\uparrow}{\underline{e}}{}^- \text{ and } \overset{\uparrow}{\underline{\mu}}{}^- \; ;$$
$$\nu^{\dagger}_{e;1,\mathcal{M}} = \nu^{\dagger}_{\mu;1,\mathcal{M}} \quad \text{for} \quad \overset{\downarrow}{\nu}_e \text{ and } \overset{\downarrow}{\nu}_\mu \; ; \qquad \nu^{\dagger}_{e;5,\mathcal{M}} = \nu^{\dagger}_{\mu;5,\mathcal{M}} \quad \text{for} \quad \overset{\uparrow}{\bar{\nu}}_e \text{ and } \overset{\uparrow}{\bar{\nu}}_\mu \; .$$

(19) suggest a minus factor (a phase?) in the relation of the algebraic forms of the creation operators for the electrons and the muons, or perhaps also for the relation of the ′electronic−neutrinos′ and the ′muonic−neutrinos′.

Similarly for the anti-muons with the anti-electrons and the corresponding uncharged leptons.

In another study [7] we have differentiated the neutrinos corresponding to the electron to the muon and to the tau particles by assigning different values to the other angle parameter, ϕ . We did so in relation to an explanation for the oscillations of the neutrinos.

III) Guided by the Quarks. Appearance of the gluons: $\{\, g \,\} \cup$ quark ?

i) A β decay: the non conservation of the parity. Madame Wu C.S. et al. (1956).

$$n \rightarrow p \;\; \overline{\nu}_e \; e^- \;\; (+\, 2\gamma) : \qquad \{g\} \cup d \;\longrightarrow\; \{g\} \cup u \;\; \overline{\nu}_e \; e^- \qquad \left(\begin{cases} \{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{d^-} \longrightarrow \{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\downarrow}{u^+} \;\; \overset{\uparrow}{\overline{\nu}}_e \, \overset{\uparrow}{e^-} \\ \{\overset{\uparrow\uparrow,\uparrow\uparrow}{g}\} \cup \overset{\downarrow}{d^-} \longrightarrow \{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{u^+} \;\; \overset{\uparrow}{e^-} \, \overset{\uparrow}{\overline{\nu}}_e \\ \{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\downarrow}{d^-} \longrightarrow \{\overset{\downarrow\downarrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{u^+} \;\; \overset{\uparrow}{e^-} \, \overset{\uparrow}{\overline{\nu}}_e \end{cases} \right).$$

And several other configurations that should be checked. Are there different configurations of gluons?

We have verified the first one algebraically within our geometrical framework:

$$g_{m,-6} \, \underline{\overset{\uparrow}{\overline{d}^-}} \, g_{m,6} \;\longrightarrow\; \left\{ g_{m,12} \overset{\downarrow}{u^+} g_{m,-12} \,,\; \overset{\uparrow\uparrow}{\mathbf{W}}{}^{\dagger}_{\overline{\nu}\,\underline{e},\,\mathcal{M}} \right\} = g_{m,12} \overset{\downarrow}{u^+} g_{m,-12} \;\; \overset{\uparrow\uparrow}{\mathbf{W}}{}^{\dagger}_{\overline{\nu}\,\underline{e},\,\mathcal{M}} + \overset{\uparrow\uparrow}{\mathbf{W}}{}^{\dagger}_{\underline{e}\,\overline{\nu},\,\mathcal{M}} \; g_{m,12} \overset{\downarrow}{u^+} g_{m,-12} \,, \tag{20}$$

with $\left\{ \overset{\uparrow\uparrow}{\mathbf{W}}{}^{\dagger}_{\overline{\nu}\,\underline{e},\,\mathcal{M}} \right\} = \overset{\uparrow}{\overline{\nu}}_e \, \overset{\uparrow}{e^-} = \nu^{\dagger}_{5,\mathcal{M}} \, l^{\dagger}_{-5,\mathcal{M}}$ and $\left\{ \overset{\uparrow\uparrow}{\mathbf{W}}{}^{\dagger}_{\underline{e}\,\overline{\nu},\,\mathcal{M}} \right\} = \overset{\uparrow}{e^-} \, \overset{\uparrow}{\overline{\nu}}_e = l^{\dagger}_{-5,\mathcal{M}} \, \nu^{\dagger}_{5,\mathcal{M}}$ in (14b) .

We write the following algebraic equality for (20):

$$(A \equiv) \quad \mathbf{R}_{om}^{-\frac{\pi}{6}} q^{\dagger}_{-5,\mathcal{M}} \mathbf{R}_{om}^{\frac{\pi}{6}} = -\left[\mathbf{R}_{om}^{\frac{\pi}{12}} q^{\dagger}_{2,\mathcal{M}} \mathbf{R}_{om}^{-\frac{\pi}{12}} \; \nu^{\dagger}_{5,\mathcal{M}} l^{\dagger}_{-5,\mathcal{M}} + l^{\dagger}_{-5,\mathcal{M}} \nu^{\dagger}_{5,\mathcal{M}} \; \mathbf{R}_{om}^{\frac{\pi}{12}} q^{\dagger}_{2,\mathcal{M}} \mathbf{R}_{om}^{-\frac{\pi}{12}} \right] \quad (\equiv B), \tag{21}$$

which we can split:

$$\begin{cases} 1,\ldots,m-1: \quad A \longrightarrow \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{6}} = \begin{cases} \mathbf{R}_{e_{M-1}}^{\frac{\pi}{3}} \, \mathbf{R}_{e_{M-1}}^{0} \, \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{2}} \longleftarrow B\,, & \left(-\frac{\pi}{6} = \frac{\pi}{3} - 0 + (-\frac{\pi}{2}) \right) \\ \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{2}} \, \mathbf{R}_{e_{M-1}}^{0} \, \mathbf{R}_{e_{M-1}}^{\frac{\pi}{3}} \longleftarrow B\,, & \left(-\frac{\pi}{6} = -\frac{\pi}{2} - 0 + \frac{\pi}{3} \right) \end{cases} \\ m: \quad \begin{cases} A \longrightarrow \left[\mathbf{R}_o^{-\frac{\pi}{6}} \left(\mathbf{R}_o^{-\frac{\pi}{12}} (\mathrm{m}^- \sigma^+) \mathbf{R}_o^{\frac{\pi}{12}} \right) \mathbf{R}_o^{\frac{\pi}{6}} \right] = \left[\mathrm{m}^- \, \mathbf{R}_o^{-\frac{\pi}{4}} \; \underline{\langle \sigma^+ \rangle} \; \mathbf{R}_o^{\frac{\pi}{4}} \right] \\ B \longrightarrow -\left[\mathbf{R}_o^{\frac{\pi}{12}} \left(\mathbf{R}_o^{\frac{\pi}{6}-\frac{\pi}{2}} (-\mathrm{m}^- \sigma^+) \mathbf{R}_o^{-\frac{\pi}{6}+\frac{\pi}{2}} \right) \mathbf{R}_o^{-\frac{\pi}{12}} \left(\mathrm{m}^- \sigma^+ \right) \left(\mathbf{R}_o^{-\frac{\pi}{4}} (\mathrm{m}^- \sigma^+) \mathbf{R}_o^{\frac{\pi}{4}} \right) + \right. \\ \quad \left. + \left(\mathbf{R}_o^{-\frac{\pi}{4}} (\mathrm{m}^- \sigma^+) \mathbf{R}_o^{\frac{\pi}{4}} \right) \left(\mathrm{m}^- \sigma^+ \right) \mathbf{R}_o^{\frac{\pi}{12}} \left(\mathbf{R}_o^{\frac{\pi}{6}-\frac{\pi}{2}} (-\mathrm{m}^- \sigma^+) \mathbf{R}_o^{-\frac{\pi}{6}+\frac{\pi}{2}} \right) \mathbf{R}_o^{-\frac{\pi}{12}} \right] = \\ \quad = 2 \left[\mathrm{m}^- \, \mathbf{R}_o^{-\frac{\pi}{4}} \left\langle \sigma^+ \mathbf{R}_o^{\frac{\pi}{4}} \, \mathbf{R}_o^{-\frac{\pi}{2}} \sigma^- \, \mathbf{R}_o^{\frac{\pi}{4}} \, \sigma^+ \right\rangle \mathbf{R}_o^{\frac{\pi}{4}} \right] \\ \quad = \left[\mathrm{m}^- \, \mathbf{R}_o^{-\frac{\pi}{4}} \; \underline{\left\langle 2\, \sigma^+ \mathbf{R}_o^{-\frac{\pi}{4}} \sigma^- \, \mathbf{R}_o^{\frac{\pi}{4}} \, \sigma^+ \right\rangle} \; \mathbf{R}_o^{\frac{\pi}{4}} \right] \longrightarrow A \end{cases} \end{cases} \tag{22}$$

Could this be interpreted with this diagram?

