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Advances in the field of stress testing and systemic risk

MEMORIA PARA OPTAR AL GRADO DE DOCTOR

PRESENTADA POR

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TESIS DOCTORAL

Avances en el campo de los test de estrés y el riesgo sistémico

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Advances in the field of stress testing and systemic risk.

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Resumen

Facultad de Ciencias Económicas y Empresariales

Grado de Doctor

Avances en el campo de los test de estrés y el riesgo sistémico

por Javier OJEA FERREIRO

La complejidad y las interconexiones del sistema financiero hacen que garantizar su estabilidad sea especialmente importante. Para ello, es esencial prevenir el riesgo sistémico, entendido como aquellos eventos extremos que pueden generar perturbaciones financieras y afectar en último término a la economía real. Con el fin de lograr este objetivo se deben considerar una serie de indicadores de riesgo que tengan en cuenta las interdependencias surgidas en escenarios extremos. Las turbulencias financieras acaecidas con posterioridad a la crisis del 2008 han puesto de relieve el papel que juegan las interconexiones entre los diferentes sectores económicos para desencadenar efectos contagio, acentuando la necesidad de crear instituciones resistentes. La relación entre variables económicas en escenarios extremos puede poner en peligro la estabilidad financiera. Esta relación es compleja, multidimensional y evoluciona a lo largo del tiempo. La identificación de los sectores o instituciones que pueden condicionar unas mayores pérdidas al sistema financiero y la monitorización de las respuestas de los sectores bajo situaciones de estrés contribuyen al fortalecimiento de la economía.

Esta tesis doctoral busca cuantificar la exposición de distintos sectores económicos a escenarios de estrés de diversa índole, caracterizando qué marco condiciona unas mayores pérdidas. El análisis de vulnerabilidades en periodos atípicos permite identificar el riesgo sistémico. El concepto de test de estrés comúnmente indica un conjunto de técnicas analíticas y ejercicios que estudian las relaciones entre variables en situaciones extremas. El primer capítulo estudia el riesgo sistémico surgido de las interconexiones entre los derivados de crédito soberano y financiero europeo, identificando endógenamente los periodos de mayor estrés y calculando estadísticamente los cambios producidos en la distribución de probabilidad del indicador de contagio. El segundo capítulo se centra en el diseño de escenarios de estrés para el petróleo y el estudio de la respuesta de la bolsa europea a estos. En este capítulo se plantean estrategias para reducir las pérdidas de una cartera de renta variable europea cuando se materializan escenarios con grandes oscilaciones en el precio del petróleo. Para ello, se tiene en cuenta la evidencia muestral acerca de no linealidades en la relación entre la bolsa y el *commodity* (petróleo). El tercer capítulo estudia el modo en que el riesgo subyacente que genera el escenario para el petróleo en euros condiciona el rendimiento de la bolsa europea. La respuesta de la bolsa europea a un shock del petróleo en euros depende de si el factor que lidera el movimiento en precios es el petróleo (en cuyo caso tendríamos riesgo de *commodity*) o el tipo de cambio euro-dólar (riesgo de tipo de cambio).

La identificación temprana de spillovers entre los sectores financiero y soberano es una cuestión crucial dada la multitud de canales de transmisión que pueden

desencadenar episodios de retroalimentación en las pérdidas de ambos sectores. El primer capítulo de la tesis estudia la evolución de indicadores de contagio crediticio entre el sector financiero y el sector soberano en la eurozona. Estos indicadores han sido construidos a partir de la diferencia en los valores numéricos de las medidas de riesgo evaluadas en escenarios adversos y en momentos de calma en el otro sector. En este caso, los momentos de calma o estrés viene definidos por sus cuantiles. Esta medición nos permite cuantificar la dependencia entre sectores que emerge en eventos extremos. Desde un punto de vista técnico se utiliza una metodología de copulas para combinar las distribuciones marginales y generar la distribución conjunta. La distribución conjunta se construye utilizando un modelo que tiene en cuenta cambios en media, varianza y dependencia a lo largo del tiempo, además de considerar la existencia de exceso de curtosis y asimetría en los datos. La distribución marginal se incorpora de forma flexible utilizando un modelo autorregresivo combinado con un modelo GARCH con efecto apalancamiento e innovaciones con distribución t de Student asimétrica. La flexibilidad en la elección de las cópulas permite reflejar rasgos como la asimetría conjunta o la dependencia en las colas que evolucionan a lo largo del tiempo. El ejercicio empírico utiliza rendimientos semanales derivados de los CDS soberanos (Austria, Bélgica, Francia, Alemania, Italia, Holanda y España) y financieros, desde 2009 hasta el 2016. Los periodos de mayor contagio entre sectores se encuentran en marzo de 2010, con el surgimiento de las complicaciones en la deuda griega, y en el verano del 2011, cuando la sostenibilidad de la deuda española e italiana fue cuestionada. Tales periodos han sido identificados a partir de un modelo de regímenes cambiantes estocásticos, mientras que el test de Kolmogorov Smirnov nos permite contrastar cambios en la distribución de los indicadores de contagio. Este capítulo contribuye a la literatura analizando la conexión entre los riesgos de crédito soberano y financiero usando medidas construidas a partir de los percentiles de la distribución condicional. El contagio desde instituciones financieras a soberanas parece finalizar después del verano de 2012, mientras que la desaparición del contagio desde el sector soberano al financiero es más gradual. Este estudio identifica los sectores dónde la intervención puede ser necesaria, facilitando la evaluación cuantitativa del riesgo sistémico.

El estudio del comportamiento de la bolsa europea cuando se materializan escenarios extremos para las materias energéticas es un campo de interés para multitud de agentes económicos. Primero, es una investigación de interés para la política monetaria, al estudiar en qué medida la existencia de niveles inestables de precios energéticos puede condicionar la distribución de rendimientos en la economía. Segundo, los reguladores de mercado pueden encontrar, en los indicadores de riesgo condicionales y en su diferencia respecto al indicador incondicional, medidas que permitan cuantificar el impacto en la bolsa de cambios bruscos en el precio de las materias energéticas. Esta información podría ayudar a establecer unos requisitos de capital suficientes para que las instituciones reguladas pudiesen sobrevivir bajo situaciones extremas, y minimizar el efecto contagio que pudiese desencadenar en eventos sistémicos. En particular, en los capítulos 2 y 3 se analiza cómo escenarios extremos en el cambio de precio del petróleo Brent medido en euros pueden condicionar el Valor en Riesgo del Eurostoxx.

El capítulo 2 estudia la respuesta de la bolsa europea y sus distintos subíndices, clasificados en función de su actividad industrial, a un mismo escenario de estrés para el petróleo en euros. Este análisis aporta información de interés para los inversores sobre diversificación del riesgo en escenarios de precios inestables del petróleo.

La modelización de los rendimientos de bolsa y petróleo debe reflejar una serie de características específicas, más allá del exceso de curtosis y la presencia de asimetría. La dependencia en las colas y la asimetría conjunta de los datos se recoge utilizando un enfoque de cópulas, mientras que las no-linealidades y los cambios estructurales vienen recogidos por una metodología de Markov switching, permitiendo además identificar endógenamente los periodos donde se producen dichos cambios. Las distribuciones marginales son modelizadas a partir de un modelo AR(1)-GARCH-GJR con innovaciones t de Student asimétricas. Utilizando datos semanales desde el 2000 hasta el 2015 se identifica un cambio estructural en 2008. Antes de 2008, la relación entre renta variable y petróleo resulta negativa, siendo esta relación más fuerte frente a bajadas en el precio del petróleo que frente a subidas. Después de 2008, la relación entre renta variable y petróleo pasa a ser positiva, con una dependencia mayor en escenarios con caídas de precios en ambos mercados. El cambio en la dependencia podría estar explicado por el ciclo económico. Durante el periodo expansivo del ciclo económico, caídas en el precio del petróleo implican mayores márgenes de ganancias para las empresas, mientras que subidas en el precio de este input básico se verán reflejadas en el precio final. Por otro lado, después de la crisis de 2008 se produce una contracción de la demanda agregada en todos los sectores, generando una relación positiva entre petróleo y bolsa europea. La diversificación entre sectores con distinta elasticidad de demanda de sus productos reduce las pérdidas de una cartera de renta variable cuando se materializan movimientos extremos en el precio del petróleo. Un ejercicio *out-of-sample* muestra que las potenciales pérdidas se reducen combinando el Eurostoxx con un sector con una demanda inelástica de sus productos, i.e. el sector de cuidados sanitarios, debido al diferente comportamiento que presentan bajo el mismo escenario para el cambio en el precio del petróleo.

El rendimiento logarítmico del petróleo en euros, empleado para generar escenarios en el capítulo 2, es la suma de dos procesos estocásticos dependientes que implican distintos tipos de riesgo. Por un lado el riesgo de tipo de cambio puede generar cambios en el precio del petróleo en euros debido a apreciaciones o depreciaciones del euro frente a la divisa de negociación para el petróleo, el dólar estadounidense. Por otro lado el riesgo de *commodity* surge de cambios en los precios de negociación en el mercado originario. Definir el papel del tipo de cambio en el escenario del petróleo en euros puede ayudar a mejorar el diseño de test de estrés y aumentar la precisión del impacto del escenario en la economía. La respuesta de la bolsa europea a un mismo escenario para el petróleo en euros se esperaría que fuese distinta dependiendo de la fuente de riesgo que lidera el escenario. El tercer capítulo afronta este reto, combinando para ello los conceptos de convolución y cópula, de manera que se refleje la dependencia entre los dos factores subyacentes que generan el petróleo en euros. Este capítulo abre una nueva línea de investigación dada la ausencia de literatura sobre la influencia del tipo de cambio en la generación de escenarios internacionales para la economía doméstica. En consecuencia, este capítulo tiene implicaciones significativas para mejorar la precisión de la respuesta de las variables de interés a un determinado escenario internacional. La presencia de cambios estructurales en la dependencia entre variables y en la estructura de volatilidad permite su modelización utilizando un modelo AR(1)- SWARCH(2,1) con una mixtura de cópulas, donde los pesos varían en función de la probabilidad de encontrarnos en cada estado. El modelo SWARCH(2,1) no sólo sirve para explicar cambios estructurales en la volatilidad, sino que también puede reflejar el exceso de curtosis en la distribución de rendimientos. Los resultados del análisis indican que las pérdidas nominales derivadas del Valor en Riesgo de los rendimientos de la bolsa europea condicionado a un escenario de estrés para el petróleo en euros puede aumentar un 30% respecto del caso bivariante estudiado en la literatura, dependiendo del factor subyacente que lidere el movimiento. La investigación resalta el problema de consistencia que puede surgir en la estimación de la respuesta de la bolsa al no definir la fuente de riesgo que genera el escenario, ya que condiciona fuertemente la distribución de los rendimientos de la bolsa europea. Por un lado, el papel dominante del riesgo de *commodity* en los escenarios donde el precio del petróleo en euros experimenta una caída puede incrementar de manera importante las pérdidas en el mercado de renta variable europea. Por otro lado, el riesgo de tipo de cambio puede acentuar las pérdidas de la bolsa europea cuando desencadena un escenario extremo en el que los precios del petróleo en euros se incrementan. El enfoque propuesto en este capítulo puede mejorar nuestro conocimiento acerca del modo en que los movimientos de tipos de cambio pueden afectar al resultado de los test de estrés en mercados internacionales. Estos resultados abogan por un diseño cuidadoso de los test de estrés, incorporando el papel que juega el tipo de cambio en la generación de escenarios globales.

En definitiva, este trabajo de investigación doctoral contribuye con avances en la caracterización de efectos de contagio entre mercados financieros, en el diseño de escenarios de test de stress y en la medición y monitorización del riesgo sistémico. Estos temas presentan una gran relevancia tanto desde el punto de vista regulatorio como desde el punto de vista profesional.

COMPLUTENSE UNIVERSITY OF MADRID

Summary

Faculty of Economics & Business

Doctor of Philosophy

Advances in the field of stress testing and systemic risk.

by Javier OJEA FERREIRO

The complexity and interconnectedness of the financial system makes particularly important to guarantee its stability. Therefore, it is essential to prevent systemic risk, understood as an extreme event that generates a disturbance in the financial markets and may end up affecting the real economy. To control systemic risk, we should consider a broad catalogue of risk measures that take into account interdependences arisen in extreme scenarios. The financial turbulences, which have occurred from the 2008 crisis on, have highlighted the role played by the interconnections between different economic sectors in triggering contagion spillovers across the financial system. From these experiences came a need to create resistant institutions to possible extreme scenarios. The stronger dependence between economic variables when extreme scenarios materialise can endanger financial stability. This relationship between variables is complex, multidimensional and evolves over time. Identifying the sectors or institutions that could cause major losses to the financial system and monitoring the responses of sectors under stress contribute to strengthen the economy.

The aim of this doctoral thesis is to quantify the exposure of different economic sectors to diverse distress scenarios, in order to identify which framework provides higher losses. The analysis of the vulnerability in atypical periods allows us to measure systemic risk. The concept of stress test commonly indicates a set of analytical techniques and exercises used to study the relationships between variables in extreme situations. The first chapter studies the systemic risk arisen from the interconnectedness between European sovereign and financial credit derivatives, and identifies endogenously the periods of greatest stress, testing statistically the changes produced in the probability distribution of the contagion indicators. The second chapter focuses on the design of stress scenarios for oil prices and the response of the European stock market. This chapter proposes strategies to reduce the losses of European equity portfolios in scenarios where large fluctuations in oil price materialise. To this end, this study takes into account the important sample evidence of non-linearity in the relationship between equities and the commodity (oil). The third chapter examines the way in which the underlying risk, which generates the scenarios for oil price in euros, conditions the performance of the European stock market. The response of the stock market depends on whether the fundamental source of shocks is oil (in which case we would have commodity risk) or the euro-dollar exchange rate (exchange rate risk).

The timely identification of spillovers between the financial system and sovereign sectors is a crucial topic due to the multiple transmission channels that can trigger feedback loops of losses in both sectors. The first chapter of the thesis studies the evolution of credit contagion indicators between the financial sector and the

sovereign sector in the euro area. These indicators have been built as the difference of the numerical values of the risk measures evaluated in adverse scenarios and in tranquil periods for the other sector. In this case, tranquil periods and distress scenarios are defined according to their quantile. This approach allows us to quantify the dependence between sectors that arises from extreme events. From a technical point of view, the copula methodology allows us to combine marginal distributions to generate the joint distribution. The joint distribution is built using a model that considers changes in mean, variance and dependence over time, in addition to the excess kurtosis and asymmetry exhibited by the financial data. The marginal distribution is fitted using an autoregressive model combined with a GARCH model with leverage effect and innovations with asymmetric Student t distribution. The choice of different copulas enables us to consider features such as joint asymmetry or tail dependence that evolve over time. The empirical exercise employes weekly returns obtained from sovereign CDS (Austria, Belgium, France, Germany, Italy, the Netherlands and Spain) and the financial sector from 2009 to 2016. The high-contagion periods are found in March 2010, coinciding with the beginning of Greek troubles, and in the summer of 2011, when the sustainability of Spanish and Italian debt began to be questioned. Such periods have been identified by a Markov switching model, while Kolmogorov Smirnov test allows us to investigate significant changes in the distribution of contagion indicators. This chapter contributes to the literature by analysing the credit risk connections between euro zone sovereign sectors and the financial system using risk measures built from percentiles of the conditional distribution. The contagion from financial institutions to sovereign seems to end after the summer of 2012, while the contagion from the sovereign to the banking sector fades more gradually. This study identifies sectors where intervention may be necessary and provides a quantitative assessment of systemic risk.

The response of the European stock market to extreme energy-related scenarios is a field of interest for a large number of economic agents. To begin with, it is useful to monetary policy since it provides information about how unstable energy prices could condition stock returns distribution. Secondly, market regulators can begin to quantify the impact of abrupt changes in energy prices by looking at the conditional risk measures and their difference from the unconditional measures. This information would help set a capital buffer that allows regulated institutions to survive under distress scenarios, minimising the contagion effect between them that could lead to a systemic event. In particular, Chapters 2 and 3 analyse how extreme changes of the Brent price denominated in euros can condition the Value at Risk of the Eurostoxx.

Chapter 2 examines the response of the European stock market and its subindices, classified according to their industrial activity, to the same distress oil-related scenario. This analysis provides useful information for investors about risk diversification in scenarios of unstable oil prices. The modelling of stock market and oil returns must consider further characteristics, beyond the excess of kurtosis and skewness. A copula approach gathers the dependence in the tail of the joint distribution and the joint skewness while a Markov switching methodology reflects non-linearities and structural changes, allowing endogenous identification of the periods where these changes occur. An AR(1)-GARCH-GJR model with skewed Student t innovations fits the marginal distribution. A structural change is identified in 2008 using weekly data from 2000 to 2015. Before 2008, the relationship between stock returns and oil returns is negative. This relationship is stronger when oil price decreases than when it increases. After 2008, the relationship between the stock market and oil returns becomes positive. This relationship presents a greater dependence when both prices fall. The change in dependence could be explained by the economic cycle. During the expansive phase of the economic cycle, falls in oil prices imply a greater profit for companies, while an increase in the price of this basic input will be reflected in the final price. On the other hand, after the 2008 crisis there is a decrease of aggregate demand in all sectors, generating a positive relationship between changes in oil prices and movements in the European stock market. Diversification between sectors with different elasticity in the demand for their products reduces the stock portfolio losses when extreme movements in oil price materialise. An out-of-sample exercise shows that combining the Eurostoxx with a sector with an inelastic demand for its products, i.e. the healthcare sector, reduces potential losses due to their different behaviour when the same oil-related scenario materialises.

The logarithmic oil returns in euros, employed in Chapter 2 to generate scenarios, is the sum of two stochastic dependent processes that involve different types of risk. On the one hand, the exchange rate risk can generate changes in oil prices denominated in euros on account of appreciations or depreciations of the euro against the trading currency for international commodities, the US dollar. On the other hand, commodity risk arises from changes in trading prices in the original market. Defining the role of the exchange rate in oil-related scenarios can help enhance the design of tailor-made stress tests and compute more accurately the impact of a distress scenario on our domestic markets. The response of the European stock market to the same oil-related scenario is expected to be different depending on whether the exchange rate plays the main role or not. The third chapter deals with this challenge by combining the concepts of convolution and copula to reflect the dependence between the two underlying factors that generate oil returns in euros. There is a lack in the literature regarding the role of the exchange rate in the generation of international scenarios for the domestic economy. Hence, this chapter creates a new line of research that can have important consequences for improving the precision in the response of the variables of interest to a given international scenario. The presence of structural changes both in the dependence between variables and in the volatility structure allows us to model it using an AR(1)- SWARCH(2,1) model with a mixture of copulas, where weights vary depending on the probability of being in each state. The SWARCH(2,1) model is not only able to explain structural changes in volatility, but it can also reflect the excess of kurtosis in the returns distribution. The results of the analysis indicate that nominal losses obtained from the Value at Risk of the European stock returns conditioned to a distress scenario for oil in euros may increase by 30% with respect to the bivariate case studied in the literature, which depends on the underlying factor leading the change. The research highlights the consistency problem that could arise in the response of the stock market when the source of risk generated by the scenario is undefined, since the triggering risk can strongly condition the distribution of the European stock returns. On the one hand, the dominant role of commodity risk in scenarios where the oil prices in euros experience a downward movement can sharply increase the losses in the European stock market. On the other hand, the exchange rate risk might exacerbate the losses in the European stock market if it triggers an extreme event where oil prices in euros increase. The approach proposed in this chapter improves our understanding of how the movements in the exchange rate could condition the stress test results in international markets. These results call for a careful design of stress tests, incorporating the role played by the exchange rate in the generation of global scenarios.

To conclude, this doctoral research contributes to advances in the characterization of contagion effects between financial markets, in the design of stress test scenarios and in the measurement and monitoring of systemic risk. These topics are of great relevance from both a regulatory and a practitioner point of view.

Chapter 1

Contagion spillovers between sovereign and financial European sector from a $\Delta CoVaR$ approach.

Abstract
I examine the evolution of contagion indices between the European financial sector and the sovereign sector (Austria, Belgium, France, Germany, Italy, Netherlands and Spain) during the European sovereign credit crisis looking at CDS returns. Contagion indices, $\Delta CoVaR$ *and* $\Delta CoES$*, reflect events associated with extreme realizations and interdependencies between defaults useful for risk management purposes. I use a copula approach with time-varying parameters to capture changes in the tail dependence between returns in the financial and the sovereign sectors. I employ a Markov Switching model to identify the most stressful moments as measured by these contagion indicators. The results point out the emergence of Greek debt crisis in March 2010 and the vulnerable situation of Spain and Italy in the summer of 2011 as the main periods where the contagion from the sovereign to the financial sector was stronger. The decrease in contagion was gradual since the speech made by the ECB on* 26 *July* 2012*. The statistical significance of the change in the contagion indicators is checked using bootstrap tests.*

1.1 Introduction

Systemic risk in biological terms is defined as a possible global disaster arising from the behaviour of a single individual of the species that coexist in the same habitat. Likewise in Economics, systemic risk is the threat of a system breakdown because the effects of the interactions among individuals are undervalued, i.e. negative externalities arise from the relationship between economic agents. Since systemic risk affects by nature all sectors, it should be evaluated not only within sectors, but also across sectors. The timely identification of spillovers between the financial and the sovereign sectors is a crucial topic to inform policies that can prevent events such as the European sovereign credit crisis. Government guarantees and bailouts have helped to build a close relationship between the financial and sovereign sectors, ultimately triggering massive damages to the welfare state (Gropp et al. 2013, Acharya and Mora 2015). Sovereign debt positions held by banks in their portfolios and the link between the ratings of the financial and the sovereign sectors worked as transmission channels for risk from the sovereign to the financial sector. This two-way feedback, as named by Acharya et al. (2014), can become an adverse feedback loop between sectors in case of crisis. While such relationship was extensively studied during the European sovereign credit crisis (Albertazzi et al. 2014, Panetta et al. 2011, Acharya et al. 2014, Gray et al. 2007, Gerlach et al. 2010, Ejsing and Lemke 2011, Dieckmann and Plank 2011), how this loop has been weakened has not been so widely studied. This article studies how the credit risk contagion between these sectors has evolved during the period $2009 - 2016$.

To shed light on this matter we have to clarify first what is meant by contagion here. There is not a unique criterion to identify a contagion event. Contagion is a sophisticated and multidimensional concept that has several features. The focus on a certain set of contagion characteristic will lead to a different methodology for building contagion indicators. For instance, defining contagion as the spread of idiosyncratic negative shocks to other institutions may lead to a Vector Autoregression (VAR) framework. Indeed, most research on this topic has been conducted in a VAR framework (Alter and Beyer (2012), Bicu and Candelon (2012), Kok and Gross (2013), Alter and Schüler (2012), Chudik and Fratzscher (2012), Candelon et al. (2011)). Following the VAR methodology, impulse response functions and variance decomposition are employed to evaluate contagion through the effects of an idiosyncratic shock on the other economic agents. The contagion measure under this approach expresses mean effects, but a measure based on the left tail returns would be more useful for risk management proposes. Moreover, not all the dependence between the sovereign and the financial sectors should be considered contagion. They are not independent sectors and a certain level of connection may be advantageous (Allen et al. 2018). These two points, i.e. the behaviour in an adverse scenario and the interdependencies between sectors different than those observed in normal times, are the key features that lead in this paper to a different proposal of contagion measure. Indeed, the proposed contagion indicators have implications for investors, who need a risk management tool to assess the exposure of their sectoral portfolios from undesired links with other sectors which are not taken into account by unconditional risk measures such as Value-at-Risk (VaR) and Expected Shortfall (ES). Looking at the change in the conditional risk measures ($CoVaR$ and $CoES$) when the conditioning sector moves from normal times to a distress scenario gives essential information concerning the capital shortfall in the conditioned institution due to the existence of dependence between sectors i.e. $\Delta CoVaR$ and $\Delta CoES$. I employ $\Delta CoVaR$ and $\Delta CoES$

as indicators of credit risk contagion between the sovereign sector on a country level and the European financial sector as a whole. The financial sector is measured on an European level due to the cross positions of sovereign debt by European financial institutions and also because of the high level of integration in the euro zone financial sector. The article uses the same methodology as Reboredo and Ugolini (2015a), however there are important differences between these studies. Firstly, this article deals with the systemic risk arisen from the link between the sovereign sector and the financial system, while Reboredo and Ugolini (2015a) focus on the sovereign sector. Secondly, the dataset is different, while Reboredo and Ugolini (2015a) use government bond for the period 200-2012, this article employs the returns derived from the credit risk (CDS) using a sample from 2009 to 2016. Thirdly, this article choose a sample period that allows us to study how the contagion between these two sectors faded out.

The methodological approach for building $\Delta CoVaR$ and $\Delta CoES$ should be flexible in order to characterize accurately marginal features such as heteroskedasticity, leverage effects, asymmetry and skewness, apart from considering different possible joint distributions and changes in dependence. A copula methodology where the copula parameter is time-varying combined with a suitable marginal model meets these criteria. This methodology is employed not only because of its straightforward decomposition of the joint distribution, but also due to computational reasons, as it is less computational expensive than other approaches that imply numerical integration such as the GARCH proposal by Girardi and Ergün (2013).
Once the contagion risk indicators have been built, I identify regimes for the level of contagion risk based on a Markov switching model. In particular, the Markov switching model points out that for most sovereign sectors the contagion to the financial sector was concentrated in two periods, the first one around March 2010, when the Greek debt crisis emerged and a second in the summer of 2011 due to a confidence crisis concerning Spanish and Italian sovereigns. In addition, the contagion from the financial sector to the sovereign sector seems to end later and more slowly than the contagion from the sovereign sector to the financial sector. Considering Mario Draghi's speech on 26 July 2012 as a breakpoint in the European credit crisis, I test a possible change in the distribution and in the mean of the contagion measures orthogonalized by its own past using a bootstraping procedure. Results show a decrease in the mean level of contagion after the ECB's speech and a smaller downside spillover between sectors after the breakpoint compared to the period before 26 July 2012.

The remainder of the article is organised as follows: Section 1.2 describes the framework where the contagion risk measures are applied. Section 1.3 suggests the copula approach for assessing $\Delta CoVaR$ and $\Delta CoES$, describing the different dependence structures considered in the paper. Section 1.4 presents the data employed for the empirical application. Section 1.5 shows the main results and robustness checks. Section 1.6 closes describing possible future research lines and some policy recommendations, pointing out $\Delta CoVaR$ and $\Delta CoES$ as suitable tools for assessing contagion from a risk management point of view.

1.2 Background

The $CoVaR$ measure was introduced by Adrian and Brunnermeier (2016) as a systemic risk measure for identifying Systemically Important Financial Institutions (SIFIs). The aim was to express the minimum returns for the conditioned institution y with some confidence level $(1-\beta)100\%$ given a quantile α of the returns distribution for conditioning institution x, i.e.,

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta)|r_{x,t} = VaR_{x,t}(\alpha)] = \beta. \tag{1.1}$$

Girardi and Ergün (2013) enhances $CoVaR$ definition to allow backtesting and improve the behaviour of $CoVaR$ as a function of the dependence between institutions (Mainik and Schaanning (2014), Zhang (2015)). The modified $CoVaR$ definition expresses the minimum returns for the conditioned institution y with some confidence level $(1-\beta)100\%$ given that the conditioning institution x is below its $\alpha100\%$ worst case scenario, i.e.

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta)|r_{x,t} \leq VaR_{x,t}(\alpha)] = \beta. \tag{1.2}$$

I employ the subscript f for representing the global European financial sector and s for the European sovereign sectors. The level α of the conditioning event is usually fixed at $\alpha = \beta$ where $\alpha, \beta \in (0,1)$, and due to the focus on the left tail returns, α and β are close to zero in a distress scenario. Employing a conditioning event as the VaR, which is independent of the level of risk of the conditioning institution x, allows us to compare $CoVaR$ given several conditioning institutions with different risk profiles.

Even though the $CoVaR$ properties improve under Equation (1.2), it still has some limitations since it looks only to a certain percentile of the conditioned institution y and consequently it is not subadditive (Acerbi and Tasche 2002). This feature can be enhanced if the Value-at-Risk dimension is moved to an Expected Shortfall framework. The Conditional Expected Shortfall, $CoES_{y|x,t}(\alpha,\beta)$, measures the average return for institution y when the returns are lower than $CoVaR_{y|x,t}(\alpha,\beta)$, i.e.

$$CoES_{y|x,t}(\alpha,\beta) = \frac{1}{\beta}\int_0^{\beta} CoVaR_{y|x,t}(\alpha,q)dq, \tag{1.3}$$

where $CoVaR_{y|x,t}(\alpha,q)$ is given by Equation (1.2).

Losses not considered in normal scenarios can trigger a systemic event because of lack of liquidity, i.e. in a normal scenario capital needs can be fulfilled without spillover effect between sectors, but in a distress scenario capital needs could lead to bankrupt and bailout processes, triggering a contagion event between the sovereign and the financial sectors. Therefore, $CoVaR$ and $CoES$ are unsatisfactory measures for assessing the contagion between sectors. Indeed, they may be enough to capture the losses in a given scenario but not the loss changes when the conditioning scenario moves. The change in the previous conditional risk measures when the conditioning variable moves from tranquil times to a distress period are known as Delta Conditional measures, i.e. $\Delta CoVaR$ and $\Delta CoES$. There is no consensus about the definition of tranquil times under $\Delta CoVaR$. Chen and Khashanah (2014) employs the unconditional VaR measure and Girardi and Ergün (2013) uses a standard deviation range around the mean value of the conditioning variable. However, the former definition responds to the importance of taking into account the conditioning variable for risk assessment proposed but it does not capture the relevance of a change in the conditioning variable from a normal period to a distress scenario for the conditioned variable. On the other hand, the latter definition for normal scenario is not fully defined for non-Gaussian marginal distributions due to the need to use higher moments, e.g. skewness and kurtosis. In this article, the normal scenario is defined as a $\beta/2$ range of quantiles around the median. Consequently, I define $\Delta CoVaR_{y|x,t}(\beta)$ as

$$\Delta CoVaR_{y|x,t}(\beta) = CoVaR_{y|x,t}(\alpha_s, \beta) - CoVaR_{y|x,t}(\alpha_n, \beta), \tag{1.4}$$

where $\alpha_s = \beta$ in Equation (1.2) , i.e.

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha_s, \beta)|r_{x,t} \leq VaR_{x,t}(\beta)] = \beta$$

and α_n are the set of quantiles between the lower percentile α^- and the upper percentile α^+ such that

$$P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha_n, \beta)|VaR_{x,t}(\alpha^-) \leq r_{x,t} \leq VaR_{x,t}(\alpha^+)] = \beta,$$

where $\alpha^+ = 0.5 + \beta/2$ and $\alpha^- = 0.5 - \beta/2$. The idea of considering an upper and a lower bound for the conditioning variable was already considered by Reboredo and Ugolini (2016). The proposed definition for a normal scenario is as accurate as the one for the distress scenario, because we are considering the same β range of quantiles, and it is fully defined in percentile terms. These features were not fulfilled by previous definitions. $\Delta CoVaR_{y|x,t}(\beta)$ expresses the undervaluation of the minimum returns measure with a confidence level $(1-\beta)100\%$ for institution y when institution x moves from normal times to an adverse scenario. $\Delta CoES$ can be computed following the same procedure as in Equation (1.4).

Delta Conditional measures do not distinguish whether the increase in the risk measure is due to causal reasons or to a common factor between both institutions. Hence, I will capture changes in the conditioned institution even in the absence of a direct link. Imagine that the financial sector f has a diversified sovereign debt portofolio where an isolated bankrupt in one country s would not cause contagion to the financial system. However, Delta Conditional measures would disclose contagion if the distress is due to a common factor of the set of countries. Although $\Delta CoVaR$ and $\Delta CoES$ do not express causality, they are directional measures, i.e. $\Delta CoVaR_{f|s,t} \neq \Delta CoVaR_{s|f,t}$.

1.3 Methodology

The model structure for $CoVaR$ can be divided into three steps: the marginal model structure that gathers individual features as heterokedasticity or kurtosis, the copula function that links marginal density functions and the copula time-varying parameter that allows changes in tail dependence. The assessment of $CoVaR$ is straightforward given these three stages.

Following Bayes' theorem and copula theory Equation (1.2) can be rewritten as a ratio of probabilities, i.e.

$$\begin{aligned} P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta)|r_{x,t} \leq VaR_{x,t}(\alpha)] &= \frac{C(u_y,\alpha;\theta_t)}{\alpha} \\ &= \beta, \end{aligned}$$

where θ_t is the copula parameter at time t, $P_{t-1}[r_{y,t} \leq CoVaR_{y|x,t}(\alpha,\beta), r_{x,t} \leq VaR_{x,t}(\alpha)] = C(u_y,\alpha;\theta_t)$ and $P_{t-1}[r_{x,t} \leq VaR_{x,t}(\alpha)] = \alpha$.
Therefore $CoVaR_{y|x,t}(\alpha,\beta)$ is obtained by identifying the value u_y^* such that $C(u_y^*,\alpha) = \alpha\beta$ and then employing the inverse cumulative distribution function of institution y's returns, i.e. $F_{r_{y,t}}^{-1}(u_y*) = CoVaR_{y|x,t}(\alpha,\beta)$.
In this section, first I describe the marginal model and the assumption distribution about the innovation, then I present a set of copulas considered to model the joint distribution between the financial and the sovereign sector and finally I establish the dynamic evolution in the copula parameter to allow time-varying dependence between sectors.

1.3.1 Marginal model

For each sector, I estimate a P-order autoregressive model ($AR(P)$) where the lag P, for parsimony reasons, is the minimum such that the innovation has no autocorrelation. I also model heterokedasticity and leverage effect by using a GJR-GARCH(1,1) representation. Finally I model skewness and kurtosis by assuming a Hansen (1994)'s skewed t distribution for the innovations. That is,

$$r_{j,t} = \underbrace{\phi_{j,0} + \sum_{k=1}^{P} \phi_{j,1} r_{j,t-k}}_{\mu_{j,t}} + \varepsilon_{j,t}, \qquad j = f, s \tag{1.5}$$

with $\varepsilon_{j,t} = \sigma_{j,t}\xi_{j,t}$ where $E\left(\varepsilon_{j,t}\varepsilon_{j,t-k}\right) = 0$ for $\forall k > 0$ and $\sigma_{j,t}^2$ is the conditional variance given by a GJR-GARCH(1,1) specification, i.e.

$$\sigma_{j,t}^2 = \omega_i + \alpha_j(1+\theta_j \mathbb{1}_{j,t-1})\varepsilon_{j,t}^2 + \beta_j\sigma_{j,t-1}^2, \tag{1.6}$$

where the indicator function $\mathbb{1}_{j,t-1}$ values 1 if $\epsilon_{j,t} < 0$ and zero otherwise and $\xi_{j,t} \sim f(\xi_{j,t};\eta_j,\lambda_j)$ where f is the probability distribution function of the skewed-t distribution, η_j denotes the number of degrees of freedom and λ_j the asymmetry parameter. The density of Hansen (1994)'s skewed-t distribution is

$$h(\xi_t|\eta,\lambda) = \begin{cases} bc(1+\frac{1}{\eta-2}(\frac{b\xi_t+a}{1-\lambda})^2)^{-(\eta+1)/2} & \xi_t < -a/b \\ bc(1+\frac{1}{\eta-2}(\frac{b\xi_t+a}{1+\lambda})^2)^{-(\eta+1)/2} & \xi_t \geq -a/b \end{cases}, \tag{1.7}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$. The constants a, b and c are given by

$$a = 4c\lambda\left(\frac{\eta-2}{\eta-1}\right), b = \sqrt{1+3\lambda^2-a^2}, c = \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})}.$$

Note that when $\lambda = 0$ Equation (1.7) reduces to the standard Gaussian distribution as $\eta \to \infty$. When $\lambda = 0$ and η finite, we obtain the standardized symmetric-t distribution.

1.3.2 Copula function

The choice of copula determines the relationship between a couple of marginal distributions. An inaccurate copula choice would imply missleading $CoVaR$ and $\Delta CoVaR$ estimates and ultimately a wrong interpretation of their values. To reduce that risk I compare a broad range of copula choices using Akaike Information Criterion ($AICc$) corrected for small sample bias as suggested by Hurvich and Tsai (1989). $AICc$ criterion has been employed for copula selection in other $CoVaR$ studies such as Reboredo and Ugolini (2015b). I consider 8 alternative copulas that are broadly employed in financial studies. Each copula implies a different tail dependence. The Clayton and the Survival Gumbel copulas allow for lower tail dependence but no upper tail dependence, whereas the opposite situation is found in Gumbel copula. The Joe-Clayton (BB7), the Student t and the Clayton-Gumbel (BB1) copulas allow for either upper and lower tail dependence. Table 1.1 presents the main tail dependence features in the set of employed copulas.

TABLE 1.1: Main tail dependence features for each copula

Family	**Lower tail dependence**	**Upper tail dependence**
Clayton	$2^{-1/\theta_t}$	$-$
Gumbel	$-$	$2-2^{1/\theta_t}$
Frank	$-$	$-$
BB7 (Joe-Clayton)	$2^{-1/\delta_t}$	$2-2^{1/\theta_t}$
Survival Gumbel	$2-2^{1/\theta_t}$	$-$
Student t	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\theta_t)}{1+\theta_t}}\right)$	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\theta_t)}{1+\theta_t}}\right)$
BB1 (Clayton-Gumbel)	$2^{-1/\theta_t\delta_t}$	$2-2^{1/\delta_t}$
Gaussian	$-$	$-$

Note: $-$ represents that there is no tail dependency.
θ_t and δ_t are parameters of the copula at time t. The number of degrees of freedom of the Student t copula is η.
Source: (Ao et al., 2017, p. 22) and Jiang (2012).

1.3.3 Time evolution in the copula parameter

I assume that the functional form of the copula remains fixed over the sample while the parameters for each copula are varying based on some equation for the time evolution. A time-varying copula parameter allows changes in tail dependence along time. As a result, the model is more flexible for tracking changes in the relationship between both sectors. I employ the approach proposed by Patton (2006). Alternative approaches for modeling time-varying copula parameter may be using rolling-windows (Aloui et al. (2013)), Generalized Autoregressive Score (GAS) (Creal et al. (2013)) or Stochastic Autoregressive copulas ($SCAR$) (Hafner and Manner (2012)). Another way to have a change in tail dependence over time is to assume different states, each of one characterized by a certain copula, a regimen-switching copulas like Rodriguez (2007). A comparative analysis of them is out of the scope of this work.

The parametric representation for the Clayton, the Gumbel and the Survival Gumbel copulas is

$$\theta_t = \Lambda_1 \left(\omega + \beta\theta_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}| \right), \tag{1.8}$$

where Λ_1 is $\exp(x)$ for the Clayton copula and $(\exp(x) + 1)$ for the Gumbel and Survival Gumbel copula to keep the values in the feasible region of the parameter space. The evolution for the parameter δ of Frank copula is represented by

$$\delta_t = \omega + \beta\delta_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}|. \tag{1.9}$$

The evolution equation for the two parameter families of non-elliptical copulas , i.e. BB1 and BB7 copulas, is based on the link between these parameters and the tail dependence, which is disclosed in Table 1.1.

$$\tau_t^K = \Lambda_2 \left(\omega_K + \beta_K \tau_{t-1}^K + \alpha_K \frac{1}{20} \sum_{k=1}^{20} |u_{s,t-k} - u_{f,t-k}| \right), \qquad K = U, L \tag{1.10}$$

where $\Lambda_2(x) \equiv (1 + \exp(-x))^{-1}$ is the logistic transformation to keep the tail dependence coefficient between 0 and 1.
For the Student t copula I assume that the number of degrees of freedom is constant (Elliott and Timmermann, 2013, p. 932 , Reboredo and Ugolini, 2016) and only the correlation parameter, i.e., ρ_t, is time-varying. The dynamics for the parameter of elliptical copulas is

$$\rho_t = \Lambda_3 \left(\omega + \beta\rho_{t-1} + \alpha \frac{1}{20} \sum_{k=1}^{20} \Phi^{-1}(u_{s,t-k}) \Phi^{-1}(u_{f,t-k}) \right),$$

where Φ^{-1} is either the inverse Gaussian cumulative distribution function in case the elliptical copula is Gaussian or the inverse Student t cumulative distribution function with η degrees of freedom for the Student t copula. The modified logistic transformation allows for a value of $\rho_t \in (-1, 1)$, i.e. $\Lambda_3(x) \equiv \frac{1-\exp(-x)}{1+\exp(-x)}$.
Table 1.2 provides a summary of the time-varying parameters representation proposed for each copula.

TABLE 1.2: Time-varying parameter representation for each copula

General model	$\Lambda\left(\omega_K + \beta_K \theta^K_{t-1} + \alpha_K \frac{1}{20}\sum_{k=1}^{20} \lvert u_{s,t-k} - u_{f,t-k}\rvert\right)$	
Copula	**Parameter** θ	**Function** $\Lambda(x)$
Clayton	θ	$\exp(x)$
Gumbel	θ	$(\exp(x)+1)$
Frank	θ	x
BB7	$\tau^L; \tau^U$	$(1+\exp(-x))^{-1}$
Survival Gumbel	θ	$(\exp(x)+1)$
Student t	ρ	$\frac{1-\exp(-x)}{1+\exp(-x)}$
BB1	$\tau^U: \tau^L$	$(1+\exp(-x))^{-1}$
Gaussian	ρ	$\frac{1-\exp(-x)}{1+\exp(-x)}$

Note:
$\tau^U, \tau^L \in (0,1)$.
For the BB7 copula $\theta = \frac{1}{\log_2(2-\tau^U)}$ and $\delta = \frac{-1}{\log_2(\tau^L)}$.
For the BB1 copula $\delta = \frac{1}{\log_2(2-\tau^U)}$ and $\theta = \frac{-\log_2(2-\tau^U)}{\log_2(\tau^L)}$.
The general model for elliptical copulas is: $\Lambda\left(\omega_K + \beta_K \theta^K_{t-1} + \alpha_K \frac{1}{20}\sum_{k=1}^{20} \Phi^{-1}(u_{s,t-k})\Phi^{-1}(u_{f,t-k})\right)$ where Φ^{-1} is the inverse Gaussian cumulative distribution function or the inverse Student t cumulative distribution function with η degrees of freedom.

The joint density function is obtained by combining the marginal probability distribution functions and the density copula function. I employ the two-step method of Inference Functions for Margins (IFM) to estimate the parameters by maximum log-likelihood, where marginal distributions and copulas are estimated separately. The computational cost of finding the optimal set of parameters is significantly reduced significantly by this approach. Joe and Xu (1996) shows that the estimated parameters using IFM method are consistent and asymptotically normal.

1.4 Data

It is widely accepted that the European sovereign debt crisis was led by a confidence crisis in the institutions. Consequently I employ a credit derivative, credit default swaps (CDS), obtained from Datastream on weekly basis from 22 May 2009 to 13 May 2016 to compute *CoVaR* measure. The total number of observations is 338.
I use the 5-year contract because it is the most liquid maturity. Concerning the restructuring event, I choose complete restructuring, also known as old reestructuring because its credit event is used mainly in Europe and it is the usual one for sovereign institutions (Anson et al., 2004, p. 62). Moreover, the CDS employed in this study are those with a senior debt underlying since it is the most traded branch of the CDS categories. I choose the same type, seniority and maturity for the financial firms' CDS.[1]
I consider sovereign CDS from Austria, Belgium, France, Germany, Italy, Netherlands and Spain.[2] A total of 25 European financial CDS meet the criteria for the considered period, 14 being financial institutions from the core European area whereas 11 are in the periphery. The number of financial institutions and their countries

[1]It is worth noting that most of the financial firms have CDS where the restructuring event is modified restructuring, so the focus in complete restructuring reduces the sample. However, using a different restructuring event could bias the results, because the hedging of the CDS against default is not perfect.

[2]Note that the CDS are not traded anymore when the underlying event occurs, i.e. when the institution experiences a bankrupt, that explains why there are no financial firms like Allied Irish Banks or Greek, Portuguese or Irish sovereign CDS in the sample.

are: Austria (2), Belgium (1), Finland (1), France (5), Germany (5), Italy (4), Netherlands(3), Portugal (1) and Spain (3).

TABLE 1.3: European financial institutions employed for building the financial system credit risk index

Name	**Country**
Banca Monte dei Paschi di Siena	Italy
Banco Comercial Português	Portugal
Banco Popular Español	Spain
Banco Santander	Spain
Bayerische Landesbk	Germany
BBVA	Spain
BNP Paribas	France
Commerzbank AG	Germany
Cooptieve Cente Rabo BA	Netherland
Credit Agricole	France
Credit Lyonnais	France
Danske Bank A/S	Finland
Deutsche bank AG	Germany
Erste Group Bank AG	Austria
ING Bank N.V.	Netherland
Intesa Sanpaolo Spa	Italy
KBCA Bank	Belgium
Lb Badenwuerttemberg	Germany
Mediobanca Spa	Italy
Natixis	France
Portigon AG	Germany
SNS Bank N.V.	Netherland
Societé Générale	France
Unicredit	Italy
Unicredit Bank AG	Austria

I build financial CDS indices for each country by taking the median CDS spread in a given country each week. Later, I transform them into returns and I obtain the common financial risk factor among CDS spread using principal component analysis[3]. According to Rodríguez-Moreno and Peña (2013), the first principal component of a CDS portfolio is the best systemic measure in the macro group. Table 1.4 shows the weight under the principal component analysis. To check for robustness, the equally weighted financial portfolio is also built with similar results.

TABLE 1.4: Weights for building the financial sector proxy.

Countries	1^{st} PCA (%)
Austria	10.83
Belgium	8.02
Finland	9.95
France	12.73
Germany	11.73
Italy	11.95
Netherland	12.62
Portugal	9.64
Spain	12.53

1^{st} PCA column expresses the weights obtained by the first principal component.
Equal indicates the equally weighted portfolio.

[3] In order to avoid giving an excessive weight to the most volatile country-level CDS returns, the PCA is performed on the correlation matrix. This approach is similar to the one employed by Chamizo and Novales Cinca (2016) for obtaining the returns of the financial system credit risk.

CDS spreads are transformed in returns following Berndt and Obreja (2010) and Ballester et al. (2016).

$$\begin{aligned} r_{i,t} &= -\Delta CDS_t A_t(T) \\ &= -\Delta CDS_t \frac{1}{4} \sum_{j=1}^{4T} \delta\left(t, \frac{j}{4}\right) q\left(t, \frac{j}{4}\right), \end{aligned} \tag{1.11}$$

where $\Delta CDS_t(T)$ is the weekly change in CDS spreads with maturity T and $A_t(T)$ is the value of a defaultable quarterly annuity over the next T years. T is equal to five years, given the selected CDS data. The risk-free discount factor for day t and s quarter is $\delta(t,s)$, fitted from Euribor rates[4]. The risk-neutral survival probability of the firm or government over the next s quarters can be written as $q(t,s) = \exp(-\lambda_t(s))$ where λ_t is the risk-neutral default intensity. λ_t is computed directly from observed CDS spreads as $\lambda_t = 4\log(1 + CDS_t/4L)$. L denotes the risk neutral expected loss given default (LGD), fixed at 60% for corporate firms and 40% for governments. Note that the change in CDS spreads is used for returns estimation preceded by a minus sign, so an increase in credit risk, i.e. a raise in CDS spreads, supposes a decrease in CDS returns whereas a reduction of credit risk implies an increase in CDS returns. Figure 1.1 shows the path of the CDS quote in basis points (red line) and the price employing an exponential function on the returns obtained from Equation (1.11) (blue line). Prices seems to react to the same shocks as CDS, although in opposite directions. The black line indicates the week of 26 July 2012 when Mario Draghi made a speech that, as can be seen in the Figure, changed the trend for Spanish and Italian CDS quotes.

Table 1.5 provides descriptive statistics for the CDS returns of the financial sector and the European countries. The excess kurtosis and the skewness in the data support the choice of the skewed t distribution for innovations. Annual volatility is around $45 - 50\%$, in line with volatility in the stock market. Furthermore, CDS returns from the financial sector have lower volatility than sovereign CDS returns and also the lowest mean return.

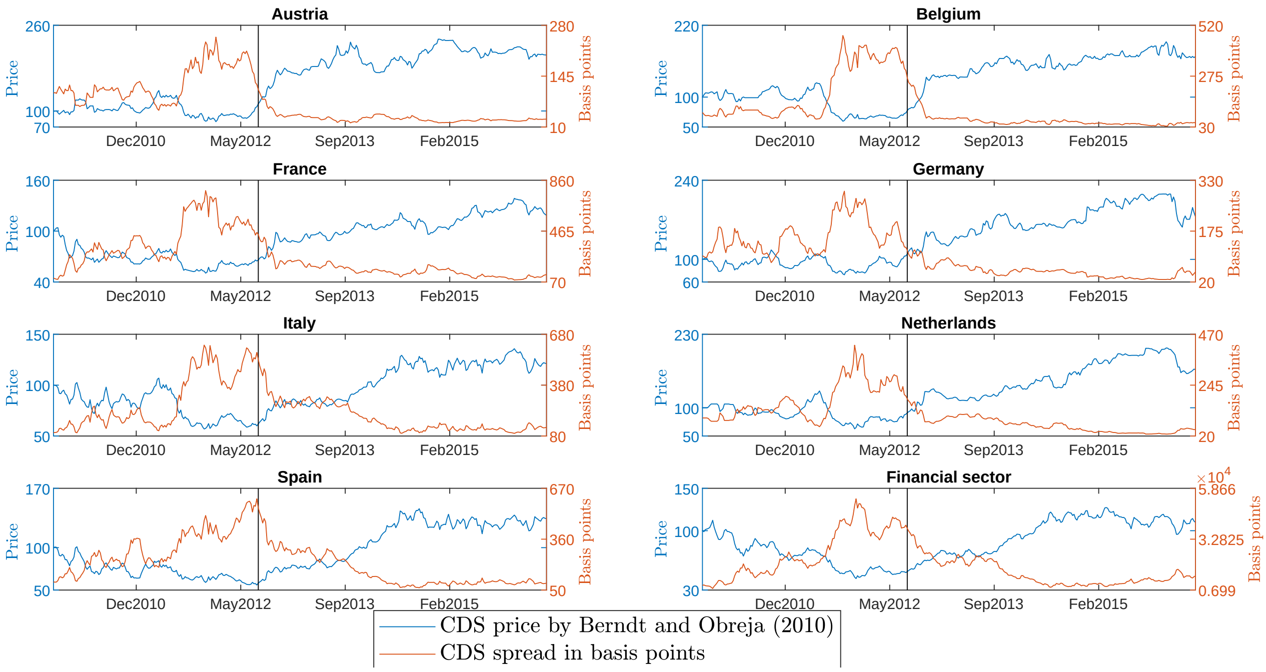
TABLE 1.5: Descriptive statistics: CDS returns.

	Financial sector	Austria	Belgium	France	Germany	Italy	Netherlands	Spain
Mean	0.0007	0.0026	0.0020	0.0010	0.0022	0.0012	0.0020	0.0014
Maximum	0.0818	0.1907	0.1102	0.1441	0.1482	0.1298	0.1398	0.1347
Minimum	-0.0866	-0.1081	-0.1162	-0.1008	-0.1151	-0.1292	-0.0998	-0.0826
Std. Dev.	0.0292	0.0310	0.0302	0.0324	0.0337	0.0338	0.0300	0.0317
Skewness	-0.1031	0.4788	-0.1247	0.3387	0.2664	0.1037	0.3422	0.2529
Kurtosis	3.2177	7.7882	5.3097	5.8765	5.1924	4.5488	5.8348	4.1274

Weekly data for the period from 22 May 2009 to 13 May 2016. Returns obtained following Equation (1.11).

[4] Euribor rates are obtained from the European Money Markets Institute (EMMI) and floored at 0%.

FIGURE 1.1: CDS spread and price following Berndt and Obreja (2010)



The red line represents CDS quotes. The blue line represents the price of an asset built using the exponential function of the returns from formula (1.11).
Both trends react with an opposite sign. The price seems to have a smoother path than the CDS quote. The value of time series is established in the initial date of the sample at 100. The vertical black line represents the week of 26 July 2012, when Mario Draghi made the speech that changed the trend of the CDS quote for Spain and Italy.

1.5 Results

I discuss the results for contagion between the financial and the sovereign sector by presenting first the results of the marginal distribution from which I obtain the inputs for the copula and from which I assess the quantile for the conditioning variable. Later I discuss the results for the copula estimations and copula choice, from which I assess the conditional quantile. Finally I discuss the Delta Conditional measures ($\Delta CoVaR$ and $\Delta CoES$), finding stress periods for these indicators using a Markov switching process. I employ bootstrap tests to check for a possible change in the distribution.

