

Creffield and Sols Reply: Benenti *et al.* [1] assert that in our Letter [2] we claimed that the ratchet current we observed for time-symmetric driving would persist indefinitely. Their assertion is false. In our Letter we clearly indicated that, in general, the ratchet currents would be transient, and indeed wrote that we estimated them only to be “stable over time scales . . . of the order of 50 driving periods.” Unfortunately, Benenti *et al.* appear not to have read our paper with sufficient care to have noted our discussion of this point, since we did not claim, or even imply, that the ratchet currents would be of infinite duration.

To arrive at our estimate for the stability of the current, we used a technique developed in Ref. [3] to estimate the Ehrenfest time of the system. In our study we considered a completely coherent time evolution, and accordingly, the current is given by a *coherent* sum

$$I(t) = \sum_{m,n} c_n^* c_m e^{it(\epsilon_n - \epsilon_m)} \int_0^{2\pi} dx \langle \phi_n(t) | p_x | \phi_m(t) \rangle, \quad (1)$$

where c_n are expansion coefficients in the Floquet basis, ϵ_n are the quasienergies, $|\phi_m(t)\rangle$ are the Floquet states, and p_x is the standard momentum operator. It is important to note the off-diagonal interference terms $\exp[it(\epsilon_n - \epsilon_m)]$. If the system were strongly chaotic, level repulsion would imply that the quasienergy separations are generally large, and so these interferences would rapidly average to zero. This yields the *approximate* formula given in Eq. (1) of Ref. [1], in which solely the diagonal terms of the current are retained, collapsing the coherent sum to an incoherent one. This strong chaoticity would correspond to a short Ehrenfest time, and so our analysis would similarly predict a short time scale for the stability of the ratchet current.

When the quasienergy spectrum contains degeneracies, the corresponding interference terms in Eq. (1) will not decay (for exact degeneracies), or will only decay extremely slowly (when the degeneracy is approximate). Although the analysis of Benenti *et al.* cannot describe this situation, our approach would simply yield a longer Ehrenfest time, indicating the enhanced stability of the current. Such a quasidegeneracy is actually present (see Fig. 1 of our supplementary material [4]) in the numerical results presented in the Comment. For a value of the asymmetry parameter $\alpha = 0.32$, a very narrow crossing appears, producing the long-lived current plotted in the inset on Fig. 1 of Ref. [1]. The conclusion of Benenti *et al.* that “no asymptotic directed transport occurs for any value of K ” is thus not generally correct—it depends on the detailed form of the quasienergy spectrum.

Benenti *et al.* correctly note that “the stroboscopic current . . . remains finite forever.” We do not dispute this point, but it is irrelevant. This would be an issue only if we had attempted to deduce the time scale for the decay of the current by making a fit of the time dependence of the stroboscopically averaged current. As we emphasize above, this was not our procedure. Even making use of the continuous time average proposed by Benenti *et al.*, in

place of the more experimentally relevant stroboscopic average plotted in Fig. 3 of Ref. [2], the conclusions of our Letter would be unaffected. In Fig. 2 of the supplementary material [4] we show the decay rates of the continuously averaged current, which clearly show that even for time-symmetric driving, significant ratchet currents are produced over time scales that are very long in comparison to typical experimental observation times [5]. Although the interacting case ($g \neq 0$) is not amenable to Floquet analysis, very similar results are numerically obtained for the values of nonlinearity considered in Fig. 3 of Ref. [2].

Benenti *et al.* further attempt to support their case by considering the behavior of the harmonic oscillator. This example is trivial; it is not even periodically driven. A more telling comparison would be with the phenomenon of dynamical localization [6]. Here a particle on a lattice, subjected to a driving potential, periodically expands and collapses when the parameters of the driving are adjusted to certain specific ratios. Viewed stroboscopically, the particle appears to be frozen. The purely stroboscopic character of this phenomenon does not prevent it from being a genuine physical effect, as reflected in the name “dynamical localization.”

In summary, in our Letter we never claimed that for time-symmetric driving a ratchet current would last forever (although in the present Reply we point out that it could be possible if exact quasienergy degeneracies existed). A stroboscopic simulation may indeed overestimate the decay time of the ratchet current. However, our decay estimate was based on general quantum chaos theory arguments. Moreover, even a continuously time-averaged current may exhibit ratchet behavior for times longer than present experimental times. The conclusions of our Letter thus remain unaffected.

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