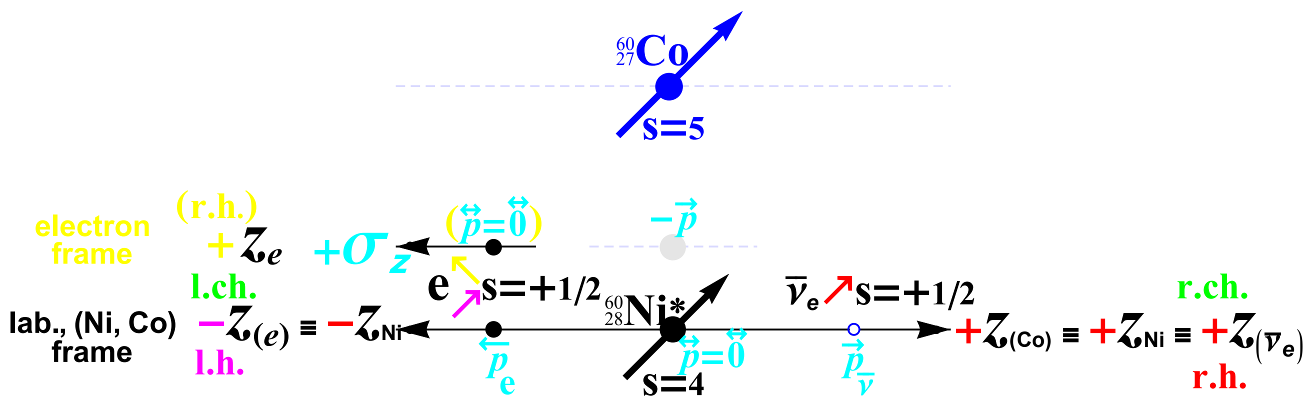

FIG. 2. Schematic of the Madame Wu experiment. Helicity and chirality in different reference frames.

The red, magenta and yellow arrows indicate the up or down spin (a half). Similarly, the big blue arrow in the **Co** (spin five) and the black one in the **Ni** (spin four). They do not represent vectors in a three dimensional space.

ii) Helicity of the neutrino. Goldhaber M.L. et al. (1957).

$$p\ e^- \to n\ \nu_e \qquad \text{or} \qquad \{g\} \cup u\ e^- \longrightarrow \{g\} \cup d\ \nu_e .$$

With our model, a possibility is:

$$g_{m,-6}\ \overset{\downarrow}{u^+}\ \overset{\downarrow}{e^-} \longrightarrow \overset{\downarrow}{\nu_e}\ \overset{\downarrow}{d^-}\ g_{m,-6}\,, \qquad \text{or}$$

$$\mathbf{R}_{om}^{-\frac{\pi}{6}}\ q^{\dagger}_{2,M}\ l^{\dagger}_{1,M} = \nu^{\dagger}_{1,M}\ q^{\dagger}_{1,M}\ \mathbf{R}_{om}^{-\frac{\pi}{6}}\,, \tag{23}$$

which, algebraically, drives to:

$$\begin{cases} 1,\ldots,m-1: & \mathbf{R}_{e_{M-1}}^{\frac{\pi}{3}}\ \mathbf{R}_{e_{M-1}}^{\frac{\pi}{2}} = \mathbf{R}_{e_{M-1}}^{0}\ \mathbf{R}_{e_{M-1}}^{\frac{\pi}{6}}\,, \qquad \left(\frac{\pi}{3}-\frac{\pi}{2}=0-\frac{\pi}{6}\right) \\ m: & \mathbf{R}_o^{-\frac{\pi}{6}}\left(\mathbf{R}_o^{\frac{\pi}{6}}\,\sigma^-\,\mathbf{R}_o^{-\frac{\pi}{6}}\right)\left(\mathbf{R}_o^{\frac{\pi}{4}}\,\sigma^-\,\mathbf{R}_o^{-\frac{\pi}{4}}\right) = \left(\sigma^-\right)\left(\mathbf{R}_o^{\frac{\pi}{12}}\,\sigma^-\,\mathbf{R}_o^{-\frac{\pi}{12}}\right)\mathbf{R}_o^{-\frac{\pi}{6}} \end{cases} . \tag{24}$$

iii) Proton − antiproton collision.

In our treatment **we do not handle the families**. We only use the $'d'$ and $'u'$.

In a very schematic way we wrote: $d\,\overline{d} \to \{\gamma, Z^0\}$; $u\,\overline{u} \to \{\gamma, Z^0\}$; (previously treated in **I)**). We add: $\{\overline{u}\,d\,,\ \overline{d}\,u\}$, with spins $\{+1\,,-1\,,0\}$ $(\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow, \downarrow\uparrow)$; including the case of the pion with its spin equal to zero $(\uparrow\downarrow\,,\ \downarrow\uparrow)$.

In this framework it is simpler to consider a *proton − antiproton collision.* Perkins, section 7.12 [5].

We study: $\overline{u}\,d \to \mathbf{W}^- \to e^-\,\overline{\nu}_e\,,$ or

$$\{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overline{u}\,d \to \boxed{\left[\left(\{\overset{\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \mathbf{W}^-_q\right) \leftrightarrow \widetilde{\mathbf{W}}^-_q\right] \to \mathbf{W}^-_l} \to e^-\,\overline{\nu}_e\,. \tag{25}$$

With our model, a possibility is:

$$\begin{cases} g_{m,-6}\ \overset{\uparrow}{\overline{u}^-}\ \overset{\uparrow}{\underline{\overline{d}^-}}\ g_{m,6} \to \overset{\uparrow}{\overline{\nu}_e}\ \overset{\uparrow}{\underline{e}^-} \\ \mathbf{R}_{om}^{-\frac{\pi}{6}}\ q^{\dagger}_{6,M}\ q^{\dagger}_{-5,M}\ \mathbf{R}_{om}^{\frac{\pi}{6}} = \nu^{\dagger}_{5,M}\ l^{\dagger}_{-5,M} \end{cases} ,$$

we have skipped the writing of $\overset{\uparrow\uparrow}{\mathbf{W}}{}^-$. It is algebraically justified with:

$$\begin{cases} \mathbf{R}_{e_{M-1}}^{\frac{\pi}{3}}\ \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{6}} = \mathbf{R}_{e_{M-1}}^{0}\ \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{2}}\,, \qquad \left(\frac{\pi}{3}-(-\frac{\pi}{6})=0-(-\frac{\pi}{2})\right) \\ \mathbf{R}_o^{-\frac{\pi}{6}}\left(\mathbf{R}_o^{\frac{\pi}{6}}\,\sigma^+\,\mathbf{R}_o^{-\frac{\pi}{6}}\right)\left(\mathbf{R}_o^{-\frac{\pi}{12}}\,\sigma^+\,\mathbf{R}_o^{\frac{\pi}{12}}\right)\mathbf{R}_o^{\frac{\pi}{6}} = \left(\sigma^+\right)\left(\mathbf{R}_o^{-\frac{\pi}{4}}\,\sigma^+\,\mathbf{R}_o^{\frac{\pi}{4}}\right) \end{cases} . \tag{26}$$

Also, for a second form:

$$\begin{cases} g_{m,-6}\ \overset{\uparrow}{\underline{\overline{d}^-}}\ \overset{\uparrow}{\overline{u}^-}\ g_{m,6} \to \overset{\uparrow}{\underline{e}^-}\ \overset{\uparrow}{\overline{\nu}_e} \\ \mathbf{R}_{om}^{-\frac{\pi}{6}}\ q^{\dagger}_{-5,M}\ q^{\dagger}_{6,M}\ \mathbf{R}_{om}^{\frac{\pi}{6}} = l^{\dagger}_{-5,M}\ \nu^{\dagger}_{5,M} \end{cases} ,$$

with:

$$\begin{cases} \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{6}}\ \mathbf{R}_{e_{M-1}}^{\frac{\pi}{3}} = \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{2}}\ \mathbf{R}_{e_{M-1}}^{0}\,, \qquad \left(-\frac{\pi}{6}-\frac{\pi}{3}=-\frac{\pi}{2}-0\right) \\ \mathbf{R}_o^{-\frac{\pi}{6}}\left(\mathbf{R}_o^{-\frac{\pi}{12}}\,\sigma^+\,\mathbf{R}_o^{\frac{\pi}{12}}\right)\left(\mathbf{R}_o^{\frac{\pi}{6}}\,\sigma^+\,\mathbf{R}_o^{-\frac{\pi}{6}}\right)\mathbf{R}_o^{\frac{\pi}{6}} = \left(\mathbf{R}_o^{-\frac{\pi}{4}}\,\sigma^+\,\mathbf{R}_o^{\frac{\pi}{4}}\right)\left(\sigma^+\right) \end{cases} . \tag{27}$$

The assignation of the values of the spin (up and down) to our hypothesized gluons (g´s) requires more research.