1.5.1 Results for the marginal models

Table 1.6 shows the estimated parameters with the z-statistics in brackets. A first order autoregressive model is employed for all the sectors except for Spanish and Italian sovereign CDS returns. A second order autoregressive model is employed considering the autocorrelation analysis and the backtesting performance for these countries . Unconditional coverage backtesting test proposed by Kupiec (1995) and the conditional one proposed by Christoffersen (1998) are used for testing the number of exceedances of a VaR with a 5% significance level. All the models pass the tests as show by their p-values, i.e. we do not reject neither that the probability of having an exceedance is a 5% nor that those exceedances are independent from each other. P-values for Ljung-Box and Engle's ARCH tests show that autocorrelation and heteroscedasticity are corrected gathered in the model.

TABLE 1.6: Estimates for the marginal distribution models.

	Financial sector	Austria	Belgium	France	Germany	Italy	Netherlands	Spain
ϕ_0	0.0007	0.0013	0.0019	0.0021	0.0017	0.0018	0.0009	0.0013
	(0.44)	(1.05)	(1.42)	(1.39)	(1.12)	(1.05)	(0.88)	(0.77)
ϕ_1	0.2542	0.2468	0.2418	0.2253	0.2603	0.2230	0.3035	0.2073
	(4.80)	(4.27)	(4.14)	(3.76)	(4.61)	(4.18)	(19.27)	(3.89)
ϕ_2	-	-	-	-	-	-0.1515	-	-0.1443
						(-2.91)		(-2.68)
ω	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0001	0.0000
	(2.21)	(0.66)	(1.33)	(1.14)	(0.47)	(0.57)	(1.56)	(1.02)
α	0.0000	0.2513	0.3164	0.2298	0.1208	0.0404	0.2297	0.0001
	(0.00)	(2.38)	(1.93)	(1.83)	(1.03)	(1.07)	(4.23)	(0.00)
β	0.8974	0.7486	0.6342	0.7070	0.8577	0.9134	0.6996	0.8980
	(70.62)	(7.57)	(4.43)	(4.75)	(8.60)	(11.36)	(12.92)	(11.42)
θ	0.0956	0.0000	0.0006	0.0000	0.0430	0.0552	0.1414	0.1073
	(1.79)	(0.00)	(0.00)	(0.00)	(0.35)	(0.62)	(6.25)	(1.33)
λ	-0.0962	0.0367	-0.0037	0.0111	0.0283	-0.0169	0.0274	-0.0750
	(-1.22)	(0.58)	(-0.06)	(0.14)	(0.40)	(-0.23)	(1.05)	(-0.95)
η	27.9570	4.6061	3.8584	4.5905	3.9826	4.7036	3.2816	7.0903
	(0.77)	(3.58)	(3.81)	(3.28)	(4.78)	(3.67)	(73.02)	(15.07)
$LogLike$	736.398	745.540	755.618	728.359	707.992	698.789	767.387	705.898
Kupiec (1995)	0.598	0.317	0.830	0.598	0.444	0.144	0.444	0.969
Christoffersen (1998)	0.131	0.760	0.786	0.935	0.111	0.065	0.469	0.177
LB	0.550	0.352	0.824	0.561	0.970	0.882	0.139	0.948
ARCH	0.783	0.340	0.291	0.624	0.839	0.751	0.983	0.451

Notes: The table provides information on maximum likelihood parameter estimates and z-statistics (in brackets) for the marginal models in Equation (1.5)-(1.6). *LogLike* stands for the log-likelihood value. Kupiec (1995) and Christoffersen (1998) denote p-values of the unconditional coverage test from Kupiec (1995) and the conditional coverage test from Christoffersen (1998) with a significance level of 5%. LB and ARCH refer to p-values of the Ljung-Box test for serial correlation with 20 lags and the Engle's Lagrange multiplier for ARCH effects in the first lag.

1.5.2 Results for the copula model

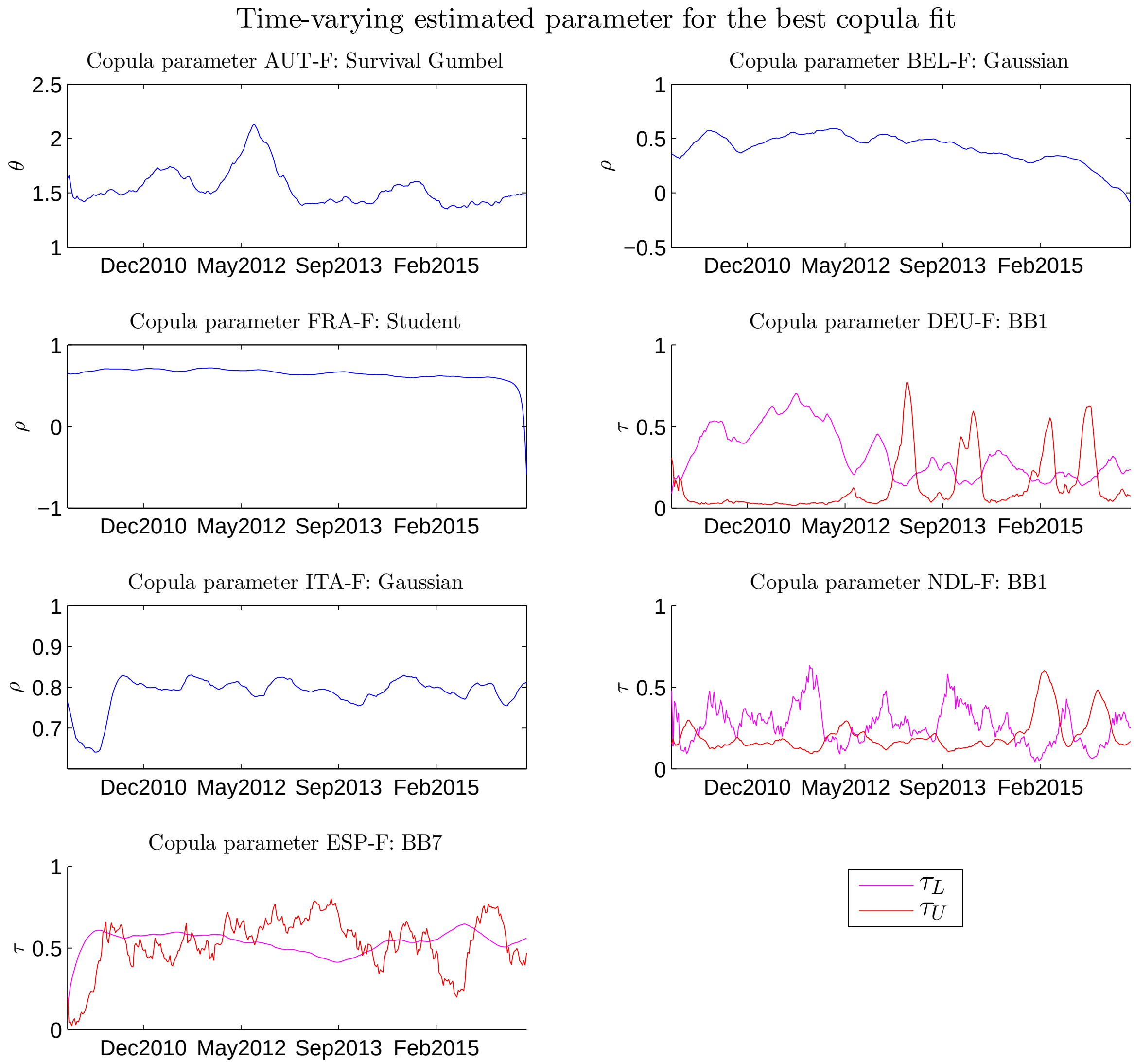
I estimate different types of copulas (see Table 1.1) using the skewed-t cumulative distribution function of the standardized residuals for each of the marginal models. Table 1.7 and 1.8 summarize the estimated parameters and the standard deviation between brackets.

The interpretation of those values is less straightforward than in a GARCH model due to the transformation done to keep the time-varying parameter in a region of the parameter space. However, the time-varying evolution of the copula parameter is plotted for the selected copulas according to the AICc criterion. Figure 1.2 shows the time-series evolution of the copula parameter between the financial sector and each one of the sovereign sectors. Austria clearly presents a peak in dependence after May 2012, whereas the sovereign sectors of France and Belgium reduce significantly their dependence parameter with the financial sector at the end of the sample.

The copula selection according to AICc criterion (see Table 1.9) could lead to choose a copula that fits very well the higher tail of the joint distribution but not so well for lower quantiles. To double check the chosen copulas, Table 1.10 presents backtesting results for the frequency of the exceedances below the 5% quantile of returns in conditioned institution y when there are exceedances below the ex-ante $VaR_x(0.5)$ of the conditioning. This corresponds to $CoVaR_{y|x,t}(0.5, 0.05)$ in Equation (1.2). Table 1.10 shows the p-values of the $CoVaR$ for the unconditional coverage test proposed by Kupiec (1995). Besides the p-values, the upper and lower bound of the non-rejection area with 5% significance level is presented jointly with the number of exceedances of the conditioned variable, i.e. the exceedance bounds out of which

we could reject the null hypothesis with a 95% confidence level. The number of exceedances for the conditioning variable is also shown in Table 1.10. Table 1.10 shows the p-values for the conditional coverage test proposed by Christoffersen (1998) to detect possible clusters in the exceedances of the conditioned variable.
Both backtesting tests, i.e. unconditioned and conditioned, are passed with a 5% significance level in both directions, i.e. $CoVaR_{s|f,t}(0.5, 0.05)$ and $CoVaR_{f|s,t}(0.5, 0.05)$. Further information about how these tests are built can be found in Apendix B.

FIGURE 1.2: Time-varying evolution of the copula parameter



Time-varying evolution of the copula parameter for the best copula fit according to AICc. AUT: Austria. BEL : Belgium. FRAN: France. DEU: Germany. ITA: Italy. NDL: Netherlands. ESP: Spain. F. European financial sector.

TABLE 1.7: Copula model estimates for financial and sovereign sectors' returns for the period 2009-2016. (I)

		Austria	Belgium	France	Germany	Italy	Netherlands	Spain
Clayton	ω	-0.33	-0.78	-0.40	-0.41	1.72	1.07	2.59
		(0.15)	(0.23)	(0.32)	(0.25)	(0.35)	(0.92)	(1.09)
	α	-2.18	-1.48	-0.82	-2.03	-6.74	-6.24	-7.43
		(0.25)	(0.41)	(0.19)	(0.11)	(0.33)	(1.07)	(0.51)
	β	0.70	1.01	0.61	0.76	-0.08	0.00	-0.54
		(0.22)	(0.29)	(0.19)	(0.14)	(0.16)	(0.16)	(0.15)
Gumbel	ω	1.95	-2.27	-1.99	-2.82	-0.60	-0.01	0.53
		(0.35)	(0.13)	(0.67)	(2.69)	(0.13)	(0.79)	(0.13)
	α	-9.39	-3.12	-0.78	-1.62	-2.04	2.13	-4.45
		(0.46)	(0.08)	(0.53)	(1.92)	(0.16)	(0.70)	(0.50)
	β	-0.48	1.39	1.05	1.60	0.49	-1.00	0.13
		(0.25)	(0.07)	(0.38)	(1.51)	(0.18)	(0.32)	(0.45)
Frank	ω	1.24	0.41	-0.28	0.36	1.96	3.95	8.95
		(0.35)	(0.33)	(0.01)	(20.13)	(0.06)	(1.06)	(0.35)
	α	-2.93	-1.10	1.01	-0.99	-3.55	4.79	-14.41
		(0.09)	(0.32)	(0.02)	(17.61)	(0.18)	(2.08)	(0.15)
	β	0.84	0.95	1.02	0.96	0.80	-0.51	-0.04
		(0.08)	(0.28)	(0.00)	(9.58)	(0.06)	(0.46)	(0.46)
BB7	ω_L	2.95	4.95	-1.69	5.81	-2.16	2.28	-2.02
		(0.06)	(0.00)	(0.20)	(0.00)	(0.11)	(1.87)	(0.06)
	α_L	-12.06	-20.30	-1.04	-24.28	0.59	-13.31	0.48
		(0.01)	(0.05)	(0.14)	(0.01)	(0.08)	(0.06)	(0.09)
	β_L	-2.59	-3.95	3.77	-4.07	4.14	-0.19	3.90
		(0.01)	(0.08)	(0.12)	(0.05)	(0.00)	(1.29)	(0.02)
	ω_U	0.77	1.78	0.60	2.74	-0.78	-3.20	2.66
		(0.08)	(0.01)	(0.05)	(0.01)	(0.12)	(1.44)	(0.18)
	α_U	-9.77	-13.89	-0.30	-12.79	-3.37	4.16	-16.17
		(0.08)	(0.07)	(0.21)	(0.01)	(0.10)	(0.46)	(0.02)
	β_U	1.13	-1.68	-2.12	-5.81	2.78	4.04	0.41
		(0.02)	(0.01)	(0.04)	(0.06)	(0.1)	(1.18)	(0.00)

Notes: The table provides information on maximum likelihood parameter estimates and standard deviation (in brackets) for the copula models in Equations (1.8), (1.9) and (1.10). Standard deviation of copula parameters has been computed following the sandwich form presented in Patton (2013).

TABLE 1.8: Copula model estimates for financial and sovereign sectors' returns for the period 2009-2016. (II)

		Austria	Belgium	France	Germany	Italy	Netherlands	Spain
Survival Gumbel	ω	-2.23	-0.70	-2.05	-2.28	0.60	-0.31	2.96
		(0.03)	(0.01)	(0.01)	(0.16)	(0.08)	(0.01)	(0.02)
	α	-1.57	-5.12	-0.58	-2.26	-4.75	-3.33	-7.27
		(0.02)	(0.03)	(0.01)	(0.01)	(0.44)	(0.01)	(0.03)
	β	1.24	0.63	1.07	1.35	0.13	0.15	-0.80
		(0.06)	(0.02)	(0.01)	(0.01)	(1.07)	(0.02)	(0.03)
Student t	η	9.46	19.69	4.16	10.73	99.67	41.62	11.33
		(0.21)	(0.00)	(0.01)	(0.00)	(0.13)	(0.00)	(0.00)
	ω	1.42	1.39	-0.53	2.27	1.25	1.35	1.93
		(0.31)	(0.00)	(0.01)	(0.01)	(0.04)	(0.01)	(0.06)
	α	0.64	-2.04	-0.37	-4.36	8.79	-0.98	3.55
		(0.24)	(0.23)	(0.03)	(0.01)	(0.19)	(0.02)	(0.02)
	β	-0.98	1.08	3.47	1.73	-4.31	0.34	-2.27
		(0.10)	(0.12)	(0.02)	(0.01)	(0.16)	(0.00)	(0.02)
BB1	ω_L	2.35	6.36	-1.77	-1.12	0.66	2.74	1.80
		(0.12)	(0.2)	(0.01)	(0.03)	(0.02)	(0.02)	(0.01)
	α_L	-10.90	-28.76	-0.96	-3.64	-6.83	-16.46	-4.95
		(0.12)	(0.20)	(0.01)	(0.03)	(0.11)	(0.01)	(0.01)
	β_L	-2.90	-4.93	3.88	3.56	0.79	-0.34	-2.33
		(0.01)	(0.12)	(0.02)	(0.01)	(0.03)	(0.02)	(0.02)
	ω_U	-0.60	1.10	-0.86	-5.74	-1.96	-3.12	0.35
		(0.07)	(0.08)	(0.01)	(0.02)	(0.03)	(0.01)	(0 (0.02)
	α_U	-5.27	-7.68	1.77	11.97	0.02	3.79	-5.86
		(0.01)	(0.07)	(0.00)	(0.04)	(0.00)	(0.01)	(0.04)
	β_U	2.36	-8.55	-0.39	4.32	3.92	4.04	1.19
		(0.01)	(0.04)	(0.01)	(0.02)	(0.07)	(0.01)	(0.00)
Gaussian	ω	-0.01	-0.05	2.24	0.16	-1.42	0.46	-0.33
		(0.03)	(0.07)	(0.08)	(0.12)	(0.03)	(0.04)	(0.02)
	α	0.20	0.13	0.87	0.25	0.11	0.17	0.28
		(0.01)	(0.09)	(0.09)	(0.02)	(0.04)	(0.01)	(0.01)
	β	2.08	2.16	-1.90	1.61	4.42	1.03	2.78
		(0.01)	(0.04)	(0.01)	(0.08)	(0.06)	(0.05)	(0.01)

Notes: The table provides information on maximum likelihood parameter estimates and standard deviation (in brackets) for the copula models in Equations (1.8), (1.9) and (1.10). Standard deviation of copula parameters has been computed following the sandwich form presented in Patton (2013).

TABLE 1.9: Value of the Akaike Information Criterion corrected for small sample bias for the considered copulas.

	Austria	Belgium	France	Germany	Italy	Netherlands	Spain
AICc							
Clayton	-106.67	-64.46	-170.21	-88.85	-242.52	-79.10	-216.72
Gumbel	-100.29	-47.99	-169.62	-74.43	-279.59	-66.11	-256.65
Frank	-107.79	-59.59	-172.07	-84.69	-270.14	-83.53	-231.55
BB7	-113.66	-64.68	-195.68	-94.36	-304.67	-82.87	**-282.81**
Survival Gumbel	**-117.45**	-65.21	-193.20	-96.62	-288.21	-84.55	-260.46
Student t	-111.81	-56.65	**-199.06**	-95.09	-306.85	-81.44	-268.17
BB1	-115.02	-67.96	-196.52	**-99.25**	-301.78	**-84.74**	-273.48
Gaussian	-116.39	**-68.54**	-184.78	-91.99	**-307.65**	-83.77	-273.13

Notes: *AICc* denotes Akaike Information Criterion corrected for small sample bias.
$AICc = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$ where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum *AICc* value (in bold) indicates the best copula fit.

TABLE 1.10: CoVaR backtesting

			Austria	Belgium	France	Germany	Italy	Netherlands	Spain
$CoVaR_{f\|s,t}(0.5, 0.05)$	Kupiek	pvalue	0.5635	0.8319	0.4980	0.5635	0.5750	0.3209	0.7244
		Lower bound	4	4	4	4	4	4	4
		Upper bound	14	14	15	14	14	14	13
		# exceedances	7	8	7	7	7	6	7
		# observations	172	172	178	172	171	174	159
	Christoffersen	pvalue	0.4394	0.3754	0.4476	0.4394	0.4380	0.5114	0.4204
		T_{00}	157	155	163	157	156	161	144
		T_{01}	7	8	7	7	7	6	7
		T_{10}	7	8	7	7	7	6	7
		T_{11}	0	0	0	0	0	0	0
$CoVaR_{s\|f,t}(0.5, 0.05)$	Kupiek	pvalue	0.4095	0.8454	0.1456	0.8755	0.8755	0.8454	0.5750
		Lower bound	4	4	4	4	4	4	4
		Upper bound	14	14	14	14	14	14	14
		# exceedances	11	8	13	9	9	8	7
		# observations	171	171	171	171	171	171	171
	Christoffersen	pvalue	0.2171	0.3740	0.1421	0.3157	0.3157	0.3740	0.4380
		T_{00}	148	154	144	152	152	154	156
		T_{01}	11	8	13	9	9	8	7
		T_{10}	11	8	13	9	9	8	7
		T_{11}	0	0	0	0	0	0	0

Kupiec refers to the unconditional coverage test from Kupiec (1995) whereas Christoffersen denotes the conditional coverage test from Christoffersen (1998).

Backtesting is drawing up employing observations where returns of the conditioning variable are lower than the minimum return according to $VaR_{x,t}(0.5)$. The number of observations that cross the threshold is showed in # *observations*. Given these observations, the backtesting for the conditioned variable is computed using those returns below the $CoVaR_{y|x,t}(0.5, 0.05)$.

Confidence interval for the null hypothesis is presented in the upper bound and lower bound rows. The actual number of exceedances is presented in # *exceedances*.

For the conditional coverage test by Christoffersen (1998) about $CoVaR$, T_{00} indicates the number of pairs of observation where no exceedance occurs neither in $t-1$ nor in t, T_{11} shows the number of pairs of observation where an exceedance occurs in $t-1$ and in t, T_{01} shows the number of pairs of observation where an exceedance occurs in t but no in $t-1$, and T_{10} indicates the number of pairs of observation where an exceedance occurs in $t-1$ but no in t.

1.5.3 Contagion indicator results

Figures 1.3 and 1.4 present weekly returns for the sovereign sector and the financial sector (in the blue lines), the Value-at-Risk assessed with a 95% confidence level (in the black lines), and the minuend and subtrahend from which is built the $\Delta CoVaR_{y|x}(\beta)$, i.e. the VaR for the conditioned institution when the conditioning institution is in normal times or $CoVaR_{y,x,t}(\alpha_n, \beta)$ (in the magenta lines) and the VaR for the conditioned institution when the conditioning institution is in distress or $CoVaR_{y,x,t}(\alpha_s, \beta)$ (in the red lines). All the risk measures are computed with a 95% confidence level, i.e., $\beta = 0.05$.

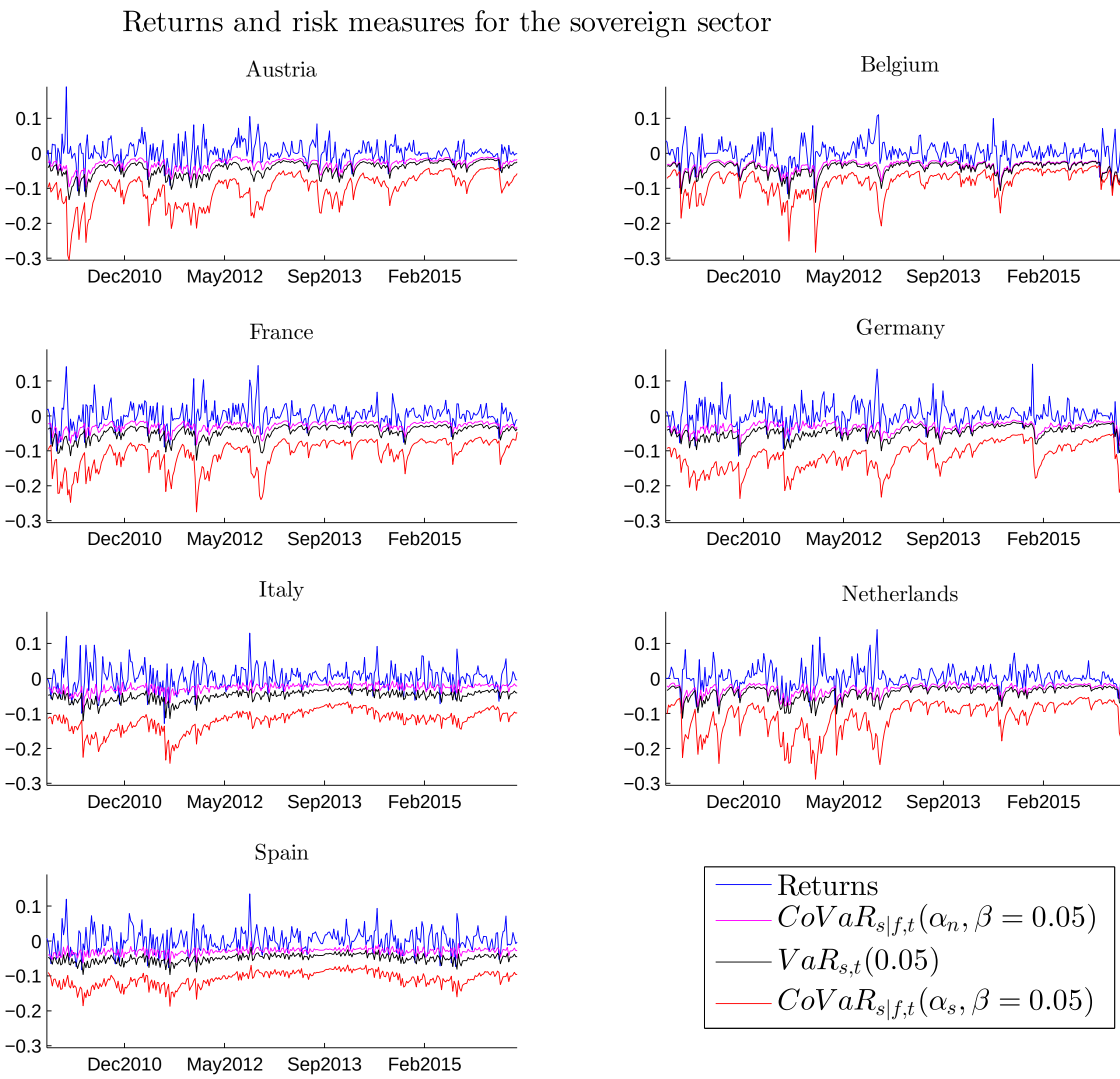
I employ a Markov switching model which endogenously identifies periods of extreme contagion for $\Delta CoVaR$ measures. The assumption that time series properties of $\Delta CoVaR$, e.g. mean and variance, are state-dependent where the transition between states occurs stochastically allows us to distinguish periods of high contagion from periods of moderate contagion. In other words, the cumulative distribution function for each $\Delta CoVaR$ measure is approximated using a mixture of normal distributions where probabilities are given by a first order Markov Chain. The number of states assumed may be two or three because of the economical interpretation of low, medium or high contagion regime. The number of states and the changes in parameters, i.e. changes in the mean parameter or also in the variance parameter, are chosen according to the values of AICc and Regimen Classification Measure (RCM) like Hollo et al. (2012). I choose the regime with the lowest mean $\Delta CoVaR$

as the distress period. Figures 1.5 and 1.6 show these periods of crises shaded for the contagion measures $\Delta CoVaR$ and $\Delta CoES$. Figures 1.5 and 1.6 show that are two main periods of stress, one around March 2010 when the Greek debt crisis arose and a second one around August 2011 when the ECB had to intervene in the Italian and Spanish debt markets using the Securities Market Program. The lack line represents 26 July 2012 when Mario Draghi made his speech[5]. Note that distress periods after that date are less frequent and with a lower length than before, but according to the Markov switching approach the contagion periods did not stop suddenly for most of the countries. Moreover, the high contagion period from the sovereign sector to the financial sector seems to end earlier than the contagion period from the financial sector to the sovereign sector.

I perform some bootstrap tests to check the robustness of the change in the contagion risk indicators since 26 July 2012. First, I delete autocorrelation in the $\Delta CoVaR$ time-series by orthogonalize it from previous month values, so the new time series can not be explained by the past trend of the indicator. The estimation of the parameters of $\Delta CoVaR$ introduces a nuisance parameter that invalidates the Kolmogorov-Smirnov (KS) test for equal distribution of the time-series before and after the breakpoint. Certainly, this nuisance parameter affects to the free-distribution of the KS test (Durbin, J. (1973)). Abadie (2002) proposes a bootstrap KS test to deal with this problem. Bernal et al. (2014) and Reboredo and Ugolini (2015a) along others use this test in the $CoVaR$ framework. I employ the bootstrap procedure for building also a mean test, obtaining the critical values through the bootstrap method. Appendix C explains in detail the procedure for building these tests.
The null hypothesis of mean before 26 July 2012 is lower or equal than after in absolute value is rejected at a 5% significance level for most contagion indices (Table 1.11). There is enough statistical evidence against the null hypothesis that the mean contagion levels between the financial and the sovereign sectors before 26 July 2012 are lower or equal than after this breakpoint.

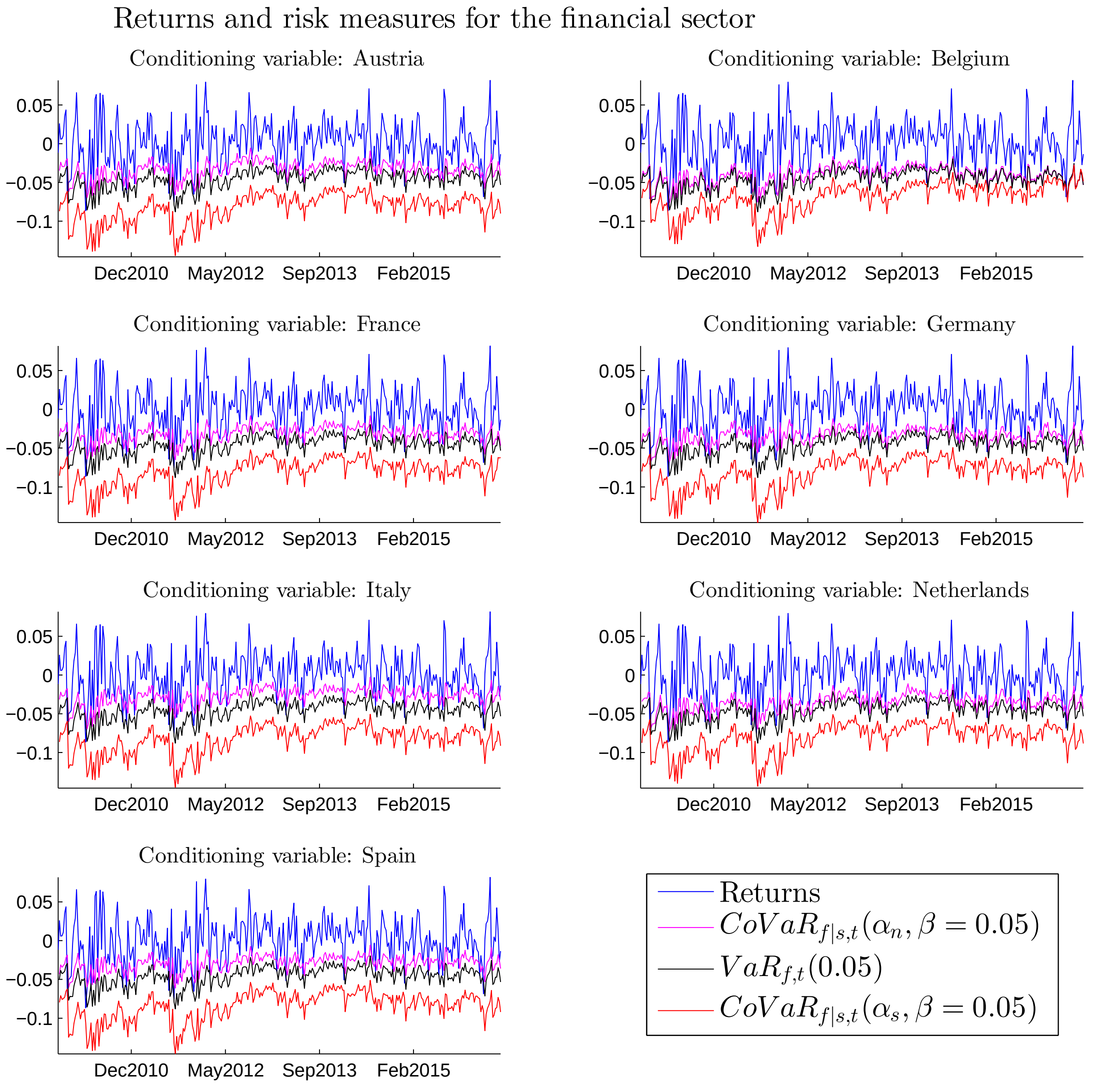
[5]This date was chosen ad hoc due to the impact in the CDS quotes and in the stock markets (Georgiadis and Gräb 2016)

FIGURE 1.3: Weekly sovereign CDS returns and risk measures



Time series plots for weekly sovereign CDS returns for the period 2009-2016 (in blue). Risk measures such as Value-at-Risk with a significance level of 5% (in black), *CoVaR* for the sovereign sector with the same significance level when the financial sector is distress (in red) and when there are normal times for the financial sector (in magenta).

FIGURE 1.4: Weekly CDS returns from financial sector and risk measures



Time series plots for weekly CDS returns from financial sector for the period 2009-2016 (in blue). Risk measures such as Value-at-Risk with a significance level of 5% (in black), *CoVaR* for the financial sector with the same significance level when the sovereign sector *s* is distress (in red) and when there are normal times for the sovereign sector *s* (in magenta).

TABLE 1.11: Boostrap pvalues

Country-Financial sector	Measure	$t1$	$t2$	$KS1$	$KS2$
Austria	F → S	0.0276	0.0135	0.0226	0.0099
	S → F	0.0104	0.0047	0.1372	0.3018
Belgium	F → S	0.0514	0.0264	0.0018	0.0011
	S → F	0.1770	0.0811	0.0002	0.0002
France	F → S	0.0832	0.0358	0.0014	0.0006
	S → F	0.1608	0.0796	0.0017	0.0007
Germany	F → S	0.0884	0.0427	0.0354	0.6281
	S → F	0.0178	0.0102	0.5345	0.3739
Italy	F → S	0.1202	0.0584	0.0014	0.0008
	S → F	0.0352	0.0188	0.0146	0.0081
Netherlands	F → S	0.0160	0.0110	0.0821	0.0377
	S → F	0.1022	0.0521	0.0536	0.0626
Spain	F → S	0.0878	0.0451	0.0039	0.0020
	S → F	0.0280	0.0138	0.0006	0.0004

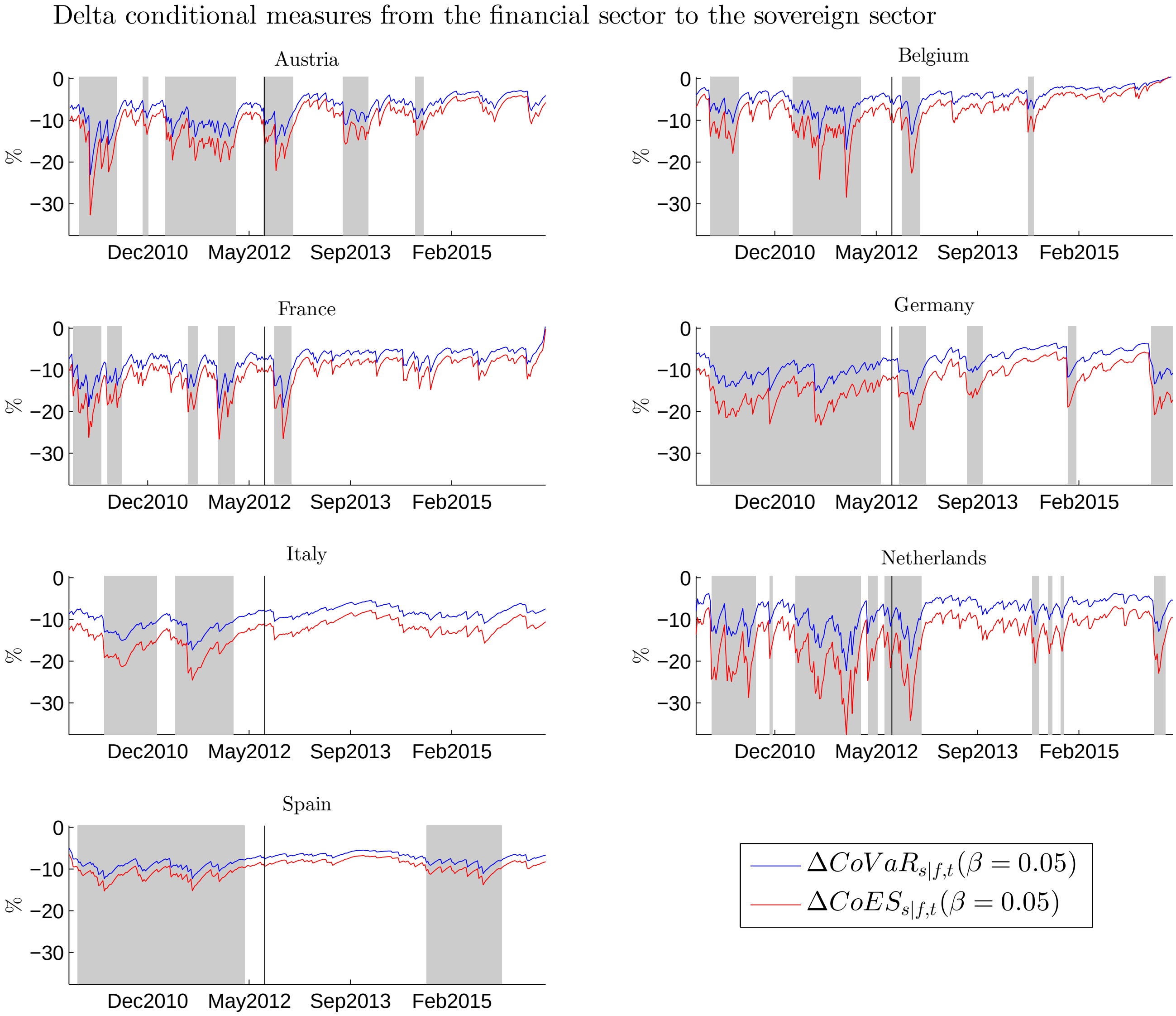
F → S stands for the orthogonalized $-\Delta CoVaR_{s|f,t}(0.05)$ whereas S → F indicates the orthogonalized $-\Delta CoVaR_{f|s,t}(0.05)$. $\Delta CoVaR_{f|s,t}(0.05)$ is multiplied by minus one in order to speak about levels of contagion, and due to orthogonalization the $\Delta CoVaR$ is not explained by its previous month. The chosen breakpoint is 26 July 2012. All p-values are obtained using a bootstrap procedure explained in C using $B = 10000$ simulations.

$t1$ shows the p-vale of t test where the null hypothesis is that the mean of level of contagion not explained by the previous month is the same before and after the breakpoint, i.e. $H_0 : \mu_B = \mu_A$ and $H_1 : \mu_B \neq \mu_A$ whereas the alternative hypothesis in $t2$ is $H_1 : \mu_B > \mu_A$.

$KS1$ shows the p-value of the Kolgomorov Smirnov test where the null and alternative hypothesis are $H_0 : F_B(z) = F_A(z) \forall z$ and $H_0 : F_B(z) \neq F_A(z) \forall z$ where B is the level of contagion not explained by the previous month before 26 July 2012 and A is the same variable after that date.

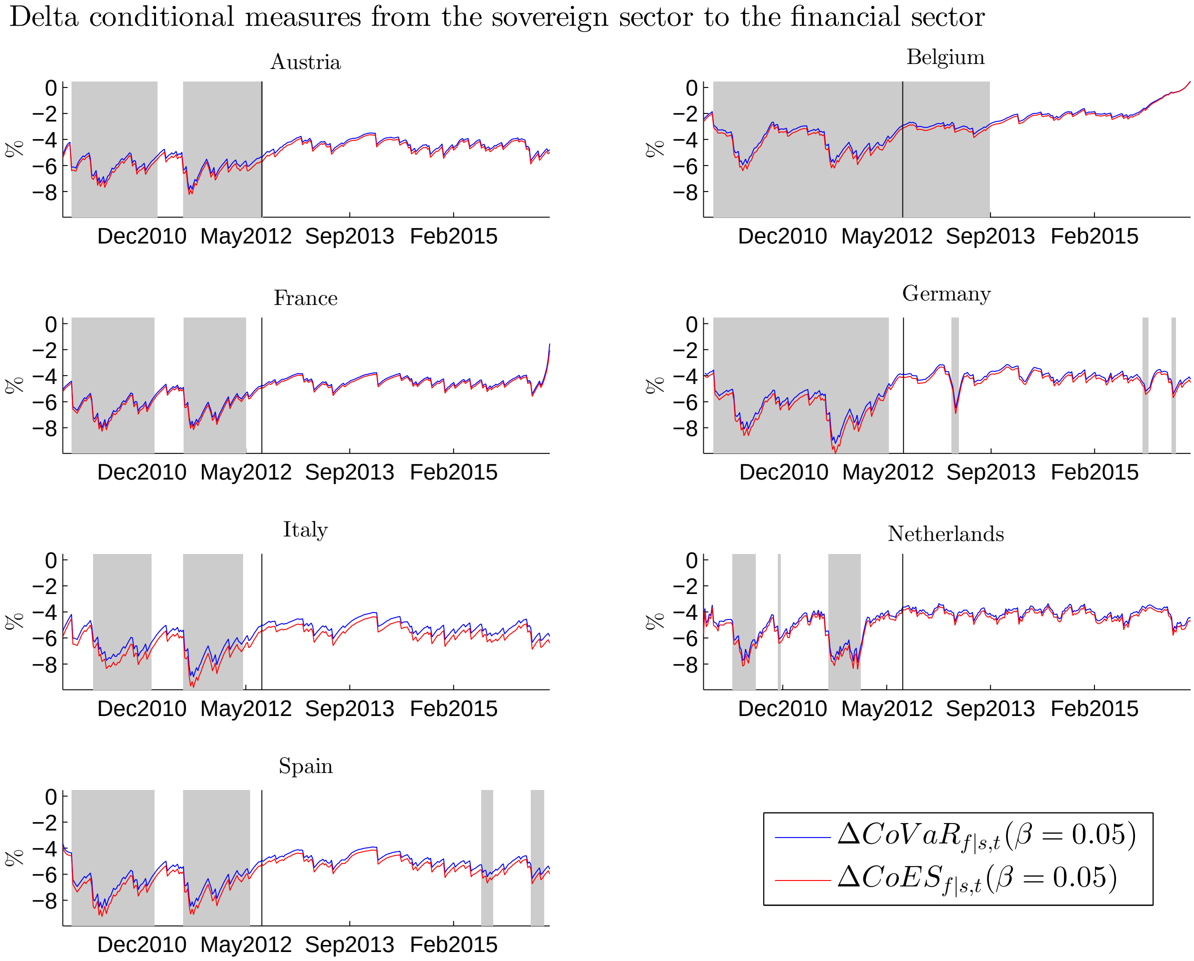
$KS2$ indicates the p-value to the Kolgomorov Smirnov test where the alternative hypothesis is the first-order stochastic dominance of the distribution of the contagion before the breakpoint over the distribution of the contagion after the breakpoint, i.e., $H_1 : F_B(z) < F_A(z) \forall z$. In order words, for any level of contagion z it would be a more extreme scenario after that before the breakpoint.

FIGURE 1.5: Time-varying evolution of the contagion measures from the financial sector to the sovereign sector



Delta conditional measures from the financial sector to the sovereign sector ($\Delta CoVaR_{s|f}(\beta = 0.05)$ and $\Delta CoES_{s|f}(\beta = 0.05)$) show the evolution of contagion from the financial sector to each European country. Distress periods (grey areas) are obtained from a Markov switching model.

FIGURE 1.6: Time-varying evolution of the contagion measures from the sovereign sector to the financial sector



Delta conditional measures from the sovereign sector to the financial sector ($\Delta CoVaR_{f|s}(\beta = 0.05)$ and $\Delta CoES_{f|s}(\beta = 0.05)$) show the evolution of contagion from each country to the European financial sector. Distress periods (grey areas) are obtained from a Markov switching model.

1.6 Conclusions

I have examined how contagion indices between the European financial sector and sovereign sector in a country level (Austria, Belgium, France, Germany, Italy, Netherlands, Spain) evolved during the European sovereign credit crisis, before and after the breakpoint stated by the ECB's speech on 26 July 2012. In this article *CoVaR* measure is employed with a copula methodology to assess the relationship between sovereign and financial credit risk. The economic literature has not employed yet this approach to deal with the spillovers between sovereign and financial credit risk. This approach is a robust way of measuring systemic risk focusing on a low quantile of the returns distribution.

The copula methodology allows us to decompose the joint distribution in an understandable way, besides of being a time-saving and less computationally expensive method than other procedures. The time-varying representation of the copula parameter allows for a flexible adaptation to the sample, capturing changes in tail dependence and contagion levels. Indeed, the copula parameter plays a key role weighting the level of stress for each assets, as measured by its volatility, by its dependence with the other sector. Future studies might try to find out how different dynamics for the time evolution of the copula parameter might affect to the numerical values of $\Delta CoVaR$ and $\Delta CoES$. These methodological improvements could increase model risk due to the high adaptability of the model. The possibility of more flexibility could mean a noisy estimation.

Using weekly CDS returns from May 2009 to May 2016, a Markov switching model estimated for $\Delta CoVaR$ indicates two main periods of contagion between both sectors. The first period would be related to the surge of Greek imbalances in March 2010 while the second period in summer 2011 might be due to the doubts concerning Spain and Italy. The contagion from sovereign to financial crisis seems to almost finish after the ECB's speech on 26 July 2012 . On the other hand, contagion from the financial to the sovereign sector seems to decrease more slowly. Several bootstrap tests are computed to check if there was a change in contagion after this breakpoint. Results show a change in the level and range of values taken by $\Delta CoVaR$.

Policy makers need indicators for assessing the effectiveness and the collateral detrimental effects that some policy measures can have on the economy. The contagion indicators built using $CoVaR$ methodology have implications for investors, who need a risk management tool to assess the exposure of their sectoral portfolios from undesired links with other sectors which are not taken into by unconditional risk measures. For instance, this measure provides information with a certain confidence level on how much could increase the maximum loss from a sovereign debt portfolio if a financial crisis occurs. $\Delta CoVaR$ and $\Delta CoES$ provide support as tools for the measurement of contagion and spillover effects, make them suitable to provide reliable information for effective and efficient risk management.

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Appendix

A Set of Copulas

The demonstration of some of the hereinbelow formulas can be seen in Karimalis and Nomikos (2014) and in Bernardi et al. (2017).
For the following equations $F_{\xi_{x,t}}(\xi_{x,t}) = u_{x,t}$ and $F_{\xi_{y,t}}(\xi_{y,t}) = u_{y,t}$.

Clayton copula. This copula allows positive dependence and asymmetric lower tail dependence. The Clayton copula has a dependence parameter $\theta \in (0, +\infty)$. When $\theta \to 0$ implies independence and when $\theta \to \infty$ implies perfect dependence. The uniform value u_y^* is given by the following formula

$$u_y^* = \left(1 + (\alpha\beta)^{-\theta} - \alpha^{-\theta}\right)^{-\frac{1}{\theta}}.$$

and the copula density function according to Equation

$$c(u_{x,t}, u_{y,t}; \theta) = (\theta + 1)\left(u_{x,t}^{-\theta} + u_{y,t}^{-\theta} - 1\right)^{-2-\frac{1}{\theta}} (u_{x,t}u_{y,t})^{-\theta-1}.$$

Gumbel copula. This copula allows positive dependence and asymmetric upper tail dependence. The Gumbel copula has a dependence parameter $\theta \in [1, +\infty)$. When $\theta = 1$ implies independence and when $\theta \to \infty$ implies perfect dependence. The analytical expression for the conditional quantile U_y^* is

$$u_y^* = \exp\left(-\left[(-\log(\alpha\beta))^{\theta} - (-\log\alpha)^{\theta}\right]^{\frac{1}{\theta}}\right).$$

and the copula density function for the log likelihood procedure is

$$\begin{aligned} c(u_{x,t}, u_{y,t}; \theta) &= (A + \theta - 1)\, A^{1-2\theta} \exp(-A) \\ &\quad (u_{x,t}u_{y,t})^{-1}(-\log u_{x,t})^{\theta-1}(-\log u_{y,t})^{\theta-1}, \end{aligned}$$

where $A = \left[(-\log u_{y,t})^{\theta} + (-\log u_{x,t})^{\theta}\right]^{\frac{1}{\theta}}$.

Frank copula. This copula allows positive and negative dependence structures without implying tail dependence. The Frank copula has a dependence parameter $\theta \in (-\infty, +\infty)\backslash\{0\}$. When $\theta \to 0$ implies independence, when $\theta \to \infty$ implies positive perfect dependence and when $\theta \to -\infty$ implies negative perfect dependence. The expression of the conditional quantile u_y^* is

$$u_y^* = -\frac{1}{\theta}\log\left(1 - \frac{(1-\exp(-\theta)) - (1-\exp(-\theta))(\exp(-\theta\beta\alpha))}{(1-\exp(-\theta\alpha))}\right).$$

and the copula density function is

$$c(u_{x,t}, u_{y,t}; \theta) = \frac{\theta(1-\exp(-\theta))\exp(-\theta(u_{x,t}+u_{y,t}))}{\left(1-\exp(-\theta)-(1-\exp(-\theta u_{x,t}))(1-\exp(-\theta u_{y,t}))\right)^2}.$$

BB7 copula. This copula is also known as Joe-Clayton copula [6]. This is a copula with parameters $\theta \geq 1$ and $\delta > 0$, where θ measures upper tail dependence and δ measures lower tail dependence. The Joe-Clayton copula captures positive dependence while it allows for asymmetric upper and lower tail dependence. When $\delta \to 0$ the Joe copula is obtained and Clayton copula is the resulted one when $\theta = 0$.
The conditioned quantile u_y^* is

$$u_y^* = \psi^{-1}\left[\psi\left(\beta\alpha\right) - \psi\alpha\right],$$

where $\psi(x;\theta,\delta) = [1-(1-x)^\theta]^{-\delta} - 1$ and $\psi^{-1}(x;\theta,\delta) = 1 - [1-(1+x)^{-\frac{1}{\delta}}]^{\frac{1}{\theta}}$.
The copula density function for the log likelihood procedure is

$$\begin{aligned} c(u_{x,t}, u_{y,t}; \theta, \delta) &= \left[T_1(u_{x,t})T_1(u_{y,t})\right]^{-1-\delta} T_2(u_{x,t})T_2(u_{y,t}) \\ &\quad L_1^{-2(1+\delta)/\delta}(1-L_1^{-1/\delta})^{1/\theta-2}\left[(1+\delta)\theta L_1^{1/\delta} - \theta\delta - 1\right], \end{aligned}$$

where $T_1(s) = 1-(1-s)^\theta$, $T_2(s) = (1-s)^{\theta-1}$ and $L_1 = T_1(v)^{-\delta} + T_1(s)^{-\delta} - 1$.

Survival Gumbel copula. The Gumbel copula has a asymmetric dependence in the tails. Actually, it has no tail dependency in the lower tail but positive dependence in the upper tail when the parameter $\theta > 1$. The opposite tail dependence is obtained if the copula is rotated 180°. If (U_x,U_y) has a copula $C_\theta(u_x, u_y)$, then ($1-U_x$,$1-U_y$) is distributed according to the survival copula $C_\delta^R(u_x, u_y)$.

$$\begin{aligned} C^R_{Gumbel}(u_x, u_y; \theta) &= u_x + u_y - 1 + C_{Gumbel}(1-u_x, 1-u_y; \theta) \\ &= u_x + u_y - 1 + \exp\{-[(-\log(1-u_x))^\theta + (-\log(1-u_y))^\theta]^{1/\theta}\}. \end{aligned}$$

The 180° rotated Gumbel copula is not an Archimedean copula so there is not closed form expression for the conditional quantile like in the Gumbel copula.
In order to estimate the parameter θ we need to employ the copula density function, which can obtained from the Gumbel copula, i.e.,

$$c^S_{Gumbel} = c_{Gumbel}(1-u_x, 1-u_y) \tag{1.12}$$

Following Reboredo and Ugolini (2015a) two additional copulas are also considered: the t-Student copula and the Clayton-Gumbel copula (BB1 copula).

Student's t copula. This copula allows positive and negative symmetric tail dependence. The parameter ρ measures correlation and the parameter η, the degrees of freedom, controls the probability mass assigned to extreme joint co-movements of

[6] See De Luca and Rivieccio (2012) for more information about this copula.

risk factors changes[7]. When $\eta \to \infty$ corresponds to the Gaussian copula[8]. Due to the fact that the t-Student copula is an implicit copula, we can not obtain a close form conditioned quantile. Given the conditional copula $C_{y|x}(\alpha|u_x)$ we can get the conditional quantile from the following formula

$$\overbrace{\int_0^{u_y^*} \underbrace{C_{x|y}(\alpha|s)}_{P[F_{\xi_{x,t}}(\xi_{x,t})<\alpha|F_{\xi_{t,t}}(\xi_{t,t})=s]} ds}^{P[F_{\xi_{x,t}}(\xi_{x,t})<\alpha, F_{\xi_{y,t}}(\xi_{y,t})<u_y^*]} = \beta\alpha,$$

where $C_{x|y}(\alpha|s;\eta,\rho) = T_{\eta+1}\left(\sqrt{\frac{\eta+1}{\eta+(T_\eta^{-1}(s))^2}}\frac{T_\eta^{-1}(\alpha)-\rho T_\eta^{-1}(s)}{\sqrt{1-\rho^2}}\right)$, T_η is the cdf of a t-Student with η degrees of freedom and T_η^{-1} represents it inverse[9].
For estimating the degrees of freedom (η) and the correlation parameter ρ the copula density function is employed, i.e.,

$$\begin{aligned} c(u_{x,t}, u_{y,t});\eta,\rho) &= K\frac{1}{\sqrt{1-\rho^2}} \\ &\left[1+\frac{T_\eta^{-1}(u_{x,t})^2 - 2\rho T_\eta^{-1}(u_{x,t})T_\eta^{-1}(u_{y,t}) + T_\eta^{-1}(u_{y,t})^2}{\eta(1-\rho^2)}\right]^{-\frac{\eta+2}{2}} \\ &\left[(1+\eta^{-1}T_\eta^{-1}(u_{x,t})^2)(1+\eta^{-1}T_\eta^{-1}(u_{y,t})^2)\right]^{\frac{\eta+1}{2}}, \end{aligned}$$

where $K = \Gamma(\frac{\eta}{2})\Gamma(\frac{\eta+1}{2})^{-2}\Gamma(\frac{\eta+2}{2})$.