In a similar way with $u\,\overline{d}$.

Let us look in detail to (25) (another way for rewriting (26)). We establish a new definition for the $\mathbf{W}$ bosons with:

$$\overset{\uparrow\uparrow}{\widetilde{\mathbf{W}}}{}^{-\dagger}_{\bar{u}\,\underline{d}\,,\mathcal{M}} \equiv g_{m,-6}\;\Big(\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{u}\,\underline{d}\,,\mathcal{M}}\Big)\;g_{m,6} = \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}\,\underline{e}\,,\mathcal{M}}\,. \tag{28}$$

With an extensive use of $(A\otimes b).(C\otimes d) = (A.C)\otimes(b.d)$ we demonstrate these relationships:

$$\left\{\begin{aligned}
&\Big(\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{u}\,\underline{d}\,,\mathcal{M}}\Big) : \ \overset{\uparrow}{\bar{u}}{}^{-}\ \overset{\uparrow}{\underline{d}}{}^{-} \longleftrightarrow q^{\dagger}_{6,\mathcal{M}}\,q^{\dagger}_{-5,\mathcal{M}} = \\
&= \Big[\mathbf{R}^{\frac{\pi}{3}}_{e_{M-1}}\ \Big(\mathbf{R}_o^{\frac{\pi}{6}}\,(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{6}}\Big)_m\Big]\Big]\,\mathbf{R}^{-\frac{\pi}{6}}_{e_{M-1}}\ \Big(\mathbf{R}_o^{-\frac{\pi}{12}}(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{\frac{\pi}{12}}\Big)_m\Big] = \\
&= \mathbf{R}^{\frac{\pi}{2}}_{o_{M-1}}\ \Big(\mathbf{R}_o^{\frac{\pi}{6}}\,(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\,(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{\frac{\pi}{12}}\Big)_m = \\
&= \mathbf{R}^{\frac{\pi}{6}}_{om}\Big[\underline{\mathbf{R}^{\frac{\pi}{2}}_{o_{M-1}}\ \Big((\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\,(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{\frac{\pi}{4}}\Big)_m}\Big]\,\mathbf{R}^{-\frac{\pi}{6}}_{om} = g_{m,6}\,\Big(\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{u}\,\underline{d}\,,\mathcal{M}}\Big)\,g_{m,-6}
\end{aligned}\right. \tag{29}$$

$$\left\{\begin{aligned}
&\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}\,\underline{e}\,,\mathcal{M}} : \ \overset{\uparrow}{\bar{\nu}}_{(e,\mu)}\ \overset{\uparrow}{\underline{e}} \longleftrightarrow \nu^{\dagger}_{5,\mathcal{M}}\ l^{\dagger}_{-5,\mathcal{M}} = \\
&= \Big[\mathbf{R}^{0}_{e_{M-1}}\ \Big((\mathfrak{m}^-\sigma^+)\Big)_m\Big]\Big[\mathbf{R}^{-\frac{\pi}{2}}_{e_{M-1}}\ \Big(\mathbf{R}_o^{-\frac{\pi}{4}}\,(\mathfrak{m}^-\sigma^+)\ \mathbf{R}_o^{\frac{\pi}{4}}\Big)_m\Big] = \\
&= \underline{\mathbf{R}^{\frac{\pi}{2}}_{o_{M-1}}\ \Big((\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\,(\mathfrak{m}^-\sigma^+)\,\mathbf{R}_o^{\frac{\pi}{4}}\Big)_m}
\end{aligned}\right. \tag{30}$$

We now study (25) looking at a possible **geometrical interpretation**. Observing the geometry in the φ variable (formal form similar to the form of the rotations):

$$\left.\begin{aligned}
q^{\dagger}_{6,\mathcal{M}} &= \mathbf{R}^{\frac{\pi}{6}}_{o_M}\ \nu^{\dagger}_{5,\mathcal{M}}\ \mathbf{R}^{-\frac{\pi}{6}}_{o_M}\\
q^{\dagger}_{-5,\mathcal{M}} &= \mathbf{R}^{\frac{\pi}{6}}_{o_M}\ l^{\dagger}_{-5,\mathcal{M}}\ \mathbf{R}^{-\frac{\pi}{6}}_{o_M}
\end{aligned}\right\}
\qquad
\left.\begin{aligned}
l^{\dagger}_{-5,\mathcal{M}} &= \mathbf{R}^{-\frac{\pi}{4}}_{o_M}\ \nu^{\dagger}_{5,\mathcal{M}}\ \mathbf{R}^{\frac{\pi}{4}}_{o_M}\\
q^{\dagger}_{-5,\mathcal{M}} &= \mathbf{R}^{-\frac{\pi}{4}}_{o_M}\ q^{\dagger}_{6,\mathcal{M}}\ \mathbf{R}^{\frac{\pi}{4}}_{o_M}
\end{aligned}\right\} \tag{31}$$

$$\mathbf{R}^{-\frac{\pi}{6}}_{om}\big(q^{\dagger}_{6,\mathcal{M}}\,q^{\dagger}_{-5,\mathcal{M}}\big)\,\mathbf{R}^{\frac{\pi}{6}}_{om} = \boxed{\mathbf{R}^{-\frac{\pi}{6}}_{om}\ \Big(\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{u}\underline{d},\mathcal{M}}\Big)\ \mathbf{R}^{\frac{\pi}{6}}_{om} = \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}\,\underline{e},\mathcal{M}}} = \nu^{\dagger}_{5,\mathcal{M}}\ l^{\dagger}_{-5,\mathcal{M}}. \tag{32}$$

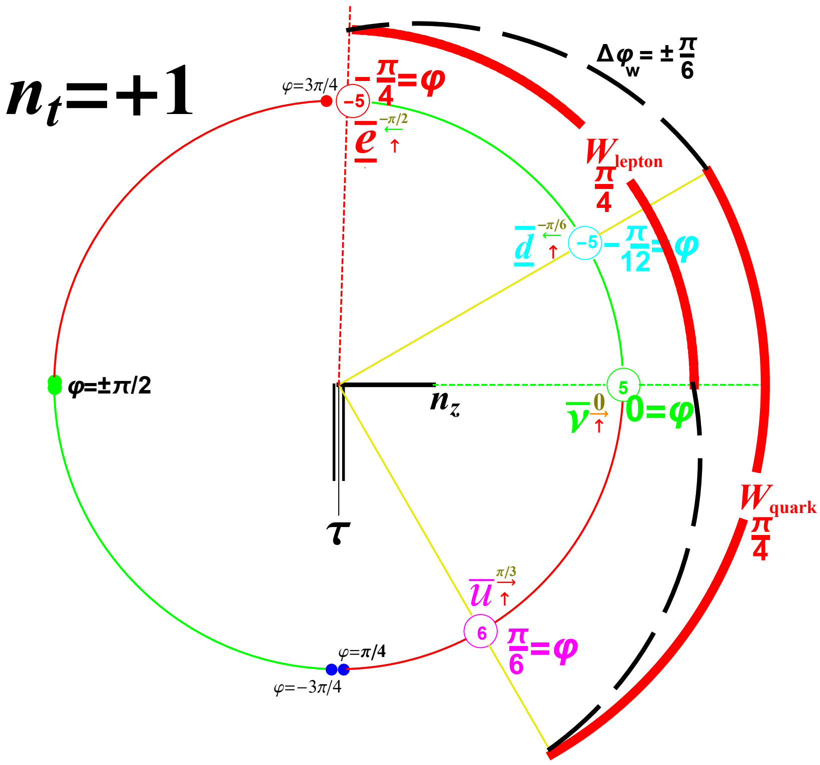

FIG. 3. W bosons: with leptons and with quarks.

Similarly with $(\,\underline{d}\,\bar{u}\,)$:
$$\overset{\uparrow\uparrow}{\widetilde{\mathbf{W}}}{}^{-\dagger}_{\underline{d}\,\bar{u}\,,\mathcal{M}} \equiv g_{m,-6}\;\Big(\overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\underline{d}\,\bar{u}\,,\mathcal{M}}\Big)\;g_{m,6} = \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\underline{e}\,\bar{\nu}\,,\mathcal{M}}\,.$$

iv) The decay of a charged pion (π^-).

The W boson in between. Afterwards the previous muon decay (equations (18)).

We exemplify one of the decays $\pi^-_{(\bar{u},\, d)} \xrightarrow{W} \mu^- \bar{\nu}_\mu$. Compare with **iii)** .

The pions, with their spins zero and without introducing gluons, could have the simple forms:

$$\overset{0}{\pi}{}^- \left\{ \begin{array}{l} \overset{\uparrow\downarrow}{\pi}{}^- \\ \overset{\downarrow\uparrow}{\pi}{}^- \end{array} \right. \longleftrightarrow \{ \bar{u}^\downarrow d^\uparrow,\ d^\uparrow \bar{u}^\downarrow,\ \bar{u}^\uparrow d^\downarrow,\ d^\downarrow \bar{u}^\uparrow \} \longrightarrow \bar{\nu}^\uparrow \mu^\downarrow .$$

It is not possible to obtain with them any algebraic equality with the leptons in the right part.

With $\bar{u}^\downarrow d^\uparrow \overset{\text{our geometry}}{\longrightarrow} [\, \overset{\downarrow}{\underline{u}}{}^- \overset{\uparrow}{\underline{d}}{}^- \,]$: $\quad \mathbf{R}_{om}^{-\frac{\pi}{3}}\ q^{\dagger}_{-2,\mathcal{M}}\ \mathbf{R}_{om}^{\frac{\pi}{3}}\ \underline{\mathbf{R}_{o,\,M-1}^{\frac{\pi}{3}}}\ \mathbf{R}_{om}^{-\frac{\pi}{6}}\ q^{\dagger}_{-5,\mathcal{M}}\ \mathbf{R}_{om}^{\frac{\pi}{6}} = \nu^{\dagger}_{5,\mathcal{M}}\ l^{\dagger}_{1,\mathcal{M}}$. The algebra:

$$\left\{ \begin{array}{ll} \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{3}}\ \underline{\mathbf{R}_{o,\,M-1}^{\frac{\pi}{3}}}\ \mathbf{R}_{e_{M-1}}^{-\frac{\pi}{6}} = \mathbf{R}_{e_{M-1}}^{0}\ \mathbf{R}_{e_{M-1}}^{\frac{\pi}{2}} , & (-\frac{\pi}{3} - \frac{\pi}{3} + \frac{\pi}{6} = 0 - \frac{\pi}{2}) \\ \left[\mathbf{R}_o^{-\frac{\pi}{3}} \left(\mathbf{R}_o^{\frac{\pi}{2} - \frac{\pi}{6}}\, \mathfrak{m}^- \sigma^+\, \mathbf{R}_o^{-\frac{\pi}{2} + \frac{\pi}{6}} \right) \mathbf{R}_o^{\frac{\pi}{6}} \left(\mathbf{R}_o^{-\frac{\pi}{12}}\, \sigma^+\, \mathbf{R}_o^{\frac{\pi}{12}} \right) \mathbf{R}_o^{\frac{\pi}{6}} \right]_m = \left[(\sigma^+) \left(\mathbf{R}_o^{-\frac{\pi}{2} + \frac{\pi}{4}}\, \mathfrak{m}^- \sigma^+\, \mathbf{R}_o^{\frac{\pi}{2} - \frac{\pi}{4}} \right) \right]_m & \end{array} \right. .$$

A possible gluon structure with the $\mathbf{R}_o^{\varphi}$, $\varphi \in \{\pm\frac{\pi}{3}, \pm\frac{\pi}{6}\}$. It is more difficult to interpret the $\underline{\mathbf{R}_{o,\,M-1}^{\frac{\pi}{3}}}$ term; perhaps this in not possible. It already appeared in (12) (with $\varphi_R = \frac{\pi}{12}$).

Let us now consider the following possible structures for the pion $\overset{0}{\pi}{}^-$, in terms of quarks and glouns (assumed):

$$\overset{0}{\pi}{}^- \longleftrightarrow \left\{ \left[\{\overset{\uparrow\uparrow,\downarrow\downarrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{\bar{u}}{}^- \overset{\uparrow}{\underline{d}}{}^- \right],\ \{\overset{\uparrow\uparrow,\downarrow\downarrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{\underline{d}}{}^- \overset{\uparrow}{\bar{u}}{}^-,\ \{\overset{\uparrow\uparrow,\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\downarrow}{d}{}^- \overset{\downarrow}{\underline{u}}{}^-,\ \{\overset{\uparrow\uparrow,\uparrow\uparrow,\downarrow\downarrow}{g}\} \cup \overset{\downarrow}{\underline{u}}{}^- \overset{\downarrow}{d}{}^- \right\} \longrightarrow \bar{\nu}^\uparrow \nu^\downarrow \qquad (33)$$

the spins of the gluons have an important role.

An algebraic justification for the first one $\overset{0}{\pi}{}^- \longleftrightarrow \left[\{\overset{\uparrow\uparrow,\downarrow\downarrow,\downarrow\downarrow}{g}\} \cup \overset{\uparrow}{\bar{u}}{}^- \overset{\uparrow}{\underline{d}}{}^- \right] \longleftrightarrow$

$$\longleftrightarrow \quad \mathbf{R}_{om}^{*}\ q^{\dagger}_{6,\mathcal{M}}\ \mathbf{R}_{om}^{a\frac{\pi}{2}}\ q^{\dagger}_{-5,\mathcal{M}}\ \mathbf{R}_{om}^{**} =$$

$$= \mathbf{R}_{om}^{*} \left(\mathbf{R}_e^{\frac{\pi}{3}} \right)_{M-1} \left(\mathbf{R}_o^{\frac{\pi}{6}} (\mathfrak{m}^- \sigma^+)\, \mathbf{R}_o^{-\frac{\pi}{6}} \right)_m \mathbf{R}_{om}^{a\frac{\pi}{2}} \left(\mathbf{R}_e^{-\frac{\pi}{6}} \right)_{M-1} \left(\mathbf{R}_o^{-\frac{\pi}{12}} (\mathfrak{m}^- \sigma^+)\, \mathbf{R}_o^{\frac{\pi}{12}} \right)_m \mathbf{R}_{om}^{**}$$

$$= \mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(\mathbf{R}_o^{*}\ \mathbf{R}_o^{\frac{\pi}{6} - \epsilon\frac{\pi}{2}} (\mathfrak{m}^+ \sigma^-)\, \mathbf{R}_o^{-\frac{\pi}{6} + \epsilon\frac{\pi}{2}}\ \mathbf{R}_o^{a\frac{\pi}{2}}\ \mathbf{R}_o^{-\frac{\pi}{12}} (\mathfrak{m}^- \sigma^+)\, \mathbf{R}_o^{\frac{\pi}{12}}\ \mathbf{R}_o^{**} \right)_m$$

$$= \mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(\mathbf{R}_o^{* + \frac{\pi}{6} - b\frac{\pi}{2}}\ \sigma^-\ \mathbf{R}_o^{(b+a)\frac{\pi}{2} - \frac{\pi}{4}}\ \sigma^+\ \mathbf{R}_o^{\frac{\pi}{12} + **} \right)_m =$$

we choose two different sets of values for $\{*, **\}$, and afterwards for the combination (ab)