BB1 copula. The BB1 copula, also known as the Clayton-Gumbel copula, allows asymmetric tail dependence. The BB1 copula has two dependence parameters: one for the Clayton behavior $\theta \in (0, +\infty)$ and another one for the Gumbel behavior $\delta \in [1, +\infty)$. When $\delta = 1$ and $\theta > 0$ we get the Clayton copula and as a consequence upper tail independence and lower tail dependence. When $\theta \to 0$ and $\delta > 0$ the Gumbel copula is obtained with upper tail dependence only. In the case of $\theta \to 0$ and $\delta = 1$ we get upper and lower tail independence[10].
The expression of the conditional quantile u_y^* is

$$u_y^* = \left[\left\{\left[(\beta\alpha)^{-\theta}-1\right]^\delta - (\alpha^{-\theta}-1)^\delta\right\}^{\frac{1}{\delta}} + 1\right]^{-\frac{1}{\theta}}.$$

and the copula density function[11] is

$$c(u_{x,t}, u_{y,t};\theta,\delta) = (u_{x,t}u_{y,t})^{-\theta-1}(ab)^{\delta-1}c^{\frac{1}{\delta}-2}d^{-\frac{1}{\theta}-1}\left\{d^{-1}c^{\frac{1}{\delta}}(1+\theta)+\theta(\delta-1)\right\} \quad (1.13)$$

where $a = u_{x,t}^{-\theta} - 1$, $b = u_{y,t}^{-\theta} - 1$, $c = a^\delta + b^\delta$ and $d = 1 + c^{\frac{1}{\delta}}$.

[7]For more information about the properties of the t-Student copula see Demarta and McNeil (2005)
[8]The Gaussian copula underestimates the probability of joint extreme co-movements in high volatility and correlation scenarios according to Aussenegg and Cech (2011)
[9]See for instance Cech (2006)
[10]See for instance Venter (2002) or Nicoloutsopoulos (2005)
[11]See Cech (2006)

Gaussian copula. This copula has a parameter ρ that gathers linear correlation, when $\rho = 1$ the tail dependence is 1, otherwise this copula doesn't present tail dependence. Due to the fact that Gaussian copula is implicit, there is not a closed form expression. The copula probability density function is equal to the density of the Gaussian multivariate distribution divided by the products of the densities of its marginals, i.e.,

$$c(u_x, u_y); \rho) = \frac{\frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{ -\frac{\Phi^{-1}(u_x)^2 - 2\rho\Phi^{-1}(u_x)\Phi^{-1}(u_y) + \Phi^{-1}(u_y)^2}{2(1-\rho^2)} \right\}}{\phi(\Phi^{-1}(u_x))\phi(\Phi^{-1}(u_y))},$$

where Φ^{-1} stands for the Gaussian inverse cumulative distribution function and ϕ represents the Gaussian probability distribution function. Given the conditional copula $C_{x|y}(\alpha|u_y)$ we can get the conditional quantile from the following formula

$$\overbrace{\int_0^{u_y} \underbrace{C_{x|y}(\alpha|s)}_{P[F_{\xi_{x,t}}(\xi_{x,t})<\alpha|F_{\xi_{y,t}}(\xi_{y,t})=s]} ds}^{P[F_{\xi_{x,t}}(\xi_{x,t})<\alpha, F_{\xi_{y,t}}(\xi_{y,t})<u_y]} = \beta\alpha,$$

where $C_{x|y}(\alpha|s;\rho) = \Phi\left(\frac{\Phi^{-1}(\alpha) - \rho\Phi^{-1}(s)}{\sqrt{1-\rho^2}}\right)$, Φ stands for the normal cumulation distribution and Φ^{-1} is its inverse. Meyer (2013) takes a in-depth look at this copula.

B Backtesting procedure on CoVaR

The proportion of exceedances over the threshold of the CoVaR should equal approximately the significance level and they should take place independently, not in clusters. Consequently to check the accuracy of the proposed model we can use the statistical tests for unconditional coverage from Kupiec (1995) and the conditional coverage from Christoffersen (1998). The null hypothesis of the unconditional and conditional coverage is performed at 5% level of significance under skewed-t margins and the best fit according to the corrected Akaike Information Criterion (AICc).

For the conditioned institution in $CoVaR_{y|x}(\alpha, \beta)$ I build the indicator function that takes the value one if past ex-post losses of x cross the ex-ante VaR forecast and zero otherwise, i.e.,

$$\mathbb{1}_{x,t} = \begin{cases} 1 & if \quad r_{x,t} \leq VaR_{x,t}(\alpha) \\ 0 & if \quad r_{x,t} > VaR_{x,t}(\alpha) \end{cases}.$$

For those days t where $\mathbb{1}_{x,t} = 1$ I use a second indicator function that values one if past ex-post losses of y cross the ex-ante $CoVaR$ forecast and zero otherwise, i.e.,

$$\mathbb{1}_{j|l,t} = \begin{cases} 1 & if \quad r_{y,t} \leq CoVaR_{y,t}(\alpha, \beta) \\ 0 & if \quad r_{y,t} > CoVaR_{y,t}(\alpha, \beta) \end{cases}.$$

For this last hit sequence I have $T_{\mathbb{1}_{x,t}=1}$ observations , i.e., the observations where $r_{x,t} \leq VaR_{x,t}$.Consequently, to build the backtesting procedure I only employ $T_{\mathbb{1}_{x,t}=1}$ observations and not all the sample as in the backtesting procedure on VaR.

Unconditional coverage test from Kupiec (1995). The proportion of exceedances over the threshold is equal to the significance level if $CoVaR_{y|x}(\alpha, \beta)$ satisfies the unconditional coverage property, i.e. $P(\mathbb{1}_{y|x,t+1} = 1) = \beta$. Consequently the null and alternative hypothesis in this test would be

$$\begin{cases} H_0 : E[\mathbb{1}_{y|x,t}] \equiv p = \beta, \\ H_1 : E[\mathbb{1}_{y|x,t}] \equiv p \neq \beta. \end{cases}$$

Let us define $X = \sum_{t=1}^{T_{\mathbb{1}_{x,t}=1}} \mathbb{1}_{y|x,t}$, then the likelihood ratio of Kupiec (1995) is given by

$$LR = \frac{p^X (1-p)^{T_{\mathbb{1}_{x,t}=1} - X}}{\left(\frac{T_{\mathbb{1}_{x,t}=1} - X}{T_{\mathbb{1}_{x,t}=1}}\right)^{T_{\mathbb{1}_{x,t}=1} - X} \left(\frac{X}{T_{\mathbb{1}_{x,t}=1}}\right)^{X}},$$

where $-2\log(LR) \sim \chi_1^2$ under the null hypothesis.

Conditional coverage test from Christoffersen (1998). The expected proportion of exceedances over the threshold at $t+1$ is independent to the proportion of exceedances at t to satisfy the conditional coverage property, $P_t(\mathbb{1}_{y|x,t+1} = 1) = \beta$. Given the assumption that $\mathbb{1}_{x|l,t}$ follows a first-order Markov sequence with transition probability matrix

$$P_1 = \begin{bmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{bmatrix},$$

where $p_{k,q}$ indicate the probability of having in t+1 $\mathbb{1}_{y|x,t+1} = q$ conditional to the scenario on t where $\mathbb{1}_{y|x,t} = k$ with $q, k = 0, 1$. The probability of a exception in t+1 doesn't depend on the fact of having an exception on t if the conditional coverage property is satisfied, i.e. $P_t(\mathbb{1}_{y|x,t+1} = 1) = P(\mathbb{1}_{y|x,t+1} = 1)$. In conclusion, the null and the alternative hypothesis are

$$\begin{cases} H_0 : E[\mathbb{1}_{y|x,t}] \equiv p = p_{01} = p_{11}, \\ H_1 : E[\mathbb{1}_{y|x,t}] \equiv p \neq p_{01} = p_{11}, \end{cases}$$

Given the fact that there are $T_{\mathbb{1}_{x,t}=1}$ observations, a total of $T^{pair}_{\mathbb{1}_{x,t}=1} \equiv T_{\mathbb{1}_{x,t}=1} - 1$ pair of observations can be obtained. The sample of pair of observations can be divided in four subsamples, i.e.

$$T^{pair}_{\mathbb{1}_{x,t}=1} = T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1},$$

where the superscripts indicate that if there was an exceedance at periods t and $t-1$ and the subscript indicate that all the observations hold $r_{x,t+1} \leq VaR_{x,t+1}$.
Defining

$$\hat{p}_{01} = \frac{T^{pair,01}_{\mathbb{1}_{x,t}=1}}{T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1}},$$

and

$$\hat{p}_{11} = \frac{T^{pair,11}_{\mathbb{1}_{x,t}=1}}{T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}},$$

H_0 holds if $\hat{p}_{01} \approx \hat{p}_{11}$, as a consequence the probability of having an exceedance in $t+1$ could be defined without taking into account the scenario in t, i.e.,

$$\hat{p} = \frac{T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}}{T^{pair,00}_{\mathbb{1}_{x,t}=1} + T^{pair,01}_{\mathbb{1}_{x,t}=1} + T^{pair,10}_{\mathbb{1}_{x,t}=1} + T^{pair,11}_{\mathbb{1}_{x,t}=1}}.$$

The likelihood ratio of Christoffersen (1998) is employed, i.e.

$$\begin{aligned} LR \quad = \quad & \left(\frac{\hat{p}}{\hat{p}_{01}}\right)^{T^{pair,01}_{\mathbb{1}_{x,t}=1}} \left(\frac{\hat{p}}{\hat{p}_{11}}\right)^{T^{pair,11}_{\mathbb{1}_{x,t}=1}} \\ & \left(\frac{1-\hat{p}}{1-\hat{p}_{01}}\right)^{T^{pair,00}_{\mathbb{1}_{x,t}=1}} \left(\frac{1-\hat{p}}{1-\hat{p}_{11}}\right)^{T^{pair,10}_{\mathbb{1}_{x,t}=1}}, \end{aligned}$$

where $-2\log(LR) \sim \chi^2_1$. The frequency with which consecutive exceedances are observed may be few due to the fact that they are rare events, as a consequence the power of this test is limited.

C Bootstrap tests

In this section I briefly present the steps followed to build the bootstrap tests. The main reason to build bootstrap tests in estimated measures is due to the introduction of a nuisance parameter in the sample distribution. We estimate the model parameters to build the systemic measure and because of that, the distribution under the null hypothesis may be different, affecting to the confidence interval and the p-values. Durbin, J. (1973) points out the effect of estimated parameters in Kolgomorov Smirnov test. Abadie (2002) employs a bootstrap procedure to build a Kolgomorov Smirnov test when there are estimated parameters and Bernal et al. (2014) employs it in the $CoVaR$ framework. I extend the bootstrap tests on $CoVaR$ framework to test a change in the mean.
The standard parametric t test assumes normality and homocedasticity in the sample distribution. In a bootstrap procedure it is not necessary to make any assumption about the sample distribution, beyond the independence of the observations, because we do not use a theoretical probability distribution but the sample distribution under the null hypothesis. The following subsections show the steps in order to obtain the bootstrap p-values.

Bootstrap t test Given two samples x and y with size n_x and n_y:

Step 1 : Substract the mean for each sample and add the joint mean, i.e., $\tilde{x} = x - \bar{x} + \bar{z}$ and $\tilde{y} = y - \bar{y} + \bar{z}$ where $\bar{x}$ is the mean of the sample x, $\bar{y}$ is the mean of the sample y and $\bar{z}$ is the mean of $z = [x; y]$.

Step 2 : Resample n_x observations for $\tilde{x}$ and n_y observations for $\tilde{y}$ obtaining two vector columns x^b and y^b.

Step 3 : Asses t statistic

$$t^b = \frac{\bar{x}^b - \bar{y}^b}{\sqrt{\frac{\sigma^2_{x^b}}{n_x} + \sigma^2_{y^b} n_y}}$$

where $\bar{x}^b$ is the mean of

Step 4 Repeat steps Step 2 - Step 3 B times.

Step 5 Compare t statistic from the original data, i.e., $t^{original}$, with the t statistic from the simulated data, i.e. t^b for $b = 1, ..., B$. Depending on the alternative hypothesis this last step is different (MacKinnon (2009)).

(A) $H_1 : \mu_x \neq \mu_y$

$$pvalue = 2 \min \left(\frac{\sum_{b=1}^{B} \mathbb{1}_{t^b > t^{original}} + 1}{B+1}, \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b < t^{original}} + 1}{B+1} \right)$$

(B) $H_1 : \mu_x > \mu_y$

$$pvalue = \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b > t^{original}} + 1}{B+1}$$

(C) $H_1 : \mu_x < \mu_y$

$$pvalue = \frac{\sum_{b=1}^{B} \mathbb{1}_{t^b < t^{original}} + 1}{B+1}$$

Bootstrap Kolgomorov Smirnov test These steps are obtained following Abadie (2002). Given two samples x and y with size n_x and n_y:

Step 1 : Resample $N = n_x + n_y$ observations for $z = [x; y]$ obtaining a vector column z^b.

Step 2 : The first n_x rows of column z^b would be x^b and the following n_y would be y^b.

Step 3 : Assess $KS^{original}$ statistic or the modified version depending if your alternative hypothesis is $F_x(z) \neq F_y(z)$, i.e. not equal distribution, or $F_x(z) < F_y(z)$, i.e., first order stochastic dominance of x over y.

Step 4 Repeat steps Step 1 - Step 3 B times.

Step 5 : Assess KS^b statistic or the modified version for the original data

Step 6 : pvalues are obtained as:

$$pvalue = \frac{1 + \sum_{b=1}^{B} \mathbb{1}_{KS^b > KS^{original}}}{B+1}$$

For the test where the alternative hypothesis is $H_1 : F_x(z) \neq F_y(z)$, the KS statistic is

$$KS = \left(\frac{n_x n_y}{n_x + n_y} \right)^{1/2} sup_{z \in \mathbb{R}} \left| F_{x,n_x}(z) - F_{y,n_y}(z) \right|$$

and when the alternative hypothesis is $H_1 : F_x(z) < F_y(z)$, i.e., dominance of x over y,

$$KS = \left(\frac{n_x n_y}{n_x + n_y} \right)^{1/2} sup_{z \in \mathbb{R}} \left(F_{x,n_x}(z) - F_{y,n_y}(z) \right).$$

Chapter 2

Structural change in the link between oil and the European stock market: implications for risk management.

Abstract

The relationship between the European stock market and the crude oil depends on the significance of the different industries in the European economy. The literature points to a structural change after the 2008 crisis without getting into details of which sectors lead this regime switch. The co-movement between oil prices and stock market is known to exhibit (1) non-linearity, (2) asymmetric tail dependence and (3) variation over time. I combine a copula approach with Markov switching models to capture this complex linkage while the CoVaR measure translates the consequences of the tail dependence into potential losses. The results indicate a change in the lower tail dependence from negative to positive association between oil and Eurostoxx, meaning a shift in the exposure of our stock portfolio to commodity risk. There is a structural change in dependence after the 2008 financial crisis led by energy-intensive sector, e.g. basic materials and consumer goods. The economic cycle and its implications for profit margin and oil demand might explain this switch. Healthcare sector responds to oil shocks in an opposite way than Eurostoxx, displaying useful features to reduce the exposure of the stock portfolio to oil spillovers.

2.1 Introduction

The relationship between stock market and oil prices is a key topic extensively studied in the literature, given the several transmission channels that connect both markets. A large number of industries employ oil and petroleum products, such as kerosene or plastic materials, as input factors, so that higher input costs will generally affect firms' returns and sales price, triggering inflationary processes. The effects of oil shocks go beyond inflation and declines in corporate profits. A shock in oil prices may also show up in aggregate measures of output and employment (Hamilton 1983; Mork 1989; Hooker 1999). On the other hand, stock market returns can be seen as a high-frequency proxy of economic growth, so that analysing the stock market exposure to oil movements allows us to evaluate short-term effects of oil swings on the economy.

The aim of this article is to check the existence of a structural break in the relationship between oil prices and European stock market, identifying the sectors that experience the change as well as those that are useful to reduce the contagion from oil to a European stock portfolio. I compute the impact of extreme scenarios for oil prices on the European stock market and its sectors, bearing in mind the non-linearity of that relationship, the possible structural changes and the asymmetric features exhibited by the co-movement between oil and stock markets. The focus on European industries plays a key role to understand the diversification advantages available during periods of great oscillations in oil prices. The study also deals with the challenge of translating the impact of oil spillover on stock markets into a quantitative measure convenient for risk management purposes. The change in the Value-at-Risk (VaR) of the stock market when oil prices move abruptly measures the effect of tail dependence on the extreme quantiles of the stock market returns. The Conditional Value at Risk ($CoVaR$) provides a simple way to summarize complex information, such as the portfolio exposure to a commodity risk. It is therefore a perfect tool to understand the consequence of risk as potential losses.

This study combines the copula methodology with a Markov switching approach to analyse the link between oil and the European stock market. The proposed methodology perspective, which has not yet been employed in this topic, allows for disclosing some hidden patterns inside the data, while shedding light on possible changes in joint tail dependence. This sophisticated technique identifies endogenously different regimes over time having a clear economic interpretation. Delle Chiaie et al. (2017) find a common factor among different set of commodities that we could identify as a proxy of the external demand and international trade stability. Hence, it has sense to model the relationship depending on an endogenous probability.

The input data employed for the empirical analysis goes from the beginning of 2000 to the end of 2015 including several periods of crisis and potential structural breaks, e.g. the dot-com crisis, the 2008 financial crisis, the European debt crisis or the 2014 oil glut period. The results are in line with the literature, finding a structural change after the financial crisis (Reboredo and Ugolini 2016) and a high sensitivity to oil shocks of industries with high-energy requirements (Moya-Martínez et al. 2014, Lee et al. 2012, Nandha and Brooks 2009). The findings indicates that the relationship between oil prices and the European stock market experienced a structural change after 2008 common in sectors with an elastic demand of their products.

The relationship between the stock market and oil returns presents a strong asymmetric tail dependence. They had an asymmetric negative association before 2008, with low quantiles of oil returns and high quantiles of stock returns having stronger dependence than high quantiles of oil returns and low quantiles of stock returns. The relationship became positive in the aftermath of the 2008 financial crisis, with lower tail dependence. The economic cycle and its implications for profit margin, oil demand and herd behaviour may explain the change in dependence. During the expansion phase of the business cycle a decrease in the price of a key production input as oil would increase the profit margin between sales price and the unit cost of production. On the other side, increases in oil prices entail a general rise in production costs which would be translated to sales prices. At the outset of the crisis firms ran into losses, the unemployment rate increased sharply and a substitution effect occurred between oil and employment (Fernández Casillas et al. 2012) leading to a decrease in the oil demand as an input factor. The study presents the change in the Value-at-Risk at 5% and 95% when oil prices experience a severe downward movement, i.e. *bearish CoVaR*, or an acute upward swing, i.e. *bullish CoVaR*. The differences between *CoVaR* and *VaR* values are found statistically significant using Kolgomorov-Smirnov (KS) bootstrapping test, specially after the 2008 financial crisis. The four-month period ahead forecast exercise using out-of-sample data from 2015 to 2018 indicates that adding the healthcare sector to the stock portfolio reduces the oil exposure.

This research has implications for investors and portfolio managers, who need risk management strategies to hedge stock portfolio against extreme movements in oil prices; for market authorities, who have to supervise stock firms' quotes, measuring the exposure and impact of oil swings on the stock market; and for policy makers, who are concerned on a sector analysis regarding the effect of extreme changes in oil prices on the markets due to its consequences on growth, employment and household income. Financial institutions also need information concerning how the exposure to oil prices could affect firm's returns directly, for instance via positions in oil derivatives, and also indirectly through investment of financial firms in stock market.

The article is laid out as follows: Section 2.2 provides a summary of the literature in this topic, Section 2.3 presents the CoVaR measure and the copula methodology using a Markov switching model to produce time-varying copulas. Section 2.4 introduces the data and performs a descriptive analysis. Section 2.5 presents the results and section 2.6 concludes.

2.2 Literature review

The oil-stock link may well be sector specific, and it would then not be convenient to try to characterise it at the level of the stock market index. Arouri and Nguyen (2010) show that the diversification opportunities to reduce the exposure to oil prices arise across industries more than across countries. Ramos and Veiga (2013) and Park and Ratti (2008) realize that oil exposure depends on the role of the country as an oil provider or consumer. Lee et al. (2012) point out that the domestic stock market index can dress up the impact of oil shocks on the economy depending on the sectoral diversification of each country. Hence, the discrepancies between countries might be due to a difference in their industrial production more than to a geographical issue. Arouri et al. (2011) suggest industry-specific factors within each region to explain differences in the same sectors across regions, such as the degree of oil consumption or the concentration in the industry. Arouri et al. (2012) also find high variability in oil exposure across sectors. The sensitivity to changes in oil prices is higher on oil-related industries (Moya-Martínez et al. 2014, Sadorsky 2001, Boyer and Filion 2007). Lee et al. (2012) indicate that energy-intensive sectors, like transportation or chemical industrials, have a great exposure to oil shocks in G7 economies. Nandha and Brooks (2009) obtain the same conclusions for the transport sector in developed countries.

The literature identifies several issues to be considered in the econometric analysis to avoid drawing misleading conclusions: non-linearities, structural breaks, asymmetric behaviour and tail dependence. Non-linearity shows up as a difference in correlation on average than on extreme scenarios. Ciner (2001) points out that overlooking this feature may lead to deny any impact from oil on stock markets (Apergis and Miller 2009, Chen et al. 1986 and Huang et al. 1996). Reboredo (2010) highlights this characteristic in his Markov switching analysis, finding structural breaks that explain a change in the dependence between oil and the stock market. A change in monetary policy, e.g. the introduction of the Euro (Moya-Martínez et al. 2014, Sukcharoen et al. 2014) or an economic crisis, e.g. the 2008 financial crisis (Reboredo and Ugolini 2016, Zhu et al. 2016) may imply a structural break in the linkage between oil and stock markets. An asymmetric pattern, i.e. the different joint behaviour in a low-quantile than in a high-quantile scenario, must also be considered when analysing extreme co-movements in the stock market and oil prices. Aloui et al. (2013) provide evidence of an increase in the co-movement between both markets during acute periods of financial stress combining a copula methodology with a rolling windows approach. Tail dependence, i.e. the probability of having very extreme realizations for stock market returns given very extreme realizations for oil returns, plays a key role in understanding the linkage under severe scenarios especially in the lower joint tail (Aloui et al. 2013, Nguyen and Bhatti 2012, Wen et al. (2012)). Investors' herd behaviour during the contraction phase in the business cycle might also explain this feature.

Methodologically, the copula approach enables us to capture the asymmetric pattern exhibited by financial data. The greater flexibility of this state-of-the-art technique explains the increasing attention that has received in the latest years to carry out the analysis of spillovers between oil and stock markets (Sukcharoen et al. 2014, Nguyen and Bhatti 2012, Wen et al. 2012, Reboredo and Ugolini 2016, Mensi et al. 2017). This approach allows for gathering more information about the distribution, which motivates its use in this study. To find possible structural changes in that linkage, the copula structure evolves according to Markov switching specification. The Markov switching approach is more robust to misspecification than other models employed to incorporate time-varying characteristics as Patton (2006).[1] Reboredo and Ugolini (2016) use the dynamics proposed by Patton (2006) for the copula, which ignore the change in sign of the relationship for some copulas. Consequently, they do not find dependence before 2008, while the results of this article point out a negative association between variables. Reboredo and Ugolini (2016) use copulas that

[1]For a comparison between different specification for the evolution of dependence using copulas see for instance Manner and Reznikova (2012).

only allow for positive association, which might overlook the change in the relationship between oil and the stock market pointed by recent literature (**?**, **?**). This potentially misleading conclusion has serious implications for risk management because both sectors may then be assumed to be independent when actually, the stock portfolio is exposed to sharp changes in oil prices. Markov switching models are ideal to capture regime switch episodes between oil shocks and economic variables, such as changes in output growth (Raymond and Rich 1997, Clements and Krolzig 2002, Holmes and Wang 2003, Manera and Cologni 2006), sector employment (Fernández Casillas et al. 2012) or stock markets (Balcilar et al. (2015), Reboredo (2010), Aloui and Jammazi (2009)).

2.3 Methodology

This section is structured in three parts. The first part presents the $CoVaR$ measure, in which the main results of the article are based on. Subsection 2.3.2 introduces the copula methodology and how to measure CoVaR using this approach. Finally subsection 2.3.3 highlights some features concerning the marginal and joint distribution model with particular attention to the time-varying specification for the dependence between variables, i.e. Markov switching model.

2.3.1 Conditional Value at Risk

The Conditional Value at Risk ($CoVaR$) measure (Adrian and Brunnermeier 2016, Girardi and Ergün 2013) indicates in this study a quantile of the stock market returns given an sharp change in oil prices. $CoVaR$ gives a conditional view of VaR measure, which is widely employed for risk management purposes and capital buffer requirements in the financial sector. $CoVaR$ translates spillovers from oil to stock market into potential losses in the stock market portfolio. The comparison between $CoVaR$ and the unconditional VaR can give us an idea of the change in the risk measure when extreme oil scenarios occurs as an indicator of dependence. This way of looking at the link between oil and stock market is very convenient for investors, who get aware of the negative consequences of oil unhedging, and policy makers and market authorities, which need a quantitative estimate of the effects of swings in oil prices on stock markets.

$CoVaR$ focuses on the tail of the distribution where non-linearities and asymmetries appear and where the effects of spillovers are more harmful. I compute four assessment of $CoVaR$ depending on the oil-related scenario and the tail of the stock markets. The variable r_o refers to the oil returns, which is the conditioning variable in this study, and r_m represents the stock market returns. The subscript t indicating the time is left out for notational convenience. I distinguish two types of scenarios for the oil returns: a *bearish* scenario where oil returns are below its $\alpha 100 - th$ quantile, i.e. $P(r_o < VaR_o(\alpha)) = \alpha$, and a *bullish* scenario where oil returns are above its highest $\alpha - th$ quantile, i.e. $P(r_o > VaR_o(1 - \alpha)) = \alpha$. The *bearish* and *bullish* $CoVaR_{m|o}(\alpha, \beta)$ are computed at a confidence level $\beta 100\%$ for the stock market. Hence, setting the β close to one and close to zero allows for analysing the effects of oil spillovers on the right and left tails of the distribution of stock market returns. The asymmetries in tail dependence and its changes over time justify the assessment of $CoVaR$ for different scenarios and confidence levels.

The *CoVaR* measure comes from the Bayes' theorem and the copula representation of the joint distribution. Next subsection presents the copula methodology and shows how we can employ it to compute CoVaR.

2.3.2 CoVaR in terms of copulas

Copulas functions provide a straightforward decomposition of the joint distribution. This property gives us higher flexibility to model complex joint distribution capturing diverse features as asymmetric dependence or strong joint tail behaviour. Sklar's theorem (Sklar 1959) states that a multivariate cumulative distribution function can be expressed as a combination of marginal cumulative distribution functions and a copula, i.e.

$$F(r_o, r_m) = C\left(F_o(r_o), F_m(r_m)\right), \tag{2.1}$$

where F_k is the cumulative distribution function of variable $k = o, m$ and $C(\dots)$ is the copula function.
Bayes' theorem allows for expressing a conditional probability as the ratio of the joint probability of seeing both scenarios to the probability of observing the conditioning scenario. Copulas and rotated copulas[2] provide us the expression for the joint probability. After solving the conditional probability equation, the *CoVaR* value is the result of using the inverse distribution function of the conditioned variable.

For instance, the bearish $CoVaR_{m|o}(\alpha, \beta)$ of the market returns m would be obtained implicitly from

$$\begin{aligned} P\left(r_m < CoVaR_{m|o} | r_o < VaR_o(\alpha)\right) &= \frac{P\left(r_m < CoVaR_{m|o}, r_o < VaR_o(\alpha)\right)}{P(r_o < VaR_o(\alpha))} \\ &= \beta, \end{aligned}$$

where $P(r_o < VaR_o(\alpha)) = \alpha$ and $P\left(r_m < CoVaR_{m|o}, r_o < VaR_o(\alpha)\right)$ can be expressed as

$$C\left(F_m(CoVaR_{m|o}), \alpha\right). \tag{2.2}$$

The quantile $F_m(CoVaR_{m|o})$ is obtained by numerical optimization.[3] Assessing CoVaR through copulas is faster and less time consuming than other approaches that imply integration methods. Then, the *CoVaR* value arises as the result of employing the inverse cumulative distribution function of the conditioned variable, i.e

$$F_m^{-1}(F_m(CoVaR_{m|o})) = CoVaR_{m|o}.$$

[2]Rotated copulas are transformations of standard copulas to reflect some empirical features in the joint cumulative distribution. For instance, you can rotate a Clayton copula to provide a negative dependence with asymmetric tail dependence between two assets. Further information about rotated copulas can be found in A.

[3]Using *MATLAB* software and for certain values of α and β, the function *fzero* is employed to get $u^* = F_m(CoVaR_{m|o})$. Note that only in case of independence between both markets $F_m(CoVaR_{m|o}) = \beta$.

The bullish $CoVaR_{m|o}(\alpha, \beta)$ of the stock market returns is obtained from

$$\begin{aligned} P\left(r_m < CoVaR_{m|o} | r_o > VaR_o(1-\alpha)\right) &= \frac{P\left(r_m < CoVaR_{m|o}, r_o > VaR_o(1-\alpha)\right)}{P(r_o > VaR_o(1-\alpha))} \\ &= \beta, \end{aligned}$$

where $P(r_o > VaR_o(1-\alpha)) = \alpha$ and $P\left(r_m < CoVaR_{m|o}, r_o > VaR_o(1-\alpha)\right)$ can be expressed as

$$F_m(CoVaR_{m|o}) - C\left(F_m(CoVaR_{m|o}), 1-\alpha\right). \tag{2.3}$$

The $CoVaR$ presents the same drawbacks than VaR as a risk measure, i.e. it is not sub-additive. The $CoES$ overcomes the shortcomings of the $CoVaR$, i.e. it is a coherent risk measure (Acerbi and Tasche 2002, Huang and Uryasev 2018), which complements the information provided by the $CoVaR$. The Conditional Expected Shortfall ($CoES$) is the mean value of the variable beyond the $CoVaR$. We would focus on the mean returns below this threshold when the interest lies in the left tail of the conditional distribution, i.e.

$$CoES_{m|o}(\alpha, \beta) = \frac{1}{\beta} \int_0^\beta CoVaR_{m|o}(\alpha, q) dq.$$

whilst if the interest is on the right tail of the conditional distribution would be

$$CoES_{m|o}(\alpha, \beta) = \frac{1}{1-\beta} \int_\beta^1 CoVaR_{m|o}(\alpha, q) dq.$$

2.3.3 Marginal distribution and joint dependence structure

This subsection introduces the model for the marginal distribution and the copula functions that make up the joint distribution.

Marginal model

We characterise the marginal densities of the stock market (m) and oil (o) returns by an AR(p) model, i.e.

$$r_{k,t} = \underbrace{\phi_{k,0} + \sum_{j=1}^{p} \phi_{k,j} r_{k,t-j}}_{\mu_{k,t}} + \epsilon_{k,t}, \quad k = m, o \tag{2.4}$$

where p is a non-negative integer, $\phi_{k,j}$ are the autoregressive (AR) parameters with $j = 0, \ldots, p$ and $\epsilon_{k,t} = \sigma_{k,t} z_{k,t}$. The dynamic of the variance of $\epsilon_{k,t}$ follows a $GJR-GARCH(1,1)$ specification, which allows for leverage effects, i.e.

$$\sigma^2_{k,t} = \omega_k + \beta_k \sigma^2_{k,t-1} + (\alpha_k + \gamma_k \mathbb{1}_{\epsilon_{k,t-1}<0}) \epsilon^2_{k,t-1}, \quad k = m, o \tag{2.5}$$

where ω_k, β_k and α_k are the GARCH parameters and $\mathbb{1}_{\epsilon_{k,t-1}<0}$ is an indicator function that values 1 if $\epsilon_{k,t-1} < 0$ and zero otherwise. γ_k captures leverage effects, i.e. negative shocks have more impact on variance than positive ones. When $\gamma_k = 0$ we have the $GARCH$ model. Furthermore, $z_{k,t}$ is a i.i.d. random variable with zero mean and unit variance that follows a Hansen (1994)'s skewed-t distribution which allows us

to capture higher moments, i.e. skewness and kurtosis.
The density of Hansen (1994)'s skewed-t distribution is

$$h(z_{k,t}|\eta_k,\lambda_k) = \begin{cases} bc(1+\frac{1}{\eta_k-2}(\frac{bz_{k,t}+a}{1-\lambda_k})^2)^{-(\eta_k+1)/2} & z_{k,t} < -a/b \\ bc(1+\frac{1}{\eta_k-2}(\frac{bz_{k,t}+a}{1+\lambda_k})^2)^{-(\eta_k+1)/2} & z_{k,t} \geq -a/b \end{cases}, \tag{2.6}$$

where $2 < \eta_k < \infty$ and $-1 < \lambda_k < 1$. The constants a, b and c are given by

$$a = 4c\lambda_k\left(\frac{\eta_k-2}{\eta_k-1}\right), b = \sqrt{1+3\lambda_k^2-a^2}, c = \frac{\Gamma(\frac{\eta_k+1}{2})}{\sqrt{\pi(\eta_k-2)}\Gamma(\frac{\eta_k}{2})}.$$

Note that when $\lambda_k = 0$ Equation (2.6) reduces to the standard Gaussian distribution as $\eta_k \to \infty$. When $\lambda_k = 0$ and η_k finite, we obtain the standardized symmetric-t distribution.

Copula specification and time-varying features

Set of copulas I initially choose five types of copulas, summarized in Table 2.3.1, as potential dependence functions to fit the data because of their tail dependence features, i.e. Gaussian, Student t, Clayton, Gumbel, BB1. Gaussian and Student copulas allow for positive and negative association, while Gaussian copula has no tail dependence, Student t copula has symmetric tail dependence. Gumbel and Clayton copulas allow only for positive asymmetric association, while Clayton copula has lower tail dependence, Gumbel copula has upper tail dependence. BB1 copula, also known as Clayton-Gumbel copula, allows only positive association, but it can be asymmetric. It has two parameters that model upper and lower tail dependence. Later on, the set of copulas is enhanced by rotating the Archimedean copulas, i.e. Gumbel, Clayton and BB1 copulas, to enable negative co-movement with implications for tail dependence. Appendix A provides further details about these copulas and their properties.

TABLE 2.3.1: Main tail dependence features for each copula

Family	τ_L	τ_U
Gaussian	$-$ (if $\rho = 1$ then 1)	$-$ (if $\rho = 1$ then 1)
Student t	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\rho)}{1+\rho}}\right)$	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\rho)}{1+\rho}}\right)$
Clayton	$2^{-1/\theta}$	$-$
Gumbel	$-$	$2-2^{1/\theta}$
BB1 (Clayton-Gumbel)	$2^{-\frac{1}{\theta\delta}}$	$2-2^{1/\delta}$

Note:
$-$ represents no tail dependence.
Source: (Ao et al., 2017, p. 22), Jiang (2012), Joe and Hu (1996), Fischer (2003) and (Joe, 1997, p. 193–204).
Let u_1 and u_2 denote two uniform-distributed variables across (0,1).
The lower tail dependence, τ_L, is defined as $\tau_L = lim_{q\to 0}P(u_2 < q|u_1 < q)$.
The upper tail dependence, τ_U is defined as $\tau_U = lim_{q\to 1}P(u_2 > q|u_1 > q)$.

Model selection. An inaccurate copula choice can have serious consequences in the joint dependence construction, leading to mistaken interpretation of the relationship between variables. To avoid choosing a wrong copula, I use analytical and graphical tools to propose a set of copulas that fit the empirical evidence as potential dependence structures. In addition to that, I consider potential variations over time in the copula parameters and in the copula itself following a Markov switching model.
I use graphical tools as contour plots, lambda functions and Tail Concentration Functions (TCF) and analytical tools as the Akaike Information Criterion Corrected for small-sample bias ($AICc$) to choose a suitable copula that fits the true data dependence. $AICc$ is the principal indicator for copula selection (Brechmann and Schepsmeier (2013), Reboredo and Ugolini (2015a), Reboredo and Ugolini (2015b), Reboredo and Ugolini (2016), Rodriguez (2007), Reboredo (2011)). The results are also analysed by looking at the confidence interval for the risk measure using bootstraping techniques.

Markov switching specification and estimation process The relationship between oil and stock market returns has been poited out to exhibit a structural change after the 2008 financial crisis according to recent literature, e.g. Reboredo and Ugolini (2016), Zhu et al. (2016). Hence, I consider a two-regime Markov switching model to replicate the evolution over time of the dependence structure, i.e. the changes in tail dependence across regimes. The switch can be limited to a change in the copula parameter or a change also in the copula itself depending on the empirical evidence provided by analytical and graphical tools. Each regime has an economic interpretation depending on the type of dependence observed within each state. The regimes are not directly observable but they can be identified from the estimation process.
The joint distribution is decompose following Equation (2.1) assuming that the copula function depends on a latent variable s_t that reflects the kind of relationship in the co-movement between oil prices and the stock market, i.e.

$$F(r_{o,t}, r_{m,t}) = C\left(F_{o,t}(r_{o,t}; \mu_{o,t}, \sigma_{o,t}, \eta_o, \lambda_o), F_{m,t}(r_{m,t}; \mu_{m,t}, \sigma_{m,t}, \eta_m, \lambda_m)); \theta_{s_t}, s_t\right). \quad (2.7)$$

where $\mu_{k,t}$ is given by Equation (2.4), $\sigma_{k,t}$ follows the dynamic in Equation (2.5) and η_k, λ_k are the parameters from the innovation process in Equation (2.6) with $k = o, m$. $C(\ldots; \theta_{s_t}, s_t)$ is the copula under the state s_t with parameter θ_{s_t}. To begin with, I constrain the copula switch to a simple change in its parameter θ_{s_t} on the assumption that the type of copula remains unchanged throughout the entire sample. Later on, this constraint is relaxed to allow for different copulas across regimes. In other words, the Markov switching approach models the dependence between oil returns and stock markets returns as a copula mixture where the weights are given by the forecasting probabilities, i.e. $P(s_t|I_{t-1})$ where I_{t-1} indicates the information set at $t-1$.

The copula of the conditional process depends on a regime, s_t, which is assumed to be stochastic and unobservable. The probability of being at each time t in each state s_t depends only on the state at $t-1$, i.e. the regime generating process follows a first order Markov chain defined by its transition probabilities

$$p_{ij} = P(s_t = j | s_{t-1} = i), \quad (2.8)$$

such that $\sum_{j=1}^{2} p_{ij} = 1$ for $i = 1, 2$.
The transition matrix defined by the Markov Chain is

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \tag{2.9}$$

where $1 - p_{11} = p_{12}$ and $1 - p_{22} = p_{21}$ refer to the probability of changing between states and p_{11} and p_{22} are the probabilities of staying in the same state. For the sake of brevity the reader can find further details about the Markov switching methodology in Appendix B.

The parameters are estimated using the full maximum likelihood method. The copula is assumed to be regime dependent, i.e. it moves according to a two-state Markov Chain ($s_t = 1$ and $s_t = 2$). This assumption keeps the model tractable for estimation purposes but, at the same time, gives a great flexibility to identify changes in the linkage. At each time t the likelihood for each observation can be written as

$$\begin{aligned} L_t(r_{o,t}, r_{m,t}; I_{t-1}, \Theta_t) &= f(r_{o,t}, r_{m,t} | \Theta_{s_t=1}, I_{t-1}) P(s_t = 1 | I_{t-1}) \\ &\quad + f(r_{o,t}, r_{m,t} | \Theta_{s_t=2}, I_{t-1}) P(s_t = 2 | I_{t-1}), \end{aligned} \tag{2.10}$$

where Θ_t stands for the set of parameters of the joint distribution at each state. This is a mixture of two joint distribution where the weights are given by the likelihood of being at each state. Deriving from Equation (2.1) we can rewrite $f(r_{o,t}, r_{m,t} | \Theta_{s_t=i})$ as

$$f_{o,t}(r_{o,t}; \mu_{o,t}, \sigma_{o,t}, \eta_o, \lambda_o) f_{m,t}(r_{m,t}; \mu_{m,t}, \sigma_{m,t}, \eta_m, \lambda_m) c\,(u_{o,t}, u_{m,t}; \theta_{s_t=i}, s_t = i) \tag{2.11}$$

where $u_{o,t} = F_{o,t}(r_{o,t}; \mu_{o,t}, \sigma_{o,t}, \eta_o, \lambda_o)$, $u_{m,t} = F_{m,t}(r_{m,t}; \mu_{m,t}, \sigma_{m,t}, \eta_m, \lambda_m)$ and $c\,(\ldots; \theta_{s_t=i}, s_t = i)$ is the copula density under the state $s_t = i$ with parameter $\theta_{s_t=i}$ with $i = 1, 2$. It is worth noting that the log-likelihood function, i.e. $\sum_t log(L_t(r_{o,t}, r_{m,t}; I_{t-1}, \Theta_t))$ has to be maximized using a non-linear method because this function depends in a non-linear way on the set of parameters.[4]

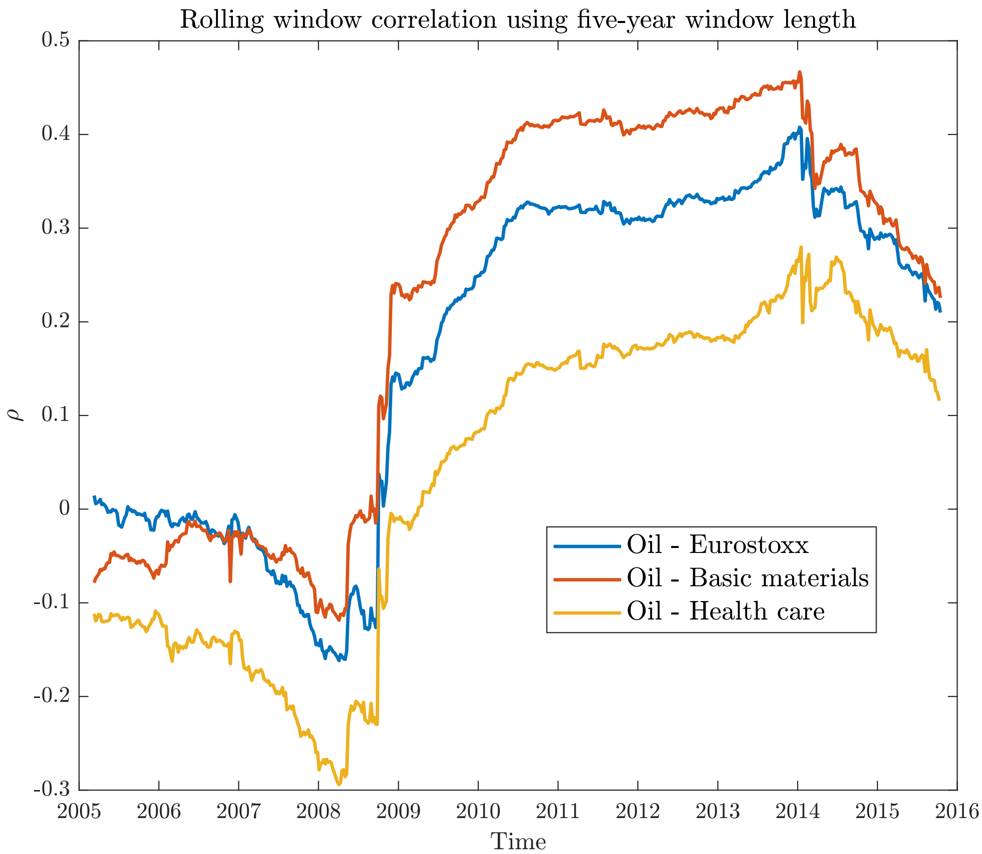
2.4 Data

I employ weekly data on stock market, exchange rate and commodity prices from 7 January 2000 to 23 October 2015, where the oil prices have experienced large oscillations.
Concerning commodity prices, I use the spot prices of Europe Brent crude oil sourced from the US Energy Information Agency (http://www.eia.doe.gov). This is the main benchmark to settle the price of light crudes and it is a better proxy for oil price series than OPEC oil quote (Sukcharoen et al. 2014). Brent crude oil is denominated in dollars per barrel, so it is transformed into Euros to perform the empirical exercise. The EUR/USD exchange rate is obtained from the European Central Bank Statistical Data Warehouse (https://sdw.ecb.europa.eu).
Regarding the European stock market variables, I employ the EUROSTOXX index and its ten-industry decomposition based on the Industry Classification Benchmark

[4] The $fminsearch$ function in $MATLAB$ software provides good estimates while some transformation of the parameters are performed to keep them in a feasible region. For instance instead of looking for values of p_{11} and p_{22}, I obtain the optimal estimate for a parameter x and y such that $1/(1 + exp(-x)) = p_{11}$ and $1/(1 + exp(-y)) = p_{22}$. Hamilton and Susmel (1994) employ this kind of transformations to estimate the parameters of its $SWARCH$ model.

(ICB) nomenclature obtained from Datastream (oil & gas, basic materials, industrials, consumer goods, healthcare, consumer services, telecommunications, utilities, financials, technology). This set of variables makes possible to measure not only the joint effect of large swings in oil prices on the European economy as a whole but also on the different productive industries.

FIGURE 2.4.1: Time-varying correlation between stock returns and oil returns.



This figures shows the evolution of the correlation between oil and Eurostoxx (blue line), basic materials (red line) and health care (yellow line). The correlation evolves over time using a rolling window on weekly returns with a five-year window length, i.e. at each time t I assess the correlation of the weekly returns between $t-260$ and t. The figures depict two evidences. First, the lower correlation between health care and oil than other stock returns. Second, the shift in the correlation after 2008-2009, which might indicate the presence of a regime switch.

Table 2.4.1 reports the main statistics for the stock market returns and the oil returns denominated in Euros. It considers the full sample and two subsamples where 15 September 2008 is established as a breakpoint after the fall of Lehman Brothers and the consequent onset of the financial crisis. The post-crisis subsample shows higher kurtosis and standard deviation. It also shows a more negative skewness and mean than the pre-crisis subsample. The p-value of Jarque Bera indicates the importance of higher moments in the distribution, supporting the choice of Hansen (1994)'s skewed t distribution for the marginals. The correlation between stock market returns and oil returns increases after the financial crisis. In some cases, as in the health care sector, the correlation moves from negative values to positive ones after the 2008 crisis. Figure 2.4.1 shows the time varying correlation between oil and Eurostoxx (blue line), oil and basic materials (red line) and oil and heath care sector (yellow line). We get the time-varying correlation using a rolling windows approach on the weekly returns with a five-year windows length. Two main conclusions are inferred from this figure. First, the health care sector presents lower correlation throughout the entire sample than basic materials or Eurostoxx. Second, around 2008-2009 the correlation between these stock returns and oil returns experiences a sharp upward movement which might be an indication of regime switch. This evidence is also shown in the empirical joint distribution of Figure 2.4.2.

Figure 2.4.2 shows an approximation to the joint distribution of Eurostoxx returns and oil returns. Axis shows the empirical cumulative distribution function, i.e. $\tilde{F}(r_{i,t}) = \frac{\sum_{t=1}^{T} \mathbb{1}_{r_{i,t} > r_{i,j}}}{T+1}$. The probability space is divided into 25 areas where each area indicates a interquintile range for each variable. The colour of each area depends on the probability mass observed, the darker colours indicate a higher clustering of data. For instance a darker colour in the bottom left corner of the graph indicates the higher density of pairwise observations when both returns are in their lowest quintile, i.e. there is a higher dependence in the lower tail.
Figure 2.4.3 shows the empirical joint distribution between the oil and the different sectors of the stock market. Left set of subfigures refer to the full period while center and right set of subfigures consider respectively the observations in the pre-crisis and post-crisis sample. The relationship between oil and basic materials or health care sectors seems to change from a negative dependence in the pre-crisis period to a positive tail dependence in the post-crisis phase. This change in linkage is ignored if we look at the full sample. There are evidences concerning a structural change during the analysed period justifying the Markov switching choice.

TABLE 2.4.1: Descriptive statistics

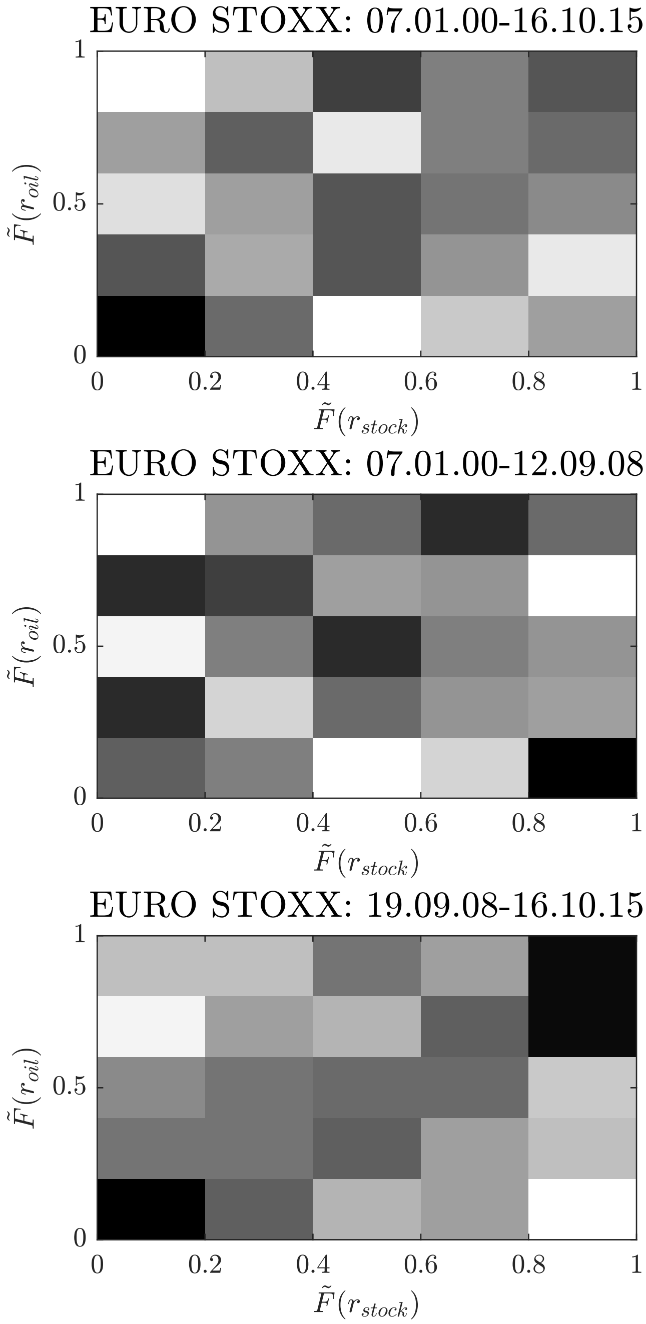
	A	B	C	D	E	F	G	H	I	J	K	L
Panel A : All Sample												
Mean	0,000	0,000	0,001	0,000	0,001	0,001	0,000	-0,001	0,000	-0,001	-0,001	0,001
Std. Dev.	0,031	0,034	0,035	0,033	0,030	0,030	0,029	0,036	0,031	0,041	0,045	0,049
Skewness	-0,935	-0,906	-0,650	-0,662	-0,431	-1,241	-0,973	-0,305	-1,501	-0,750	-0,340	-0,604
Kurtosis	9,321	10,038	7,479	6,710	7,576	11,686	9,661	7,062	17,276	8,612	5,958	4,689
Quantile 5	-0,051	-0,051	-0,059	-0,053	-0,047	-0,041	-0,046	-0,060	-0,047	-0,065	-0,076	-0,088
Quantile 95	0,042	0,048	0,049	0,048	0,044	0,042	0,042	0,051	0,043	0,058	0,067	0,070
J-B	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001
ARCH	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
Linear Corr.	0,171	0,363	0,165	0,169	0,104	0,025	0,089	0,067	0,167	0,123	0,104	-
Rank Corr.	0,089	0,217	0,090	0,087	0,062	-0,008	0,052	0,036	0,076	0,046	0,070	-
Panel B : Pre-crisis sample												
Mean	-0,001	0,000	0,001	0,000	0,000	0,000	-0,002	-0,002	0,001	0,000	-0,002	0,002
Std. Dev.	0,027	0,030	0,032	0,029	0,027	0,029	0,030	0,038	0,024	0,032	0,052	0,052
Skewness	-0,280	-0,440	-0,256	-0,625	-0,676	-0,396	-0,539	-0,046	-0,453	-0,377	-0,160	-0,478
Kurtosis	4,817	5,984	5,946	4,620	5,836	4,822	7,020	4,992	4,419	7,058	4,930	3,766
Quantile 5	-0,049	-0,050	-0,058	-0,050	-0,048	-0,043	-0,049	-0,068	-0,035	-0,056	-0,090	-0,091
Quantile 95	0,040	0,044	0,046	0,045	0,039	0,043	0,042	0,056	0,036	0,045	0,079	0,077
J-B	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001
ARCH	0,000	0,000	0,000	0,000	0,000	0,002	0,000	0,001	0,009	0,000	0,063	0,000
Linear Corr.	-0,007	0,269	-0,036	-0,008	-0,059	-0,128	-0,054	-0,012	0,001	-0,088	0,001	-
Rank Corr.	0,010	0,181	-0,004	0,008	-0,033	-0,071	-0,014	-0,027	0,003	-0,058	0,013	-
Panel C : Post-crisis sample												
Mean	0,000	-0,001	0,001	0,001	0,002	0,002	0,001	0,000	-0,001	-0,001	0,001	-0,001
Std. Dev.	0,035	0,039	0,039	0,038	0,033	0,030	0,028	0,033	0,038	0,051	0,035	0,044
Skewness	-1,309	-1,136	-0,924	-0,684	-0,282	-2,223	-1,604	-0,761	-1,624	-0,779	-0,831	-0,899
Kurtosis	10,666	11,023	8,007	7,092	8,209	19,646	14,121	11,263	16,045	7,064	7,391	6,514
Quantile 5	-0,055	-0,056	-0,060	-0,056	-0,047	-0,038	-0,041	-0,049	-0,057	-0,076	-0,063	-0,081
Quantile 95	0,049	0,057	0,058	0,060	0,048	0,041	0,042	0,046	0,049	0,073	0,047	0,062
J-B	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001
ARCH	0,096	0,002	0,000	0,000	0,000	0,001	0,060	0,000	0,005	0,001	0,002	0,002
Linear Corr.	0,385	0,488	0,418	0,381	0,313	0,247	0,318	0,204	0,334	0,329	0,339	-
Rank Corr.	0,212	0,282	0,234	0,211	0,212	0,103	0,163	0,135	0,177	0,177	0,175	-

Pre-crisis sample goes from 7 January 2000 to 12 September 2008. Post-crisis sample goes from 19 September 2008 to 23 October 2015.