$$= \left\{ \begin{array}{l} \mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(\mathbf{R}_o^{\frac{\pi}{12} + b\frac{\pi}{2} + \frac{\pi}{6} - b\frac{\pi}{2}}\ \sigma^-\ \mathbf{R}_o^{(b+a)\frac{\pi}{2} - \frac{\pi}{4}} \sigma^+ \mathbf{R}_o^{\frac{\pi}{12} - \frac{\pi}{12}} \right)_m = \\ \qquad = \underline{\mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(-(ab)\, \mathbf{R}_o^{\frac{\pi}{4}}\ \sigma^-\ \mathbf{R}_o^{-\frac{\pi}{4}}\ \sigma^+ \right)_m} = (ab)\ \overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{l\bar{\nu},\mathcal{M}} \qquad \left(\leftarrow \overset{\downarrow\uparrow}{\tilde{\mathbf{W}}}{}^{-\dagger}_{\overset{(\uparrow)}{\bar{u}}\,\overset{(\uparrow)}{\underline{d}}{}^-} \ \overset{\uparrow\uparrow,\downarrow\downarrow,\downarrow\downarrow}{\underset{g}{\leftsquigarrow}}\ \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\overset{\uparrow}{\bar{u}}\,\overset{\uparrow}{\underline{d}}{}^-} \right) \\ \text{or also} \\ \mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(\mathbf{R}_o^{-\frac{\pi}{6} + b\frac{\pi}{2} + \frac{\pi}{6} - b\frac{\pi}{2}}\ \sigma^-\ \mathbf{R}_o^{(b+a)\frac{\pi}{2} - \frac{\pi}{4}} \sigma^+ \mathbf{R}_o^{\frac{\pi}{12} + \frac{\pi}{6}} \right)_m = \\ \qquad = \underline{\underline{\mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(-(ab)\, \sigma^-\ \mathbf{R}_o^{-\frac{\pi}{4}} \sigma^+ \mathbf{R}_o^{\frac{\pi}{4}} \right)_m}} = (ab)\ \overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\nu \underline{l},\mathcal{M}} \qquad \left(\leftarrow \overset{\downarrow\uparrow}{\tilde{\mathbf{W}}}{}^{-\dagger}_{\overset{(\uparrow)}{\bar{u}}\,\overset{(\uparrow)}{\underline{d}}{}^-} \ \overset{\uparrow\uparrow,\downarrow\downarrow,\downarrow\downarrow}{\underset{g}{\leftsquigarrow}}\ \overset{\uparrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\overset{\uparrow}{\bar{u}}\,\overset{\uparrow}{\underline{d}}{}^-} \right) \end{array} \right. , \qquad (34)$$

where we have inserted $\{a, b\} \in \{-1, +1\}$ $((ab) \in \{-1, +1\})$. See **iii)**. And in a similar form the other ones in (33).

And the lepton production, with:

$$\overset{0}{\pi}{}^{-} \xrightarrow{W} \begin{cases} \overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{l\bar{\nu},\,\mathcal{M}} = \underline{\mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(-\,\mathbf{R}_{o}^{\frac{\pi}{4}}\,\sigma^{-}\,\mathbf{R}_{o}^{-\frac{\pi}{4}}\,\sigma^{+} \right)_{m}} = l^{\dagger}_{1,\mathcal{M}}\,\nu^{\dagger}_{5,\mathcal{M}} \longleftrightarrow \overset{\downarrow}{\mu}{}^{-}\,\overset{\uparrow}{\bar{\nu}}_{\mu} \\ \overset{\uparrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{\bar{\nu}l,\,\mathcal{M}} = \mathbf{R}_{o_{M-1}}^{-\frac{\pi}{2}} \left(-\,\sigma^{+}\,\mathbf{R}_{o}^{\frac{\pi}{4}}\,\sigma^{-}\,\mathbf{R}_{o}^{-\frac{\pi}{4}} \right)_{m} = \nu^{\dagger}_{5,\mathcal{M}}\,l^{\dagger}_{1,\mathcal{M}} \longleftrightarrow \overset{\uparrow}{\bar{\nu}}_{\mu}\,\overset{\downarrow}{\mu}{}^{-} \\ (\not{L}\,)\ \ \text{Lepton number violating modes}\ (< 10^{-2}\%)\ (\textbf{without antineutrinos}): \\ \overset{\downarrow\uparrow}{\mathbf{W}}{}^{-\dagger}_{\nu\underline{l},\,\mathcal{M}} = \underline{\underline{\mathbf{R}_{o_{M-1}}^{\frac{\pi}{2}} \left(-\,\sigma^{-}\,\mathbf{R}_{o}^{-\frac{\pi}{4}}\,\sigma^{+}\,\mathbf{R}_{o}^{\frac{\pi}{4}} \right)_{m}}} = \nu^{\dagger}_{1,\mathcal{M}}\,l^{\dagger}_{-5,\mathcal{M}} \longleftrightarrow \overset{\downarrow}{\nu}_{\mu}\,\underline{\overset{\uparrow}{\bar{\mu}}}{}^{-} \\ \overset{\uparrow\downarrow}{\mathbf{W}}{}^{-\dagger}_{\underline{l}\nu,\,\mathcal{M}} = \mathbf{R}_{o_{M-1}}^{-\frac{\pi}{2}} \left(-\,\mathbf{R}_{o}^{-\frac{\pi}{4}}\,\sigma^{+}\,\mathbf{R}_{o}^{\frac{\pi}{4}}\,\sigma^{-} \right)_{m} = l^{\dagger}_{-5,\mathcal{M}}\,\nu^{\dagger}_{1,\mathcal{M}} \longleftrightarrow \underline{\overset{\uparrow}{\bar{\mu}}}{}^{-}\,\overset{\downarrow}{\nu}_{\mu} \end{cases} \quad . \qquad (35)$$

The first and the third are equal to the first and the second, respectively, in (34) with $(ab) = 1$. The other two (the second and the fourth) with another of the terms in (33).

Afterwards, for the muon decay see **II)**.

The negative charged lepton (a muon) and the antinueutrino have the same right helicity in the rest frame of the pion, although the weak interaction permits only the left chirality part of the negatively charged leptons. Main point in the interpretation of the relation of the helicity and the chirality in the Dirac equation (Greiner page 16) and with the experiments themselves. See, Perkins, sections 3.3.1, 7.10 [5]; Griffiths, section 9.4 [8]; and Greiner, pages 15, 16 and 214 [9]. We present a research on this point in our Study **IV** [7].

Could this be interpreted with this diagram?

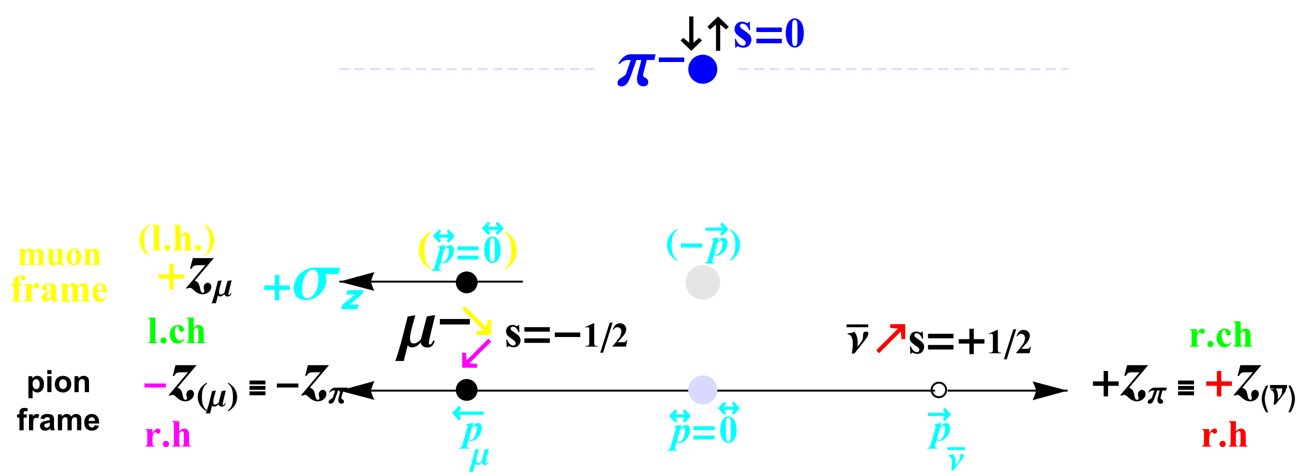

FIG. 4. Helicity and chirality in different reference frames.

The red, magenta and yellow arrows indicate the spin up or down. They do not represent vectors in the three dimensional space.

The elements for the study of the decay of the $\overset{0}{\pi}{}^{0}$ (or $\overset{\uparrow\downarrow}{\pi}{}^{0}$) already in **I**), (relate with the equations in (8)).

II. APPENDIX A: C-ROTATION AND C-REFLECTION MATRICES – VECTORS.

A basis for the linear space of $2x2$ matrices, the canonical basis:

$$\left\{ \hat{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \check{\sigma} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

We define:

$$\mathbf{R}_o^{\varphi}(\phi) = \begin{pmatrix} \cos\varphi & -\sin\varphi\, \mathrm{e}^{-\mathrm{i}\phi} \\ \sin\varphi\, \mathrm{e}^{\mathrm{i}\phi} & \cos\varphi \end{pmatrix}, \qquad \mathbf{R}_e^{\varphi}(\phi) = \begin{pmatrix} \cos\varphi & \sin\varphi\, \mathrm{e}^{-\mathrm{i}\phi} \\ \sin\varphi\, \mathrm{e}^{\mathrm{i}\phi} & -\cos\varphi \end{pmatrix}.$$

With these matrices-vectors the Pauli basis is:

$$\left\{ \mathbb{1} = \mathbf{R}_o^{0}(\phi), \quad \sigma^z = \mathbf{R}_e^{0}(\phi), \quad \sigma^x = \mathbf{R}_e^{\frac{\pi}{2}}(0), \quad \sigma^y = \mathbf{R}_e^{\frac{\pi}{2}}(\frac{\pi}{2}) \right\}.$$

Properties:

$$\begin{cases} \mathbf{R}_o^{\varphi\dagger} = \mathbf{R}_o^{\varphi -1} = \mathbf{R}_o^{-\varphi}, & \mathbf{R}_e^{\varphi\dagger} = \mathbf{R}_e^{\varphi -1} = \mathbf{R}_e^{\varphi} \\ \mathbf{R}_e^{\varphi} = \mathbf{R}_o^{\varphi}\sigma^z = \sigma^z\mathbf{R}_o^{-\varphi}, & \mathbf{R}_e^{\varphi}\,\mathbf{R}_o^{\varphi} = \mathbf{R}_o^{-\varphi}\,\mathbf{R}_e^{\varphi} = \sigma^z \\ \sigma^z\,\mathbf{R}_o^{\varphi}\,\sigma^z = \mathbf{R}_o^{-\varphi}, & \sigma^z\,\mathbf{R}_e^{\varphi}\,\sigma^z = \mathbf{R}_e^{-\varphi} \\ \det[\,\mathbf{R}_o^{\varphi}\,] = 1, & \det[\,\mathbf{R}_e^{\varphi}\,] = -1 \end{cases}.$$

$$\begin{cases} \mathbf{R}_o^{\varphi_1}\,\mathbf{R}_o^{\varphi_2} = \mathbf{R}_o^{\varphi_1+\varphi_2} \\ \mathbf{R}_e^{\varphi_1}\,\mathbf{R}_e^{\varphi_2} = \mathbf{R}_o^{\varphi_1-\varphi_2} \\ \mathbf{R}_o^{\varphi_1}\,\mathbf{R}_e^{\varphi_2} = \mathbf{R}_e^{\varphi_1+\varphi_2} \\ \mathbf{R}_e^{\varphi_1}\,\mathbf{R}_o^{\varphi_2} = \mathbf{R}_e^{\varphi_1-\varphi_2} \end{cases}, \qquad \begin{cases} \mathbf{R}_o^{\varphi 2} = \mathbf{R}_e^{\varphi}\,\mathbf{R}_e^{-\varphi} = \mathbf{R}_o^{2\varphi} \\ \mathbf{R}_e^{\varphi 2} = \mathbf{R}_o^{\varphi}\,\mathbf{R}_o^{-\varphi} = \mathbf{R}_o^{0} = \mathbb{1} \\ \mathbf{R}_o^{\varphi}\,\mathbf{R}_e^{\varphi} = \mathbf{R}_e^{\varphi}\,\mathbf{R}_o^{-\varphi} = \mathbf{R}_e^{2\varphi} \\ \mathbf{R}_e^{\varphi}\,\mathbf{R}_o^{\varphi} = \mathbf{R}_o^{\varphi}\,\mathbf{R}_e^{-\varphi} = \mathbf{R}_e^{0} = \sigma^z \end{cases}.$$

It is clear that:

$$\mathbf{R}_o^{\frac{\pi}{2}} \equiv \mathbf{R}_o^{\varphi=\frac{\pi}{2}} = \begin{pmatrix} 0 & -\mathrm{e}^{-\mathrm{i}\phi} \\ \mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix} = -\mathbf{R}_o^{-\frac{\pi}{2}}, \qquad \mathbf{R}_e^{\frac{\pi}{2}} \equiv \mathbf{R}_e^{\varphi=\frac{\pi}{2}} = \begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\phi} \\ \mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix} = -\mathbf{R}_e^{-\frac{\pi}{2}}, \quad \text{and}$$

$$\mathbf{R}_o^{0} = -\mathbf{R}_o^{\pm\pi} = -\{\mathbf{R}_o^{\pm\frac{\pi}{2}}\}^2 = \mathbb{1}, \quad \{\mathbf{R}_e^{\pm\frac{\pi}{2}}\}^2 = \mathbb{1}, \quad \mathbf{R}_e^{0} = -\mathbf{R}_e^{\pm\pi} = \sigma^z, \quad \mathbf{R}_e^{\frac{\pi}{2}}\,\mathbf{R}_o^{\frac{\pi}{2}} = -\mathbf{R}_o^{\frac{\pi}{2}}\,\mathbf{R}_e^{\frac{\pi}{2}} = \sigma^z,$$

so that:

$$\mathbf{R}_o^{\varphi}(\phi) \equiv \cos\varphi\,\mathbf{R}_o^{0} + \sin\varphi\,\mathbf{R}_o^{\frac{\pi}{2}} = \cos\varphi\,\mathbb{1} + \sin\varphi\,\mathbf{R}_o^{\frac{\pi}{2}} = \mathrm{e}^{\varphi\mathbf{R}_o^{\frac{\pi}{2}}} = \cos\varphi\,\mathbb{1} + \sin\varphi\,(\sin\phi\,\mathrm{i}\sigma^x - cos\phi\,\mathrm{i}\sigma^y) \in \mathbb{R}^{[1],(i)[2]}$$

$$\mathbf{R}_e^{\varphi}(\phi) \equiv \cos\varphi\,\mathbf{R}_e^{0} + \sin\varphi\,\mathbf{R}_e^{\frac{\pi}{2}} = \cos\varphi\,\sigma^z + \sin\varphi\,\mathbf{R}_e^{\frac{\pi}{2}} = \mathbf{R}_o^{\varphi}(\phi)\,\sigma^z = \cos\varphi\,\sigma^z + \sin\varphi\,(cos\phi\,\sigma^x + \sin\phi\,\sigma^y) \in \mathbb{R}^{[3]}.$$

Acting over diagonal and antidiagonal matrices:

$$\begin{cases} \mathbf{R}_o^{\frac{\pi}{2}}\,\mathbf{D}\,\mathbf{R}_o^{-\frac{\pi}{2}} = \mathbf{R}_o^{-\frac{\pi}{2}}\,\mathbf{D}\,\mathbf{R}_o^{\frac{\pi}{2}} = \begin{pmatrix} 0 & -\mathrm{e}^{-\mathrm{i}\phi} \\ \mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\phi} \\ -\mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix} = \begin{pmatrix} b & 0 \\ 0 & a \end{pmatrix}; \\ \mathbf{R}_o^{\frac{\pi}{2}}\,\mathbf{A}\,\mathbf{R}_o^{-\frac{\pi}{2}} = -\mathbf{R}_e^{\frac{\pi}{2}}\,\mathbf{A}\,\mathbf{R}_e^{\frac{\pi}{2}} = \begin{pmatrix} 0 & -\mathrm{e}^{-\mathrm{i}\phi} \\ \mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix}\begin{pmatrix} 0 & \mathfrak{m}^- c \\ \mathfrak{m}^+ d & 0 \end{pmatrix}\begin{pmatrix} 0 & \mathrm{e}^{-\mathrm{i}\phi} \\ -\mathrm{e}^{\mathrm{i}\phi} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \mathfrak{m}^- d \\ \mathfrak{m}^+ c & 0 \end{pmatrix} \end{cases},$$

in particular

$$\begin{cases} \mathbf{R}_o^{\pm\frac{\pi}{2}}\,\sigma^z\,\mathbf{R}_o^{\mp\frac{\pi}{2}} = \mathbf{R}_e^{\pm\frac{\pi}{2}}\,\sigma^z\,\mathbf{R}_e^{\pm\frac{\pi}{2}} = \mathbf{R}_e^{\pm\pi} = -\sigma^z \\ \mathbf{R}_o^{\pm\frac{\pi}{2}}\,\mathfrak{m}^-\sigma^+\,\mathbf{R}_o^{\mp\frac{\pi}{2}} = -\mathbf{R}_e^{\pm\frac{\pi}{2}}\,\mathfrak{m}^-\sigma^+\,\mathbf{R}_e^{\pm\frac{\pi}{2}} = -\mathrm{e}^{\,\mathrm{i}2\phi}\,\mathfrak{m}^-\sigma^- = \mathfrak{m}^+\sigma^- \\ \mathbf{R}_o^{\pm\frac{\pi}{2}}\,\mathfrak{m}^+\sigma^-\,\mathbf{R}_o^{\mp\frac{\pi}{2}} = -\mathbf{R}_e^{\pm\frac{\pi}{2}}\,\mathfrak{m}^+\sigma^-\,\mathbf{R}_e^{\pm\frac{\pi}{2}} = -\mathrm{e}^{-\mathrm{i}2\phi}\,\mathfrak{m}^+\sigma^+ = \mathfrak{m}^-\sigma^+ \end{cases}.$$