$J-B$ stands for the Jarque Bera test for normality. *ARCH* represents the Engle's LM test for heteroskedasticity using 20 lags. I show the p-values for the statistic tests.

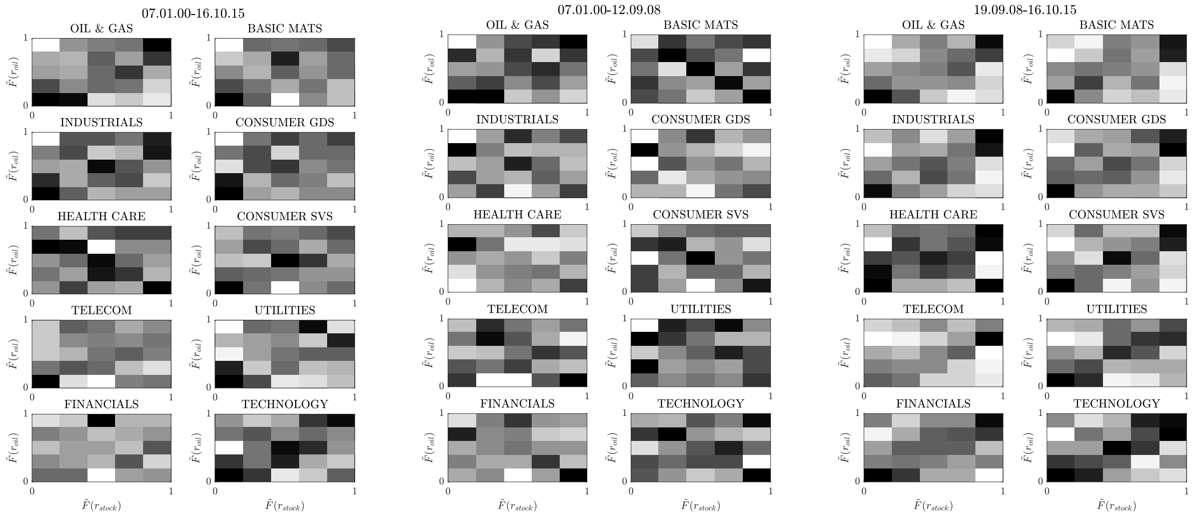
A: *EUROSTOXX*. B: *OIL&GAS*. C: *BASICMATS*. D: *INDUSTRIALS*. E: *CONSUMERGDS*. F: *HEALTHCARE*. G: *CONSUMERSVS*. H: *TELECOM*. I:*UTILITIES* . J: *FINANCIALS*. K: *TECHNOLOGY*. L: *OIL*

FIGURE 2.4.2: Empirical joint distribution for returns of EUROSTOXX and Brent oil denominated in Euros.



Top figure shows the empirical joint distribution function for all the sample, middle graphs represents the pre-crisis subsample and the bottom one the post-crisis subsample. Darker colours indicate a higher data clustering on certain part of the distribution. Looking at the entire sample and assessing a constant copula parameter may overlook the complex dependence evolution within the data.

FIGURE 2.4.3: Empirical joint distribution for returns of EUROSTOXX sector portfolios and Brent oil denominated in Euros.



(a) All sample (b) Pre-crisis (c) Post-crisis

Left set of subfigures show the empirical joint distribution function for all the sample, centre set of graphs represents the pre-crisis subsample and the right one the post-crisis subsample. Darker colours indicate a higher data clustering on certain part of the distribution. Looking at the entire sample and assessing a constant copula parameter may overlook the complex dependence evolution within the data.

2.5 Results

This section is divided in two parts. The first subsection focuses on the estimation results. It identifies the final model that shows the structural changes in the comovement between oil and stock market, paying special attention to the swing in tail dependence, and its impact on model risk. Appendix D provides several robustness checks in the model selection and the graphical tools employed during the process. The second subsection studies the implications of this change on the joint behaviour for risk measures. I employ the $CoVaR$ measure to quantify the effects of extreme movements in oil prices on the VaR estimates of the stock market. To close this section, I perform an out-of-sample forecast exercise building a stock portfolio without tail dependence with oil returns.

2.5.1 Estimation results

Figures 2.5.1a, 2.5.1b and 2.5.1c shed light on the possible structural break in the joint dependence. the figure shows the 90% confidence interval (grey area) obtained in a bootstrapping procedure, apart from the smoothed probabilities under the best copula fit according to Table 2..5 in Appendix D. 500 paths of length 2000 are simulated following the algorithm in Appendix C, which I employ to re-estimate the parameters of the model. The set of new estimates are employed to generate a confidence interval regarding the point estimates. The point estimates using the original data are not always within the grey area due to the procedure employed. According to the smoothed probabilities, the periods associated with the regime of negative or null dependence are between 2003 and 2008. The wider confidence intervals before 2008 also indicates a higher uncertainty for some sectors. The estimates and the standard deviation for the new models are presented in Tables 2.5.1a, 2.5.1b and 2.5.1c.

TABLE 2.5.1A: Parameter estimates for the joint distribution using a mixture of copulas where the weights are given by the forecast probability of each state

	A	L	B	L	C	L	D	L
ϕ_0	0.00	-0.00	0.00	0.00	0.00 *	-0.00	0.00 *	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	-0.07 **	0.03	-0.05 *	0.05 *	-0.06 **	0.03	-0.08 **	0.04
	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
ω	0.00 **	0.00 *	0.00 ***	0.00 **	0.00 ***	0.00 **	0.00 **	0.00 **
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.00	0.05 **	0.03	0.04 **	0.00	0.04 **	0.00	0.04 *
	(0.04)	(0.03)	(0.07)	(0.02)	(0.04)	(0.02)	(0.03)	(0.03)
β	0.84 ***	0.90 ***	0.80 ***	0.91 ***	0.80 ***	0.90 ***	0.83 ***	0.89 ***
	(0.07)	(0.03)	(0.07)	(0.03)	(0.08)	(0.03)	(0.07)	(0.03)
γ	0.22 ***	0.08 **	0.17 ***	0.06 **	0.23 ***	0.07 **	0.22 ***	0.09 **
	(0.07)	(0.04)	(0.07)	(0.03)	(0.07)	(0.04)	(0.08)	(0.04)
λ	-0.37 ***	-0.27 ***	-0.28 ***	-0.25 ***	-0.30 ***	-0.26 ***	-0.36 ***	-0.26 ***
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
η	12.13 ***	11.99 ***	9.70 ***	15.38 ***	14.03 ***	14.01 ***	16.38 ***	13.98 ***
	(0.46)	(0.89)	(0.57)	(0.47)	(0.47)	(0.61)	(0.52)	(0.47)
	(a)		(b)		(a)		(c)	
$\theta_{s_t=1}$	0.3877 ***		τ^L	0.1477 **	$\theta_{s_t=1}$	0.1139 **	τ^L	0.1055 *
	(0.16)			(0.07)		(0.07)		(0.07)
$\theta_{s_t=2}$	0.2742 ***		τ^U	0.1636 ***	$\theta_{s_t=2}$	0.3577 ***	τ^U	0.0783 *
	(0.06)			(0.05)		(0.09)		(0.05)
p_{11}	0.9838 ***		p_{11}	0.9866 ***	p_{11}	0.9979 ***	p_{11}	0.9982 ***
	(0.01)			(0.01)		(0.00)		(0.00)
p_{22}	0.9956 ***		p_{22}	0.9978 ***	p_{22}	0.9986 ***	p_{22}	0.9979 ***
	(0.00)			(0.00)		(0.00)		(0.00)
LL	3104.40		LL	3024.09	LL	2980.38	LL	3031.68

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the best copula mixture according to Table 2..5. *LL* is the log-Likelihood value.

$***/**/*$ indicates statistical significance at 1/5/10%

The copula parameter is assumed to evolve according to a two-state Markov switching specification. p_{ii} indicates the probability of remaining in state i, i.e. $s_t = i$, given that we have been in the same state in the previous period where $i = 1, 2$

Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: *OIL*). x could be A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

Copula mixtures: (a) 90R Clayton- Clayton; (b) Independence- BB1; (c) BB1- Independence; (d) Student-Gaussian; (e) Gaussian-Clayton; (f) Independence- Clayton; (g) R90 Gumbel-BB1.

TABLE 2.5.1B: Parameter estimates for the joint distribution using a mixture of copulas where the weights are given by the forecast probability of each state

	E	L	F	L	G	L	H	L
ϕ_0	0.00	-0.00	0.00 **	0.00	0.00	0.00	-0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	-0.04	0.03	-0.10 ***	0.04	-0.06 **	0.03	-0.03	0.02
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
ω	0.00 ***	0.00 *	0.00	0.00 *	0.00 **	0.00 *	0.00 *	0.00 *
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.00	0.05 **	0.04 *	0.04 **	0.00	0.04 *	0.04 **	0.04 **
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
β	0.82 ***	0.90 ***	0.88 ***	0.90 ***	0.87 ***	0.90 ***	0.91 ***	0.90 ***
	(0.05)	(0.03)	(0.07)	(0.03)	(0.04)	(0.03)	(0.03)	(0.03)
γ	0.22 ***	0.08 **	0.06	0.07 **	0.17 ***	0.09 **	0.06 **	0.07 **
	(0.07)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)
λ	-0.25 ***	-0.28 ***	-0.21 ***	-0.23 ***	-0.25 ***	-0.26 ***	-0.08 *	-0.27 ***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
η	11.90 ***	11.12 ***	8.77 ***	13.98 ***	8.02 ***	11.50 ***	7.26 ***	12.84 ***
	(0.88)	(1.43)	(0.86)	(0.89)	(0.57)	(1.45)	(0.53)	(0.72)

	(a)		(d)		(e)		(a)
$\theta_{s_t=1}$	0.6372 **	ρ	-0.2788 ***	ρ	-0.4848 ***	$\theta_{s_t=1}$	0.2465 ***
	(0.28)		(0.09)		(0.10)		(0.09)
$\theta_{s_t=2}$	0.2397 ***	ν	6.7021 ***	θ	0.2382 ***	$\theta_{s_t=2}$	0.2314 ***
	(0.06)		(1.54)		(0.07)		(0.07)
p_{11}	0.9523 ***	ρ	0.1279 **	p_{11}	0.9343 ***	p_{11}	0.9952 ***
	(0.03)		(0.06)		(0.03)		(0.00)
p_{22}	0.9903 ***	p_{11}	0.9793 ***	p_{22}	0.9862 ***	p_{22}	0.9977 ***
	(0.01)		(0.01)		(0.01)		(0.00)
LL	3104.28	p_{22}	0.9890 ***	LL	-3130.98	LL	2957.72
			(0.01)				
		LL	3086.64				

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the best copula mixture according to Table 2..5. *LL* is the log-Likelihood value.
$***/**/*$ indicates statistical significance at $1/5/10\%$
The copula parameter is assumed to evolve according to a two-state Markov switching specification. p_{ii} indicates the probability of remaining in state i, i.e. s_i, given that we have been in the same state in the previous period where $i = 1, 2$
Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: *OIL*). x could be A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.
Copula mixtures: (a) 90R Clayton- Clayton; (b) Independence- BB1; (c) BB1- Independence; (d) Student-Gaussian; (e) Gaussian-Clayton; (f) Independence- Clayton; (g) R90 Gumbel-BB1.

TABLE 2.5.1C: Parameter estimates for the joint distribution using a mixture of copulas where the weights are given by the forecast probability of each state

	I	L	J	L	K	L
ϕ_0	0.00	0.00	0.00	-0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	-0.03	0.03	-0.03	0.03	-0.05 *	0.06 *
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
ω	0.00 **	0.00 *	0.00 ***	0.00 *	0.00 **	0.00 **
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.00	0.05 **	0.00	0.05 **	0.03	0.02
	(0.04)	(0.03)	(0.04)	(0.03)	(0.02)	(0.02)
β	0.80 ***	0.90 ***	0.85 ***	0.90 ***	0.92 ***	0.90 ***
	(0.08)	(0.03)	(0.04)	(0.03)	(0.02)	(0.03)
γ	0.19 ***	0.07 **	0.24 ***	0.07 **	0.07 **	0.11 ***
	(0.07)	(0.04)	(0.05)	(0.04)	(0.03)	(0.04)
λ	-0.20 ***	-0.25 ***	-0.34 ***	-0.27 ***	-0.17 ***	-0.26 ***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)
η	7.91 ***	14.73 ***	11.23 ***	11.80 ***	11.32 ***	13.83 ***
	(0.66)	(0.56)	(0.51)	(0.68)	(0.43)	(4.91)
	(f)		(a)		(g)	
θ	0.3325 ***		$\theta_{s_t=1}$	0.4767 ***	θ	1.4572 ***
	(0.09)			(0.15)		(0.20)
p_{11}	0.9967 ***		$\theta_{s_t=2}$	0.2089 ***	τ^L	0.0137
	(0.00)			(0.06)		(0.03)
p_{22}	0.9976 ***		p_{11}	0.9848 ***	τ^U	0.0840 **
	(0.00)			(0.01)		(0.04)
LL	-3093.82		p_{22}	0.9936 ***	p_{11}	0.9287 ***
				(0.00)		(0.05)
			LL	2918.88	p_{22}	0.9896 ***
						(0.01)
					LL	-2800.53

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the best copula mixture according to Table 2..5. *LL* is the log-Likelihood value.

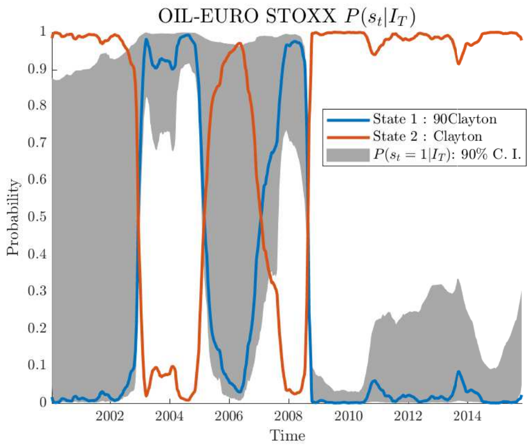
$***/**/*$ indicates statistical significance at 1/5/10%

The copula parameter is assumed to evolve according to a two-state Markov switching specification. p_{ii} indicates the probability of remaining in state i, i.e. $s_t = i$, given that we have been in the same state in the previous period where $i = 1, 2$

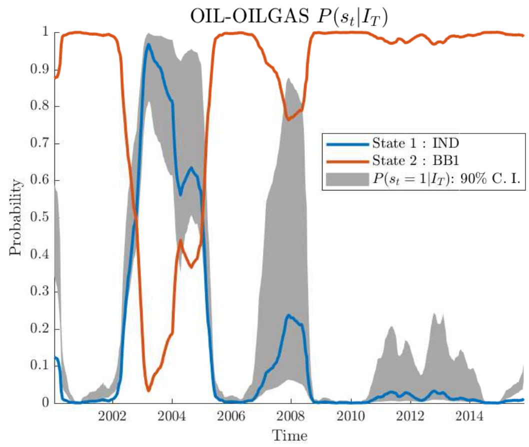
Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: *OIL*). x could be A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

Copula mixtures: (a) 90R Clayton- Clayton; (b) Independence- BB1; (c) BB1- Independence; (d) Student-Gaussian; (e) Gaussian-Clayton; (f) Independence- Clayton; (g) R90 Gumbel-BB1.

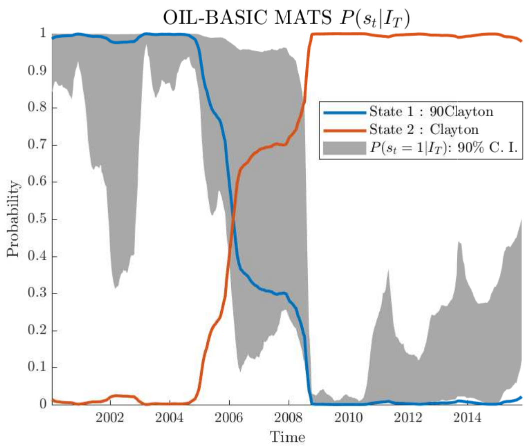
FIGURE 2.5.1A: Smoothed probabilities assuming different copula type across regimes



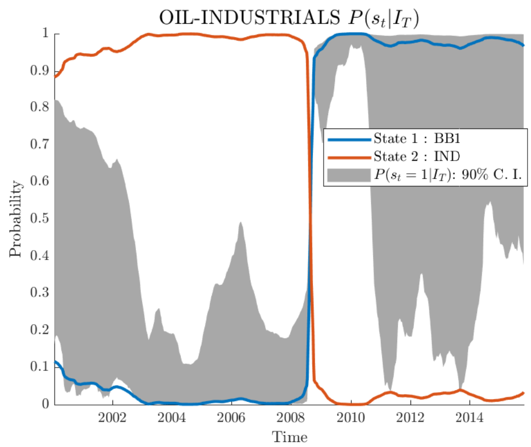
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



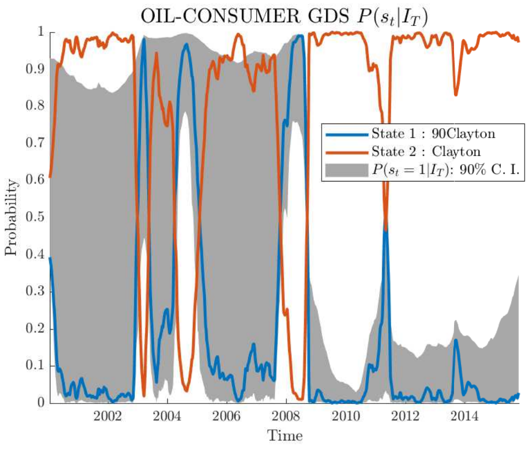
(c) Oil - BASIC MATS sector



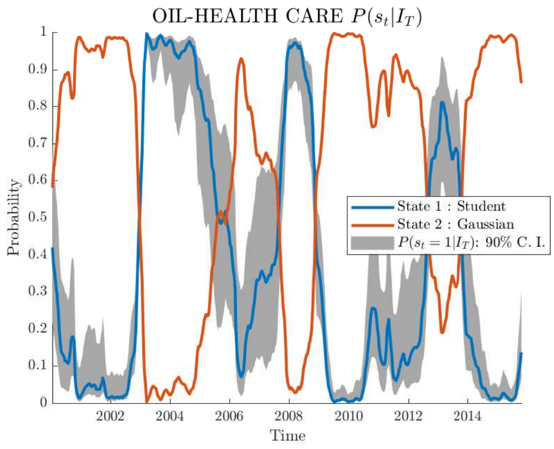
(d) Oil - INDUSTRIALS sector

Smoothed probabilities are obtained using Kim (1994)'s algorithm. Further information about this algorithm is provided in Appendix B. The legend box indicates the chosen copula according to Table 2..5. Solid lines indicate the states under the best mixture of copulas. Grey area indicates the 90% confidence interval of the smoothed probability of state 1 following a Monte Carlo technique explained in Appendix C. Generally speaking there is a clear regime after 2008 that implies positive tail dependence between oil and stock market while in the period 2003-2008 predominates a regime that implies negative tail dependence, although with higher uncertainty.

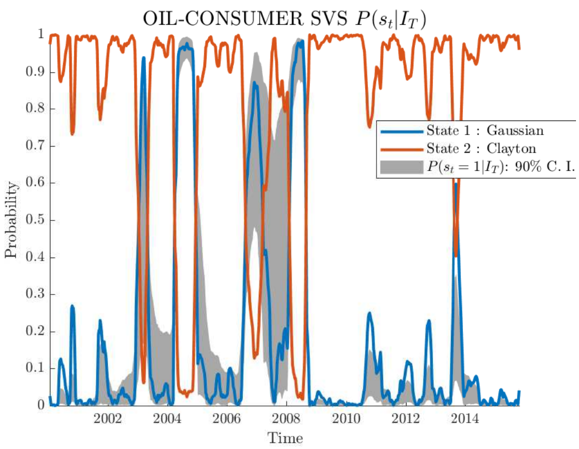
FIGURE 2.5.1B: Smoothed probabilities assuming different copula type across regimes



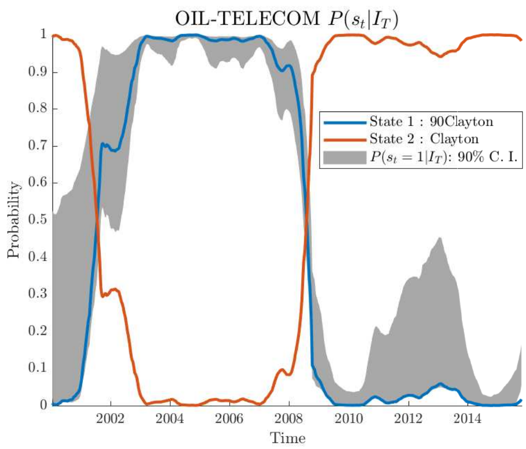
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



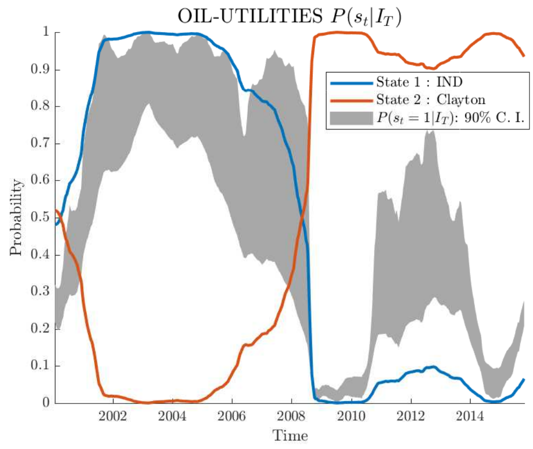
(g) Oil- CONSUMER SVS sector



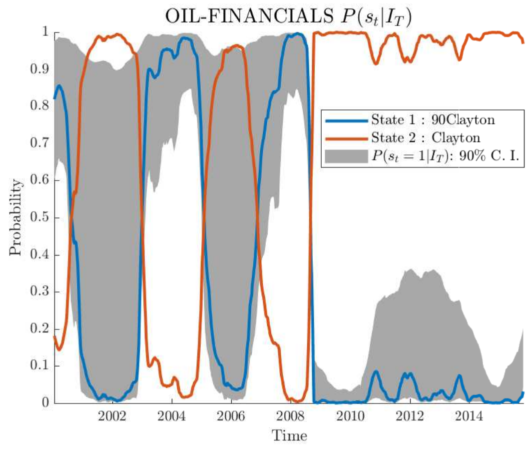
(h) Oil - TELECOM sector

Smoothed probabilities are obtained using Kim (1994)'s algorithm. Further information about this algorithm is provided in Appendix B. The legend box indicates the chosen copula according to Table 2..5. Solid lines indicate the states under the best mixture of copulas. Grey area indicates the 90% confidence interval of the smoothed probability of state 1 following a Monte Carlo technique explained in Appendix C. Generally speaking there is a clear regime after 2008 that implies positive tail dependence between oil and stock market while in the period 2003-2008 predominates a regime that implies negative tail dependence, although with higher uncertainty.

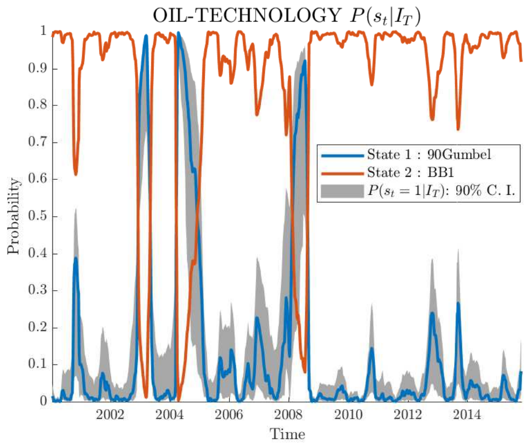
FIGURE 2.5.1C: Smoothed probabilities assuming different copula type across regimes



(i) Oil - UTILITIES sector



(j) Oil - FINANCIALS sector



(k) Oil - TECHNOLOGY sector

Smoothed probabilities are obtained using Kim (1994)'s algorithm. Further information about this algorithm is provided in Appendix B. The legend box indicates the chosen copula according to Table 2..5. Solid lines indicate the states under the best mixture of copulas. Grey area indicates the 90% confidence interval of the smoothed probability of state 1 following a Monte Carlo technique explained in Appendix C. Generally speaking there is a clear regime after 2008 that implies positive tail dependence between oil and stock market while in the period 2003-2008 predominates a regime that implies negative tail dependence, although with higher uncertainty.

Health care sector presents a regime with strong negative tail dependence while the other regime has positive but small correlation. This feature indicates that adding the healthcare to a stock portfolio can decrease the exposure to oil spillovers at extreme quantiles.

A likelihood ratio test is performed to test if the model with time-varying dependence is statistically better than the model with constant dependence. The model with constant dependence represents the restricted model, while the model with time-varying copula represents the unrestricted model. The likelihood ratio is distributed as

$$-2(log(Likel_R) - log(Likel_{UR})) \sim X_{k_{UR}-k_R}$$

where k refers to the number of parameters for the model while R is the restricted (constant) model and UR is the (time-varying) unrestricted model.

Note that the constant model is nested in the time-varying model. However the transition probabilities of the Markov process are not identified under the null hypothesis, so regularity conditions justifying the χ^2 approximation to the likelihood ratio test are not held. Following Cai (1994), I replicate 500 series of returns with a sample length of 1000 under the constant model using Monte Carlo simulations. Then, the Markov switching model is fitted for each generated series. Finally, I calculate the likelihood ratio statistic of each Monte Carlo simulation getting the distribution of the likelihood ratio statistic under the null hypothesis, which I use to obtain the p-value.

The results in Table 2.5.2 indicate that the null hypothesis is not rejected for oil & gas, industrial and technology sector when the unrestricted model only allows for a change in the parameter of the copula. Nevertheless, the probability of each state gives useful information concerning a potential change. Allowing for a change not only in the parameter but also in the copula itself improves the model, rejecting the null hypothesis that constant and time-varying models are equivalent.

TABLE 2.5.2: Likelihood ratio test between the constant and the time-varying model

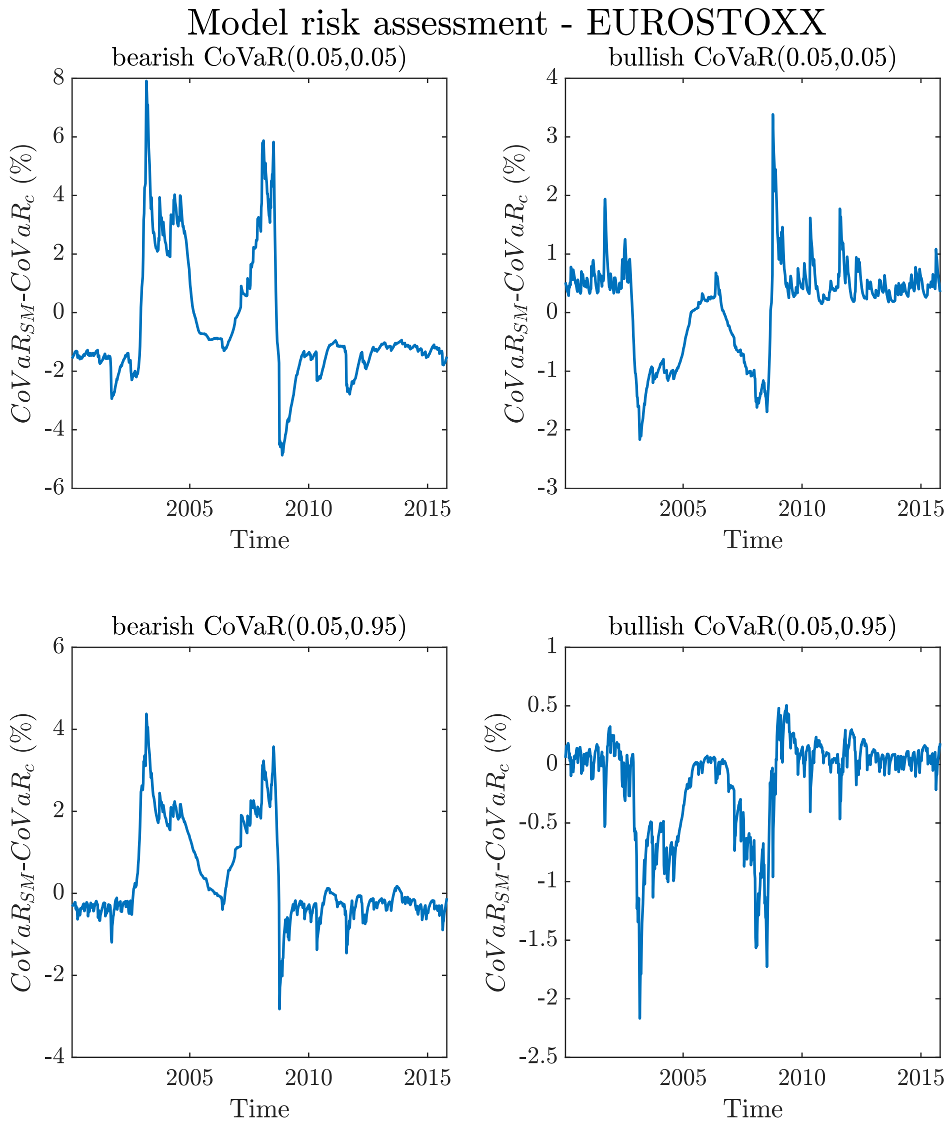
	A	B	C	D	E	F	G	H	I	J	K
LR - (a)	7,523	6,637	10,932	7,219	12,581	11,188	6,765	6,704	9,326	6,770	3,413
p-value - (a)	0,056	0,160	0,040	0,120	0,006	0,026	0,064	0,088	0,050	0,074	0,466
LR - (b)	13,703	15,966	15,139	7,270	15,571	10,342	16,946	16,288	9,326	17,338	18,088
p-value - (b)	0,002	0,000	0,000	0,016	0,002	0,018	0,000	0,004	0,012	0,008	0,000

This tables shows in the top row the likelihood ratio, i.e. $LR = -2(log(Likelihood_R) - log(Likelihood_{UR}))$, where the distribution under the null hypothesis is obtained by a Monte Carlo simulation.
The two first rows (a) indicate the likelihood ratio test where the restricted model is the constant model and the unrestricted model is the model that only allow for changes in the copula parameter. The last two rows (b) indicate the likelihood ratio test where the restricted model is the constant model and the unrestricted model is the model that allow for a change in the copula itself. The copulas employed is the best one according $AICc$ in tables 2..3 and 2..5.
LR shows the statistic of the likelihood ratio.
The $p - value$ indicates the p-value for the Likelihood test where the null hypothesis indicates that the restricted model (constant) and the unrestricted model (time-varying model) are not statistically different between them, while the alternative hypothesis is the unrestricted model is statistically better than the restricted model.
Note that for some cases where we can not reject under the case (a) we can do it under case (b) due to a better fitting and a lower number of parameters, e.g. for the cases where the copula is combined with the independence case.
A: $EUROSTOXX$; B: $OIL\&GAS$; C: $BASICMATS$; D: $INDUSTRIALS$; E: $CONSUMERGDS$; F: $HEALTHCARE$; G: $CONSUMERSVS$; H: $TELECOM$; I:$UTILITIES$; J: $FINANCIALS$; K: $TECHNOLOGY$.

The fitting improvement using the model with time-varying dependence can be quantify in terms of the estimation of a percentile for Eurostoxx depending on the scenario for oil prices. Figure 2.5.2 shows the difference between the estimation of the same percentile under the same oil-related scenario over the analysed period for the Eurostoxx using the time-varying copula (minuend) and the constant dependence model (subtrahend). The model risk of assuming a constant dependence across assets implies an overestimation by 8% and a underestimation by 4% of the same risk measure depending on the period, scenario and chosen percentile.

Appendix E provides the same type of figures for the stock sectors. They indicate a higher model risk in the estimation of low quantiles of stock returns than high percentiles, in particular under scenarios where oil prices experience a downward movement.

FIGURE 2.5.2: Model risk assessment when we assume a constant dependence compared to the time-varying model.



These figures show the difference between the estimation of the same percentile of Eurostoxx returns under the same oil-related scenario using the Markov switching model that allows for changes in the copula against the constant model. Top figures focuses on a low quantile ($\beta = 5\%$) whilst bottom graphs estimate the high quantile ($\beta = 95\%$). Left figures show a percentile of stock returns under a bearish scenario for oil returns, i.e. oil returns below its percentile 5%, and right figures show a percentile of stock returns under a bullish scenario for oil returns, i.e. oil returns above its percentile 95%. These charts indicate the higher model risk in the lower tail than in the upper tail when assuming a constant dependence across markets.

Next subsection computes the effects of extreme changes in oil prices on the Value-at-Risk for Eurostoxx and for the different European industrial subsectors at 5% and 95% confidence level.

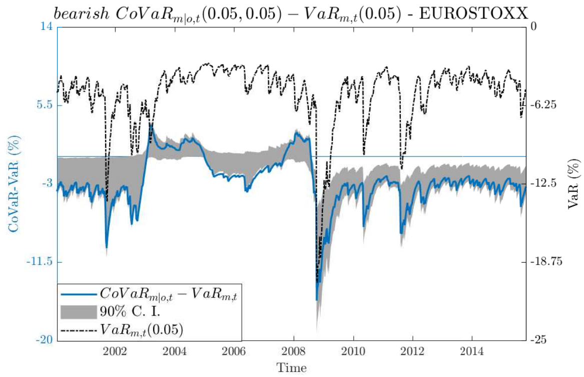
2.5.2 Implications of structural changes in joint tail dependence for risk management

This section presents first the results of the $CoVaR(0.05, 0.05)$ and $CoVaR(0.05, 0.95)$ for each stock sector conditioned to a bearish or bullish scenario for oil returns. Second, a table summarizes different quantiles of the change in the VaR measure once considered the oil scenario before and after the financial crisis. Third, a Kolgomorov-Smirnov bootstrap test is performed to check that the risk of overlooking the oil scenario is statistically significant. Fourth, I build a portfolio using Eurostoxx and healthcare to minimize (in absolute terms) the mean Conditional Value-at-Risk at 5% confidence level. I rebalance the portfolio every four months from 23 October 2015, the last day of the in-sample period, to 27 July 2018.

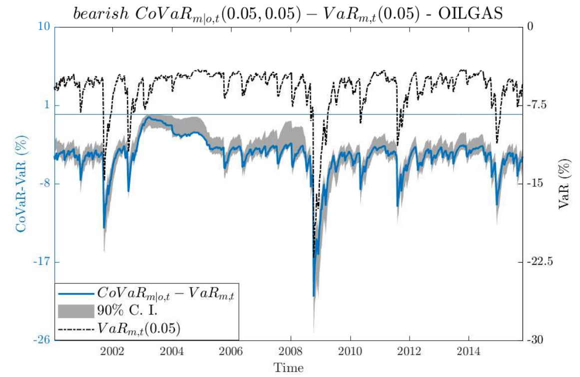
Figures 2.5.3a, 2.5.3b, 2.5.3c, 2.5.4a, 2.5.4b and 2.5.4c plot on the right axis the VaR estimates, which is depicted by the dash-dotted black line. These figures present on the right axis the difference $CoVaR - VaR$ for the best copula (solid blue line) according to AICc values from Table 2..5 and its 90% confidence interval computed by Monte Carlo simulations (see Appendix C).

Figures 2.5.3a, 2.5.3b and 2.5.3c show a bearish oil scenario for the $CoVaR_{m|o,t}(0.05, 0.05)$. The oil & gas sector has always a $CoVaR_{m|o,t}(0.05, 0.05)$ more negative than $VaR_{m,t}$ although this difference is smaller during the period 2003-2005. During period 2003-2005 the difference $CoVaR_{m|o,t}(0.05, 0.05) - VaR_{m,t}$ is positive for most of the sectors (Eurostoxx, financial, healthcare, basic materials, utilities, telecommunications, consumer goods, consumer services). There is no change in the $VaR_{m,t}$ for the industrial sector before 2008. The period 2007-2008 also presents $CoVaR_{m|o,t}(0.05, 0.05)$ less negative than $VaR_{m,t}$. The healthcare sector looks quite insensitive in its lower tail to negative shocks on the oil price with a maximum negative difference between 2% and 7% depending on the copula choice while for most of the sectors the difference reaches two-digit numbers.

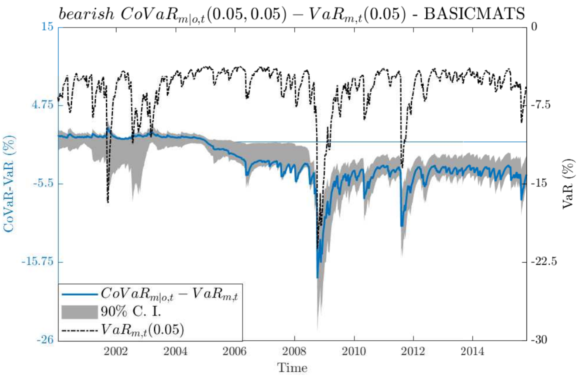
FIGURE 2.5.3A: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



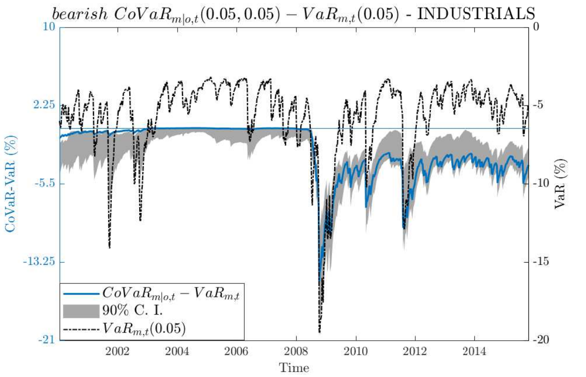
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



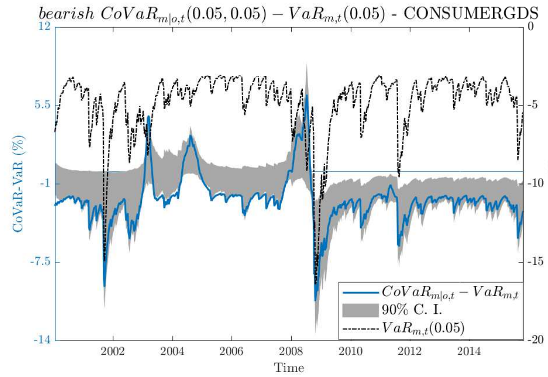
(c) Oil - BASIC MATS sector



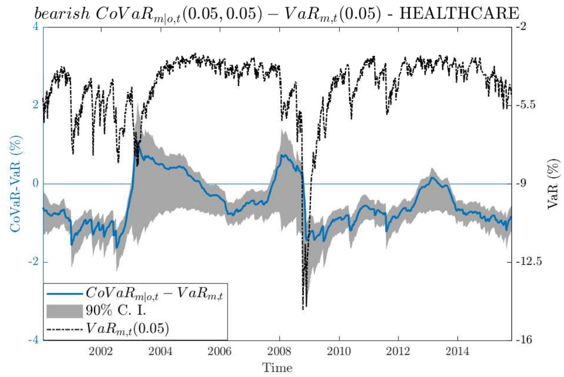
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05)$ of the returns for a certain sector under a extreme downward movement in oil price, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

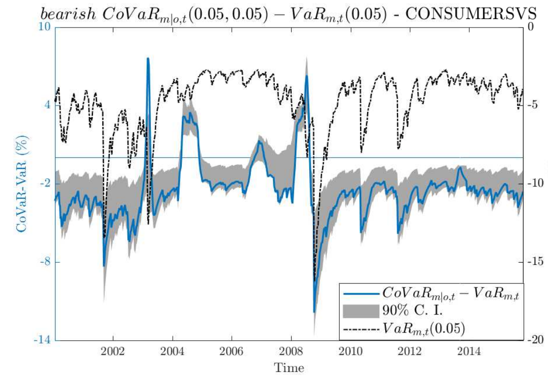
FIGURE 2.5.3B: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



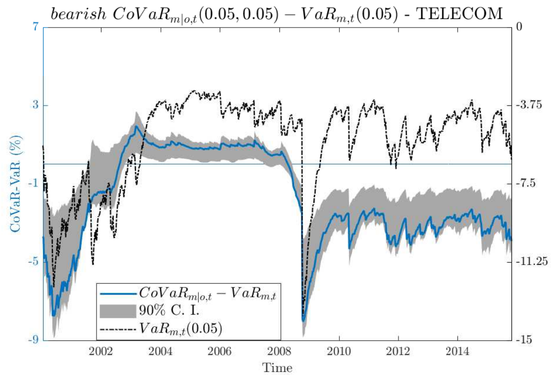
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



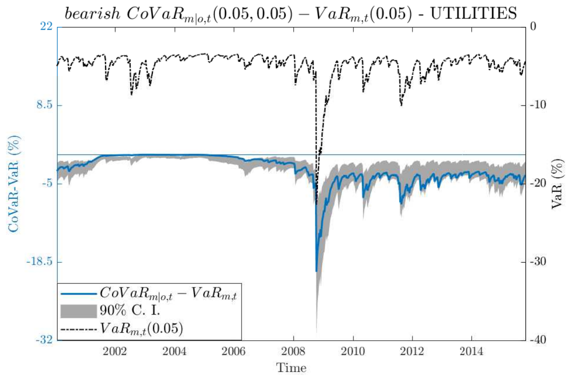
(g) Oil- CONSUMER SVS sector



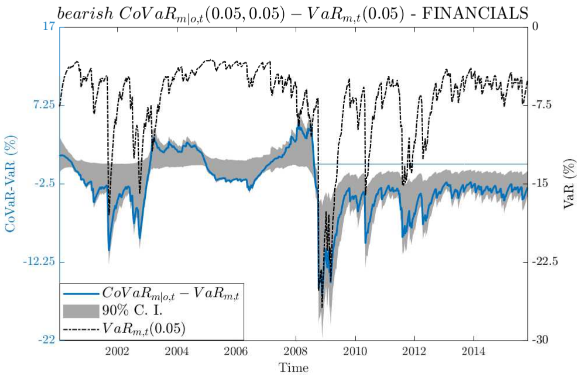
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05)$ of the returns for a certain sector under a extreme downward movement in oil price, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

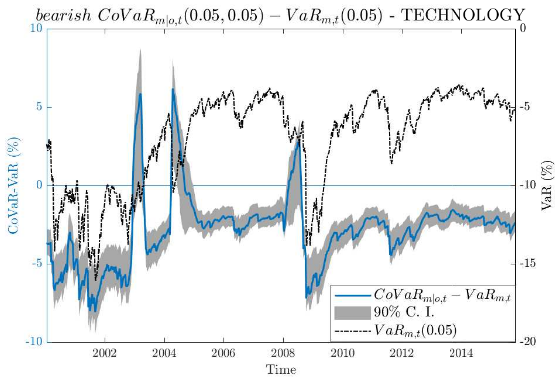
FIGURE 2.5.3C: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



(i) Oil - UTILITIES sector



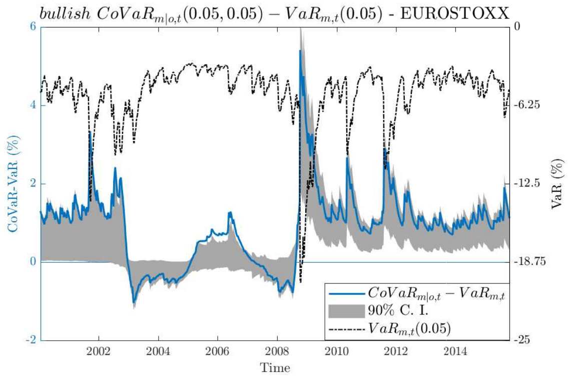
(j) Oil - FINANCIALS sector



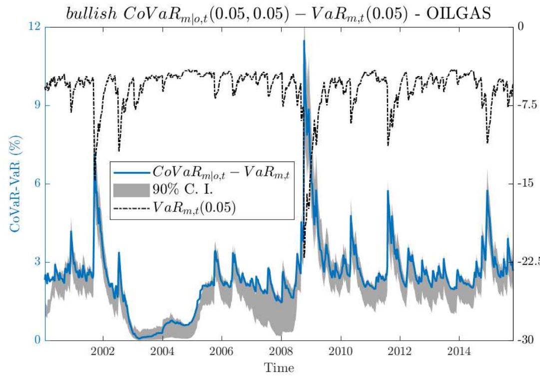
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoVaR_{m|o,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme downward movement in oil price, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m,t}(0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

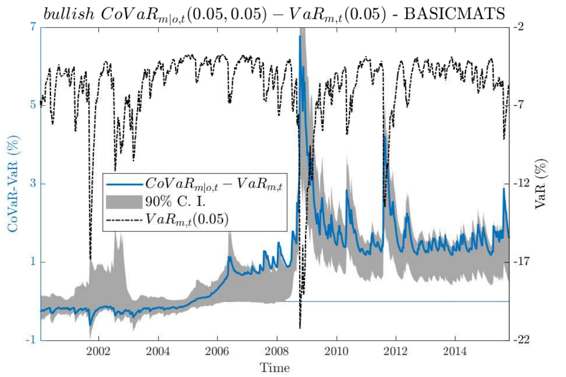
FIGURE 2.5.4A: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



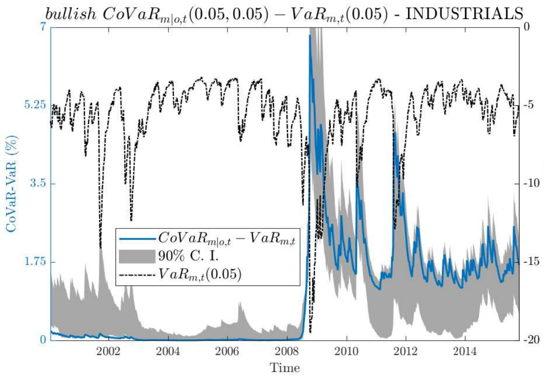
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



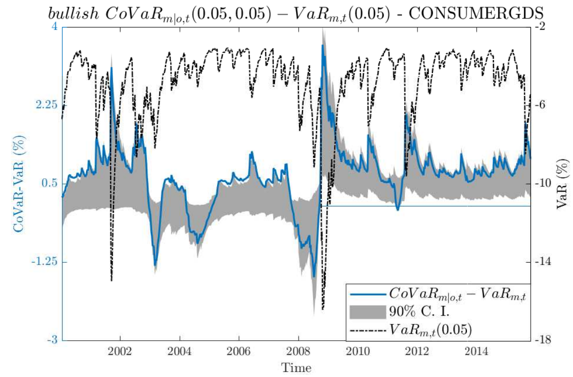
(c) Oil - BASIC MATS sector



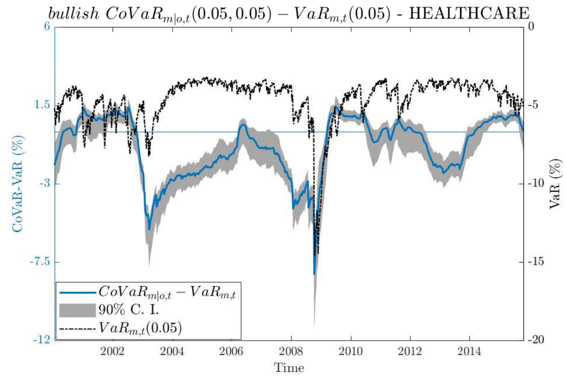
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoVaR_{m|o,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.05, 0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

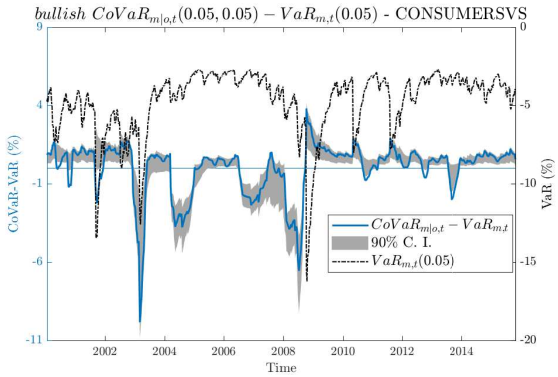
FIGURE 2.5.4B: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



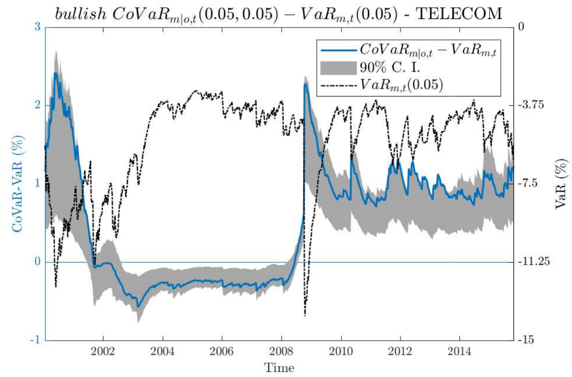
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



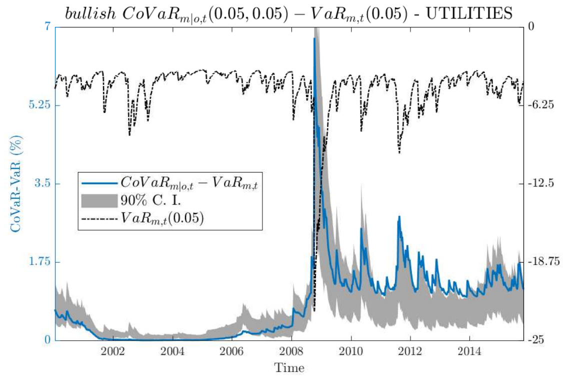
(g) Oil- CONSUMER SVS sector



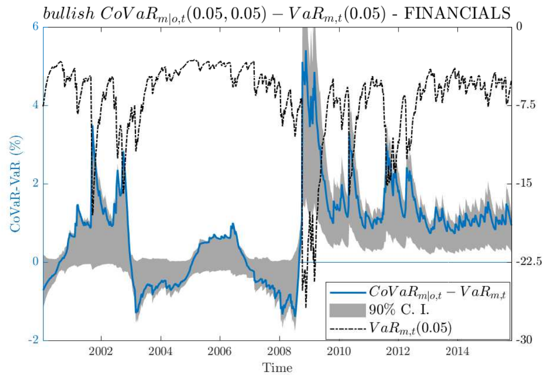
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoVaR_{m|o,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.05, 0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

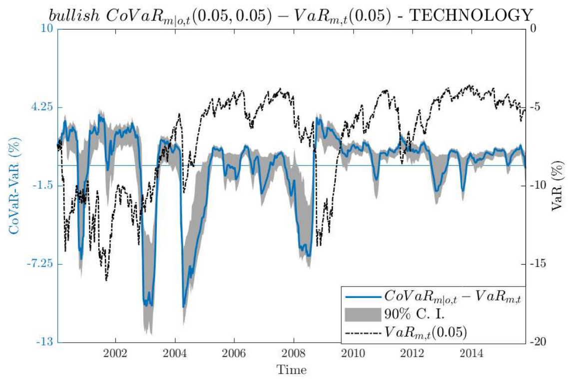
FIGURE 2.5.4C: $CoVaR_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



(i) Oil - UTILITIES sector



(j) Oil - FINANCIALS sector



(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoVaR_{m|o,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.05)$ and $VaR_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the VaR level over time.