σ^z diagonalizes $\mathbf{R}_e^{2\varphi} = \mathfrak{n} = n_z\sigma^z + (n_x\sigma^x + n_y\sigma^y) = n_z\sigma^z + \tau\,\mathbf{R}_e^{\frac{\pi}{2}}$:

$$\mathbf{R}_o^{\varphi}\,\sigma^z\,\mathbf{R}_o^{-\varphi} = \mathbf{R}_e^{\varphi}\,\sigma^z\,\mathbf{R}_e^{\varphi} = \mathbf{R}_o^{\varphi}\,\mathbf{R}_e^{\varphi} = \mathbf{R}_e^{\varphi}\,\mathbf{R}_o^{-\varphi} = \mathbf{R}_e^{2\varphi} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi\,\mathrm{e}^{-\mathrm{i}\phi} \\ \sin 2\varphi\,\mathrm{e}^{\mathrm{i}\phi} & -\cos 2\varphi \end{pmatrix} = \mathfrak{n}(\varphi).$$

Symmetries in φ, in λ, in τ: $\mathfrak{n}(-\varphi) = \mathbf{R}_e^{-2\varphi} = \mathbf{R}_o^{-\varphi}\,\sigma^z\,\mathbf{R}_o^{\varphi} = n_z\sigma^z - (n_x\sigma^x + n_y\sigma^y) = n_z\sigma^z - \tau\,\mathbf{R}_e^{\frac{\pi}{2}} = n_z\sigma^z + \tau\,\mathbf{R}_e^{-\frac{\pi}{2}}$.

APPENDIX B: GEOMETRY VERSUS ALGEBRA. GEOMETRY AND PHYSICS.

There are specially insightful paragraphs concerning the rotations in the Volume I of *The theory of the top* (1897) of Klein and Sommerfeld [10], in two different senses:

(a) it is different *a rotation*, as a relation between two geometrical positions, and *a spinning*, as an action of rotation [11]. Klein and Sommerfeld pointed out the significance of defining a rotation as the composition of two halves rotations, in such a way that the angle is measured "modulo 4π", instead of modulo 2π. "$SU(2)$ glimpsed" (see the note (34) in the page 227 [10]). A composition, one *after* the other? A *time*? Look that this means to assign a sense of rotation in a spinning once imposed a *semi-axis of rotation*. But,

(b) *(angle of rotation) x (semi-axis of rotation)* $\equiv$ *(− angle of rotation) x (− semi-axis of rotation).*
Therefore, still an indeterminacy: the *semi-axis of rotation*.

An example: $$\left(e^{\,i\frac{\pi}{2}}\right)\left(e^{\,i\frac{\pi}{2}}\right) = (+i)\,(+i) \overset{\text{algebra.}}{=} -1 \overset{\text{algebra.}}{=} (-i)\,(-i) = \left(e^{-i\frac{\pi}{2}}\right)\left(e^{-i\frac{\pi}{2}}\right),$$

the expressions in the left side are different to the expressions in the right side; geometry: they represent different rotations in $\mathbb{C}$.

Or in a more general way: $$\left(e^{\,i\frac{1}{2}\alpha}\right)\left(e^{\,i\frac{1}{2}\alpha}\right) = \left(e^{\,i\alpha}\right) \overset{\text{algebra.}}{=} \left(e^{i(\alpha-2\pi)}\right) = \left(e^{\,i\,(\frac{1}{2}\alpha-\pi)}\right)\left(e^{\,i\,(\frac{1}{2}\alpha-\pi)}\right).$$

In $\mathbb{R}^3$ with:
$$\mathbb{n}(\lambda\varphi) = \mathbf{R}_e^{2\lambda\varphi} = \mathbf{R}_O^{\lambda\varphi}\,\sigma^z\,\mathbf{R}_O^{-\lambda\varphi} = n_z\sigma^z + \lambda\,(n_x\sigma^x + n_y\sigma^y) = n_z\sigma^z + \lambda\,\tau\,\mathbf{R}_e^{\frac{\pi}{2}} = n_z\sigma^z + \tau\,\mathbf{R}_e^{\lambda\frac{\pi}{2}}.$$

Once privileged a $+n_z$ (σ^z), we can define a *a right* and *a left*, the way we choose σ^x and σ^y.

And a rotation in dimension three:
$$\mathbb{w}' \equiv \mathcal{R}(2\,\alpha\,\mathbb{n})\,[\,\mathbb{w}\,] = e^{i\,\alpha\,\mathbb{n}}\;\mathbb{w}\;e^{-i\,\alpha\,\mathbb{n}} = e^{\,i\,\mathbf{R}_O^{\varphi}\,\alpha\,\sigma^z\,\mathbf{R}_O^{-\varphi}}\;\mathbb{w}\;e^{-i\,\mathbf{R}_O^{\varphi}\,\alpha\,\sigma^z\,\mathbf{R}_O^{-\varphi}} =$$
$$= \mathbf{R}_O^{\varphi}\;e^{\,i\,\alpha\,\sigma^z}\,\mathbf{R}_O^{-\varphi}\;\mathbb{w}\;\mathbf{R}_O^{\varphi}\,e^{-i\,\alpha\,\sigma^z}\;\mathbf{R}_O^{-\varphi} = \mathbf{R}_O^{\varphi}\;(\,\widetilde{\mathbb{w}}\,)\;\mathbf{R}_O^{-\varphi}\,.$$

Our framework with $\mathbb{C}^4$. The determinants of $\mathbb{w}$ and of $\widetilde{\mathbb{w}}$ could be equal or different to zero. The $\varphi > 0$ or $\varphi < 0$ (in (2) and (3)) related with the handedness of the rotation with respect to the previously mentioned privileged semi-direction.

Several important algebraic equalities:

(a) in (4) : $\mathbf{R}_O^{\epsilon\frac{\pi}{2}}\,(\,\mathbb{m}^{\mp}\sigma^{\pm})\,\mathbf{R}_O^{-\epsilon\frac{\pi}{2}} = \mathbb{m}^{\pm}\sigma^{\mp}$, applied in order to obtain (5) and in (13),

(b) in (5) : $l^{\dagger}_{5,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{-7,\mathcal{M}}$, $l^{\dagger}_{-5,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{7,\mathcal{M}}$, $l^{\dagger}_{1,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{-3,\mathcal{M}}$, $l^{\dagger}_{-1,\mathcal{M}} \overset{\text{algebra}}{=} l^{\dagger}_{3,\mathcal{M}}$,

but: $l^{\dagger}_{5,\mathcal{M}}\uparrow \overset{\text{geom.-phys.}}{\neq} \downarrow l^{\dagger}_{-7,\mathcal{M}}$, $l^{\dagger}_{-5,\mathcal{M}}\uparrow \overset{\text{geom.-phys.}}{\neq} \downarrow l^{\dagger}_{7,\mathcal{M}}$, $l^{\dagger}_{1,\mathcal{M}}\downarrow \overset{\text{geom.-phys.}}{\neq} \uparrow l^{\dagger}_{-3,\mathcal{M}}$, $l^{\dagger}_{-1,\mathcal{M}}\downarrow \overset{\text{geom.-phys.}}{\neq} \uparrow l^{\dagger}_{3,\mathcal{M}}$.

We have justified these definitions in the studies II and IV) [1] [7], with the spins and the chiralities (the physical properties) and the development of tiny variations in the magnetic moment (anomalous magnetic moment) producing tiny variations in the coordinates.

(c) in (10) : $\mathbf{R}_O^{\epsilon\frac{\pi}{2}}\,(\left\{\begin{matrix}\hat{\sigma}\\ \check{\sigma}\end{matrix}\right\})\,\mathbf{R}_O^{-\epsilon\frac{\pi}{2}} = \left\{\begin{matrix}\check{\sigma}\\ \hat{\sigma}\end{matrix}\right\}$, without implications concerning the spin (a physical property),

(d) $\left(-\mathbb{n}(2\varphi_R)\right)_{M-1} = \left(-\mathbb{n}(2\varphi_R)\right)_1\otimes\cdots\otimes\left(-\mathbb{n}(2\varphi_R)\right)_{m-1} \overset{\text{algebra}}{=} (-1)^{m-1}\left(\mathbb{n}(2\varphi_R)\right)_1\otimes\cdots\otimes\left(\mathbb{n}(2\varphi_R)\right)_{m-1} = (-1)^{m-1}\left(\mathbb{n}(2\varphi_R)\right)_{M-1}$,

with four local vectors: $\left\{\{\mathbb{n}(2\varphi_R),\,-\mathbb{n}(2\varphi_R)\},\,\{\mathbb{n}(-2\varphi_R),\,-\mathbb{n}(-2\varphi_R)\}\right\}$ or $\left\{\{(\mathbf{R}_e^{2\varphi_R}),\,(-\mathbf{R}_e^{2\varphi_R})\},\,\{(\mathbf{R}_e^{-2\varphi_R}),\,(-\mathbf{R}_e^{-2\varphi_R})\}\right\}$, two different opposites, and only one opposite (with two opposites) for $2\varphi_R = 0$ and for $2\varphi_R = \frac{\pi}{2}$.

If we assume for $m-1$ an even value, it is: $\left(-\,\mathbb{n}(\lambda\varphi_R)\right)_{M-1} = \left(\,\mathbb{n}(\lambda\varphi_R)\right)_{M-1}$ but geometrically, beneath, we represent the vectors for opposite 3-dimensional rotations, also opposite vectors-spin (physics), at different locations in a chain.

What distinguishes a *marked* $-\mathbb{n}$ of a *marked* $\mathbb{n}$? The physics in a local time-space.

If $m-1$ is odd, it appears a minus factor and the distinction of the vectors-spin remains. But, we also have to consider the sign of the m term for the algebraic distinction of the creation operators. On the other hand, with m odd we still distinguish algebraically (with a minus sign) the creation operators: $\mathbf{U}^{\dagger}_{\lambda 1,\mathcal{M}}(\varphi_R) = -\mathbf{U}^{\dagger}_{\lambda 7,\mathcal{M}}(\varphi_R)$ and $\mathbf{U}^{\dagger}_{\lambda 5,\mathcal{M}}(\varphi_R) = -\mathbf{U}^{\dagger}_{\lambda 3,\mathcal{M}}(\varphi_R)$.

Algebra vs (and) geometry and physics:

Algebra is a very powerful instrument; but, at least, as presented here, it does not fulfill the geometrical and physical requirements.

With our Jordan and Wigner type construction, we have defined a chain with:

1) $'m'$ locations (displacements of each other), a non-local time-space character,

2) each location with a $\mathbb{C}^4$ space (a local character) and a generalization of the rotations of the real three dimensional space, in 'a la Rodrigues and Hamilton' form.

Let us consider two aspects:

- at each location in the chain. This is the proper place for our above mentioned paragraphs of Klein and Sommerfeld. Is it possible to fix absolutely a direction in a local time space? Our suggestion is:

there is a correspondence between a local geometry and a lepton or a quark (at each k). **Leptons and quarks are geometry**,

- with the displaced locations in the chain (the 1 to m). We do not have an answer. This involves the global structure of the time-space and therefore of the way we discern it, including special relativity with the Minkowski metric.

Perhaps, in a first step, we need to define a *geometrical direct product*, in order to account for the specific rotations at each displaced position ($1 \leq k \leq m$). This means that we can not use the algebraic property represented by $\left(-\mathbb{m}\right)_{M-1} = (-1)^{m-1}\left(\mathbb{m}\right)_{M-1}$. This is so in accordance with the previous fixing of absolute local directions.

A view for a possible model and a schema of research. We suggest:

matter is a local (each k) dynamic discretization $\left(\left\{ \left[(n_t, n_z, \ \tau) \leftrightarrow (\varphi_R) \ , \ (\text{also } \ \phi_l) \right] \cup \alpha_t = \alpha = \frac{\pi}{4} \right\}^M_{\lambda,1357} \right)$

of a continuous time space $\left(\{ t \, , \, z \, , \, x \, , \, y \} \in \mathbb{C}^4 \cup \{ \alpha_{\text{rotation}} \, , \, \rho_{\text{boost}} \} \right)$.

The interactions 'adjust' the 'geometry' of every individual lepton or quark, which involve its type and its momentum. Do the leptons and quarks, which are 'geometry', **'absorb and emit geometry'?** Has this to do with the concept of field?

We have tried to clarify part of these statements in this set of *studies* (Appendix C).

APPENDIX C: PROGRAM OF THE *STUDIES* CONTAINING THIS RESEARCH.

On the fermionization of the XYZ spin Heisenberg chain (algebra). (5*pages*).
(2022) https://hdl.handle.net/20.500.14352/71645 *Study -2)*

The JordanWigner transformations and the fermionization of the XYZ spin Heisenberg chain. (14*pages*).
Algebra, geometry and physics? (2022) https://hdl.handle.net/20.500.14352/71970 *Study -1)*

A tentative model of creation and annihilation operators for neutrinos. (7*pages*).
(2021) https://hdl.handle.net/20.500.14352/8066 *Study 0)*

Geometry of the time and the space. (Pending of a final wording). *Study I)*

Expression of the 3- and 4-dimensional vectors in total polar exponential form. (17*pages*).
(2021) https://hdl.handle.net/20.500.14352/8183 *Study I,1)*

Vectors. Dimensions 4 and 8. (31*pages*).
(2023) https://hdl.handle.net/20.500.14352/72871 *Study I,2)*

Geometry of the symmetries in dimension 4=(1+[1)+“2”], and general Time-Space-Spin vectors. (21*pages*, 6*fig*.).
(2023) https://hdl.handle.net/20.500.14352/72872 *Study I.3)*

Geometry and Physics of the Elementary Fermions. (On pride of Jordan Wigner Pauli Weyl Dirac).**1**. (33*pages*, 15*figs*.).
(2021) https://hdl.handle.net/20.500.14352/4587 *Study II)*

Geometry and Physics of the Fermions. **2.** (Interactions). Towards a boson structure $\{ \gamma \, , \, Z^0 \, , \, W^{\pm} \, , g \}$. (17 pages, 4 fig.).
THIS STUDY (2024) https://hdl.handle.net/20.500.14352/**** *Study II.1)*

Geometry and Physics of the Fermions. **2.** (Pending of a final wording). *Study II.2)*

Axial vector magnetic charge and magnetic moment. Maxwell’s equations and Lorentz force law. (23*pages*, 4*fig*.).
(2021) https://hdl.handle.net/20.500.14352/4586 *Study III)*

Leptons: charged and uncharged (neutrinos). Inside a proposal for a geometrical model. (27*pages*, 4*fig*.).
Some considerations. (2024) https://hdl.handle.net/20.500.14352/ **** Study IV)

Addenda. (Pending of a final wording). *Study II.3)*

[1] E. Olmedilla. *Geometry and Physics of the Elementary Fermions. (On pride of Jordan Wigner Pauli Weyl Dirac).* **1**. (2021) https://hdl.handle.net/20.500.14352/4587 .

[2] M. Thomson. *Modern Particle Physics.* Cambridge University Press (2013). (Section 15.3).

[3] L. A. Takhtadzhan, L. D. Faddeev. (1979) *The quantum method of the inverse problem and the Heisenberg XYZ model.* Russian Math. Surveys 34, 11.

[4] W. Pauli. *Pauli lectures in Physics. Volume 6. Selected Topics in Field Quantization.* Edited by C.P. Enz. The MIT press. (1977).

[5] D.H. Perkins. *Introduction to High Energy Physics, 4th ed.* Cambridge University Press (2000). (Section 15.3). (Chapters 5, 6 and 8).

[6] M. E. Peskin and D. V. Schroeder. *An Introduction to Quantum Field Theory.* Perseus, Reading MA. (1995). (Section 20.2, page 702).

[7] E. Olmedilla. *Leptons: charged and uncharged (neutrinos). Inside a proposal for a geometrical model. Some considerations).* (2024) https://hdl.handle.net/20.500.14352/ ****

[8] D. Griffiths. *Introduction to Elementary Particles.* 2nd rev. ed. . Wiley-Vch (2008).

[9] W. Greiner . B. Müller. *Gauge Theory of Weak Interactions.* Springer (2000).

[10] F. Klein, A. Sommerfeld. *The theory of the Top, Vol I* (R.J. Nagem, G. Sandri, translators.) Birkhäuser, Boston. (2008). (Page 36). Original: *Theorie des Kreisels, (1897 - 1910).*

[11] S. L. Altmann. *Rotations, Quaternions and Double Groups.* Dover Publications, Dover, New York. (2005). (Page 23).