Figures 2.5.4a, 2.5.4b and 2.5.4c show a bullish oil scenario for $CoVaR_{m|o,t}(0.05, 0.05)$, where the unconditional $VaR_{m,t}$ seems to be overestimating the losses. The healthcare sector is one exception where the $CoVaR_{m|o,t}(0.05, 0.05)$ has higher losses than the $VaR_{m,t}(0.05)$ during the period 2003-2008. Moreover, healthcare sector shows this pattern after the 2008 financial crisis, during 2012-2014, moving in the opposite direction of Eurostoxx or oil & gas sector given extreme upward movements in oil prices. This feature makes it a potential good asset to reduce the tail dependence of a stock portfolio with the oil returns.

Figures for the upper tail of the stock market sectors, i.e. $VaR_{m,t}(0.95)$, and $CoVaR_{m,t}(0.05, 0.95) - VaR_{m,t}(0.95)$ show a much lower change than in the lower tail, where two-digit values are reached. Figures for the upper tail along with the $CoES$ charts are provided in Appendix E. $CoES$ figures provide a robustness check and give information the quantile of the $CoVaR$. The $CoES$ plots support the main conclusions shown by the $CoVaR$.

Table 2.5.3 displays the p-values of the Kolgomorov-Smirnov (KS) bootstrap test for the cumulative distribution function of $CoVaR$ and VaR in the pre-crisis and post-crisis sample.[5] Within the pre-crisis sample, the null hypothesis of equal cumulative distribution function of $CoVaR$ and VaR can not be rejected for the industrial sector, but within the post-crisis period it is rejected for all the oil scenarios and confidence levels. The KS null hypothesis can not be rejected either for the consumer services under a bullish oil scenario for $CoVaR_{m|o,t}(0.05, 0.05)$ in the pre-crisis sample, but it is rejected in the post-crisis period. The utilities sector presents the same results under a bullish oil scenario for $CoVaR_{m|o,t}(0.05, 0.95)$.

Table 2.5.4 shows the $75-th$, $50-th$ and $25-th$ quantiles for the difference $CoVaR_{m|o,t}(0.05, 0.05) - VaR_{m,t}(0.05)$ and $CoVaR_{m|o,t}(0.05, 0.95) - VaR_{m,t}(9.05)$ given a upward or downward movement in oil prices for the pre-crisis and post-crisis sample. The red cells indicates an underestimation of the VaR measure given a certain scenario higher than 2% while the green cells represents an overestimation of the VaR measure when a certain oil-related scenario occurs higher than 2%. The change in $VaR_{m,t}(0.05)$ occurs in more sectors than in the $VaR_{m,t}(0.95)$ given a bearish oil scenario. This change also increases after the financial crisis getting even in the highest 25-th quantile an excess of losses greater than 2% for all sectors with the exception of the healthcare.

[5]The main reason to build bootstrap tests in estimated measures is due to the introduction of a nuisance parameter in the sample distribution. The distribution under the null hypothesis might be different because of the estimated parameters, affecting to the confidence interval and the p-values. See Abadie (2002), Bernal et al. (2014) and Ojea Ferreiro (2018) for further details about Kolgomorov-Smirnov bootstrap test.

TABLE 2.5.3: Kolgomorov-Smirnov bootstrap p-values

		A	B	C	D	E	F	G	H	I	J	K
BE05	Pre-crisis	0,000	0,000	0,000	0,255	0,000	0,001	0,000	0,000	0,000	0,000	0,000
	Post-crisis	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
BU05	Pre-crisis	0,000	0,000	0,000	1,000	0,000	0,000	0,109	0,000	0,009	0,000	0,001
	Post-crisis	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000
BE95	Pre-crisis	0,000	0,000	0,000	0,991	0,000	0,000	0,015	0,001	0,000	0,009	0,001
	Post-crisis	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000
BU95	Pre-crisis	0,000	0,000	0,071	0,986	0,041	0,007	0,000	0,000	0,678	0,000	0,000
	Post-crisis	0,004	0,000	0,000	0,000	0,058	0,000	0,030	0,000	0,000	0,078	0,000

This table shows the p-values of the Kolgomorov-Smirnov bootstrap test using 2000 simulations to compare the distribution of $CoVaR$ with the values of VaR before and after the 2008 financial crisis. For more information about the Kolmogorov Smirnov bootstrap test see for instance Abadie (2002), Bernal et al. (2014), Ojea Ferreiro (2018).
A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.
BE05: bearish $CoVaR_{m|o,t}(0.05, 0.05)$, BU05: bullish $CoVaR_{m|o,t}(0.05, 0.05)$, BE95: bearish $CoVaR_{m|o,t}(0.05, 0.95)$, BU95: bullish $CoVaR_{s^U,o^U}(0.05, 0.95)$. BE05 and BU05 are compared with the $VaR_{m,t}(0.05)$ while BE95 and BU95 are compared with $VaR_{m,t}(0.95)$.
Pre-crisis subsample goes from January 7^{th}, 2000 to September 12^{th}, 2008. Post-crisis subsample goes from 19^{th} September 2008 to 23^{rd} October 2015.

TABLE 2.5.4: Quantiles of the $CoVaR - VaR$ distribution before and after the 2008 financial crisis.

bearish $CoVaR_{m|o,t}(0.05, 0.05) - VaR_{m,t}(0.05)$ (%)

	Q	A	B	C	D	E	F	G	H	I	J	K
Pre-crisis	75	0,8	-2,8	0,6	0,0	-1,5	0,4	-1,4	1,0	0,0	1,6	-2,0
	50	-2,1	-4,1	0,4	-0,1	-2,2	-0,4	-2,5	0,8	-0,3	-1,0	-2,7
	25	-3,5	-4,8	-2,6	-0,3	-2,7	-0,8	-3,6	-1,5	-1,4	-2,7	-5,2
Post-crisis	75	-2,8	-4,3	-3,6	-3,1	-2,2	-0,6	-2,3	-2,7	-3,5	-3,2	-2,1
	50	-3,2	-4,8	-4,1	-3,7	-2,6	-0,8	-2,6	-3,0	-3,9	-3,7	-2,5
	25	-4,3	-5,9	-5,1	-4,8	-3,3	-0,9	-3,3	-3,6	-4,8	-5,4	-3,2

bearish $CoVaR_{m|o,t}(0.05, 0.95) - VaR_{m,t}(0.95)$ (%)

	Q	A	B	C	D	E	F	G	H	I	J	K
Pre-crisis	75	1,6	-0,5	1,1	0,0	1,8	1,7	1,1	2,6	0,0	2,3	0,2
	50	0,4	-1,5	0,9	0,0	0,2	0,9	-0,2	2,2	0,0	1,4	-0,8
	25	-0,7	-1,8	0,0	0,0	-0,4	-0,1	-0,6	1,9	-0,2	-0,2	-1,7
Post-crisis	75	-0,5	-1,7	-1,1	-1,0	-0,3	0,6	-0,3	-0,8	-1,0	-0,4	-0,7
	50	-0,7	-1,9	-1,2	-1,1	-0,6	-0,1	-0,5	-0,9	-1,1	-0,6	-0,9
	25	-0,9	-2,3	-1,5	-1,5	-0,8	-0,5	-0,7	-1,1	-1,4	-0,9	-1,2

bullish $CoVaR_{m|o,t}(0.05, 0.05) - VaR_{m,t}(0.05)$ (%)

	Q	A	B	C	D	E	F	G	H	I	J	K
Pre-crisis	75	1,2	2,5	0,7	0,1	0,8	0,2	0,8	0,0	0,3	0,7	1,7
	50	0,5	2,1	-0,2	0,0	0,6	-1,3	0,2	-0,2	0,1	0,0	0,4
	25	-0,4	0,9	-0,2	0,0	0,0	-2,5	-1,8	-0,3	0,0	-0,6	-2,2
Post-crisis	75	1,5	3,2	1,9	2,2	1,1	0,8	0,9	1,1	1,6	1,7	1,3
	50	1,1	2,6	1,6	1,7	0,8	0,1	0,7	0,9	1,3	1,2	1,0
	25	1,0	2,3	1,4	1,5	0,7	-0,9	0,4	0,8	1,1	1,0	0,5

bullish $CoVaR_{m|o,t}(0.05, 0.95) - VaR_{m,t}(0.95)$ (%)

	Q	A	B	C	D	E	F	G	H	I	J	K
Pre-crisis	75	0,2	2,3	0,1	0,1	0,2	0,5	0,2	-0,6	0,1	0,1	3,5
	50	0,0	2,0	-0,3	0,0	0,1	0,3	0,1	-0,7	0,0	-0,3	1,8
	25	-0,6	1,3	-0,4	0,0	-0,3	-0,3	-0,4	-0,8	0,0	-0,9	1,3
Post-crisis	75	0,3	2,9	0,4	1,5	0,2	0,6	0,2	0,3	0,4	0,3	2,2
	50	0,2	2,3	0,3	1,2	0,2	0,5	0,2	0,3	0,3	0,2	1,7
	25	0,2	2,1	0,3	1,0	0,1	0,4	0,1	0,2	0,3	0,2	1,4

The table shows different quantiles (75-th, the median and the 25-th) of the $CoVaR - VaR$ distribution in the pre-crisis and post-crisis samples. Values that implies an underestimation (overestimation) of the VaR higher than 2% are in red (green).
A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

The following subsection performs a forecast exercise to test the healthcare sector's features as a hedging asset against extreme oil movement.

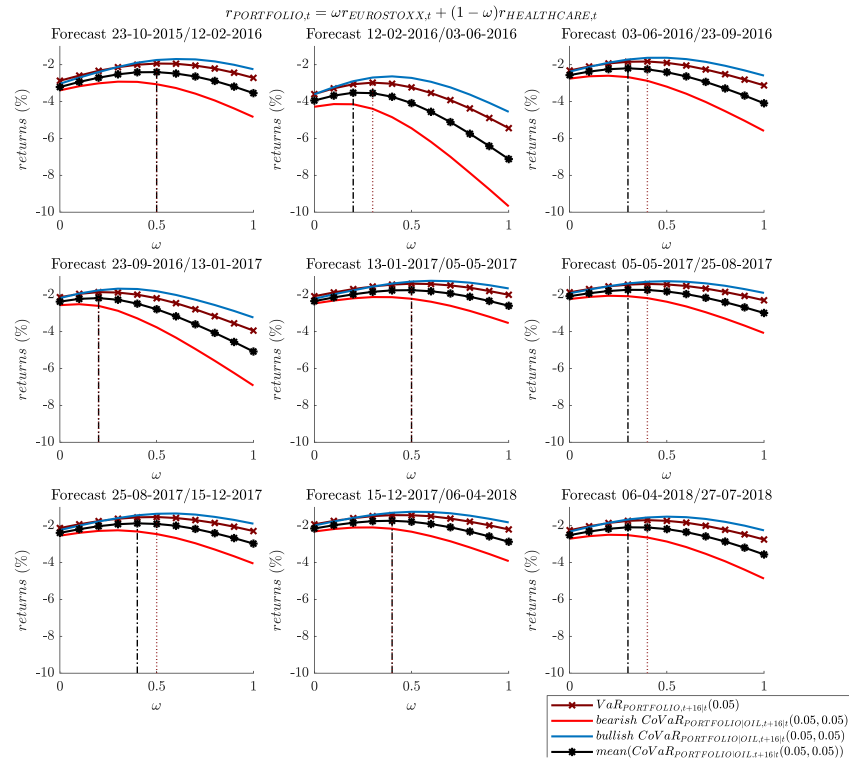
2.5.3 Portfolio exercise using an out-of-sample period

Results in previous sections suggest that the healthcare sector may have useful features to reduce the exposure of the stock portfolio to extreme movements in oil prices. To test the hedging possibilities, I build a portfolio given the information available up to the end of the in-sample period, i.e. 23 October 2015. The portfolio is rebalanced every four months until 27 July 2018. It consists of long positions in a benchmark stock market index, the Eurostoxx, and the healthcare sector. I use Monte Carlo simulations to obtain the weekly returns of Eurostoxx and healthcare sector four months ahead[6]. The same seed is used for all the simulations of the oil quantile to be sure that the paths for Eurostoxx and healthcare returns are consistent, i.e. obtained under the same oil scenario. Then, I compute the VaR, the bullish $CoVaR$ and the bearish $CoVaR$ for different weights of the portfolio. The top subgraph from Figure 2.5.5 presents the forecast for the nine rebalancing periods of VaR in the brown line, the bearish $CoVaR(0.05, 0.05)$ in the red line and the bullish $CoVaR(0.05, 0.05)$ in the blue line. Eurostoxx returns have lower bearish $CoVaR(0.05, 0.05)$ than healthcare sector but higher bullish $CoVaR(0.05, 0.05)$ for most of the forecast periods. To weight equally the extreme downward and upward movements in oil prices I assess the mean $CoVaR(0.05, 0.05)$ as the mean between the bearish and bullish $CoVaR(0.05, 0.05)$ in the black line. Not surprisingly, the optimal weight to minimize VaR (vertical dash-dotted brown line) and the optimal weights to minimize mean $CoVaR(0.05, 0.05)$ (vertical dash-dotted black line) coincide for most of the periods. Due to the simulation procedure, Eurostoxx and healthcare returns are conditionally independent, i.e. they are dependent only through the common exposure to oil returns. Hence, decreasing the exposure of the portfolio to extreme movements in oil prices implies a reduction in the degree of dependence between both assets. The lower subgraph from Figure 2.5.5 presents a performance measure where expected return is weighted by its risk, defined by its 5% lowest return in absolute value. The higher this ratio is, the lower is the risk to be faced by the investor for the same expected return. Note that the relationship between this ratio and the weights of the portfolio is not linear, and its sign depends on the oil scenario. This result indicates potential advantages introducing healthcare into the stock portfolio. Finally, I show the joint distribution of oil, Eurostoxx, healthcare sector and our optimal portfolio in Figure 2.5.6 for the out-of-sample period. The scatter plot combines with the histograms of the marginal distribution in the axes, smoothed by a kernel function. The x-axis shows the marginal distribution of the oil returns. The yellowish area indicates the scenario where oil prices experience a bullish period (top subplot) or a bearish period (bottom subplot). The y-axis indicates the conditional distribution histogram (smoothed by a kernel function), where the portfolio behaviour is closer to the distribution of the returns of the Eurostoxx or healthcare sector depending on the oil scenario. This features gives to our portfolio a better performance in terms of $CoVaR$[7].

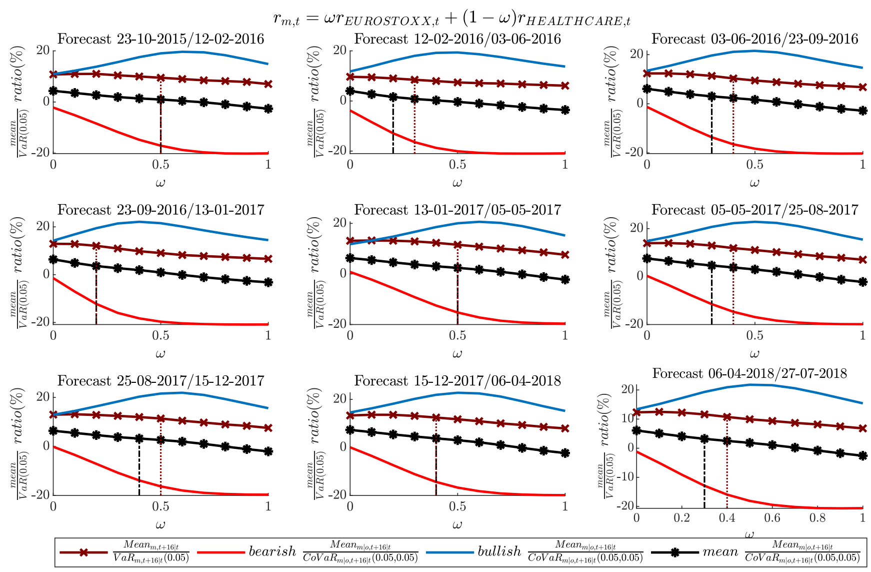
[6]Detailed information about the simulation process is provided in Appendix C

[7]The CoVaR is employed taking the 10% lowest returns of the equity portfolios given that the oil returns is above its 10% best case scenario or below its 10% worst case scenario

FIGURE 2.5.5: Forecast exercise : building a portfolio without tail dependence using Eurostoxx and health sector assets



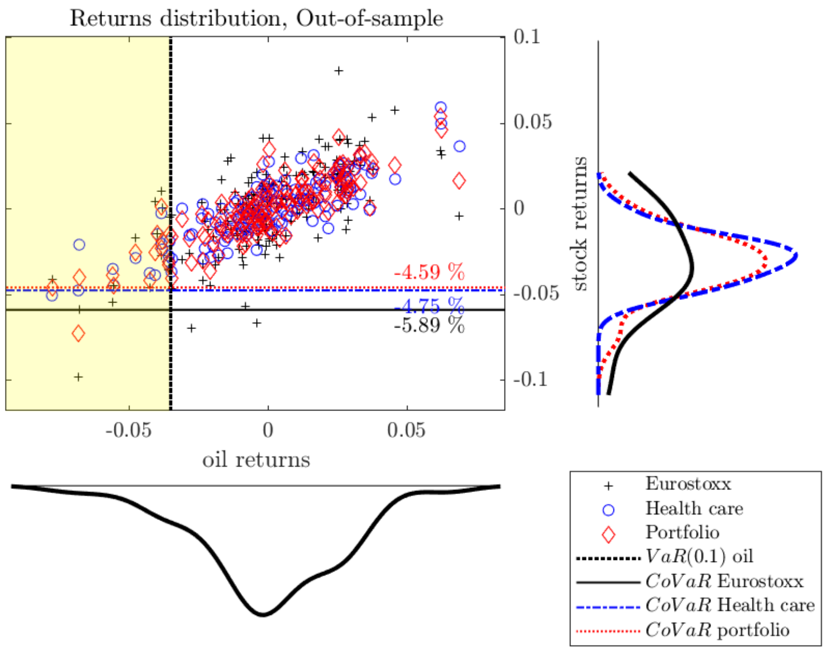
(l) Forecast $CoVaR(0.05, 0.05)$ and optimum weights between EUROSTOXX and HEALTH CARE sector



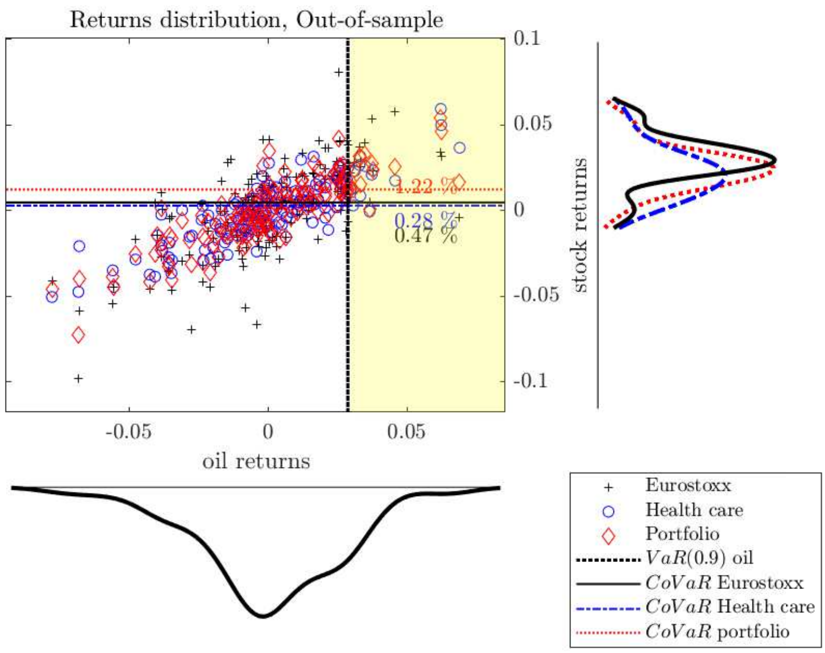
(m) Ratio between the median return of the portfolio and its 95% higher losses

A four-month period forecast for the bearish and bullish $CoVaR(0.05, 0.05)$, and the $VaR(0.05)$ is calculated out-of-sample using Monte Carlo simulations (W=100000) for a portfolio of Eurostoxx and health care. Top subfigure shows the forecast estimation for the different four-month periods and the optimal portfolio weights to minimize the mean $CoVaR(0.05, 0.05)$, i.e. the mean between the bullish and the bearish $CoVaR(0.05, 0.05)$. Lower subfigure shows a performance measure, which consists of the ratio between the mean returns of the portfolio and its $CoVaR(0.05, 0.05)$ in absolute values. The higher this ratio is, the lower risk has to be faced by the investor for the same expected return.

FIGURE 2.5.6: Empirical joint distribution during the out-of-sample period, where the current oil scenario is shaded in yellow



(a) Y-axis shows the conditional distribution for the stock portfolios given a downward movement in oil prices



(b) Y-axis shows the conditional distribution for the stock portfolios given a upward movement in oil prices

The scatter plot shows the joint distribution between the returns of oil and equity. The x-axis shows the unconditional marginal distribution of the oil returns in the out-of-sample period. The y-axis shows the conditional distribution of the returns of Eurostoxx, healthcare sector and our optimal portfolio given the oil scenario, defined as being below the 10% or above the 90% percentile. The yellowish are indicates these scenarios. The $CoVaR$ shows the 10% worst case scenario for the Eurostoxx, healthcare sector and our portfolio.

2.6 Conclusions

This article quantifies the spillovers from oil returns to the European stock market using *CoVaR* measures during the period 2000-2015. The Conditional Value-at-Risk helps us to understand better the unexpected link arising on extreme scenarios, i.e. it provides a more robust estimation to outliers than mean response results where non-linear spillovers and asymmetric tail dependence might be overlooked. Furthermore, the *CoVaR* measure is coherent with the risk aversion of economic agents, who are more interested on realising how adverse the portfolio behaviour can become than on knowing how its performance can be on average. The relationship between oil and stock markets is known to be characterised by non-linearities, asymmetric dependence and structural changes. A Markov switching approach is then useful because it can identify hidden patterns in dependence across regimes and it can capture non-linear features and asymmetries that evolve over time. Allowing for a change not only in the copula parameter but in the copula itself provides us with an econometric analysis that can produce very significant results for risk management. I show that such flexibility in the model prevents us from mistaking negative dependence for null dependence.

The model identifies a switch from negative to positive lower tail dependence. During the period 2003-2008, lower quantiles in oil returns imply higher quantiles in stock returns while, after the financial crisis, lower quantiles in stock returns are associated with lower oil returns. This structural change in the oil-stock market relationship is led by the oil-intensive sectors, e.g. basic materials or consumer goods. The dependence switch might be closely related to the economic cycle. Indeed, the dependence on the negative lower tail indicates that a decrease in oil prices would generally increase the margin between final prices and the unit cost of production before the crisis, while a rise in oil prices may have usually been reflected in the final price. The increase in prices would not necessarily lead to a lower demand because the income increases in the expansion phase of the business cycle. This could explain the negative lower tail dependence and the absence of upper tail dependence. The 2008 crisis led to a credit crunch and losses for European companies that implied a drop in oil demand, which in turn led to a decrease in its price explaining the positive dependence on the lower tail. The link in the co-movement between the economic sectors and oil prices through the business cycle is already pointed out by Andreopoulos (2009) for the US economy.

The results are relevant for investors, who want to reduce the oil exposure of their stock portfolios. The healthcare sector helps to decrease the exposure of stock market portfolio to oil shocks without taking a short position in commodities. Such strategy decreases the dependence between the stock portfolio and oil returns reducing the maximum loss provided by the unconditional Value-at-Risk. The market authorities need a quantitative analysis of the impact of oil shifts on the stock market to properly monitor the behaviour of companies in the stock exchange. The existence of market failure, when an extreme scenario for oil prices materialises, motivates policy actions taken by the regulator. *CoVaR* provides useful information to place in trading halt a certain quote of the stock exchange, or to update the variation margin for some stock derivatives. Sector analysis has implications for policy makers who are concerned about the effect on stock markets of extreme changes in oil prices, as their consequences can be felt in terms of growth and employment. In fact, stakeholders can use CoVaR as a short-term quantitative assessment of the effects of sharp movements in oil prices on the economic sectors.

Further research should analyse the potential role of exchange rate to mitigate the negative effects of abrupt movements in oil prices on the stock market. Since oil prices are denominated in dollars that are then translated into Euros, a transitory shock in oil prices can be alleviated by the right movement in the exchange rate. Additional robustness checks can be performed by slightly modifying the model. As Reboredo (2010) claims, volatility may be the trigger conditioning the type of relationship between oil and the stock market. This hypothesis could be checked by building a Markov switching model where the transition probabilities of the copula function depend on the transition probabilities of the volatility of the stock market.

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Appendix

A Bivariate Copula set

Gaussian and Student copula are elliptical copulas, i.e., the bivariate joint density under these copulas has elliptic isodensities.
Gumbel, Clayton and BB1 are Archimedean copulas, which implies that can be expressed as a function of the generate function ϕ and its inverse ϕ^{-1}, i.e. $C(u_1, u_2, \theta) = \phi^{-1}[\phi(u_1;\theta) + \phi(u_2;\theta);\theta]$ where θ is the copula parameter.
To enhance the features of copulas that only allow for positive dependence, they are rotated to capture negative tail dependence. Table **??** shows the tail dependence for the 90° rotated copulas. The 90° rotated copulas are built modifying slightly the standard copula, i.e.

$$C_{90}(u_1, u_2) = u_2 - C(1 - u_1, u_2)$$

Gaussian copula. This copula has a parameter ρ that gathers linear correlation. When $\rho = 1$ the tail dependence is 1, otherwise this copula does not present tail dependence. There is not a closed form expression due to the fact that Gaussian copula is an implicit copula. Meyer (2013) takes a in-depth look at this copula.
The copula probability density function is

$$c(u_1, u_2; \rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left\{-\frac{\rho^2\Phi^{-1}(u_1)^2 - 2\rho\Phi^{-1}(u_1)\Phi^{-1}(u_2) + \rho^2\Phi^{-1}(u_2)^2}{2(1-\rho^2)}\right\},$$

where Φ^{-1} stands for the Gaussian inverse cumulative distribution function.
The conditional copula $C_{2|1}(u_2|u_1;\rho)$ is

$$\Phi\left(\frac{\Phi^{-1}(u_2) - \rho\Phi^{-1}(u_1)}{\sqrt{1-\rho^2}}\right).$$

Student copula. This copula allows for positive and negative symmetric tail dependence. The parameter ρ measures correlation and the parameter η, the number of degrees of freedom, controls the probability mass assigned to extreme joint co-movements of risk factors changes.[8] When $\eta \to \infty$ corresponds to the Gaussian copula.[9] Student copula has not a closed form because it is a implicit copula.

[8]For more information about the properties of the t-Student copula see Demarta and McNeil (2005)
[9]The Gaussian copula underestimates the probability of joint extreme co-movements in high volatility and correlation scenarios (see Aussenegg and Cech (2011))

The copula probability density function is

$$\begin{aligned} c(u_1, u_2; \eta, \rho) &= K \frac{1}{\sqrt{1-\rho^2}} \\ &\quad \left[1 + \frac{T_\eta^{-1}(u_1)^2 - 2\rho T_\eta^{-1}(u_1) T_\eta^{-1}(u_2) + T_\eta^{-1}(u_2)^2}{\eta(1-\rho^2)}\right]^{-\frac{\eta+2}{2}} \\ &\quad \left[(1 + \eta^{-1} T_\eta^{-1}(u_1)^2)(1 + \eta^{-1} T_\eta^{-1}(u_2)^2)\right]^{\frac{\eta+1}{2}}, \end{aligned}$$

where $K = \Gamma(\frac{\eta}{2})\Gamma(\frac{\eta+1}{2})^{-2}\Gamma(\frac{\eta+2}{2})$.
The conditional copula $C_{2|1}(u_2|u_1; \rho, \eta)$ is

$$T_{\eta+1}\left(\sqrt{\frac{\eta+1}{\eta + (T_\eta^{-1}(u_1))^2}} \frac{T_\eta^{-1}(u_2) - \rho T_\eta^{-1}(u_1)}{\sqrt{1-\rho^2}}\right)$$

where T_η is the cdf of a t-Student with the numbers of degrees of freedom equal to η and T_η^{-1} represents its inverse footnoteSee for instance Cech (2006)

Clayton copula. This copula allows positive dependence and asymmetric lower tail dependence. The Clayton copula has a dependence parameter $\theta \in (0, +\infty)$. When $\theta \to 0$ implies independence and when $\theta \to \infty$ implies perfect dependence. The Clayton copula is

$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta},$$

and the copula probability density function is

$$c(u_1, u_2; \theta) = (\theta + 1)\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-2-\frac{1}{\theta}} (u_1 u_2)^{-\theta-1}.$$

The conditional copula $C_{2|1}(u_2|u_1; \theta)$ is

$$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1+\theta}{\theta}} u_1^{-\theta-1}$$

Gumbel copula. This copula allows for positive dependence and asymmetric upper tail dependence. The Gumbel copula has a dependence parameter $\theta \in [1, +\infty)$. When $\theta = 1$ implies independence and when $\theta \to \infty$ implies perfect dependence. The Gumbel copula is

$$C(u_1, u_2; \theta) = \exp\left(-\left\{(-\log u_1)^\theta + (-\log u_2)^\theta\right\}^{1/\theta}\right),$$

and the copula probability density function is

$$\begin{aligned} c(u_1, u_2; \theta) &= (A + \theta - 1)\, A^{1-2\theta} \exp(-A) \\ &\quad (u_1 u_2)^{-1} (-\log u_1)^{\theta-1} (-\log u_2)^{\theta-1}, \end{aligned}$$

where $A = \left[(-\log u_1)^\theta + (-\log u_2)^\theta\right]^{\frac{1}{\theta}}$.
The conditional copula $C_{2|1}(u_2|u_1;\theta)$ is

$$\exp\left(-\left\{(-\log u_1)^\theta + (-\log u_2)^\theta\right\}^{1/\theta}\right)\left\{(-\log u_1)^\theta + (-\log u_2)^\theta\right\}^{1/\theta-1}(-\log u_1)^{\theta-1}\frac{1}{u_1}$$

BB1 copula. The BB1 copula, also known as the Clayton-Gumbel copula, allows asymmetric tail dependence. The BB1 copula has two dependence parameters: one for the Clayton behaviour $\theta \in (0, +\infty)$ and another one for the Gumbel behaviour $\delta \in [1, +\infty)$. When $\delta = 1$ and $\theta > 0$ we get the Clayton copula and as a consequence upper tail independence and lower tail dependence. When $\theta \to 0$ and $\delta > 0$ the Gumbel copula is obtained with upper tail dependence only. In the case of $\theta \to 0$ and $\delta = 1$ we get upper and lower tail independence.[10]
The BB1 copula is

$$C(u_1, u_2; \theta, \delta) = \left(1 + \left[(u_1^{-\theta} - 1)^\delta + (u_2^{-\theta} - 1)^\delta\right]^{1/\delta}\right)^{-1/\theta},$$

and the copula probability density function is

$$c(u_1, u_2; \theta, \delta) = (u_1 u_2)^{-\theta-1}(ab)^{\delta-1} c^{\frac{1}{\delta}-2} d^{-\frac{1}{\theta}-1}\left\{d^{-1}c^{\frac{1}{\delta}}(1+\theta) + \theta(\delta - 1)\right\},$$

where $a = u_1^{-\theta} - 1$, $b = u_2^{-\theta} - 1$, $c = a^\delta + b^\delta$ and $d = 1 + c^{\frac{1}{\delta}}$.
The conditional copula $C_{2|1}(u_2|u_1;\theta,\delta)$

$$\left(1 + \left[A^\delta + B^\delta\right]^{1/\delta}\right)^{-\frac{1+\theta}{\theta}}\left[A^\delta + B^\delta\right]^{\frac{1}{\delta}-1} A^{\delta-1} u_1^{-\theta-1}$$

where $A = (u_1^{-\theta} - 1)$ and $B = (u_2^{-\theta} - 1)$

[10] See for instance Venter (2002) or Nicoloutsopoulos (2005)

TABLE 2..1: Tail dependence for the 90° rotated copulas

	$\tau_{L\|U}$	$\tau_{U\|L}$
90° R Clayton	$2^{-1/\theta}$	-
90° R Gumbel	-	$2-2^{1/\theta}$
90° R BB1	$2^{\frac{-1}{\theta\delta}}$	$2-2^{1/\delta}$

θ and δ are the copula parameters from the original copula. Further information about the rotated copula can be found in Brechmann and Schepsmeier (2013), Cech (2006), Georges et al. (2001) and Luo (2010).

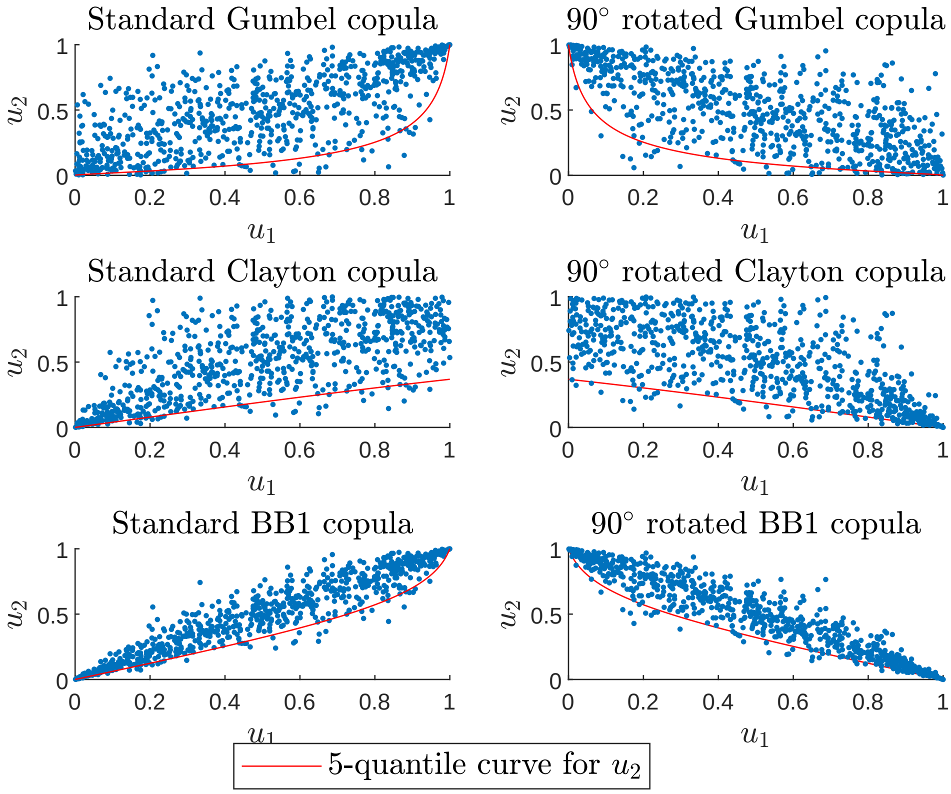
Let u_1 and u_2 denote two variables uniformly distributed across (0,1).

The negative lower tail dependence, $\tau_{L|U}$, is defined as $\tau_{L|U} = lim_{q\to 0}P(u_2 < q|u_1 > 1-q)$.

The negative upper tail dependence, $\tau_{U|L}$ is defined as $\tau_{U|L} = lim_{q\to 1}P(u_2 > q|u_1 < 1-q)$.

Figure 2..1 shows an example of how change the distribution and the tail joint behaviour when the 90° rotated copula is employed. See Zhang (2008) for further details about negative tail dependence.

FIGURE 2..1: Rotated copulas employed to capture negative tail dependence



This figure shows 800 simulations from the same seed but under different copula assumptions. Rotating 90 degrees allows us to capture negative upper tail dependence (90° rotated Gumbel), negative lower tail dependence (90° rotated Clayton) or negative asymmetric tail dependence (90° rotated BB1). The red line indicates the threshold below which the 5% of the u_2 are found given the values taken by u_1. Gumbel and Clayton copula has a copula parameter $\theta = 2$. BB1 has copula parameters $\theta = 2$ and $\delta = 2.5$.

B Markov switching specification for modelling joint dependence

This appendix sums up briefly the Markov switching estimation procedure of the joint distribution using a copula approach.
Given the Sklar (1959)'s representation of the joint distribution in terms of copulas, we can assumed that the dependence between variables depends on an unobservable regime. This assumption means rewriting Equation (2.1) as Equation (2.7). Equation (2.11) shows the result of deriving Equation (2.7) from its inputs. We obtain the joint density function as the product of the marginal density functions and the copula density function.
Let us define Ψ as a vector 2x1 that gathers the conditional copula density between $r_{o,t}$ and $r_{m,t}$ for each of the two different regimes, i.e.

$$\Psi = \begin{bmatrix} c\left(u_{o,t}, u_{m,t}; \theta_{s_t=1}, s_t = 1\right) \\ c\left(u_{o,t}, u_{m,t}; \theta_{s_t=2}, s_t = 2\right) \end{bmatrix}, \tag{2.12}$$

where θ_{s_t} is the copula parameter under the regime s_t and $u_{o,t}$ and $u_{m,t}$ refer to the marginal cumulative distribution function of oil and stock market returns.
I assume that these conditional densities depend only on the current regime s_t but

not on previous regimes, i.e.

$$c\left(u_{o,t}, u_{m,t}; I_{t-1}, s_t = j; \theta_{s_t=j}\right) = c\left(u_{o,t}, u_{m,t}; I_{t-1}, s_t = j, s_{t-1} = i, ...; \theta_{s_t=j}\right)$$

for $i, j = 1, 2$ and I_{t-1} refers to the information set at $t-1$. I assume that the evolution of s_t follows a first order Markov chain independent from past observations for oil and stock market returns, i.e.

$$p_{ij} = P(s_t = j | s_{t-1} = i) = P(s_t = i | s_{t-1} = j, s_{t-2} = k, I_{t-1}), \tag{2.13}$$

for $i, j, k = 1, 2$.
The transition matrix is shown in Equation (2.9) where each column i indicates the probability of remaining on the state i (p_{ii}) or moving to state j (p_{ij}) conditioned to the fact that we are currently at state i for $i, j = 1, 2$ and $i \neq j$. Obviously, $p_{ii} + p_{ij} = 1$ because we only consider two states. That is the reason why Equation (2.9) presents p_{ij} as $1 - p_{ii}$. We can obtain two concepts with significant economic implications from the transition matrix: the expected duration and the unconditional probabilities of each state.
The expected length for state i can be assessed as

$$\frac{1}{1 - p_{ii}},$$

for $i = 1, 2$. (Hamilton, 1994, Chapter 22). The expected length for each state can give us an idea about the persistence in the dependence given by each regime, which is extremely useful from an economic point of view.
The unconditional probability of each state is the results of computing the ergodic probabilities. These probabilities make up the eigenvector of the matrix P from Equation (2.9) associated to the unit eigenvalue, such that its elements sums one, i.e.

$$P\pi = \pi,$$

where $\pi = (\alpha, 1-\alpha)'$ Hence, $p_{11}\alpha + (1 - p_{22})(1 - \alpha) = \alpha$ so the unconditional probability of being in the state 1 is $\alpha = \frac{1-p_{22}}{2-p_{11}-p_{22}}$ and for the state 2 is $1 - \alpha = \frac{1-p_{11}}{2-p_{11}-p_{22}}$.

Let us assume that the marginal distribution of the variables employed as input in the copula and the set of copula parameters θ are known. Let us gather the probability assigned to the observation at time t of being the result of regime j, i.e. $P(s_t = j | I_t; \theta)$, in a vector $\hat{\xi}_{t|t}$,

$$\hat{\xi}_{t|t} = \left[P(s_t = 1 | I_t; \theta), P(s_t = 2 | I_t; \theta)\right]'.$$

$\hat{\xi}_{t|t}$ comprises the inference about the regime at time t given the information available at that period. The probability assigned to the observation at time $t+1$ of being the result of regime j given the information at time t is collected in vector $\hat{\xi}_{t+1|t}$,

$$\hat{\xi}_{t+1|t} = \left[P(s_t = 1 | I_{t-1}; \theta), P(s_t = 2 | I_{t-1}; \theta)\right]'.$$

$\hat{\xi}_{t+1|t}$ is the probability forecast of being in the next period $t+1$ at each regime given the information available at t.
The link between $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ is obtained by the updated probabilities. The updated probabilities include the new available information through Bayes' theorem,

i.e.

$$P(s_t = j|I_t;\theta) = \frac{P(s_t = j|I_{t-1};\theta) f(r_{o,t}, r_{m,t}|I_{t-1};\Theta_{s_t=j})}{L_t(r_{o,t}, r_{m,t}; I_{t-1}, \Theta)},$$

where $f(r_{o,t}, r_{m,t}|I_{t-1};\Theta_{s_t=j})$ is given by Equation (2.11) and $L_t(r_{o,t}, r_{m,t}; I_{t-1}, \Theta)$ is the likelihood function in Equation (2.10). Observe that in Equation (2.10) we are multiplying the joint density of $r_{o,t}$ and $r_{m,t}$ conditioned to the occurrence of each possible state at t by its probability at t given the information set at $t-1$. Assuming that the marginal behaviour of each variable does not depend on the state and only the dependence changes across states we can rewrite the previous equation that connects $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ in a matrix form as

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \odot \Psi}{1_2'(\hat{\xi}_{t|t-1} \odot \Psi)},$$

where Ψ was defined in Equation (2.12) while the forecast probability for the next period is obtained as the product of the inference probability by the transition matrix, i.e.

$$\hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t}.$$

To start the iteration we need a value for $\hat{\xi}_{1|0}$, for which I use the unconditional probabilities of each state that can be expressed in a matrix form as

$$\hat{\xi}_{1|0} = \pi = (A'A)^{-1}A'(0,0,1)'$$

where

$$A = \begin{bmatrix} I_2 - P \\ 1_2' \end{bmatrix} = \begin{bmatrix} 1-p_{11} & p_{22}-1 \\ p_{11}-1 & 1-p_{22} \\ 1 & 1 \end{bmatrix}.$$

To finish this appendix I present the Kim (1994)'s algorithm to obtain smoothed inferences, which are used to present the probabilities of being in each state at each time t given the complete sample T, i.e.

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \left\{P' \left[\hat{\xi}_{t+1|T}(\div)\hat{\xi}_{t+1|t}\right]\right\},$$

where $\odot$ and $(\div)$ represents the element-wise product and division respectively.

C Algorithms employed to simulate returns under the data generating process

Algorithm 1 Simulation of the dependence under a copula and a dynamic following a two-state Markov switching in dimension N=2 over a time period T .

procedure SIM-DEPENDENCE($\Theta_{s_t=1}, \Theta_{s_t=2}, \xi_{1|0}(s_t = 1), p_{11}, p_{22}$)
  for $\omega \leftarrow 1, \ldots, W$ **do**
    for $t \leftarrow T$ **do**
      $u_{t,\omega,1} = rand$
      if $rand < \xi_{1|0}(s_t = 1)$ **then**
        $state = 1$
      else
        $state = 2$
      end if
      $u_{t,\omega,2} = C^{-1}_{2|1}(rand|u_{t,\omega,1}; \Theta_{s_t=state,12})$
      if $state = 1$ **then**

        if $rand < p_{11}$ **then**
          $\xi_{1|0}(s_t = 1) = 1$
        else
          $\xi_{1|0}(s_t = 1) = 0$
        end if
        else if $state = 2$ **then**

          if $rand < p_{22}$ **then**
            $\xi_{1|0}(s_t = 1) = 0$
          else
            $\xi_{1|0}(s_t = 1) = 1$
          end if
          end if
        end for t
      end for ω
      Return u
    end procedure

$\Theta_{s_t=1}$ and $\Theta_{s_t=2}$ are the set of parameters for the copula structure under state 1 and 2. $\xi_{1|0}(s_t = 1)$ is the unconditional probability of being in state 1, i.e.
$\xi_{1|0}(s_t = 1) = \frac{1-p_{22}}{2-p_{11}-p_{22}}$.
p_{11} and p_{22} are the diagonal values from the transition matrix (see Equation (2.9)).
rand refers to an uniform-distributed random realization.
The *OUTPUT* u is a uniform-distributed matrix that has the joint dependence presented in the model of size $TxWx2$

Algorithm 2 Simulation from a AR(1)-TGARCH(1,1) over a time period T and skewed t distribution assumption for the innovation process.

procedure SIM-PATH($u, \phi_0, \phi_1, \nu, \lambda, \omega, \alpha, \beta, \gamma$)
    for $n \leftarrow 1, \ldots, 2$ **do**
        for $w \leftarrow 1, \ldots, W$ **do**
            $r_{1,w,n} = \phi_{0,i} / (1 - \phi_{1,i})$
            $\sigma^2_{1,w,n} = \omega_{0,i} / (1 - \alpha_i - \beta_i - \gamma F_{\nu_i,\lambda_i}(0))$
            for $t \leftarrow 1, \ldots, T$ **do**
                $\epsilon_{t,w,n} = F^{-1}_{\nu_i,\lambda_i}(u_{t,w,n})$
                $\sigma^2_{t+1,w,n} = \omega_i + (\alpha + \mathbb{1}_{\epsilon_{t,w,n}<0})\epsilon^2_{t,w,n} + \beta\sigma^2_{t,w,n}$
                $r_{t+1,w,n} = \phi_{0,i} + \phi_{1,i} r_{t,w,n} + \epsilon_{t,w,n}\sigma_{t,w,n}$
            end for t
        end for w
    end for n
    Return r
end procedure

u is a three-dimension matrix ($TxWx2$) obtained from Algorithm 1.
ϕ_0 and ϕ_1 are vectors of parameters of length $N = 2$ that drive the dynamic in Equation (2.4).
ω, α, β and γ are vectors of parameters of length $N = 2$ that drive the dynamic in Equation (2.5).
ν and λ are vectors of length N gathering the values of asymmetry and number of degrees of freedom from Equation (2.6).
The *OUTPUT* r is a three dimension matrix ($(T+1)xWx2$) of W simulated paths of length $T + 1$ for the $N = 2$ returns.

Algorithm 3 Assessment of the risk measures of the stock market returns at time t under a certain scenario for oil returns.

procedure RISKMEASURET($W, \Theta_{s_t=1}, \Theta_{s_t=2}, \xi_{t|T}(s_t = 1), \mu_t, \sigma_t, \nu, \lambda$)
  $u = rand(W, 3)$
  $u_{1:W,1} = \alpha(u_{1:W,1})$ ▷ $u_{1:W,1} = 1 - \alpha(u_{1:W,1})$ would be a bullish scenario
  for $\omega \leftarrow 1, \ldots, W$ **do**
    if $u_{\omega,3} < \xi_{t|T}(s_t = 1)$ **then**
      $state = 1$
    else
      $state = 2$
    end if
    $u_{\omega,2} = C^{-1}_{2|1}\left(u_{\omega,2}|u_{\omega,1}; \Theta_{s_t=state,12}\right)$
  end for ω
  $q = u_{1:W,2}$
  $r_t = \mu_t + \sigma_t F^{-1}_{\nu,\lambda}(q)$
  $CoVaR_t = max\left(r_{\omega,t} \quad such \quad that \quad \#\{r_{\omega,t} \leq CoVaR_t\} = W\gamma\right)$
  $CoES_t = \frac{\sum_{\omega=1}^{W} r_{\omega,t} \mathbb{1}_{r_{\omega,t}<CoVaR_t}}{\sum_{\omega=1}^{W} \mathbb{1}_{r_{\omega,t}<CoVaR_t}}$ ▷ The upper tail $CoES_t$ would be $\frac{\sum_{\omega=1}^{W} r_{\omega,t} \mathbb{1}_{r_{\omega,t}>CoVaR_t}}{\sum_{\omega=1}^{W} \mathbb{1}_{r_{\omega,t}>CoVaR_t}}$
  Return $CoVaR_t$, $CoES_t$
end procedure

$\Theta_{s_t=1}$ and $\Theta_{s_t=2}$ are the set of parameters for the copula structure under state 1 and 2. $\xi_{t|T}(s_t = 1)$ is the smoothed probability of being in state 1 at time t.
μ_t refers to the conditional mean at time t obtained from Equation (2.4).
σ_t is the conditional standard deviation at time t obtained from Equation (2.5).
ν and λ gather the values of asymmetry and number of degrees of freedom from Equation (2.6).
$rand(W, N)$ refers to a matrix of W uniform-distributed random realizations for N variables.
The *OUTPUT* contains the *CoVaR* and the *CoES* measures.

D Robustness checks

This subsection shows three set of results where the constraints and assumptions are gradually relaxed.
First, I present the results of the estimates with constant dependence parameters for the considered set of copulas. I study a possible change in dependence using different graphical tools as lambda functions and Tail Concentration Functions (TCF).
Second, I relax the constraint regarding the fixed dependence over time while assuming an unchanged type of copula. This change is justified by the results of the likelihood ratio test, which indicate a change in the dependence with oil returns for most of the stock market industries.
Finally, the previous constraint is also removed, allowing for a change in the copula over time. I select the set of potential copulas for the structural break given the information contained in the data analysis from Section 2.4.

Estimates assuming constant dependence parameters

Tables 2..2a and 2..2b present the results of the joint models for the best copula fit. The hypothesis of lack of autocorrelation and homocedasticity in the residuals given by an AR(1)-GJR-GARCH(1,1) model are not rejected, supporting the choice done for the marginal model. The hypothesis that the residuals come from a Hansen (1994)'s skewed t distribution cannot be rejected either. The marginal distribution is well-specified according to the results of the joint distribution with constant dependence.

TABLE 2..2A: Parameter estimates for the joint distribution using the best copula fit

	A	L	B	L	C	L	D	L	E	L
ϕ_0	0.00	0.00	-0.00	0.00	0.00 *	-0.00	0.00 *	0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	-0.07 **	0.03	-0.08 ***	0.02	-0.07 **	0.04	-0.09 ***	0.03	-0.04	0.04
	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
ω	0.00 ***	0.00 **	0.00 ***	0.00 **	0.00 ***	0.00 ***	0.00 ***	0.00 **	0.00 ***	0.00 **
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.00	0.05 **	0.10	0.05 **	0.00	0.05 **	0.00	0.05 **	0.00	0.05 **
	(0.04)	(0.03)	(0.25)	(0.02)	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
β	0.82 ***	0.90 ***	0.82 ***	0.90 ***	0.79 ***	0.89 ***	0.82 ***	0.88 ***	0.80 ***	0.89 ***
	(0.06)	(0.03)	(0.13)	(0.03)	(0.06)	(0.02)	(0.06)	(0.03)	(0.05)	(0.03)
γ	0.25 ***	0.07 **	0.06	0.06 **	0.25 ***	0.08 ***	0.23 ***	0.08 **	0.26 ***	0.08 **
	(0.07)	(0.04)	(0.17)	(0.03)	(0.07)	(0.03)	(0.07)	(0.04)	(0.06)	(0.04)
λ	-0.36 ***	-0.26 ***	-0.29 ***	-0.27 ***	-0.29 ***	-0.26 ***	-0.36 ***	-0.26 ***	-0.23 ***	-0.27 ***
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.06)	(0.05)	(0.05)	(0.05)
η	14.78 ***	13.18 ***	10.42 ***	15.31 ***	15.01 ***	12.84 ***	18.20 ***	13.23 ***	13.77 ***	12.99 ***
	(0.24)	(0.37)	(0.35)	(0.31)	(0.70)	(0.95)	(0.59)	(0.47)	(0.27)	(0.23)
LM	0.44	0.92	0.15	0.81	0.88	0.88	0.95	0.83	0.78	0.88
KS	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
LJ	0.88	0.47	0.66	0.42	0.95	0.48	0.81	0.46	0.86	0.48
Copula model	Clayton		BB1		Clayton		BB1		Clayton	
θ		0.14 ***	τ_L	0.14 ***	θ	0.18 ***	τ_L	0.04 *	θ	0.11 ***
		(0.04)		(0.04)		(0.05)		(0.03)		(0.04)
LL		3100.63	τ_U	0.14 ***	LL	2974.91	τ_U	0.00	LL	3097.99
				(0.06)				(0.01)		
			LL	3020.77			LL	3028.07		

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the parameters of the optimal copula according to AICc values. LL is the log-Likelihood value.
$***/**/*$ indicates statistical significance at 1/5/10%
LJ stands for the Ljung-Box test for serial correlation computed with 20 lags. LM represents the Engle's LM test for heteroskedasticity using 20 lags. KS stands for the Kolmogorov-Smirnov test where the null hypothesis is a skewed t distribution with the number of degrees and the skewness parameter obtained by estimation. I show the p-values of the statistic tests.
Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: OIL). x can be A: $EUROSTOXX$; B: $OIL\&GAS$; C: $BASICMATS$; D: $INDUSTRIALS$; E: $CONSUMERGDS$; F: $HEALTHCARE$; G: $CONSUMERSVS$; H: $TELECOM$; I:$UTILITIES$; J: $FINANCIALS$; K: $TECHNOLOGY$.

TABLE 2..2B: Parameter estimates for the joint distribution using the best copula fit

	F	L	G	L	H	L	I	L	J	L	K	L
ϕ_0	0.00 **	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	-0.09 ***	0.03	-0.06 *	0.03	-0.03	0.04	-0.04	0.03	-0.03	0.04	-0.05 *	0.04
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)
ω	0.00 **	0.00 *	0.00 ***	0.00 **	0.00 **	0.00 *	0.00 ***	0.00 *	0.00 ***	0.00 *	0.00 **	0.00 *
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α	0.04 *	0.04 *	0.00	0.05 **	0.04 **	0.04 **	0.00	0.05 **	0.00	0.05 **	0.02	0.04 **
	(0.03)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.02)	(0.02)
β	0.89 ***	0.90 ***	0.86 ***	0.89 ***	0.90 ***	0.90 ***	0.80 ***	0.90 ***	0.84 ***	0.90 ***	0.91 ***	0.91 ***
	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.06)	(0.03)	(0.04)	(0.03)	(0.02)	(0.03)
γ	0.06 *	0.07 **	0.19 ***	0.08 **	0.07 **	0.08 **	0.18 ***	0.07 **	0.26 ***	0.07 **	0.09 ***	0.07 **
	(0.04)	(0.03)	(0.05)	(0.04)	(0.04)	(0.04)	(0.06)	(0.04)	(0.05)	(0.04)	(0.03)	(0.04)
λ	-0.20 ***	-0.25 ***	-0.24 ***	-0.26 ***	-0.08 *	-0.26 ***	-0.20 ***	-0.26 ***	-0.34 ***	-0.26 ***	-0.15 ***	-0.27 ***
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
η	9.06 ***	14.59 ***	9.10 ***	13.26 ***	7.90 ***	13.42 ***	8.60 ***	13.95 ***	13.16 ***	13.79 ***	13.11 ***	12.63 ***
	(0.23)	(0.46)	(0.21)	(0.24)	(0.54)	(0.31)	(0.56)	(0.34)	(0.35)	(0.78)	(1.33)	(0.42)
LM	0.58	0.99	0.97	0.88	0.84	0.90	0.91	0.99	0.71	0.93	0.64	0.96
KS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
LJ	0.76	0.48	0.36	0.48	0.91	0.48	0.88	0.48	0.65	0.48	0.88	0.50
Copula	Student		Clayton		Clayton		Clayton		Clayton		BB1	
ρ	-0.02		θ	0.09 **	θ	0.07 **	θ	0.16 ***	θ	0.08 **	τ_L	0.00
	(0.04)			(0.04)		(0.04)		(0.04)		(0.04)		(0.01)
ν	15.33 ***		LL	3127.60	LL	2954.36	LL	3089.16	LL	2915.50	τ_U	0.00
	(0.19)											(0.01)
LL	3081.05										LL	2798.82

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the parameters of the optimal copula according to AICc values. LL is the log-Likelihood value.

$***/**/*$ indicates statistical significance at 1/5/10%

LJ stands for the Ljung-Box test for serial correlation computed with 20 lags. LM represents the Engle's LM test for heteroskedasticity using 20 lags. KS stands for the Kolmogorov-Smirnov test where the null hypothesis is a skewed t distribution with the number of degrees and the skewness parameter obtained by estimation. I show the p-values of the statistic tests.

Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: OIL). x can be A: $EUROSTOXX$; B: $OIL\&GAS$; C: $BASICMATS$; D: $INDUSTRIALS$; E: $CONSUMERGDS$; F: $HEALTHCARE$; G: $CONSUMERSVS$; H: $TELECOM$; I:$UTILITIES$; J: $FINANCIALS$; K: $TECHNOLOGY$.

Table 2..3 shows the AICc values of the stock-oil model using the initial set of copulas. The Clayton copula depicts the best fit for most of the sectors followed by the BB1 and Student t copula. Clayton copula implies lower tail dependence while BB1 and Student t copula imply upper and lower tail dependence. Oil & gas, industrial and technology sectors obtain a better fit under the BB1 copula, which has asymmetric tail dependence, i.e. the joint behaviour with oil returns is different at low quantiles than at high quantiles. The Student t explains better the relationship between healthcare and oil.

TABLE 2..3: AICc values for the models with constant copula parameter

AICc	A	B	C	D	E	F	G	H	I	J	K
Gaussian	-6164,8	-5995,9	-5909,1	-6017,7	-6160,8	-6123,3	-6218,2	-5870,0	-6135,6	-5792,4	-5553,7
Student	-6163,7	-5998,6	-5911,2	-6017,8	-6159,0	**-6125,2**	-6216,3	-5868,3	-6136,5	-5792,8	-5558,4
Clayton	**-6166,5**	-5988,3	**-5915,0**	-6018,8	**-6161,2**	-6124,7	**-6220,4**	**-5873,9**	**-6143,5**	**-5796,2**	-5554,2
Gumbel	-6160,4	-5993,4	-5903,3	-6015,1	-6156,5	-6123,3	-6215,0	-5868,8	-6130,8	-5791,7	-5555,0
BB1	-6165,6	**-6004,6**	-5913,0	**-6019,2**	-6159,1	-6122,6	-6218,3	-5871,8	-6141,5	-5794,1	**-5560,8**

This table shows the values of the Akaike Information Criterion corrected for small sample bias (AICc) (Hurvich and Tsai 1989).
$AICc = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$, where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum $AICc$ value in bold letters indicates the best copula fit.
A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

To motivate the need of time-varying parameters for dependence I present the results of two graphical tools, i.e. lambda function and Tail Concentration Functions (TCF), employed to describe co-movement and tail dependence.
The lambda function (Genest and Rivest 1993, Aas et al. 2009, Brechmann and Schepsmeier 2013, Schepsmeier 2010) is a useful graphical tool that comes from the difference between the quantile q and the Kendall function $K(q, \theta)$, i.e.

$$\lambda(q, \theta) = q - K(q, \theta)$$

where $K(q, \theta)$ is the Kendall function, which represent the probability associated to the joint distribution, i.e. $K(q, \theta) = P(C(u_1, u_2; \theta) \leq q)$.[11] If $\lambda = 0$ the variables has a perfect positive dependence. The top subfigure in Figure 2..2 presents the lambda function for the estimated copula function in the solid cyan line, while the grey area depicts the 90% confidence interval obtained by bootstrapping. The red line shows the empirical lambda function for the full sample using the residuals from the marginal models. The dotted black lines are the bands of the lambda function for perfect positive dependence (flat line) and independence (curve line). The solid green and the dashed blue lines are the pre-crisis and post-crisis empirical lambda functions. The pre-crisis empirical lambda is close to the independence case while the post-crisis empirical lambda shows a lower curvature, which indicates an increase in dependence.

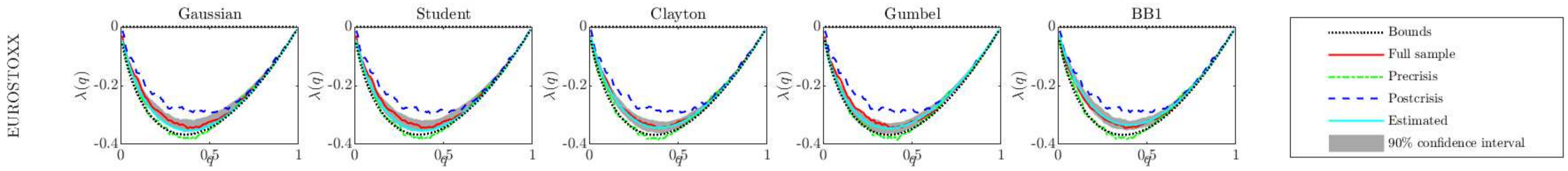
The Tail Concentration Function (TCF) is proposed by Pappadà et al. (2018) to quantify tail dependence features at a finite scale, where the weight for each tail depends on the selected quantile, i.e.

$$TCF(q) = \frac{C(q,q)}{q} \mathbb{1}_{q \leq 0.5} + \frac{1 - 2q + C(q,q)}{1-q} \mathbb{1}_{q > 0.5},$$

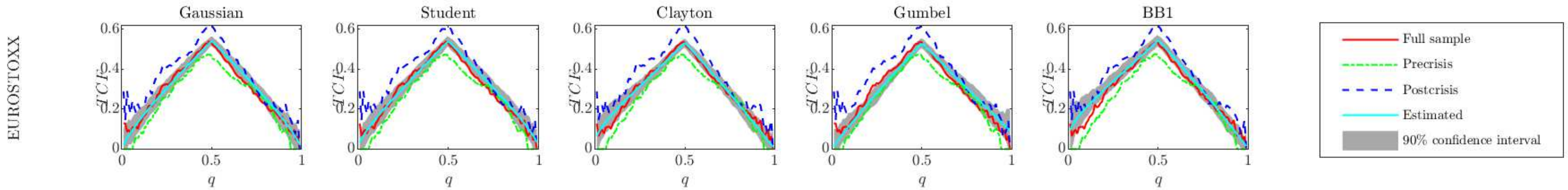
[11] Further details about the Kendall function can be found in (Cherubini et al., 2016, p. 8–11,25–26).

where q represents a certain quantile, $C(q, q)$ is the copula function and $\mathbb{1}$ is the indicator function.
The bottom subfigure in Figure 2..2 shows the TCF between the stock market returns and the oil returns. The cyan line is the TCF for the estimated copula while the grey area represents its 90% confidence interval obtained by bootstrapping. Red line is the empirical TCF for the full sample while the solid green line and the dashed blue line are the empirical TCF for the pre-crisis and post-crisis samples.

FIGURE 2..2: Graphical tools to capture dependence for the full, pre-crisis and post-crisis samples



(a) Lambda function: Oil- EUROSTOXX index



(b) Tail Concentration function (TCF): Oil- EUROSTOXX index

[TOP FIGURE] The dotted black lines represent the bands corresponding to independence (curve line) and perfect comonotonicity (flat line). The solid cyan line is the fitted lambda for the corresponding copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The empirical lambda function for the full sample is represented by the solid red line. The empirical lambda function for the pre-crisis sample is represented by the dash-dotted green line and for the post-crisis sample is represented by the dashed blue line. The closer is the empirical lambda function to the flat line, the higher is the associated Kendall's τ.

[BOTTOM FIGURE] The solid cyan line is the fitted TCF for the corresponding estimated copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The TCF quantifies the tail dependence features giving an idea about how well the parametric copula fits the empirical joint distribution for extreme quantiles. The higher values reached by the post-crisis sample indicates an increase in the lower tail dependence. For further details about the TCF and its applications see Pappadà et al. (2018).

These results show an increase in the lower tail dependence after the 2008 financial crisis, which is translated into higher TCF values for $q \leq 0.5$. Therefore, there are grounds for believing that the tail dependence between oil and stock market changed after the 2008 financial crisis. Figure 2..2 focuses on the Eurostoxx. The figures for the productive sectors are provided in Appendix E.

The next subsection relaxes the assumption of constant dependence over time identifying changes in the degree of dependence between stock market and oil returns.

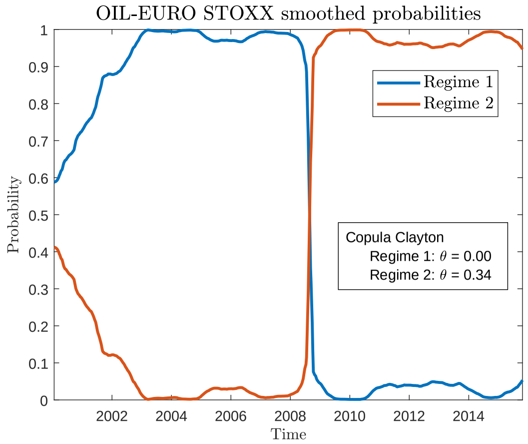
Estimates allowing for time-varying dependence parameters while keeping unchanged the type of copula

This section introduces the estimate results of the joint distribution function presented in Equation (2.7). The model assumes that the optimal copula according to Table 2..3 keeps unchanged but its parameters can vary across regimes.

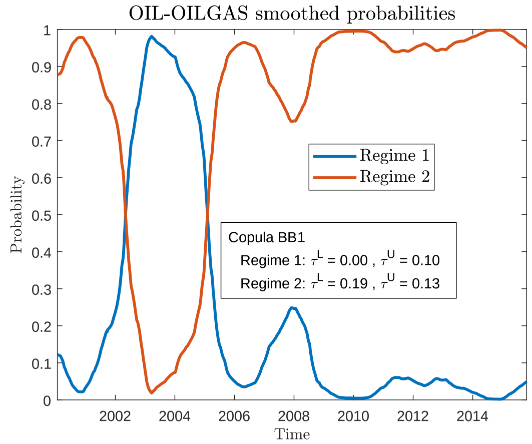
Tables 2..4a and 2..4b present the estimates of the joint distribution and their standard deviations where the copula parameter evolves according to a two-regime Markov Chain. Figures 2..3a and 2..3b show the smoothed probabilities using Kim (1994)'s algorithm. The scope of these probabilities is to determine if and when regime switch occurs. Figure 2..3a points to a structural change in the dependence between oil returns and Eurostoxx index after the 2008 financial crisis in line with recent results in the literature like Reboredo and Ugolini (2016). The sectors that lead the switch in the co-movement between the Eurostoxx index and the oil returns are basic materials, consumer goods, consumer services, telecommunications, utilities and technology. All of them present a similar change in dependence around 2008 following the same copula specification, i.e. Clayton copula. There seems to be an increase in dependence after 2008 between these sector returns and oil returns. Although the financial sector has the same copula specification and presents a rise in dependence after 2008, it seems a transitory change. Actually, the expected duration for the high dependence regime in the financial sector is around two years and for the low dependence regime is four years. The expected duration of the regimes in the Eurostoxx is four years longer than in the financial sector. The healthcare sector exhibits an increase in the correlation but a decrease in tail dependence across regimes. Actually, the great increase in the number of degrees of freedom might indicate that the dependence between oil and healthcare sector is better explained by a Student t-Gaussian mixture. The dependence in basic materials or technology with oil returns might arise from a mixture between Clayton copula and a copula that allows negative co-movement, due to the pattern inferred from the pre-crisis sample in Figure 2.4.3 and Table 2.4.1.

The following section discloses the hidden patterns in the type of dependence between oil and stock market according to a two-state Markov switching specification, where the type of copula can change across states.

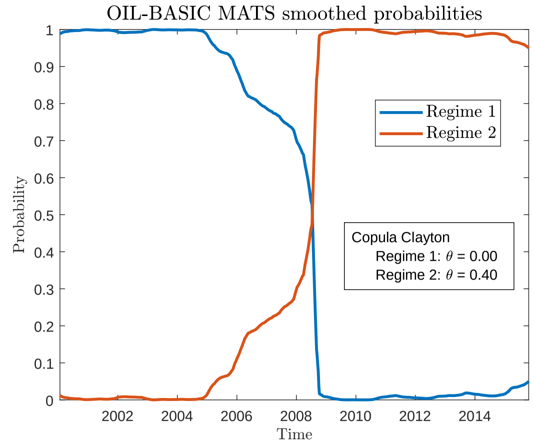
FIGURE 2..3A: Smoothed probabilities assuming same copula type but different copula parameter across regimes



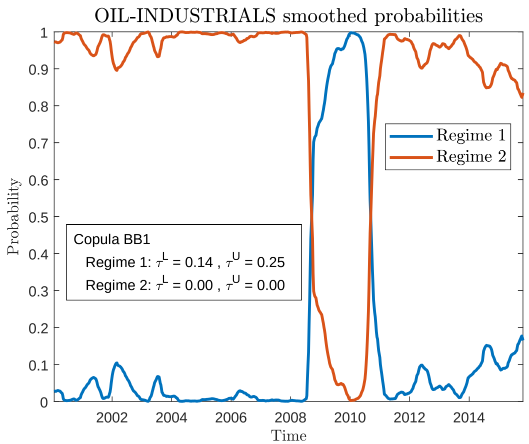
(a) Oil- EUROSTOXX index



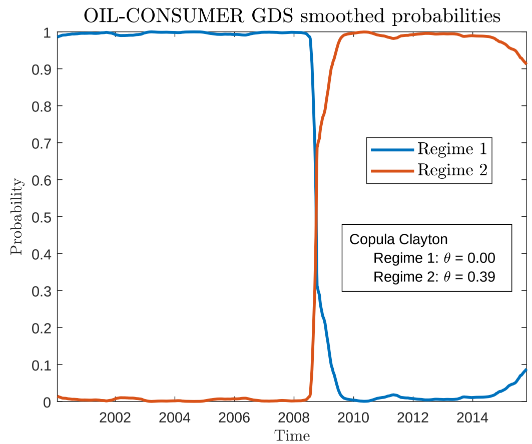
(b) Oil - OIL & GAS sector



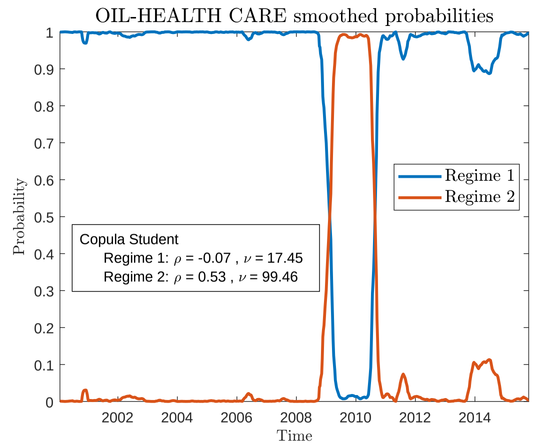
(c) Oil - BASIC MATS sector



(d) Oil - INDUSTRIALS sector



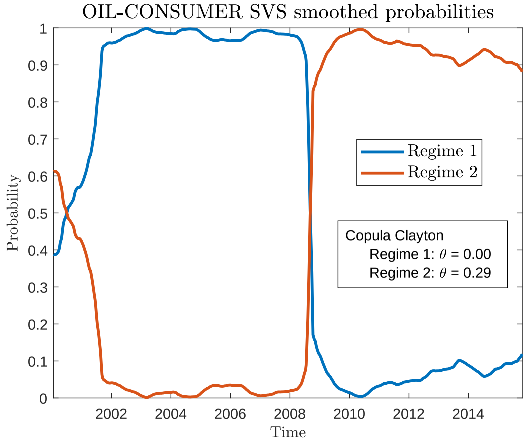
(e) Oil - CONSUMER GDS sector



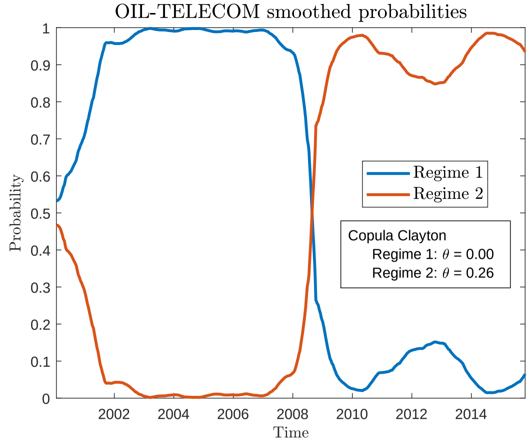
(f) Oil - HEALTH CARE sector

Smoothed probabilities are obtained using Kim (1994)'s algorithm. Further information about this algorithm is provided in Appendix B. The text box indicates the chosen copula according to Table 2..3 and the optimal parameters depending on the regime. Across regimes the copula parameter changes but the type of copula is assumed to be constant over time.

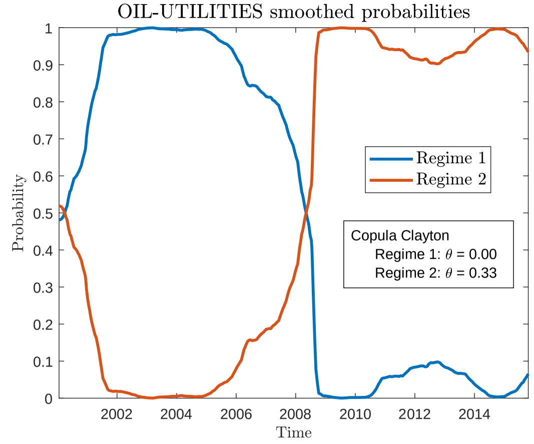
FIGURE 2..3B: Smoothed probabilities assuming same copula type but different copula parameter across regimes



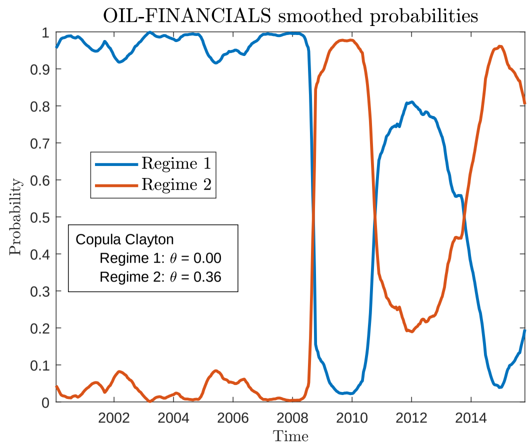
(g) Oil- CONSUMER SVS sector



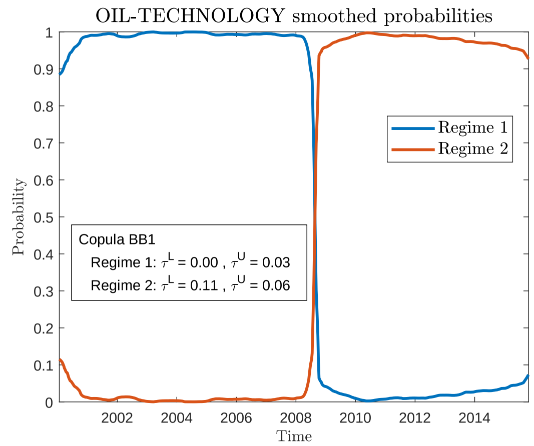
(h) Oil - TELECOM sector



(i) Oil - UTILITIES sector



(j) Oil - FINANCIALS sector



(k) Oil - TECHNOLOGY sector

Smoothed probabilities are obtained using Kim (1994)'s algorithm. Further information about this algorithm is provided in Appendix B. The text box indicates the chosen copula according to Table 2..3 and the optimal parameters depending on the regime. Across regimes the copula parameter changes but the type of copula is assumed to be constant over time.

Table 2..4A: Parameter estimates for the joint distribution using time-varying copula specification that follows a first order Markov switching model

	A	L		B	L		C	L		D	L		E	L
ϕ_0	0.00	0.00	ϕ_0	0.00	-0.00	ϕ_0	0.00 *	-0.00	ϕ_0	0.00 *	-0.00	ϕ_0	0.00	-0.00
	(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)
ϕ_1	-0.07 **	0.03	ϕ_1	-0.07 **	0.05 *	ϕ_1	-0.06 **	0.04	ϕ_1	-0.08 **	0.04	ϕ_1	-0.03	0.04
	(0.04)	(0.04)		(0.03)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)
ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***
	(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)
α	0.00	0.04 **	α	0.08 **	0.05 ***	α	0.00	0.04 **	α	0.00	0.04 **	α	0.00	0.04 **
	(0.03)	(0.02)		(0.05)	(0.02)		(0.03)	(0.02)		(0.02)	(0.02)		(0.02)	(0.02)
β	0.82 ***	0.90 ***	β	0.82 ***	0.90 ***	β	0.79 ***	0.90 ***	β	0.84 ***	0.89 ***	β	0.81 ***	0.89 ***
	(0.01)	(0.00)		(0.00)	(0.00)		(0.01)	(0.00)		(0.01)	(0.00)		(0.01)	(0.00)
γ	0.25 ***	0.08 **	γ	0.08	0.05 **	γ	0.25 ***	0.08 ***	γ	0.21 ***	0.08 ***	γ	0.26 ***	0.09 ***
	(0.05)	(0.03)		(0.06)	(0.03)		(0.05)	(0.03)		(0.04)	(0.03)		(0.05)	(0.03)
λ	-0.37 ***	-0.26 ***	λ	-0.26 ***	-0.27 ***	λ	-0.29 ***	-0.26 ***	λ	-0.36 ***	-0.27 ***	λ	-0.24 ***	-0.27 ***
	(0.04)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)
η	12.38 ***	13.66 **	η	10.10 ***	39.33 ***	η	13.48 ***	13.06 ***	η	15.29 ***	13.83 ***	η	12.36 ***	12.55 ***
	(3.98)	(5.93)		(2.92)	(12.72)		(4.98)	(5.33)		(6.44)	(5.80)		(4.59)	(4.89)

	Clayton		BB1		Clayton		BB1		Clayton
$\theta_{s_t=1}$	0.0000	$\tau^L_{s_t=1}$	0.0010	$\theta_{s_t=1}$	0.0034	$\tau^L_{s_t=1}$	0.1405	$\theta_{s_t=1}$	0.0000
	(0.00)		(0.03)		(0.06)		(0.21)		(0.00)
$\theta_{s_t=2}$	0.3380 ***	$\tau^U_{s_t=1}$	0.0966	$\theta_{s_t=2}$	0.4048 ***	$\tau^U_{s_t=1}$	0.2497 *	$\theta_{s_t=2}$	0.3945 ***
	(0.09)		(0.10)		(0.11)		(0.19)		(0.09)
p_{11}	0.9972 ***	$\tau^L_{s_t=2}$	0.1872 ***	p_{11}	0.9982 ***	$\tau^L_{s_t=2}$	0.0002	p_{11}	0.9981 ***
	(0.00)		(0.05)		(0.00)		(0.04)		(0.00)
p_{22}	0.9977 ***	$\tau^U_{s_t=2}$	0.1341 **	p_{22}	0.9985 ***	$\tau^U_{s_t=2}$	0.0000	p_{22}	0.9983 ***
	(0.00)		(0.06)		(0.00)		(0.00)		(0.00)
LL	3104.40	p_{11}	0.9900 ***	LL	2980.38	p_{11}	0.9814 ***	LL	3104.28
			(0.01)				(0.02)		
		p_{22}	0.9981 ***			p_{22}	0.9970 ***		
			(0.00)				(0.01)		
		LL	3024.09			LL	3031.68		

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the parameter of the best copula according to Table 2..3. *LL* is the log-Likelihood value.

$***/**/*$ indicates statistical significance at 1/5/10%

The copula parameter is assumed to evolve according to a two-state Markov switching specification. p_{ii} indicates the probability of remaining in state i, i.e. $s_t = i$, given that we have been in the same state in the previous period where $i = 1, 2$

Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: *OIL*). x could be A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

TABLE 2..4B: Parameter estimates for the joint distribution using time-varying copula specification that follows a first order Markov switching model

	F	L		G	L		H	L		I	L		J	L		K	L
ϕ_0	0.00 **	0.00	ϕ_0	0.00	-0.00	ϕ_0	-0.00	0.00	ϕ_0	0.00	0.00	ϕ_0	0.00	0.00	ϕ_0	0.00	-0.00
	(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)
ϕ_1	-0.09 ***	0.04	ϕ_1	-0.06 *	0.03	ϕ_1	-0.03	0.03	ϕ_1	-0.03	0.03	ϕ_1	-0.03	0.03	ϕ_1	-0.04	0.04
	(0.04)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)		(0.04)	(0.04)
ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***	ω	0.00 ***	0.00 ***
	(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)
α	0.04 **	0.04 **	α	0.00	0.04 **	α	0.04 **	0.04 **	α	0.00	0.05 **	α	0.00	0.04 **	α	0.07 ***	0.04 **
	(0.02)	(0.02)		(0.02)	(0.02)		(0.02)	(0.02)		(0.03)	(0.02)		(0.03)	(0.02)		(0.02)	(0.02)
β	0.89 ***	0.90 ***	β	0.86 ***	0.89 ***	β	0.90 ***	0.90 ***	β	0.80 ***	0.90 ***	β	0.85 ***	0.90 ***	β	0.92 ***	0.90 ***
	(0.01)	(0.00)		(0.01)	(0.00)		(0.00)	(0.00)		(0.01)	(0.00)		(0.01)	(0.00)		(0.00)	(0.00)
γ	0.06 **	0.08 ***	γ	0.18 ***	0.08 ***	γ	0.07 **	0.08 ***	γ	0.19 ***	0.07 **	γ	0.26 ***	0.07 **	γ	0.00	0.08 **
	(0.03)	(0.03)		(0.04)	(0.03)		(0.03)	(0.03)		(0.05)	(0.03)		(0.05)	(0.03)		(0.03)	(0.03)
λ	-0.20 ***	-0.25 ***	λ	-0.25 ***	-0.26 ***	λ	-0.08 *	-0.26 ***	λ	-0.20 ***	-0.25 ***	λ	-0.35 ***	-0.26 ***	λ	-0.15 ***	-0.26 ***
	(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)		(0.05)	(0.05)
η	8.99 ***	13.79 **	η	8.50 ***	13.58 ***	η	7.42 ***	13.74 ***	η	7.91 ***	14.73 **	η	11.01 ***	13.97 **	η	11.28 ***	13.93 **
	(2.22)	(5.96)		(2.03)	(5.71)		(1.51)	(5.86)		(1.67)	(6.55)		(3.08)	(6.08)		(3.89)	(6.95)

	Student		Clayton		Clayton		Clayton		Clayton		BB1
ρ_{s_1}	-0.0677 *	$\theta_{s_t=1}$	0.0000	$\theta_{s_t=1}$	0.0000	$\theta_{s_t=1}$	0.0000	$\theta_{s_t=1}$	0.0000	$\tau^L_{s_t=1}$	0.0000
	(0.04)		(0.00)		(0.00)		(0.08)		(0.00)		(0.01)
$\nu_{s_t=1}$	17.4475	$\theta_{s_t=2}$	0.2879 ***	$\theta_{s_t=2}$	0.2552 ***	$\theta_{s_t=2}$	0.3325 ***	$\theta_{s_t=2}$	0.3614 **	$\tau^U_{s_t=1}$	0.0291 *
	(19.22)		(0.08)		(0.09)		(0.09)		(0.18)		(0.02)
$\rho_{s_t=2}$	0.5333 ***	p_{11}	0.9966 ***	p_{11}	0.9970 ***	p_{11}	0.9967 ***	p_{11}	0.9956 ***	$\tau^L_{s_t=2}$	0.1052
	(0.09)		(0.00)		(0.00)		(0.00)		(0.00)		(0.10)
$\nu_{s_t=2}$	99.4614 ***	p_{22}	0.9972 ***	p_{22}	0.9973 ***	p_{22}	0.9976 ***	p_{22}	0.9905 ***	$\tau^U_{s_t=2}$	0.0623
	(22.29)		(0.00)		(0.00)		(0.00)		(0.01)		(0.09)
p_{11}	0.9978 ***	LL	3130.98	LL	2957.72	LL	3093.82	LL	2918.88	p_{11}	0.9983 ***
	(0.00)										(0.00)
p_{22}	0.9806 ***									p_{22}	0.9981 ***
	(0.02)										(0.00)
LL	3086.64									LL	2800.53

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (2.4),(2.5) and (2.6) and for the parameter of the best copula according to Table 2..3. *LL* is the log-Likelihood value.

$***/**/*$ indicates statistical significance at 1/5/10%

The copula parameter is assumed to evolve according to a two-state Markov switching specification. p_{ii} indicates the probability of remaining in state i, i.e. $s_t = i$, given that we have been in the same state in the previous period where $i = 1, 2$

Each pair of columns $x - L$ represents a full estimated model for the joint distribution between a stock market sector (x) and oil returns (L: *OIL*). x could be A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

Estimates allowing for changes in the type of copula

The empirical joint density function in Figures 2.4.1, 2.4.2 and 2.4.3 point out a change in the link from negative to positive association for Eurostoxx and most of its sectors. Assuming an unchanged type of copula over time is a strong constraint given the fact that some copulas only enable positive association. In fact, Clayton, Gumbel and BB1 copulas only allow for positive tail dependence. These copulas cannot detect potential changes in the sign of the relationship. This entails significant problems for risk management because might claim that hedging against extreme movements in oil prices is not required when a dependence between oil and stock market returns exists.
Hence, I compare the results from previous section, where the copula choice was assumed constant over time, with the estimation of a model where not only the copula parameter but the copula itself can change across regimes.

TABLE 2..5: AICc values for the models with different copula

	EUROSTOXX	BASICMATS	CONSUMERGDS	CONSUMERSVS	TELECOM	UTILITIES	FINANCIALS
Clayton-Clayton	*-6163,5*	-5915,4	-6163,2	-6216,6	-5870,1	-6142,3	-5792,4
Ind -Clayton	-6165,6	*-5917,5*	*-6165,3*	-6218,8	-5872,2	**-6144,4**	-5794,6
Gaussian-Clayton	-6165,0	-5915,5	-6164,1	**-6226,8**	*-5875,9*	*-6142,4*	*-5799,1*
Student-Clayton	-6162,7	-5913,9	-6161,6	*-6224,5*	-5873,4	-6140,0	-5796,7
R90Gumbel-Clayton	*-6163,5*	-5915,4	-6163,2	-6218,5	-5870,2	-6142,3	-5794,2
R90Clayton-Clayton	**-6169,6**	**-5919,6**	**-6166,2**	*-6224,9*	**-5879,7**	*-6143,1*	**-5803,0**
R90BB1-Clayton	-6167,5	*-5917,5*	*-6165,5*	-6222,8	*-5877,6*	-6141,0	*-5800,9*
	OILGAS		INDUSTRIALS		HEALTHCARE		TECHNOLOGY
BB1-BB1	-5998,6	BB1-BB1	-6013,8	Student-Student	*-6123,7*	BB1-BB1	-5551,5
Ind-BB1	**-6012,2**	BB1-Ind	**-6018,1**	Ind-Gaussian	*-6124,4*	Ind-BB1	-5554,8
Gaussian-BB1	*-6001,1*	BB1-Gaussian	*-6017,9*	Student-Gaussian	**-6125,0**	Gaussian-BB1	*-5566,9*
Gumbel-BB1	-5997,1	BB1-R90Gumbel	*-6016,0*			Student-BB1	-5561,4
Clayton-BB1	*-6011,0*					Gumbel-BB1	-5559,8
						R90Clayton-BB1	*-5563,3*
						R90Gumbel-BB1	**-5568,3**

This table shows the values of the Akaike Information Criterion corrected for small sample bias (AICc) (Hurvich and Tsai 1989).
$AICc = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$, where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum $AICc$ value in bold letters indicates the best copula fit. The second and the third best copulas are shown in italic letters.

TABLE 2..6: Kolgomorov Smirnov one sample test against the uniform distribution

	A	B	C	D	E	F	G	H	I	J	K
$C(u_{oil}\|u_{stock})$	0,9897	0,6849	0,9500	0,9709	0,9715	0,9966	0,9896	0,9805	0,9888	0,9735	0,9958
$C(u_{stock}\|u_{oil})$	0,8220	0,9562	0,9816	0,2642	0,6991	0,6096	0,7387	0,6401	0,5957	0,3388	0,4302

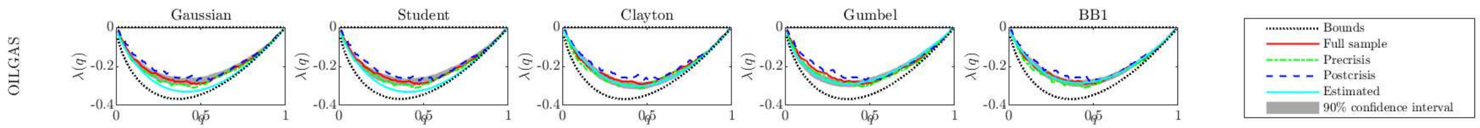
This table shows the p-values of the Kolgomorov Smirnov test comparing the conditional copula distribution from the best model according to Table 2..5 with the uniform distribution.
A: *EUROSTOXX*; B: *OIL&GAS*; C: *BASICMATS*; D: *INDUSTRIALS*; E: *CONSUMERGDS*; F: *HEALTHCARE*; G: *CONSUMERSVS*; H: *TELECOM*; I:*UTILITIES*; J: *FINANCIALS*; K: *TECHNOLOGY*.

The set of copula combinations considered in Table 2..5 depends on the results from the previous subsection and the descriptive analysis from Section 2.4. For instance, the Clayton copula is combined with the independence copula, i.e. the product of the marginal distributions, for the sectors that have presented lower tail dependence. This combination allow us to check if the low-dependence regime means that before 2008 oil returns and those sectors were independent. The limitations of the model to detect negative dependence might lead to misinterpretations of the low-dependence regime. The Clayton copula is, consequently, combined with copulas that express a negative relationship without tail dependence (Gaussian), symmetric tail dependence (Student), lower tail dependence (90° rotated Clayton), upper tail dependence (90° rotated Gumbel) or asymmetric tail dependence (90° rotated BB1).

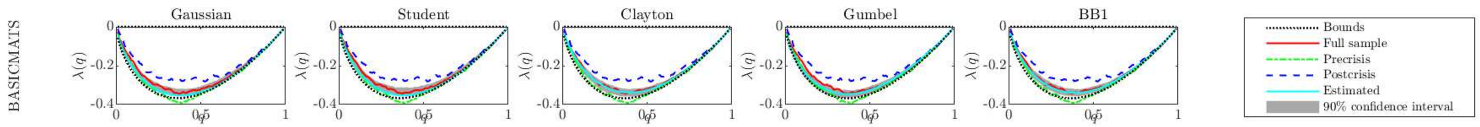
Further details about the use of rotated copulas are provided in Appendix A. For these sectors Table 2..5 shows a change from a negative to positive lower tail dependence, i.e. R90Clayton-Clayton, with the exception of consumer services which does not seem to have tail dependence before 2008 but it also presents a negative behaviour. These results are in line with the intuition behind Figure 2.4.2. Utilities is the only sector where the low-dependence regime from previous subsection is identified with the independent copula. Table 2..6 displays the p-values of the one-sample Kolgomorov-Smirnov test of the conditional copula against the uniform distribution where no indication of misleading copula is found. The conditional copula comes from the derivation of the copula function from one input variable, e.g. $C(u_2|u_1) = \partial C(u_1, u_2)/\partial u_1$. $C(u_2|u_1)$ indicates the distribution of u_2 given the realization of u_1. If the copula gathers properly the dependence between variables, the conditional probability of u_2 must be uniformly distributed (see Rodriguez 2007).

E Extra figures

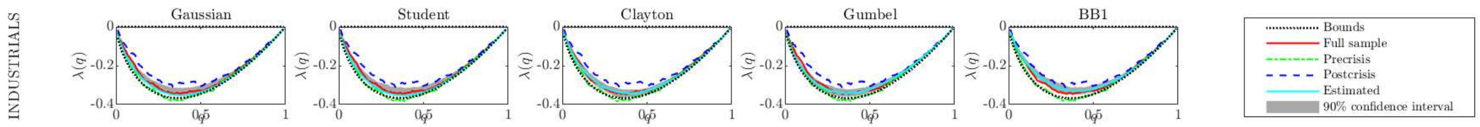
FIGURE 2..4A: Lambda function for the initial set of copulas and the empirical lambda function for the full, pre-crisis and post-crisis samples



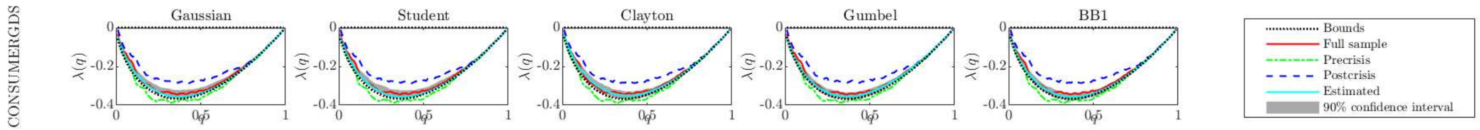
(a) Oil - OIL & GAS sector



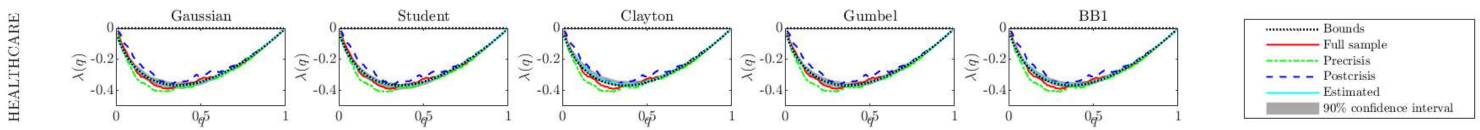
(b) Oil - BASIC MATS sector



(c) Oil - INDUSTRIALS sector



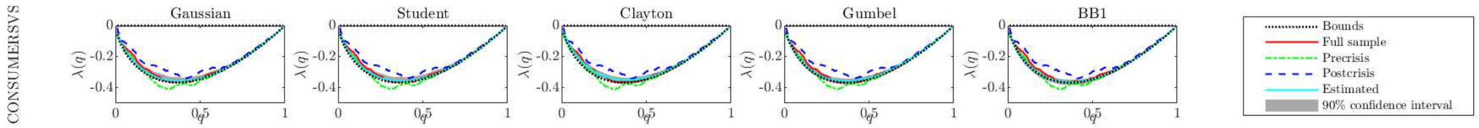
(d) Oil - CONSUMER GDS sector



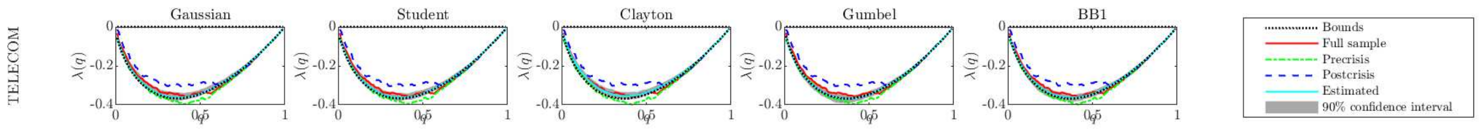
(e) Oil - HEALTH CARE sector

The dotted black lines represent the bands corresponding to independence (curve line) and perfect comonotonicity (flat line). The solid cyan line is the fitted lambda for the corresponding copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The empirical lambda function for the full sample is represented by the solid red line. The empirical lambda function for the pre-crisis sample is represented by the dash-dotted green line and for the post-crisis sample is represented by the dashed blue line.

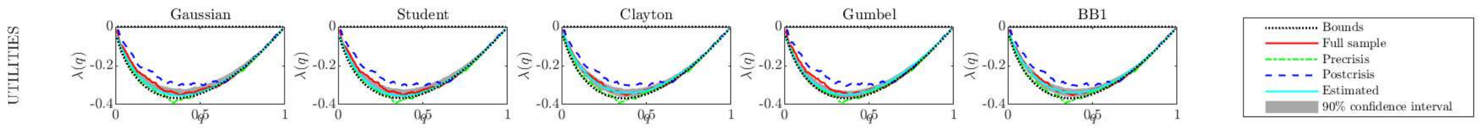
FIGURE 2..4B: Lambda function for the initial set of copulas and the empirical lambda function for the full, pre-crisis and post-crisis samples



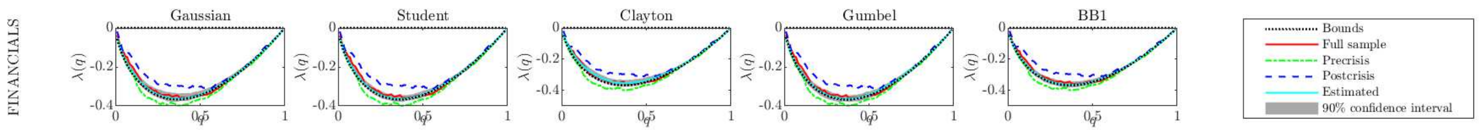
(f) Oil- CONSUMER SVS sector



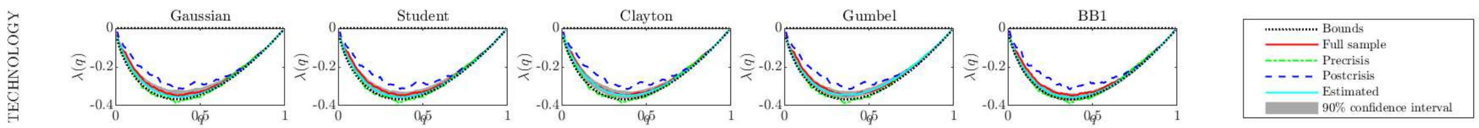
(g) Oil - TELECOMsector



(h) Oil - UTILITIES sector



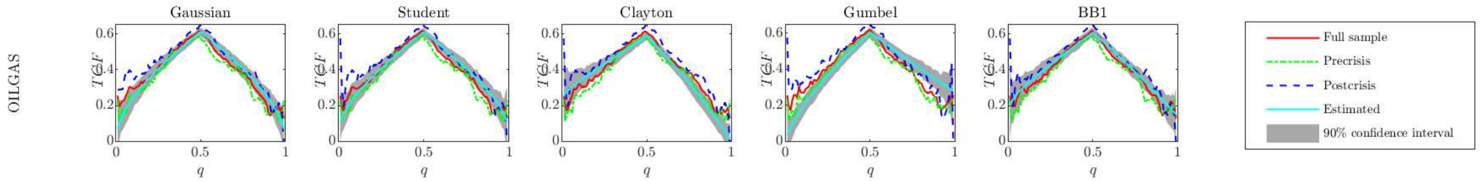
(i) Oil - FINANCIALS sector



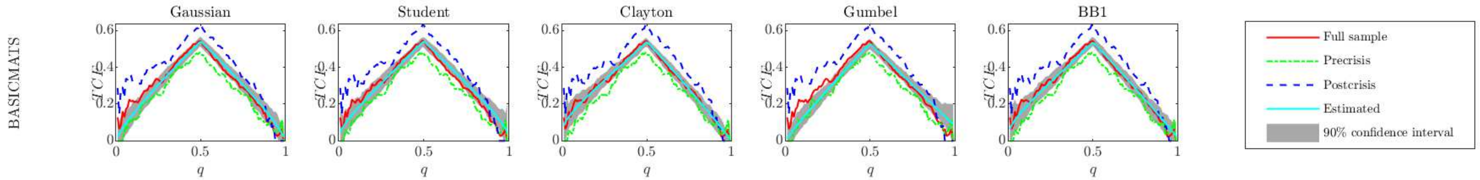
(j) Oil - TECHNOLOGY sector

The dotted black lines represent the bands corresponding to independence (curve line) and perfect comonotonicity (flat line). The solid cyan line is the fitted lambda for the corresponding copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The empirical lambda function for the full sample is represented by the solid red line. The empirical lambda function for the pre-crisis sample is represented by the dash-dotted green line and for the post-crisis sample is represented by the dashed blue line.

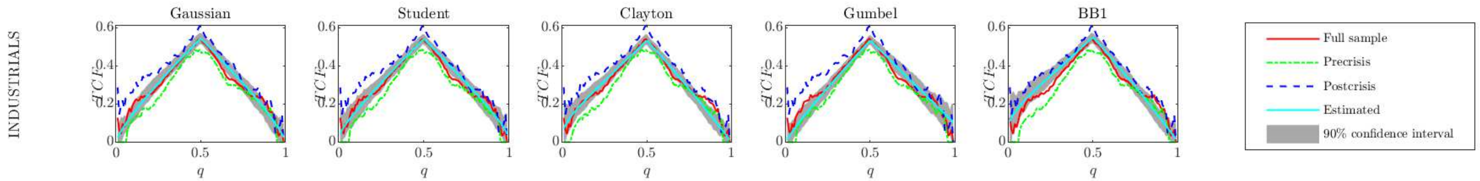
FIGURE 2..5A: Tail concentration function (TCF) for the initial set of copulas and the empirical TCF function for the full, pre-crisis and post-crisis samples



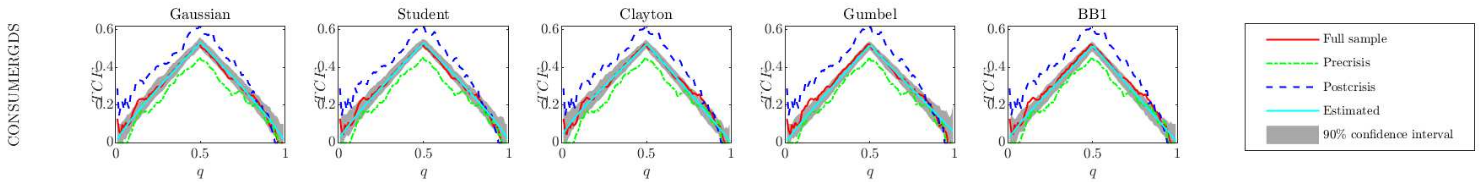
(a) Oil - OIL & GAS sector



(b) Oil - BASIC MATS sector



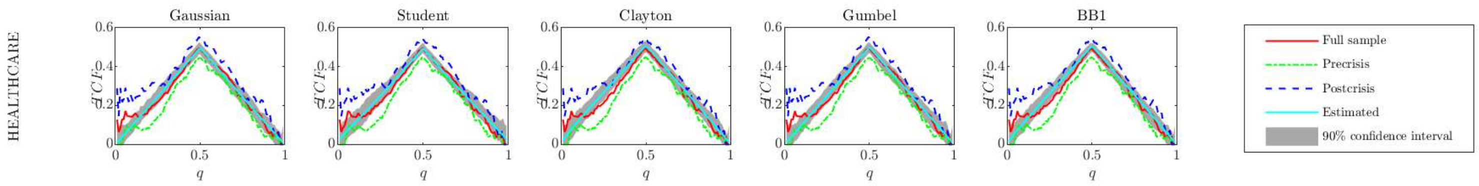
(c) Oil - INDUSTRIALS sector



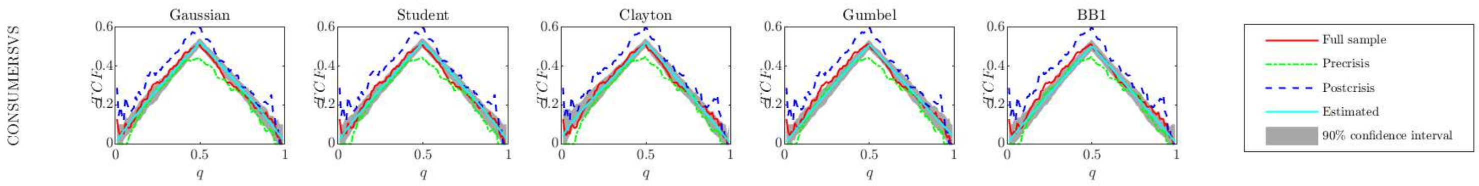
(d) Oil - CONSUMER GDS sector

The solid cyan line is the fitted TCF for the corresponding estimated copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The TCF quantifies the tail dependence features giving an idea about how well the parametric copula fits the empirical joint distribution for extreme quantiles (Pappadà et al. (2018)).

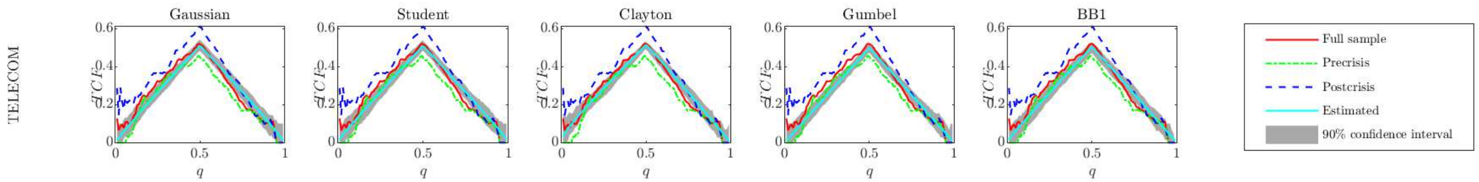
FIGURE 2..5B: Tail concentration function (TCF) for the initial set of copulas and the empirical TCF function for the full, pre-crisis and post-crisis samples



(e) Oil - HEALTH CARE sector



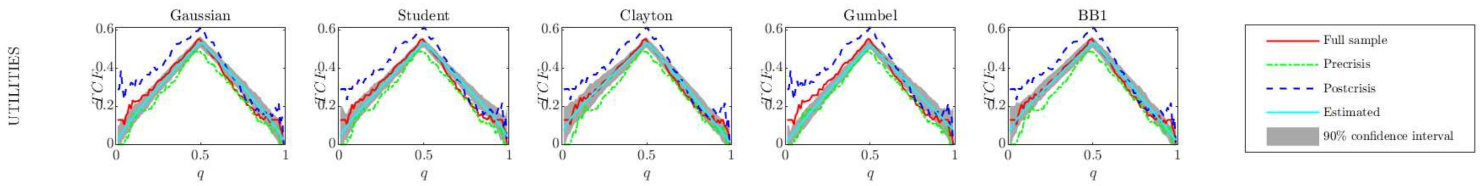
(f) Oil- CONSUMER SVS sector



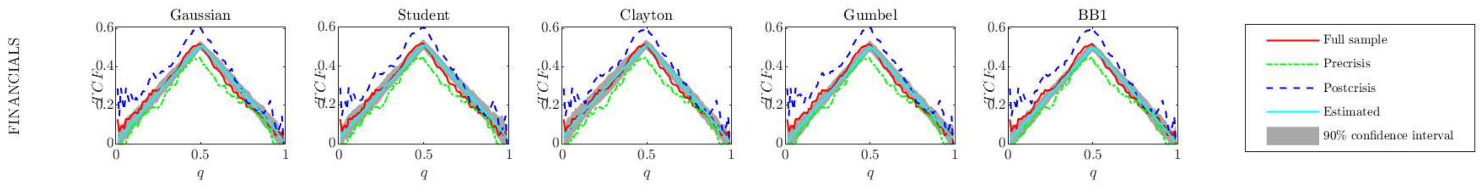
(g) Oil - TELECOMsector

The solid cyan line is the fitted TCF for the corresponding estimated copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The TCF quantifies the tail dependence features giving an idea about how well the parametric copula fits the empirical joint distribution for extreme quantiles (Pappadà et al. (2018)).

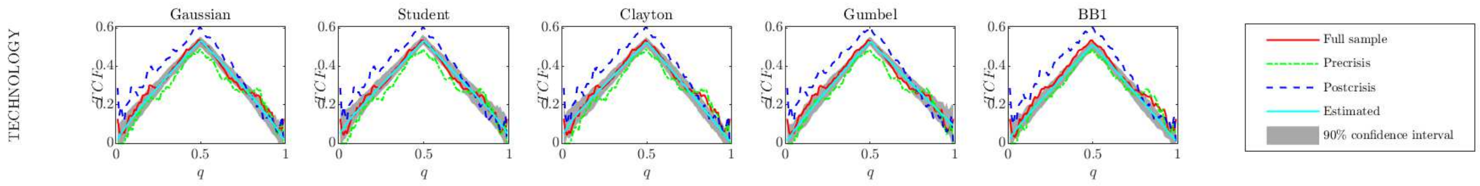
FIGURE 2..5C: Tail concentration function (TCF) for the initial set of copulas and the empirical TCF function for the full, pre-crisis and post-crisis samples



(h) Oil - UTILITIES sector



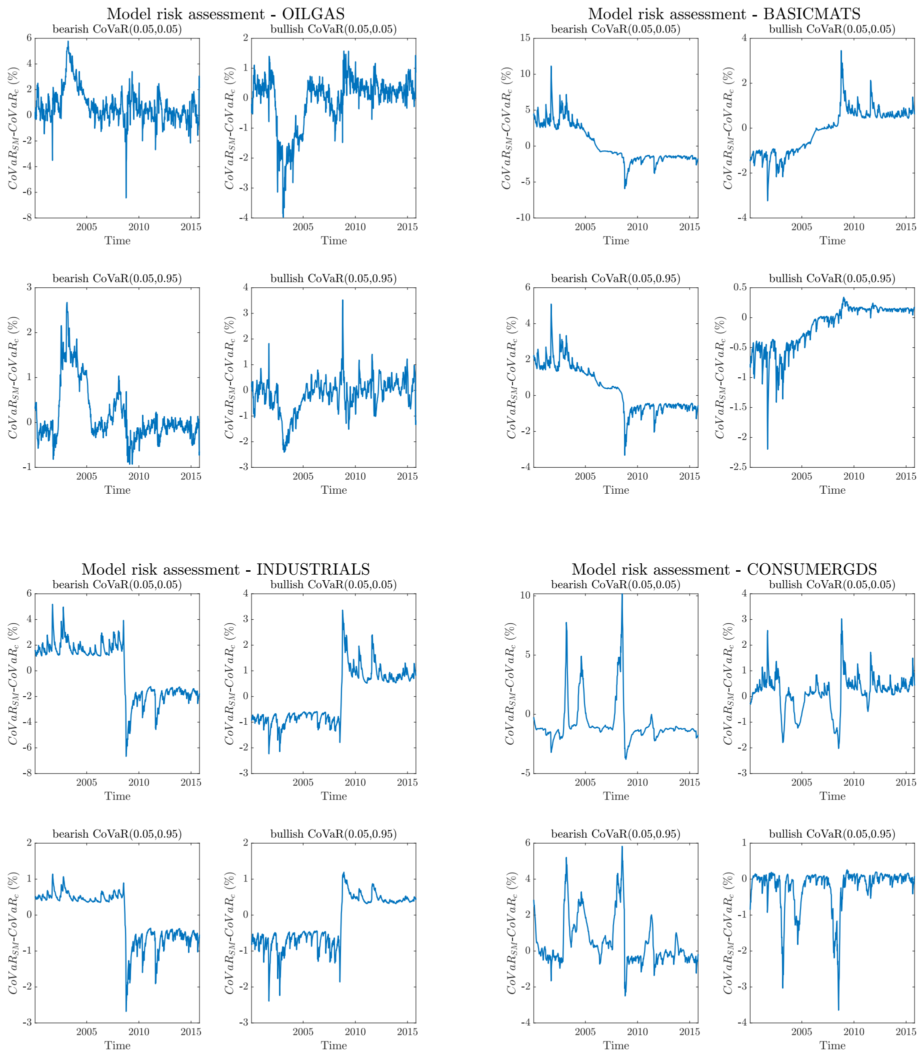
(i) Oil - FINANCIALS sector



(j) Oil - TECHNOLOGY sector

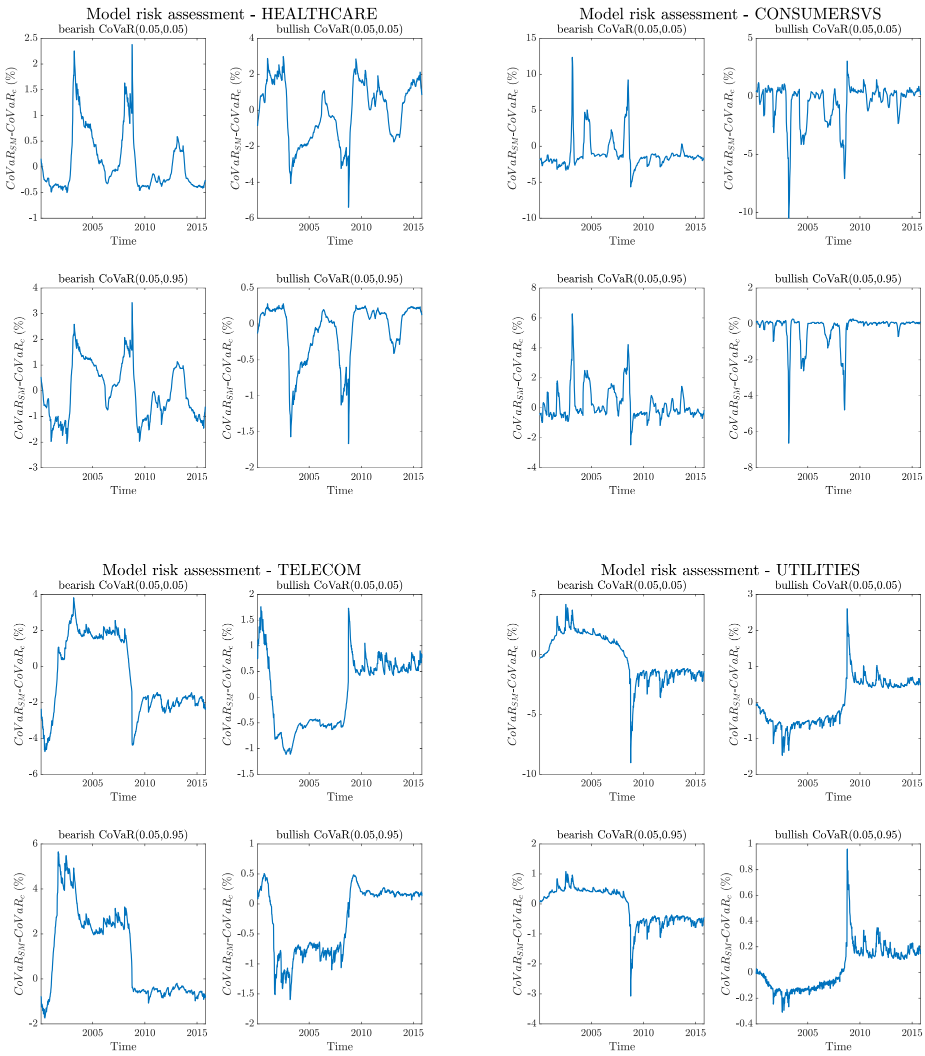
The solid cyan line is the fitted TCF for the corresponding estimated copula shown in the subtitle and the grey area is its 90% confidence interval obtained by bootstrapping. The TCF quantifies the tail dependence features giving an idea about how well the parametric copula fits the empirical joint distribution for extreme quantiles (Pappadà et al. (2018)).

FIGURE 2..6A: Model risk assessment when we assume a constant dependence compared to the time-varying model.



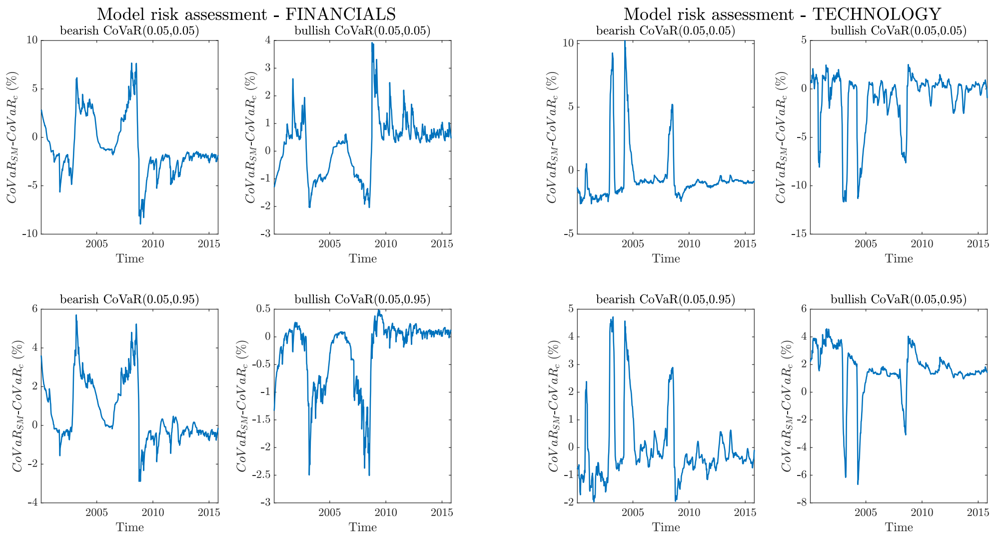
These figures show the difference between the estimation of the same percentile of stock sector returns under the same oil-related scenario using the Markov switching model that allows for changes in the copula against the constant model. Top figures focuses on a low quantile ($\beta = 5\%$) whilst bottom graphs estimate the high quantile ($\beta = 95\%$). Left figures show a percentile of stock returns under a bearish scenario for oil returns, i.e. oil returns below its percentile 5%, and right figures show a percentile of stock returns under a bullish scenario for oil returns, i.e. oil returns above its percentile 95%. These charts indicate the higher model risk in the lower tail than in the upper tail when assuming a constant dependence across markets.

FIGURE 2..6B: Model risk assessment when we assume a constant dependence compared to the time-varying model.



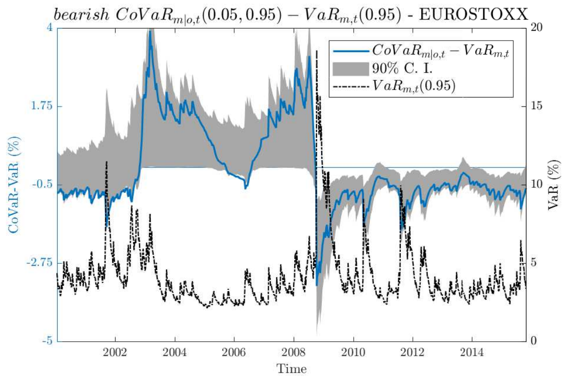
These figures show the difference between the estimation of the same percentile of stock sector returns under the same oil-related scenario using the Markov switching model that allows for changes in the copula against the constant model. Top figures focuses on a low quantile ($\beta = 5\%$) whilst bottom graphs estimate the high quantile ($\beta = 95\%$). Left figures show a percentile of stock returns under a bearish scenario for oil returns, i.e. oil returns below its percentile 5%, and right figures show a percentile of stock returns under a bullish scenario for oil returns, i.e. oil returns above its percentile 95%. These charts indicate the higher model risk in the lower tail than in the upper tail when assuming a constant dependence across markets.

FIGURE 2..6C: Model risk assessment of when we assume a constant dependence compared to the time-varying model.

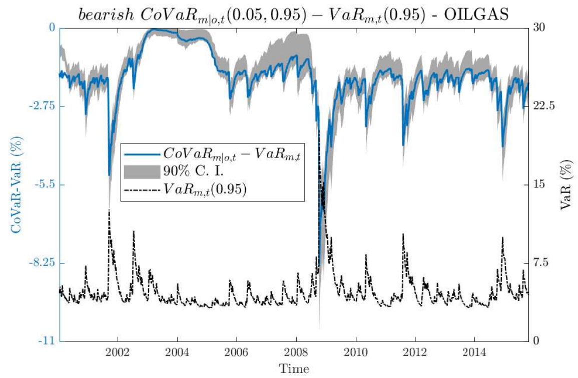


These figures show the difference between the estimation of the same percentile of stock sector returns under the same oil-related scenario using the Markov switching model that allows for changes in the copula against the constant model. Top figures focuses on a low quantile ($\beta = 5\%$) whilst bottom graphs estimate the high quantile ($\beta = 95\%$). Left figures show a percentile of stock returns under a bearish scenario for oil returns, i.e. oil returns below its percentile 5%, and right figures show a percentile of stock returns under a bullish scenario for oil returns, i.e. oil returns above its percentile 95%. These charts indicate the higher model risk in the lower tail than in the upper tail when assuming a constant dependence across markets.

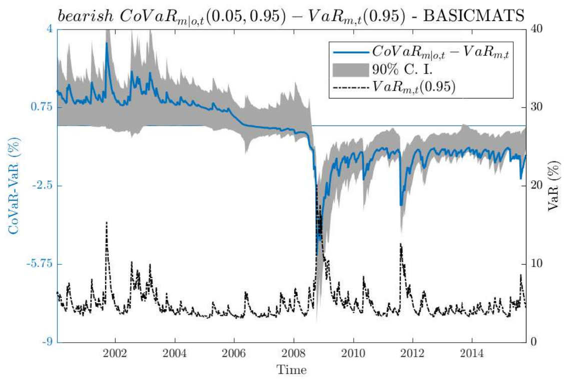
FIGURE 2..7A: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



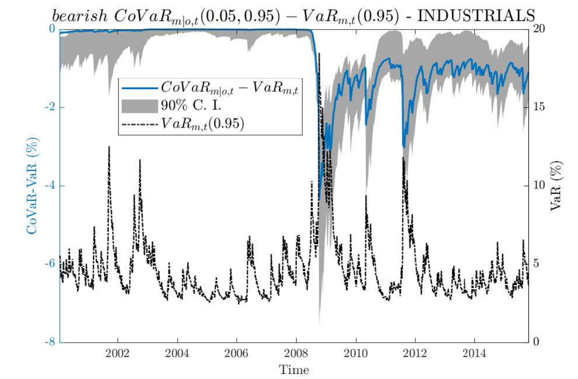
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



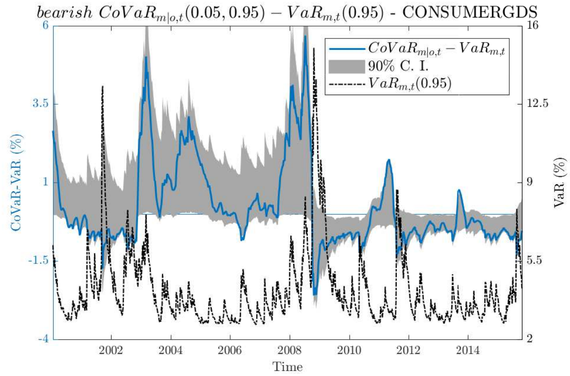
(c) Oil - BASIC MATS sector



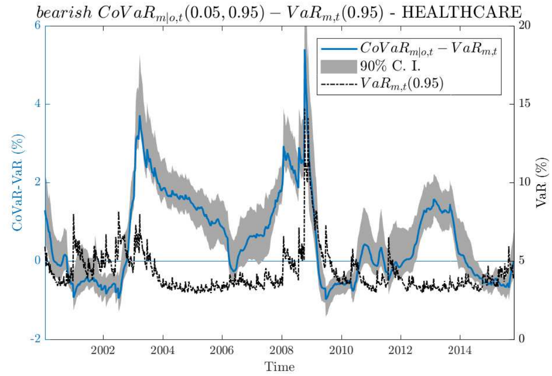
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

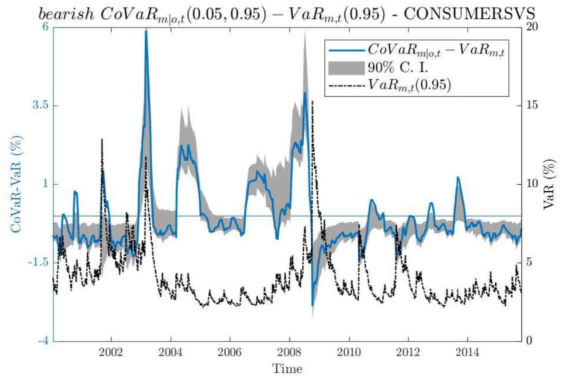
FIGURE 2..7B: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



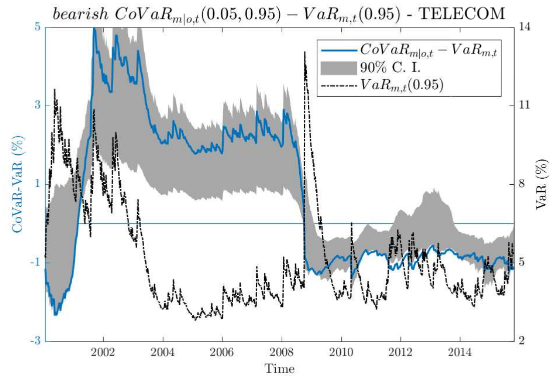
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



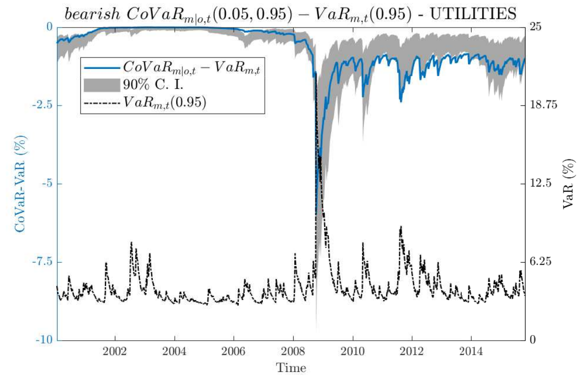
(g) Oil- CONSUMER SVS sector



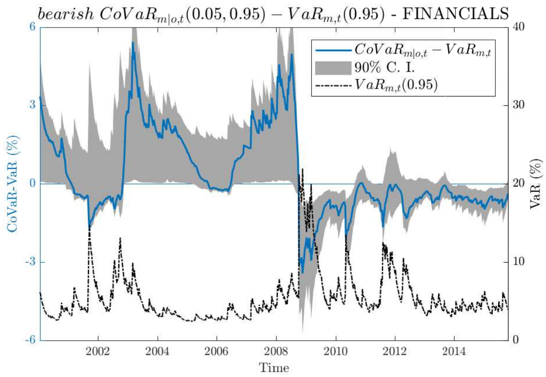
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

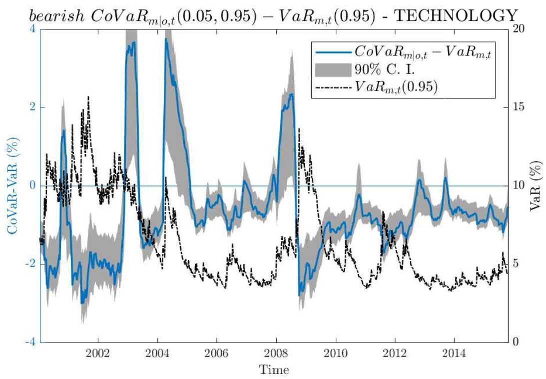
FIGURE 2..7C: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



(i) Oil - UTILITIES sector



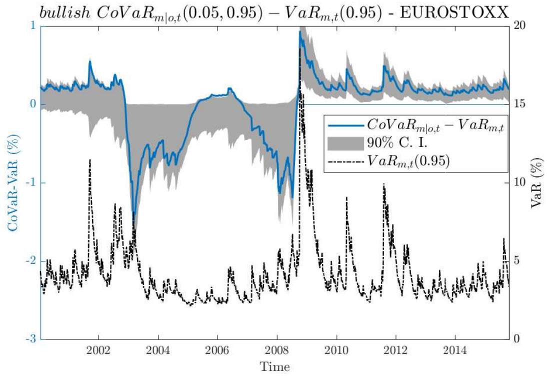
(j) Oil - FINANCIALS sector



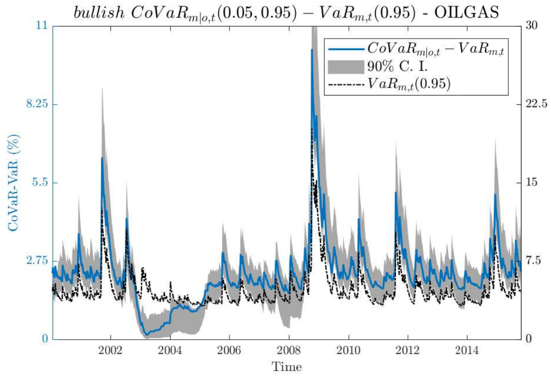
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

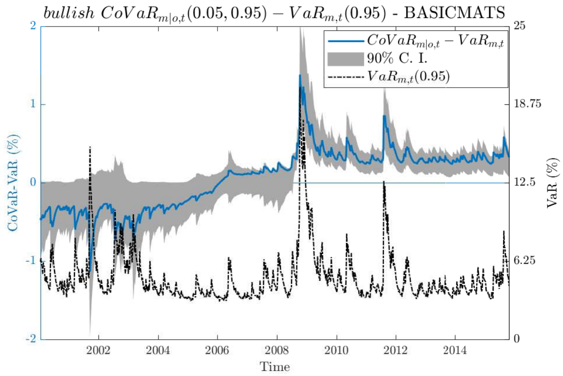
FIGURE 2..8A: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bullish scenario for oil prices



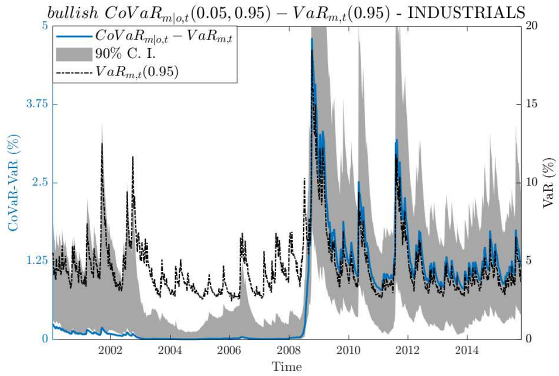
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



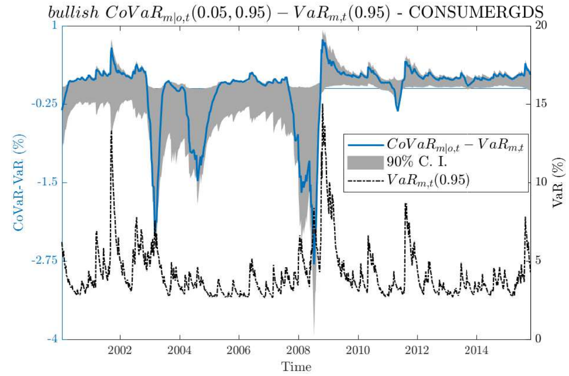
(c) Oil - BASIC MATS sector



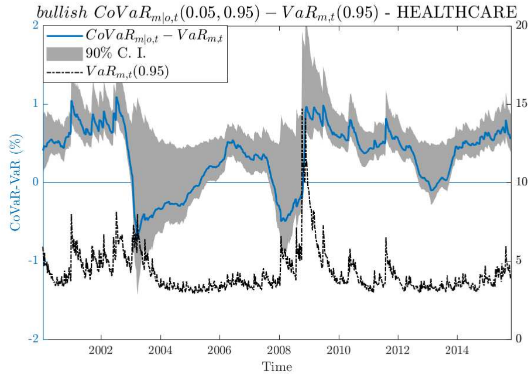
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

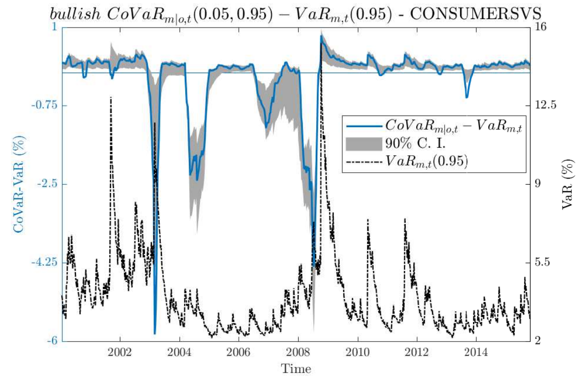
FIGURE 2..8B: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bullish scenario for oil prices



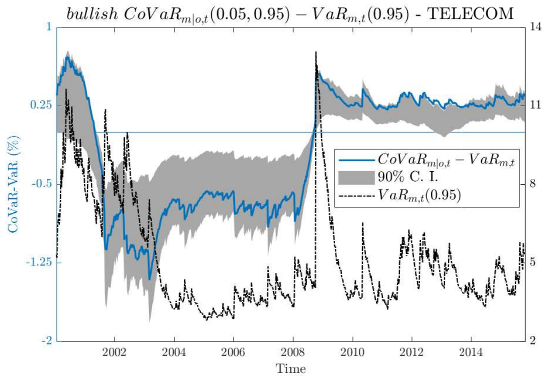
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



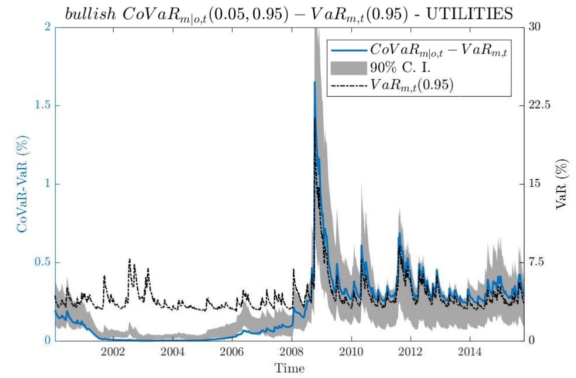
(g) Oil- CONSUMER SVS sector



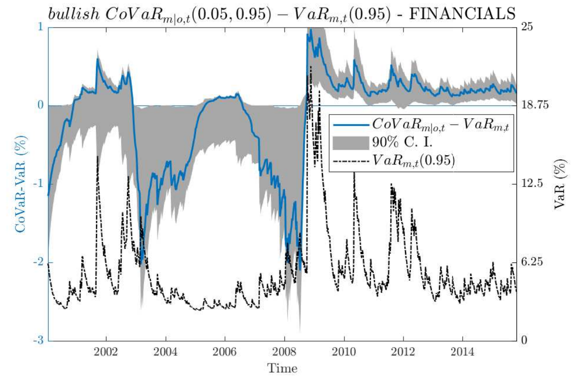
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5.Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

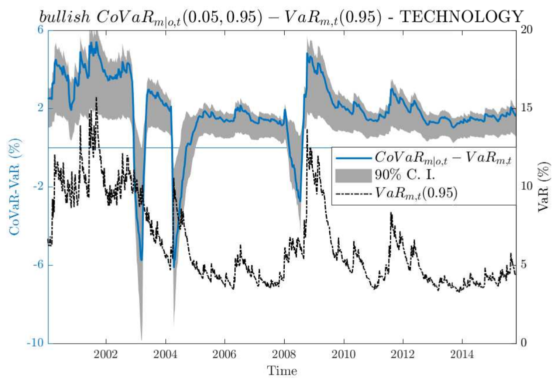
FIGURE 2..8C: $CoVaR_{m|o,t}(0.05, 0.95)$ for a certain sector given a bullish scenario for oil prices



(i) Oil - UTILITIES sector



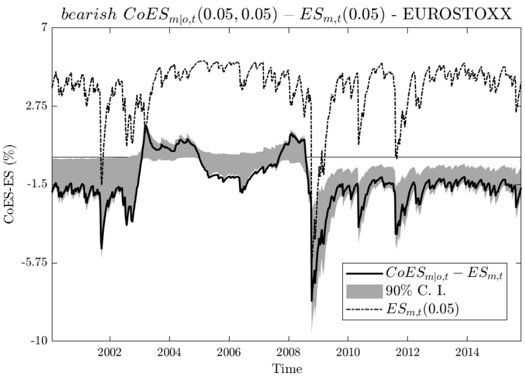
(j) Oil - FINANCIALS sector



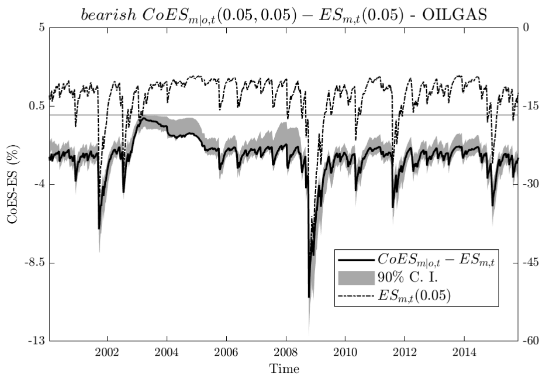
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoVaR_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $VaR_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5.Grey area indicates the 90% confidence interval of the difference between $CoVaR_{m|o,t}(0.05, 0.95)$ and $VaR_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the VaR level over time.

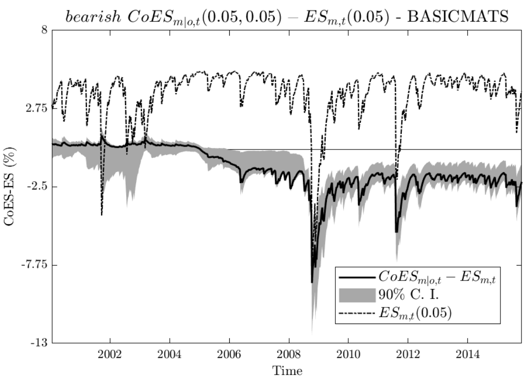
FIGURE 2..9A: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



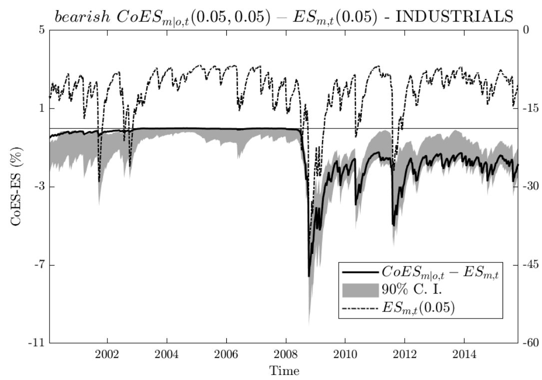
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



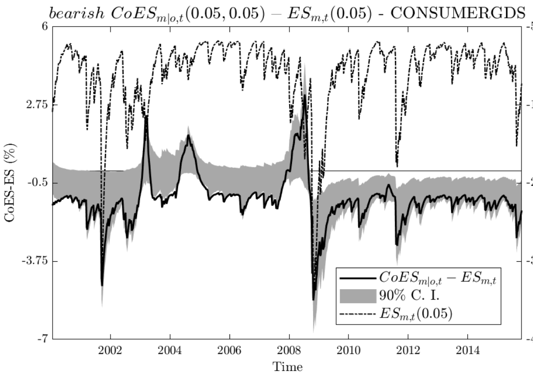
(c) Oil - BASIC MATS sector



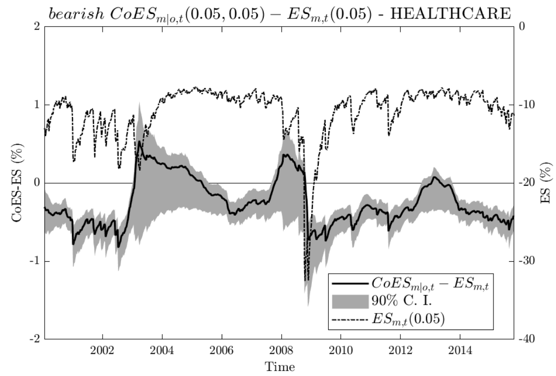
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

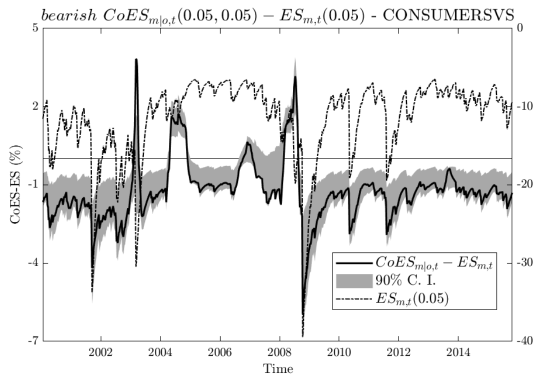
FIGURE 2..9B: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



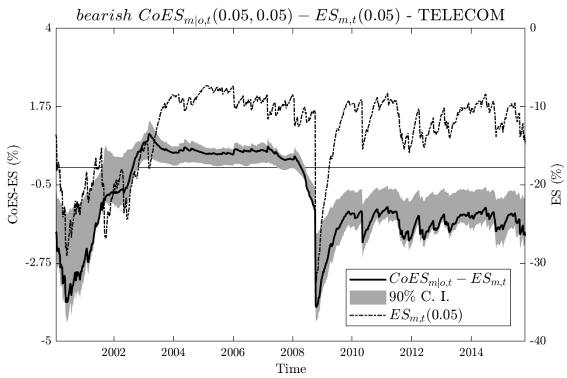
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



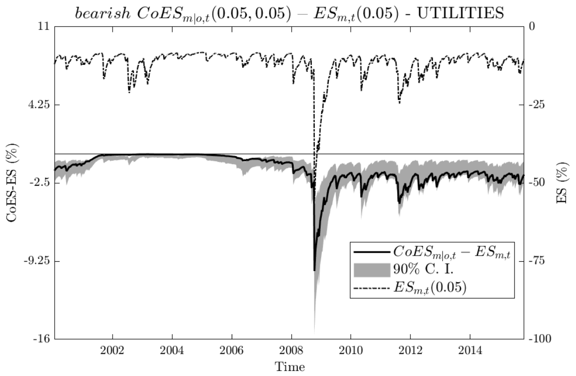
(g) Oil- CONSUMER SVS sector



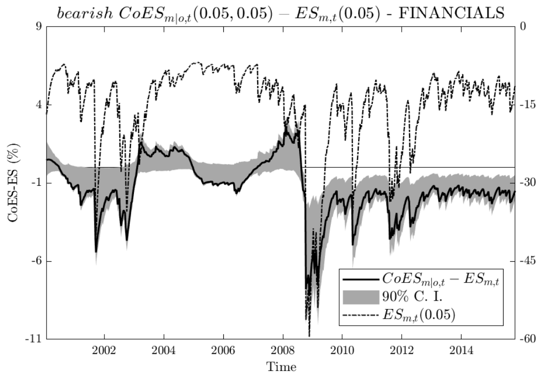
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $VaR_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

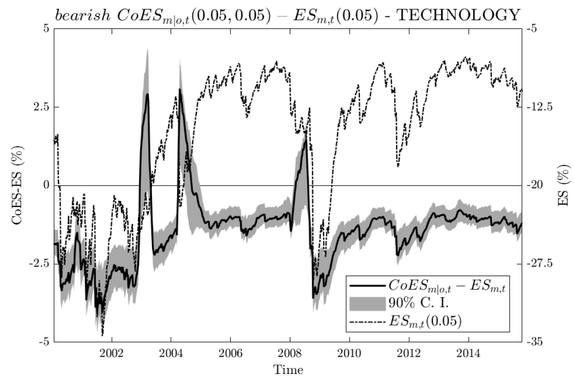
FIGURE 2..9C: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bearish scenario for oil prices



(i) Oil - UTILITIES sector



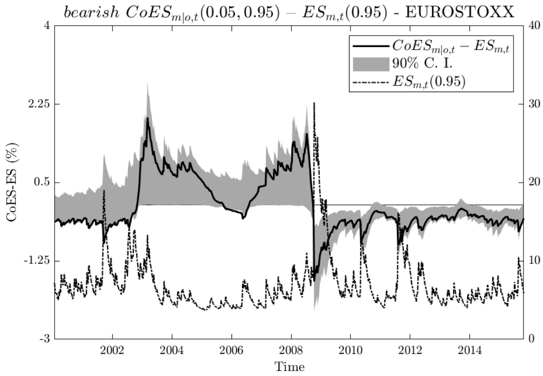
(j) Oil - FINANCIALS sector



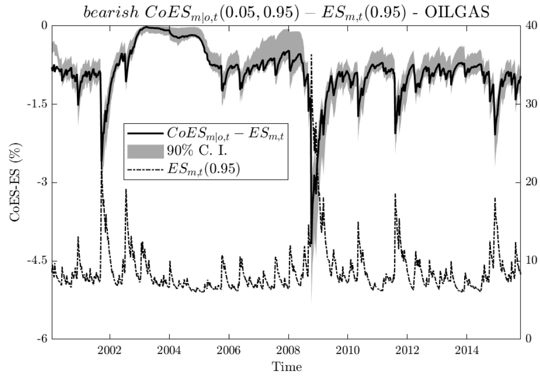
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

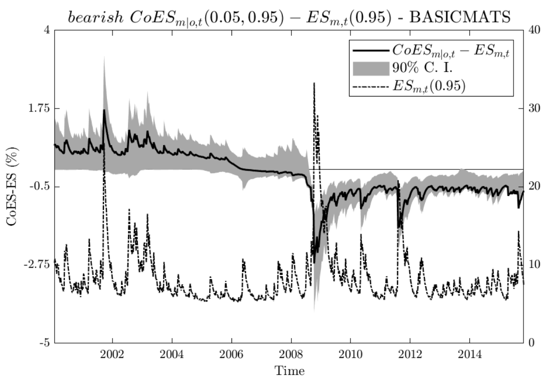
FIGURE 2..10A: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



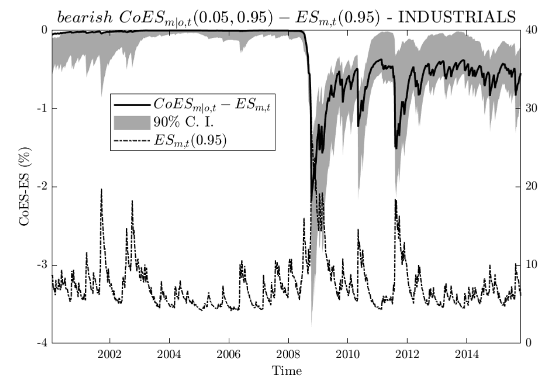
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



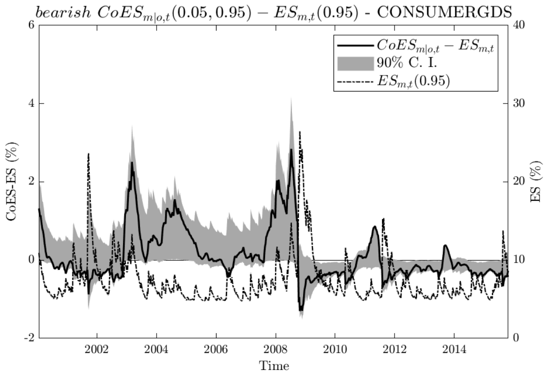
(c) Oil - BASIC MATS sector



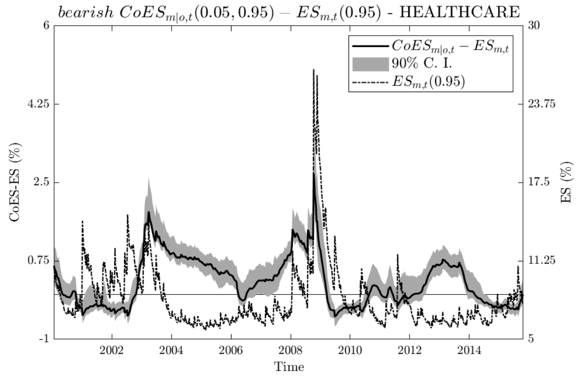
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

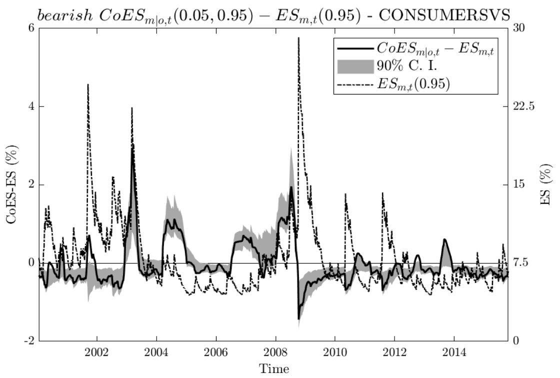
FIGURE 2..10B: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



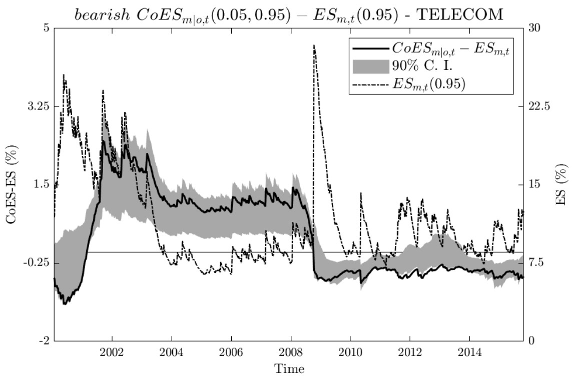
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



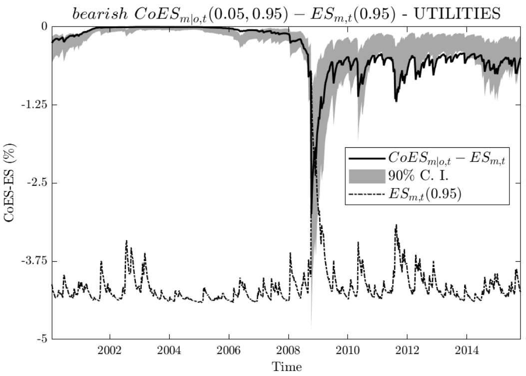
(g) Oil- CONSUMER SVS sector



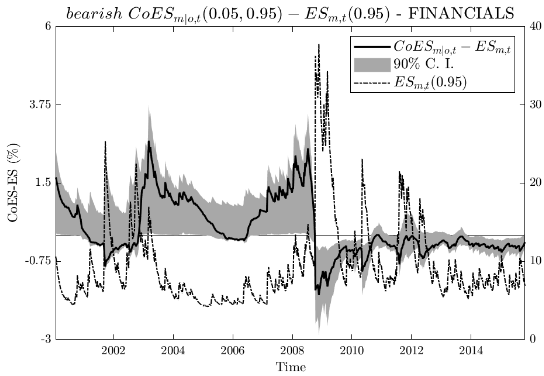
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

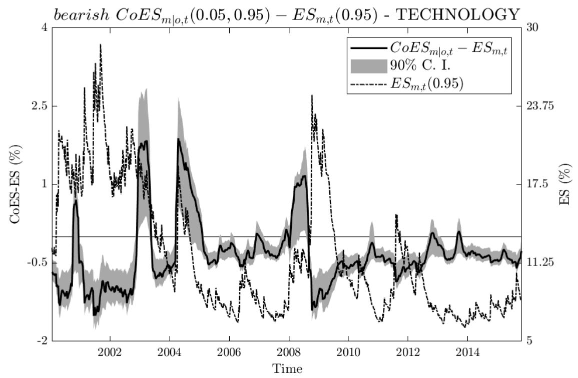
FIGURE 2..10C: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



(i) Oil - UTILITIES sector



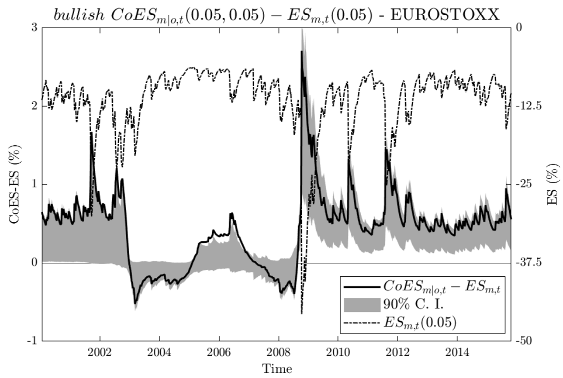
(j) Oil - FINANCIALS sector



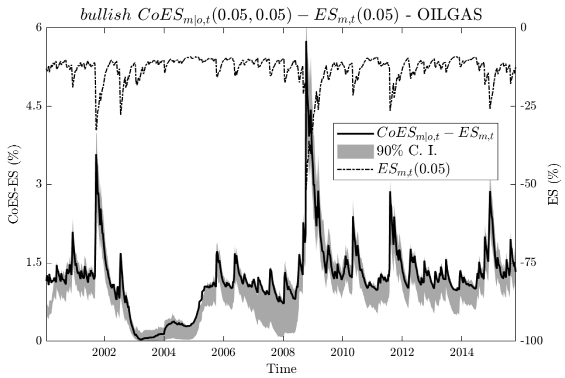
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

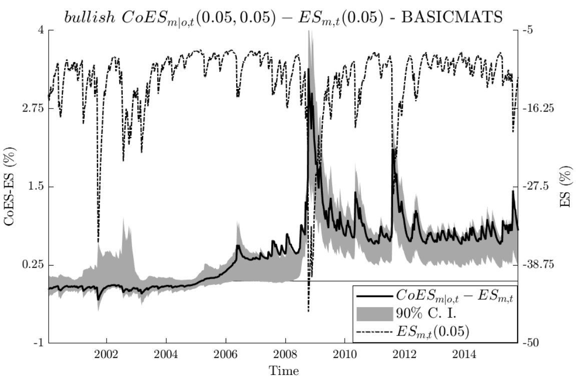
FIGURE 2..11A: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



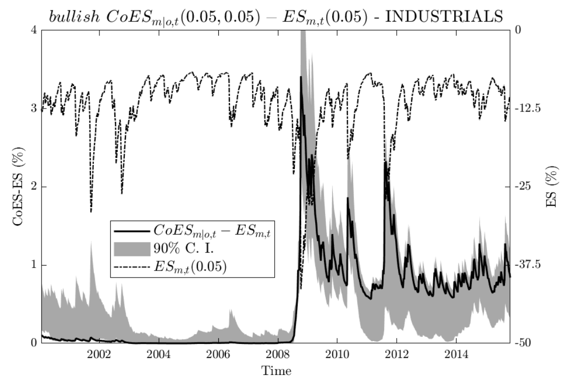
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



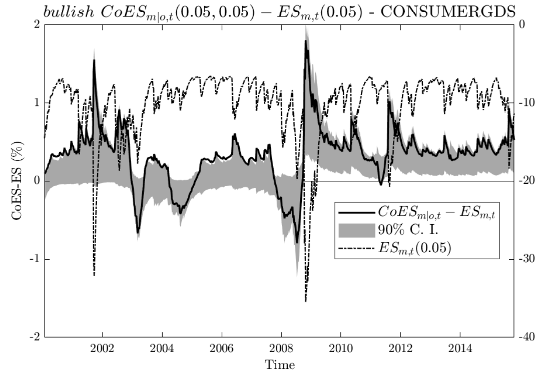
(c) Oil - BASIC MATS sector



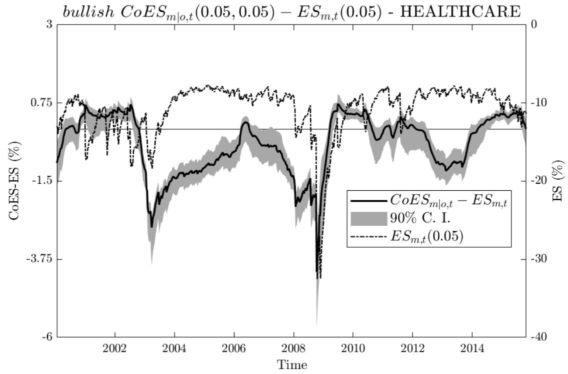
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

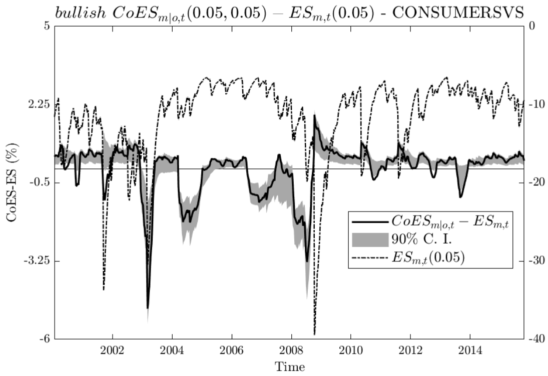
FIGURE 2..11B: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



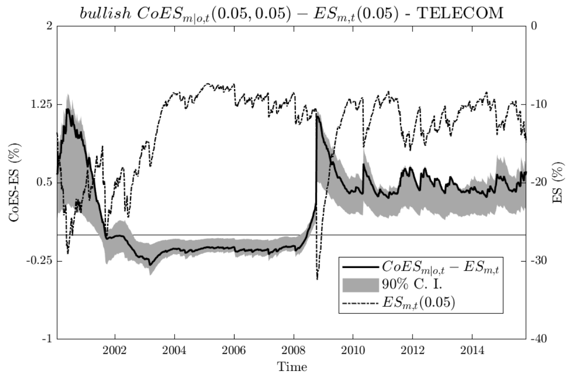
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



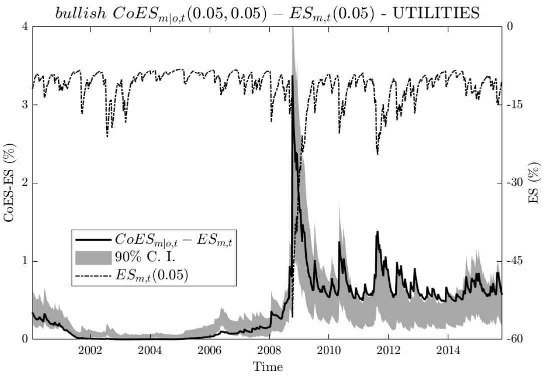
(g) Oil- CONSUMER SVS sector



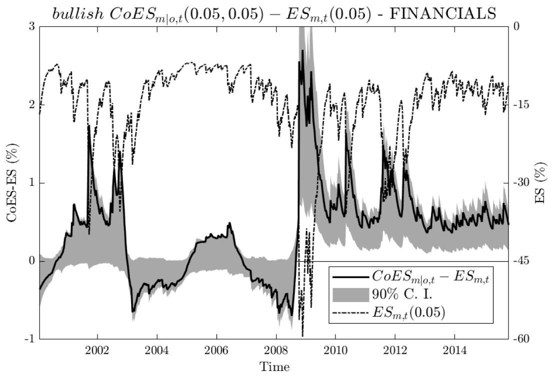
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5.Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

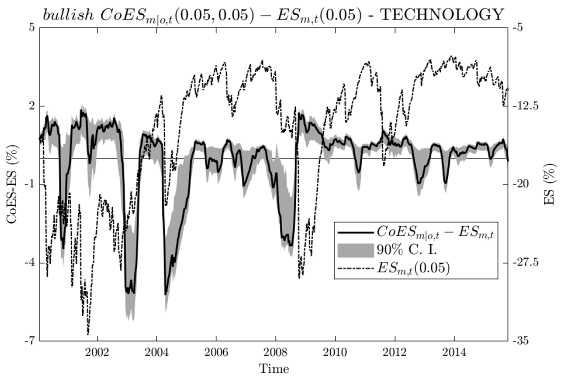
FIGURE 2..11C: $CoES_{m|o,t}(0.05, 0.05)$ for a certain sector given a bullish scenario for oil prices



(i) Oil - UTILITIES sector



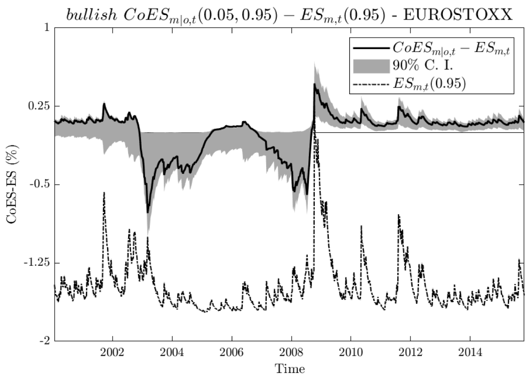
(j) Oil - FINANCIALS sector



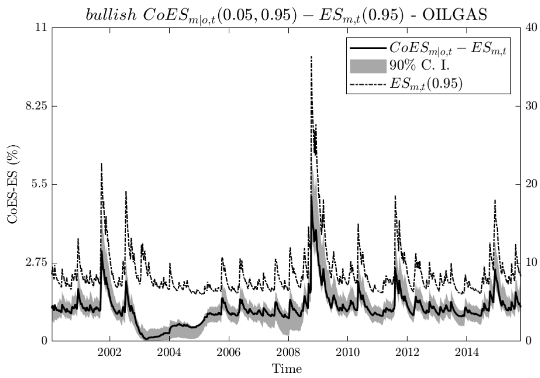
(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.05)$ of the returns for a certain sector under a extreme upward movement in oil prices, i.e. above its 95-th quantile, and its unconditional $ES_{m,t}(0.05)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5.Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.05)$ and $ES_{m,t}(0.05)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.05)$ in percentage. The dash-dotted black line indicate the ES level over time.

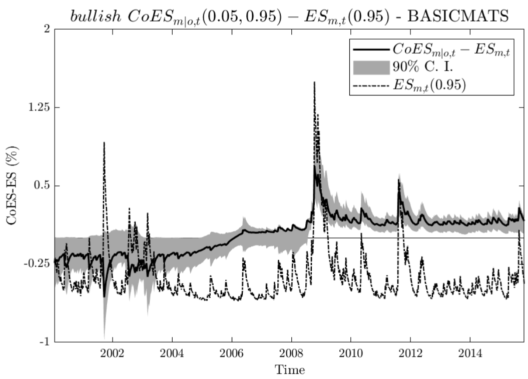
FIGURE 2..12A: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



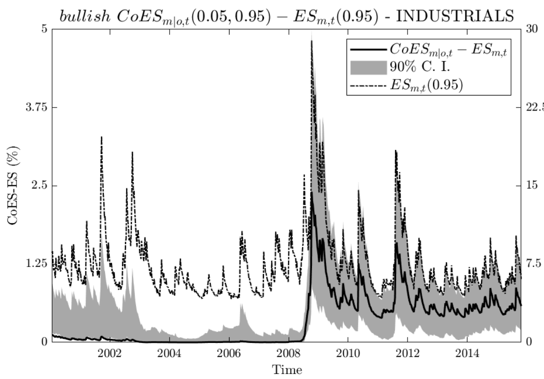
(a) Oil- EUROSTOXX index



(b) Oil - OIL & GAS sector



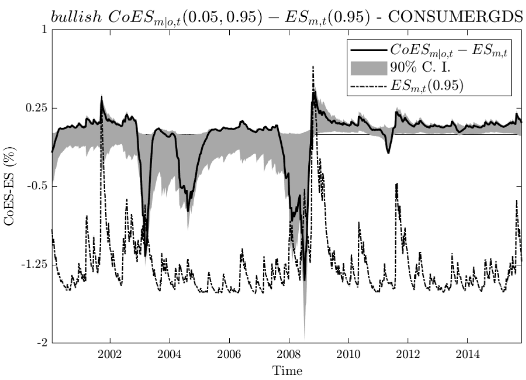
(c) Oil - BASIC MATS sector



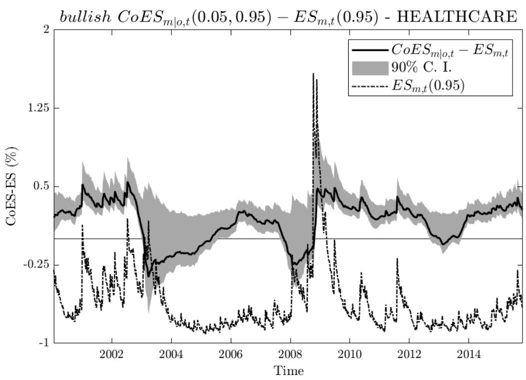
(d) Oil - INDUSTRIALS sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

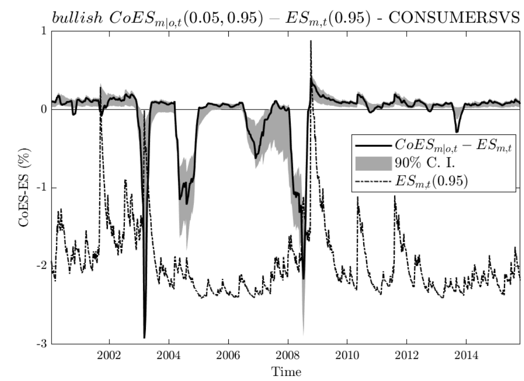
FIGURE 2..12B: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



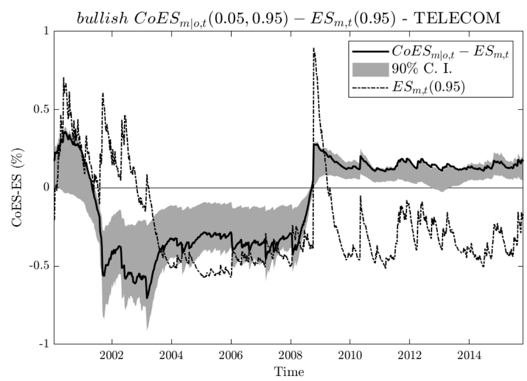
(e) Oil - CONSUMER GDS sector



(f) Oil - HEALTH CARE sector



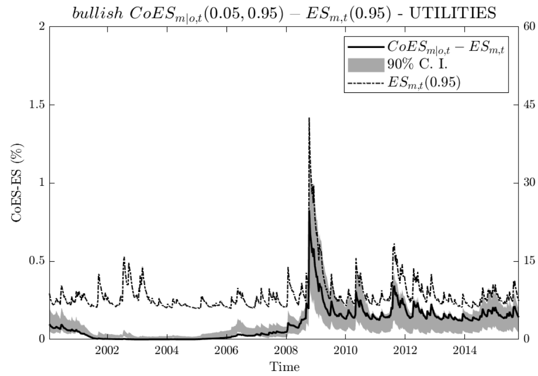
(g) Oil- CONSUMER SVS sector



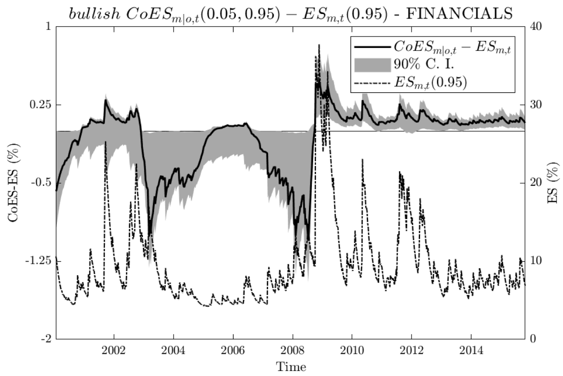
(h) Oil - TELECOM sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

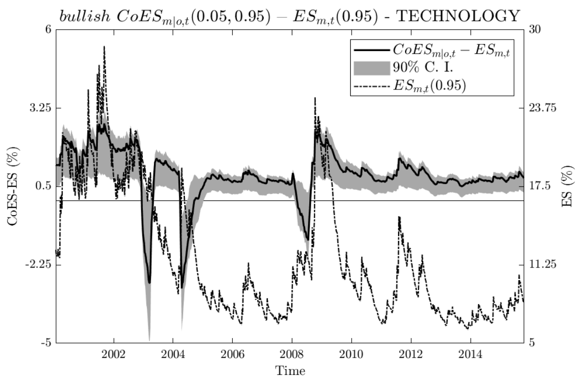
FIGURE 2..12C: $CoES_{m|o,t}(0.05, 0.95)$ for a certain sector given a bearish scenario for oil prices



(i) Oil - UTILITIES sector



(j) Oil - FINANCIALS sector



(k) Oil - TECHNOLOGY sector

Left axis shows the difference in percentage between the $CoES_{m,t}(0.05, 0.95)$ of the returns for a certain sector under a extreme downward movement in oil prices, i.e. below its 5-th quantile, and its unconditional $ES_{m,t}(0.95)$. Solid blue line shows this difference given the best copula mixture according to Table 2..5. Grey area indicates the 90% confidence interval of the difference between $CoES_{m|o,t}(0.05, 0.95)$ and $ES_{m,t}(0.95)$ following a Monte Carlo technique explained in Appendix C. The right axes shows the value of the $ES_{m,t}(0.95)$ in percentage. The dash-dotted black line indicate the ES level over time.

Chapter 3

Disentangling the role of the exchange rate in oil-related scenarios for the European stock market

Abstract
Until now, stock market responses to a distress scenario for oil prices have been analysed considering prices in domestic currency. This assumption implies merging the commodity risk with the exchange rate risk when oil and stocks are traded in different currencies. This article proposes incorporating explicitly the exchange rate, using the convolution concept, to assess how could change the stock market response depending on the source of risk that moves oil prices. I apply this framework to study the change in the 10th lowest percentile of the European stock market under an oil-related stress scenario, without overlooking the role of the exchange rate.
The empirical exercise shows that the same stress oil-related scenario in euros could generate an opposite impact in the European stock market depending on the source of risk. The source of risk is not incorporated when performing a bivariate analysis, which suggests ambiguous estimates of the stock response. This framework can improve our understanding of how the exchange rate interacts in global markets. Also, it contributes to reduce the inaccuracy in the impact assessment of foreign shocks where the exchange rate plays a relevant role.

3.1 Introduction

Stress test analyses provide a deeper understanding of the interconnections across international markets in distress scenarios. The knowledge about the behaviour of financial variables in extreme scenarios is a fundamental cornerstone to build a resilient financial system and prevent contagion spillovers. The exchange rate acts as a primary channel through which international markets connect to domestic economies. Financial variables must be denominated in the same currency to perform a stress test analysis due to magnitude issues, reflecting the actual price paid by domestic producers and consumers. This transformation implies merging two different sources of risk which may have an opposite effects on the domestic economy, increasing model risk in the stress test design. Taking into account the interaction with the exchange rate could prevent from misleading conclusions about the response of the domestic economy while improving the design of tailor-made scenarios.[1]

In this paper, I estimate the conditional distribution of the European stock returns on a distress scenario for oil prices in euros. The conditioning scenario is the result of two dependent stochastic processes that could trigger the distress event but might condition the response of the stock market in a different way. The goal is to disentangle how the response of the European stock market to the same scenario for oil prices in euros could change depending on the degree of stress in the foreign exchange market. The response is evaluated looking at the 10th percentile of the conditioned stock returns distribution, i.e. the so-called Conditional Value-at-Risk ($CoVaR$). The focus on the tail of the distribution provides two main advantages compared to other statistical measures, e.g. conditioned mean response. First, it provides a more robust estimation to outliers than mean response results. Second, a focus on low percentiles is consistent with the assumption that economic agents are risk-averse, hence they are more interested in realising how adverse the behaviour of the portfolio could become than in knowing how its performance may be on average.

I use the copula vine approach to get the multivariate joint distribution between oil, the European stock market and the USDEUR exchange rate, while a Markov switching technique allows for structural changes in this relationship. The convolution concept allows us to consider alternative combinations of events for the USDEUR exchange rate and oil returns that lead to the same scenario in terms of oil returns in euros, evaluating the stock market implications of those alternative combinations. As we will see, the source of risk in the scenario for oil prices denominated in euros strongly conditions the response of the stock market.

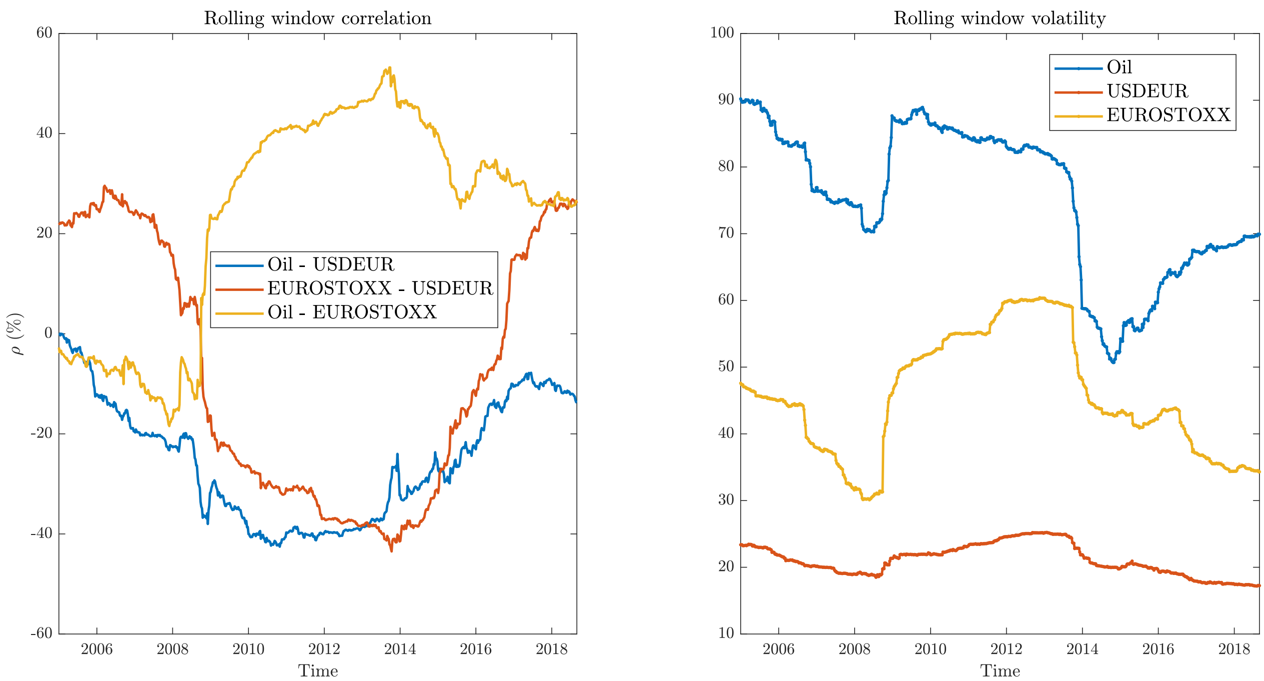
I propose using the convolution concept to incorporate the role of the exchange rate when estimating the response of the stock market to a distress scenario for oil markets denominated in domestic currency. I consider co-movements between oil and exchange rate returns when designing the stress test scenario by combining the convolution concept with the copula approach. The complex network of connections between oil, exchange rates and stock markets implies the need of considering

[1] For instance, BCBS (2013) recommends analysing the bank position on a currency-by-currency basis for stress test purposes.

the simultaneous dependence between them.[2] Overlooking one of these variables from the analysis could lead to misleading conclusions on the stock market exposure due to the fuzzy transmission channel. I use the copula vine approach to get the multivariate joint distribution between oil, the European stock market and the USDEUR exchange rate. To my knowledge, Aloui and Ben Aïssa (2016) is the only article that considers the multivariate relationship between stock market, oil and exchange rate simultaneously. They employ a vine copula approach to estimate the joint distribution between the US stock market, the US-trade weighted exchange rate and oil returns using daily data. Their results from the Bai and Perron (2003) test indicate the presence of a structural change during the 2008 financial crisis. Reboredo and Ugolini (2016) and Ojea Ferreiro (2019) also find a structural change in the bivariate relationship between the stock market and oil returns, using the Kolmogorov Smirnov test and Markov switching models. Wang et al. (2013) point to a structural break in the relationship between stock markets and exchange rates. Figure 3.1.1 provides two pieces of evidence about the existence of structural change in the data during the period 2000-2018. A rolling windows analysis using a five-year length window on the weekly returns of Brent oil, Eurostoxx and the USDEUR exchange rate depicts a general shift in correlation across the variables between the period $2009-2014$ that coincides with a general change in the volatility level of those markets. These pieces of evidence indicate that a Markov switching model, where variance and dependence move together across regimes, could explain the dynamic shown by the data. Also, a discrete switch in variance might explain the excess of kurtosis and the presence of left skewness shown by Figure 3.1.2.

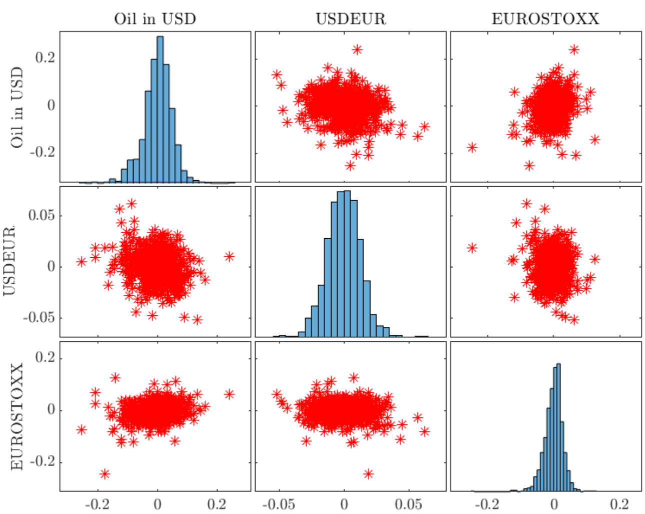
[2]Several studies state that oil price movements are partially due to the currency movements (Basher and Sadorsky 2006, Samii and Clemenz 1988, Zhang et al. 2008) and also that stock market swings may be caused by exchange rate movements (Dominguez and Tesar 2006, Francis et al. 2006, He and Ng 1998, Jorion 1990). Likewise, extreme movements in oil prices could trigger trade imbalances leading to adjustments in exchange rates (Golub 1983, Krugman 1983) while oil spillovers to stock markets may appear due to the change in production cost and indirect effects on inflation rates (Arouri et al. 2011, Lee et al. 2012, Ojea Ferreiro 2019).

FIGURE 3.1.1: Time-varying correlation and volatility.



These figures show the evolution of the correlation and volatility using a rolling window with a window length of five years, i.e. at each time t I assess the correlation and the volatility of the weekly returns between $t-260$ and t. The figures depict two set of evidence. First, there is a general shift in correlation across the variables between the period $2009-2014$. Second, this period coincides with a general change in the volatility level of those markets. This evidence indicates that a Markov switching model, where variance and dependence move together across regimes, might explain the dynamic shown by the data. Volatility value is obtained annualizing the standard deviation shown in percentage, i.e. standard deviation is multiplied by $\sqrt{52}100$.

FIGURE 3.1.2: Histogram and scatter plots for the bivariate relationships.



This figure shows the histogram for each variable and the scatter plot between each pair of variables. Concerning the histograms, they indicate an excess of kurtosis and the presence of left skewness which could be explained by a discrete switch in variance.

Results indicate that the composition of the scenario for oil prices in euros strongly conditions the response of the European stock market. On the one side, when a downward movement in oil prices materialises, highest 10% losses in the stock market could increase up to 20% if the oil market triggers the scenario compared to the scenario where the source of risk is unknown. On the other side, when an upward movement in oil prices materialises, losses in the European stock market could sharply increase up to 30% if the exchange rate triggers the scenario, compared to the same oil-related scenario where the triggering source is undefined. The findings indicate higher losses in the Value at Risk of the EUROSTOXX when a bearish oil-related scenario materialises compared to its unconditional Value at Risk. Nevertheless, the impact of a bullish oil-related scenario on the European stock market depends on the source of risk.

Empirical evidence shows an increase in the volatility of global markets jointly with a higher degree of co-movement and tail dependence across financial variables. The study identifies these periods: firstly, before 2003 at the same time of early 2000s recession; secondly, from 2008 to 2011, coinciding with the financial crisis and the beginning of European sovereign debt crisis; lastly, between 2014 to 2016, when 2010s oil glut occurs.

These findings have implications: firstly, for risk management, investors and traders, who are interested in portfolio strategies that reduce the exposure of their stock positions to commodity and exchange rate risk; secondly, for monetary and supervisory authorities, who need to build tailor-made stress test scenarios taking into account the role played by exchange rates; thirdly, for policy makers, who wish to understand the interactions between the main variables that drive the economy. Analysing the consequences of a distress scenario for international commodities in euros, rather than in US dollars, has also implications for the stability of prices for euro area producers and consumers.

The remainder of the article is laid out as follows: Section 3.2 presents three parts concerning the estimation. First Subsection 3.2.1 presents the copula concept and introduces the idea of convolution copula. Second, Subsection 3.2.2 refers to the modelling choice for marginal and joint distribution, paying special attention to the time-varying structure. Third, Subsection 3.2.3 focuses on the conditional quantile under a distress scenario, also known as Conditional Value at Risk ($CoVaR$). Section 3.3 presents the data employed for the empirical exercise in Section 3.4. Finally, Section 3.5 concludes.

3.2 Methodology

This section is divided in three parts. First, Subsection 3.2.1 presents a general and brief introduction of the copula and convolution concepts. The copula methodology is the backbone to model the joint dependence. This approach provides a great flexibility to model the joint distribution between oil, stock market and exchange rate, capturing tail behaviour and asymmetric dependence. Second, Subsection 3.2.2 studies the structure model that better fits the data. Recent literature points to a change in the dependence between these variables over time (Reboredo and Ugolini 2016, Ojea Ferreiro 2019, Reboredo 2012, Zhu et al. 2016, Aloui et al. 2013). A Markov switching approach helps us to identify potential structural changes in volatility and dependence.[3] A procedure similar to the one employed by Rodriguez (2007) and Hamilton and Susmel (1994) allows us to link the marginal behaviour for each variable to potential changes in the joint dependence in line with the evidence shown in Figure 3.1.1. Finally, Subsection 3.2.3 introduces how the conditional quantile under a distress scenario, i.e. the Conditional Value at Risk (*CoVaR*), is built. This risk measure indicates the quantile of the variable of interest in a stress test, where the triggering event is defined by a distress scenario for another variable. This assessment translates the complex linkages and connections between variables into potential losses.

3.2.1 Copula and convolution copula

The copula methodology allows for modelling marginal features and joint characteristics separately, which entails higher flexibility to gather complex patterns exhibited by financial data, like asymmetric relationship, joint tail dependence and nonlinearities.[4] The Sklar (1959)'s theorem states that the joint cumulative probability can be expressed as the combination of the marginal cumulative distribution function and the copula function, which gathers the dependence characteristics across variables, i.e.

$$F(x,y) = C\left(F_X(x), F_Y(y)\right), \tag{3.1}$$

where F_k is the marginal cumulative distribution function of variable $k = X, Y$ and $C(\dots)$ is the copula function.

The conditional copula $C_{y|x}\left(F_Y(y)|F_X(x) = x_k\right)$ expresses the conditional distribution function of a variable Y given a realization for variable X (Joe 1996). Conditional copulas are essential for the simulation process and for the construction of complex models, such as vine copulas. The conditional copula is the results of the partial derivative of the copula function with respect to one of its input factors, i.e.

$$\begin{aligned} F(y|X = x) &= C_{y|x}\left(u_y|u_x\right) \\ &= \frac{\partial C\left(u_x, u_y\right)}{\partial u_x}, \end{aligned} \tag{3.2}$$

where $u_x = F_X(x)$ and $u_y = F_Y(y)$.

The concept of copula convolution (C-convolution) appears when the interest of the analysis lies in the distribution of a variable $Z = X + Y$, where X and Y are not independent (Cherubini et al. 2004). The distribution of Z in terms of the joint distribution of X and Y is

$$F_Z(z) = \int_0^1 C_{y|x}\left(F_Y\left(z - F_X^{-1}(u)\right)|u\right) du, \tag{3.3}$$

where marginal characteristics and dependence features are combined to get the distribution of $Z = X + Y$. We can use $F_Y \overset{C}{*} F_X$ to express that the distribution of variable Z (F_Z) is the results of the convolution of the distributions of X (F_X) and Y

[3] A Markov switching approach employed to reflect time-varying dependence is robust to misspecification issues (Manner and Reznikova 2012) and provides higher flexibility than other dynamic models (Ojea Ferreiro 2019).

[4] See, for instance, Joe et al. (2010), Nikoloulopoulos et al. (2012), Kim et al. (2013).

(F_Y).

Cherubini et al. (2004) show that the C-convolution is closed with respect to mixtures of copula functions. If $C(u_x, u_y) = \pi A(u_x, u_y) + (1-\pi)B(u_x, u_y)$ where A, B are copula functions and $\pi \in [0, 1]$, then

$$\begin{aligned} F_Y \overset{C}{*} F_X &= F_Y \overset{\pi A+(1-\pi)B}{*} F_X \\ &= \pi F_Y \overset{A}{*} F_X + (1-\pi) F_Y \overset{B}{*} F_X. \end{aligned} \quad (3.4)$$

The implications for modelling the time-varying dependence given by a Markov switching process are direct. The copula and the marginal distributions functions in Equation (3.3) are assumed to be absolutely continuous, so the probability density function of variable $Z = X + Y$ is

$$f_Z(z) = \int_0^1 c_{X,Y}\left(u, F_Y\left(z - F_X^{-1}(u)\right) f_y\left(z - F_X^{-1}(u)\right)\right) du, \quad (3.5)$$

where f_y refers to the probability density function of variable Y and $c_{X,Y}(\dots)$ is the density copula between X and Y, i.e. the derivative of the copula function with respect to all its inputs.

The oil log-return denominated in euros is the sum of the logarithmic change of the oil denominated in US dollars and the logarithmic change in the exchange rate USDEUR[5]. Hence, the financial variable of oil denominated in euros is the result of the convolution of two dependent stochastic processes. The goal of this article is to assess how the conditional distribution of the European stock market returns could change when the same scenario for oil in euros materialises but the source of risk that leads the movement is different, i.e. commodity risk or exchange rate risk.

3.2.2 Model and estimation

This section is divided into two stages. First, I present the marginal model and the dependence structure across variables. Then, in a second stage, the focus is on the estimation process. The marginal model takes into account a possible switch in the market stability, using a $SWARCH$ model to gather potential structural breaks, i.e. the transition probability to move between a tranquil and a distress state is the same for all the assets but their parameters are not. This assumption is supported by the Figure3.1.1, where a simple rolling windows approach shows an increase in volatility between 2009 and 2014 for all the assets, while their correlation drastically changed. I impose a two-state model, which keeps the model tractable and makes easier the interpretation of the state. Changes in dependence across variables would happen together with volatility switches in the marginal models. The high-volatility state could be seen as an instability period for trade, which would lead to a change in the relationship between markets.[6] This way of linking the states between the

[5]The euro is the quote currency and the US dollar is the base currency.

[6]There is evidence in literature regarding the link between the change between low-volatile periods and high-volatile periods and the shift in dependence across assets. Edwards and Susmel (2001) find evidence of volatility co-movements across Latin American countries, Boyer et al. (2006) link high-volatility periods to an increase in co-movement across markets and Baele (2005) indicates a contagion effect between US market and European equity indices during high-volatility periods.

marginal distributions and the dependence structure allows us to reduce significantly the numbers of parameters providing a parsimonious model, making easier the estimation of a high-dimensional model.

Marginal model. The aim of this section is to select a parsimonious representation for the model, allowing for changes across possible regimes while keeping the model tractable. A specification in which all the parameters change with each regime would be numerically unwieldy and over-parametrized. I consider potential structural changes in key parameters for the marginal distribution, i.e. changes in variance, which would be related to changes in dependence.
I characterise the marginal densities of the stock (s), oil (o) and exchange rate (c) returns by an $ARMA(p,q)$ model, i.e.

$$r_{k,t} = \underbrace{\phi_{k,0} + \sum_{j=1}^{p} \phi_{k,j} r_{k,t-j} + \sum_{i=1}^{q} \psi_{k,i} \epsilon_{k,t-i}}_{\mu_{k,t}} + \epsilon_{k,t}, \quad k = s, o, c \tag{3.6}$$

where p and q are non-negative integers, $\phi_{k,j}$ and $\psi_{k,i}$ are respectively the autoregressive (AR) and the moving average (MA) parameters and $\epsilon_{k,t} = \sigma_{k,t} z_{k,t}$. $z_{k,t}$ is a Gaussian variable with zero mean and unit variance, i.e. the probability density function of $z_{k,t}$ is

$$f(z_{k,t}) = \frac{1}{\sqrt{2\pi}} \exp(-z_{k,t}^2/2). \tag{3.7}$$

The variance of $\epsilon_{k,t}$ has dynamics given by a Markov Switching Autoregressive Conditional Heteroskedasticity model ($SWARCH(K,Q)$)[7]. The presence of structural breaks in variance might explain the high persistence found in ARCH models (Lamoureux and Lastrapes 1990, Hwang and Valls Pereira 2008). The structural changes during the estimation period might explain also the kurtosis presented in the financial returns (Leon Li and Lin 2004). I employ the model specification by Hamilton and Susmel (1994) where the variance of $\epsilon_{k,t}$ can be divided into two components, i.e.

$$\sigma_{k,t}^2 = \kappa_{k,s_t} h_{k,t}, \tag{3.8}$$

where κ_{k,s_t} is a scale parameter of the variance depending on the state at time t. $s_t = l$ refers to the regime l at time t where $l = 1, \dots K$. The regimes are not directly observable but the probability of being on them can be implicitly estimated. The probability of switching across regimes evolves according to a first order Markov Chain of size K where K represents the number of states or regimes. κ_{k,s_t} is normalized at unity at state 1 ($s_t = 1$) while for the remainder states is higher than one. $h_{k,t}$ follows a ARCH(q) process, i.e.

$$h_{k,t} = \alpha_{k,0} + \sum_{q=1}^{Q} \alpha_{k,q} \left(\frac{\epsilon_{k,t-q}^2}{\kappa_{k,s_{t-q}}} \right) \tag{3.9}$$

where $\alpha_{k,0}$ and $\alpha_{k,q}$ are the ARCH parameters, which must be higher than zero. Note that when $s_t = 1$, $\kappa_{k,s_t} = 1 \quad \forall k$, i.e. the combination of a low-volatility regime in one market and a distress state in another market is not allowed.

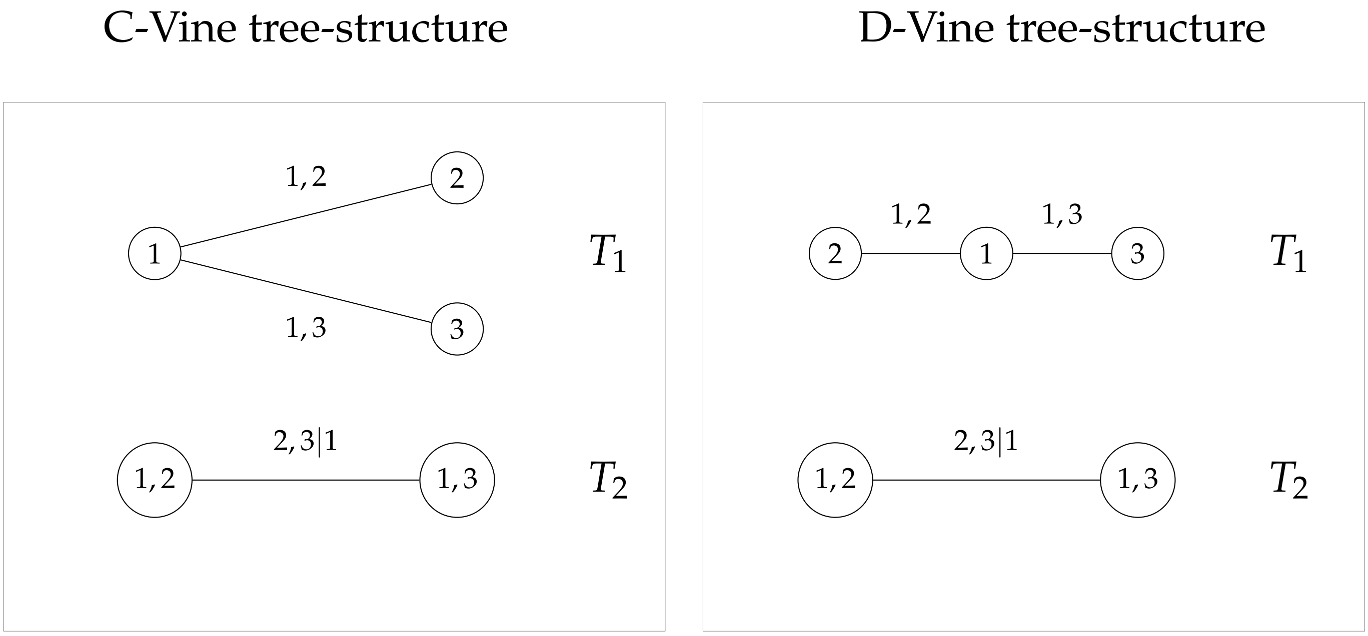
[7] K refers to the number of states and Q indicates the lags of the $ARCH(Q)$ model.

I assume two states to keep the model tractable, i.e. $K = 2$, while $Q = 1$ so a $SWARCH(2,1)$ is employed to model the variance of the financial returns. It is worth noting that there are $K(Q+1)$ potential realizations of the variance at time t, because Equation (3.9) depends on the Q most recent $\epsilon^2_{k,t-q}$ standardized by κ_{k,s_t-q} for $q = 1,\ldots,Q$. Each state of the Markov switching process has an economic interpretation. State 1 indicates a period of low volatility, which can be linked to tranquil periods. On the other side, State 2 presents a high-volatility period, where there is uncertainty about the future performance of assets. The uncertainty would lead to a change in the relationship between the variables, i.e. co-movement in distress periods would present stronger tail dependence due to contagion across assets, while in tranquil times the relationship might be diverse. Appendix D provides further information about the Markov switching specification that rules the shift in the variance of each variable and the joint dependence.

Dependence structure. Complex multivariate data can be modelled using bivariate copula in a hierarchical way like bricks of a more elaborate building. The graphical representation of these constructions are the vines. Depending on the pair-copula decomposition we could talk about Canonical vine copulas (C-Vine) or Drawable vine copulas (D-Vine) . C-Vine copulas have a star structure while D-Vine copulas have a path structure.

Figure 3.2.1 represents the graph-based tree structure of the copula decomposition of three assets (1, 2 and 3). The left figure shows the construction under a C-Vine copula while the right figure represents a D-Vine copula structure. As a matter of fact, in a three-dimensional case the copula decomposition is both a C-Vine and D-Vine. Note that the tree under the left copula structure is equivalent to the right panel in Figure 3.2.1.
I start modelling the joint dependence as a truncated vine, assuming that the joint dependence could be explained through a common exposure to the exchange rate. This structure for the vine copula is based on the key role that the exchange market plays between the stock market and the international commodity market. Indeed, an foreign exchange market is a *conditio sine qua non* for the stability in international trade markets and the economic growth in stock markets. Oil and stock returns are assumed conditionally independent once the dependence through the exchange rate is taken into account. Following Figure 3.2.1, this assumption implies that the link in T_2 step does not exist. In a second stage this assumption is relaxed, studying the complete vine structure as a natural extension of the truncated vine approach. This is the expected way to study the relationship because the structure chosen in the T_2 step depends on the structure in the T_1 step.

FIGURE 3.2.1: Example of a three-dimensional C- (left-top panel), D-vine (right-top panel) with edge indices.



Structure graphs gives the representation of the joint probability density function in the form of a nested set of trees (T_1, T_2). Each node corresponds to a density distribution, each edge corresponds to a pair-copula density and the edge label corresponds to the subscript of the pair-copula density. distribution. Note that C-Vine and D-Vine in this example show the same way of decomposing the density. Under the vine structure, variable 1 is connected to variable 2 and 3 in a first stage (T_1). Variable 2 and 3 are connected through the relationship that both have with variable 1 in T_1, and conditioned to the value of variable 1 they present an additional link between them in the second stage (T_2). Note that if the model is limited up to T_1, variable 2 and 3 would be unconditionally dependent through variable 1 but conditioned independent given a realization of variable 1.

Let us consider a three dimensions vector with joint distribution $F(x_1, x_2, x_3)$ to motivate how to model the multivariate structure. The Sklar (1959)'s theorem from Equation (3.1) can be rewritten in a three-dimension space as

$$F(x_1, x_2, x_3) = C\left(F(x_1), F(x_2), F(x_3)\right), \tag{3.10}$$

where subscripts of the cumulative distribution functions were omitted to avoid cumbersome notation. The joint density function expressed in terms of copulas and marginal densities is

$$f(x_1, x_2, x_3) = c\left(F(x_1), F(x_2), F(x_3)\right) f(x_1) f(x_2) f(x_3), \tag{3.11}$$

where factorizing recursively we obtain

$$f(x_1, x_2, x_3) = f(x_1) f(x_2|x_1) f(x_3|x_1, x_2), \tag{3.12}$$

where the subscripts of density functions were also omitted. Equation (3.12) can be rewritten using Bayes' theorem as

$$\begin{aligned} f(x_1,x_2,x_3) &= f(x_1)\frac{f(x_2,x_1)}{f(x_1)}\frac{f(x_3,x_2|x_1)}{f(x_2)} \\ &= f(x_1)c\left(F(x_2),F(x_1)\right)f(x_2)c\left(F(x_3|x_1),F(x_2|x_1)\right)f(x_3|x_1) \end{aligned} \quad (3.13)$$

where $f(x_3|x_1) = \frac{f(x_3,x_1)}{f(x_1)}$, which in terms of copulas is $f(x_3|x_1) = c\left(F(x_3),F(x_1)\right)f(x_3)$. Joe (1996) demonstrates that $F(x_j|x_k) = P(X_j < x_j|X_k = x_k) \quad \text{for} \quad j,k = 1,2,3 \quad j \neq k$ is expressed by the conditional copula, i.e. $C\left(F(x_j)|F(x_k)\right) = \frac{\partial C\left(F(x_j),F(x_k)\right)}{\partial F(x_k)}$.
To sum up, the joint density distribution under the vine approach can be expressed as

$$f(x_1,x_2,x_3) = \underbrace{c\left(F(x_2),F(x_1)\right)c\left(F(x_3),F(x_1)\right)c\left(F(x_3|x_1),F(x_2|x_1)\right)}_{c(F(x_1),F(x_2),F(x_3))}f(x_1)f(x_2)f(x_3) \quad (3.14)$$

In the current study, x_1 represents the returns of the exchange rate $USDEUR$ (r_c), while x_2 and x_3 represent oil and stock returns respectively (r_o, r_s). Observe that $c\left(F(x_3|x_1),F(x_2|x_1)\right) = 1$ in the case of a truncated vine approach. I choose between a set of copulas that present different features in terms of tail dependence, i.e. the probability of having very extreme realizations for one market given very extreme realizations for another market. Gaussian copula does not present tail dependence but it allows for positive and negative association, Student t copula also allows for positive and negative association but it presents symmetric tail dependence. Gumbel and Clayton copulas allow only for positive asymmetric association, while Clayton copula has lower tail dependence, Gumbel copula has upper tail dependence. The 90 degrees rotated version of Clayton and Gumbel allows for gathering negative association and asymmetric tail dependence. Further information about these copulas is provided in Appendix B.

TABLE 3.2.1: Main tail dependence features for each copula

Family	τ_L	τ_U
Gaussian	$-$ (if $\rho = 1$ then 1)	$-$ (if $\rho = 1$ then 1)
Student t	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\rho)}{1+\rho}}\right)$	$2t_{\eta+1}\left(-\sqrt{\frac{(\eta+1)(1-\rho)}{1+\rho}}\right)$
Clayton	$2^{-1/\theta}$	$-$
Gumbel	$-$	$2-2^{1/\theta}$

Note:
$-$ represents no tail dependence.
Source: (Ao et al., 2017, p. 22), Jiang (2012), Joe and Hu (1996), Fischer (2003) and (Joe, 1997, p. 193–204).
Let u_1 and u_2 denote two uniform-distributed variables across (0,1).
The lower tail dependence, τ_L, is defined as $\tau_L = lim_{q\to 0}P(u_2 < q|u_1 < q)$.
The upper tail dependence, τ_U is defined as $\tau_U = lim_{q\to 1}P(u_2 > q|u_1 > q)$.

I use graphical tools as bivariate histograms and analytical tools as the Akaike

Information Criterion Corrected for small-sample bias ($AICC$) to choose a suitable copula structure that fits the true data dependence. $AICC$ is chosen because of being the principal indicator for selection copulas in the conditional risk measure literature[8], i.e

$$AICC = 2k\frac{T}{T-k-1} - 2\log(\hat{L}),$$

where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum $AICC$ value indicates the best copula fit. Appendix E presents some robustness check concerning the model selection.

I propose to use a EM algorithm (Hamilton 1990) for the estimation process, which allows for decompositing the optimization problem in a set of simpler problems where the transition probability of the Markov Chain and the parameters within each regime are not estimated at the same time. The EM algorithm simplifies the computational challenge of maximizing numerically an likelihood surface plagued with multiple local optimum as happens in switching models.

Estimation procedure. I employ the EM algorithm, proposed by Hamilton (1990), to obtain the maximum likelihood estimates for our model, which are subject to a discrete shift. There are several reasons that motivate the use of EM algorithm instead of using the full maximum likelihood estimation. First, the maximization of a likelihood function with respect to a great number of unknown parameters implies a computational challenge due to the possible existence of multiple local optimum, specially in switching models. Second, It provides numerical robustness over other methods of optimization like Newton-Raphson where, if the likelihood surface is not concave, might arrive to a local maxima/minima (Dempster et al. 1977). The EM algorithm is numerically stable as the result of dividing the optimization problem into a sequence of simpler optimization problems where the probabilities of switching between regimes and the estimates within each regime are not jointly estimated. I use a large number of starting values for the EM algorithm to ensure an improvement in efficiency. The EM algorithm has been employed already in copula-based models with Markov switching dynamics by Stöber and Czado (2014) and Chollete et al. (2009).

To implement the EM algorithm, first compute the smoothed probabilities (Expectation step or $E - step$) as shown by Kim (1994)'s algorithm. Then, employ these probabilities to reweigh the observed data and maximize the reweighed log-likelihood to generate new estimates (Maximization step or $M - step$). Employ the new estimates to reassess the smoothed probabilities in an iterative process. The EM algorithm is an analytic solution to a sequence of optimization problems, where the solution in the $n + 1$ iteration increases the value of the log-likelihood function in relation to the estimates in the n iteration, achieving in the limit a optimum of the log-likelihood function.[9]

[8] Among others Brechmann and Schepsmeier (2013), Reboredo and Ugolini (2015a), Reboredo and Ugolini (2015b), Reboredo and Ugolini (2016), Rodriguez (2007), Reboredo (2011) and Ojea Ferreiro (2018)

[9] Alternatively, we can see the new estimates in the following iteration of the EM algorithm as the results of the sum of the weighted conditions over all possible states. In other words, the EM algorithm "replaces" the unobserved scores by their expectation given the estimated parameter vector in the previous iteration.

Steps to perform the EM algorithm

- $E - Step$: Inference the expected values of the state process given the observation vector, i.e. assess the conditional probabilities for the process being in a certain regimen at time t and $t-1$ given the full sample. Equation (3.28) provides
$$P(s_t = j, s_{t-1} = i|I_T) \quad \text{for } i, j = 1, 2$$
- $M - step$: Maximize the expected log-likelihood function using the smoothed probabilities to obtain new and more exact ML estimates, i.e. instead of maximize $\sum_{t=1}^{T} log\left(L_t(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, \Theta)\right)$ where $L_t(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, \Theta)$ is given by equation (3.27) , we maximize
$$\sum_{t=1}^{T}\sum_{j=1}^{2}\sum_{i=1}^{2} log\left(f(r_{o,t}, r_{s,t}, r_{c,t}|\Theta_{s_t=j, s_{t-1}=i})\right) P(s_t = j, s_{t-1} = i|I_T),$$
where $P(s_t = j, s_{t-1} = i|I_T)$ was obtained in the previous step. Notice that we are maximizing the expected conditional log-likelihood, but not the log-likelihood. We use the new estimates to update the smooth probabilities and the expected conditional log-likelihood to be maximized, we repeat the iterative algorithm until some convergence criteria are meet, e.g. in terms of the new estimates
$$|\Theta^{n+1} - \Theta^{n}| < \varepsilon,$$
where ε has a small value, e.g. $\varepsilon = 10^{-4}$.

The EM algorithm prevents from estimating at the same time the parameters within each state and the transition matrix between states, which simplifies the maximization problem. Reparametrizations are used to guarantee that all iterates are in the parameter space. For instance instead of looking for values of $\kappa_{k,s_t=2}$, I obtain the optimal estimate for a parameter x such that $exp(x) + 1 = \kappa_{k,s_t=2}$. Hamilton and Susmel (1994) also employ this kind of transformations to estimate the parameters of its $SWARCH$ model. The transition probabilities between states for iteration n are obtained from

$$p_{ij}^{n} = \frac{\sum_{t=2}^{T} P(s_t = j, s_{t-1} = i|I_T; \Theta^{n-1})}{\sum_{t=2}^{T} P(s_{t-1}|I_T; \Theta^{n-1})}, \tag{3.15}$$

Further information regarding the EM algorithm for Markov switching models can be found in Hamilton (1990) and Janczura and Weron (2012) among others.

3.2.3 Untangling the oil shock to the European stock market into commodity and exchange rate risk

The actual oil price that European firms have to cope with is the product of the oil price in USD by the exchange rate $USDEUR$[10]. The actual exposure to swings in oil prices is the sum of the logarithmic changes in oil and in the exchange rate. The convolution of the distribution of oil and exchange rate log-returns is the distribution of the oil log-returns denominated in euros.

[10] Note that $USDEUR$ indicates how many euros are exchanged by one US dollar.

Ojea Ferreiro (2019) analyses the impact of a oil shock denominated in euros into an extreme quantile of the European stock market using the Conditional Value-at-Risk (*CoVaR*) (Adrian and Brunnermeier (2016), Girardi and Ergün (2013)). The *CoVaR* measure indicates a percentile of the distribution of the European stock market returns given a sharp change in oil prices. The change in oil prices denominated in euros (r_{oe}) may come from different sources, i.e. commodity risk, exchange rate risk or a combination of both. For instance, an increase in oil price denominated in euros might be due to the depreciation of Euro or due to market-related reasons. In the first case, not only oil but every single imported product would be more expensive while exports become more competitive. The second case would be related to demand and supply reasons in the commodity. Depending on the variable that triggers the change in oil prices in euros, we could expect a different conditional distribution for the stock market returns. The existence of two underlying stochastic processes in the scenario design for oil prices in euros has been overlooked by the literature, which might condition the response of the stock market.

Following Ojea Ferreiro (2019), the bearish $CoVaR_{s|oe}(\alpha, \beta)$ of the stock returns would be obtained implicitly from

$$\begin{aligned} P\left(r_s < CoVaR_{s|oe}|r_{oe} < VaR_{oe}(\alpha)\right) &= \frac{P\left(r_s < CoVaR_{s|oe}, r_{oe} < VaR_{oe}(\alpha)\right)}{P(r_{oe} < VaR_{oe}(\alpha))} \\ &= \beta, \end{aligned} \tag{3.16}$$

where $P(r_{oe} < VaR_{oe}(\alpha)) = \alpha$.
Following Equation (3.3), $r^*_{oe,t} = VaR_{oe,t}(\alpha)$ is obtained from

$$\begin{aligned} F_{oe,t}(r^*_{oe,t}) &= \int_0^1 C_{o|c,t}\left(F_{o,t}\left(r^*_{oe,t} - F_c^{-1}(u)\right)|u\right) du \\ &= \alpha. \end{aligned} \tag{3.17}$$

We have infinitive combinations of exchange rate returns and oil returns denominated in US dollars such that

$$r^*_{oe} = r_c + r_o,$$

but notice that not all the combinations are equally probable[11] nor their implications for the conditional distribution of stock returns would be the same. Given a quantile q_c of the distribution of the exchange rate returns, there is a unique quantile q_o of the oil returns in US dollars such that $VaR_{oe}(\alpha) = F_c^{-1}(q_c) + F_o^{-1}(q_o)$. Actually, conditioning to the oil returns in euros being below a quantile α and the exchange rate *USDEUR* being below a percentile q_c is the same than conditioning to the exchange rate returns being below a quantile q_c and to the oil denominated in US dollars such that its convolution would be below the quantile α, i.e.

$$r^*_{oe} \geq F_c^{-1}(q_c) + r_o,$$

hence oil returns denominated in dollars should be below

$$r_o \leq r^*_{oe} - F_c^{-1}(q_c)$$

[11]This would be only in the case of independent variables.

which in terms of quantiles would be

$$P\left(r_o \leq r^*_{oe} - F_c^{-1}(q_c)\right) = F_o(r^*_{oe} - F_c^{-1}(q_c)) = q_o. \quad (3.18)$$

Consequently, a different response of the stock market returns might occur given the same scenario for oil returns in euros but different distress in the exchange rate returns. Incorporating the role of the exchange rate in the oil-related scenario helps us to generate tailor-made stress test where the distress in global market is tangled with the evolution of exchange markets.
$CoVaR_{s|oe}(\alpha, \beta)$ in Equation (3.16) transforms into $CoVaR_{s|oe,c}(\alpha, q_c, \beta)$ when the exchange rate is also considered in the scenario, getting

$$P\left(r_s < CoVaR_{s|oe}|r_{oe} < VaR_{oe}(\alpha), r_c < VaR_c(q_c)\right) = \frac{P\left(r_s < CoVaR_{s|oe,c}, r_o < VaR_{oe}(\alpha), r_c < VaR_c(q)\right)}{P(r_{oe} < VaR_{oe}(\alpha), r_c < VaR_c(q_c))} = \beta,$$

Equation (3.18) implies an equivalence between $CoVaR_{s|oe,c}(\alpha, q_c, \beta)$ and $CoVaR_{s|o,c}(q_o, q_c, \beta)$. Using this equivalence we can obtain at each time t the upper threshold of the quantile of the oil returns in US dollars such that for a certain upper threshold of the quantile of the exchange rate, the sum of returns is at or below the quantile α of the oil denominated in euros. Setting a scenario for the exchange rate to compute *CoVaR* provides additional information that can conditions significantly the response of the stock market.

Vine structure We could express the $CoVaR_{s|oe,c}(\alpha, q_c, \beta)$ given the chosen vine structure as

$$\frac{\int_0^{q_c} C_{s,o|c}\left(C_{s|c}(F_s(CoVaR_{s|oe,c})|u), C_{o|c}(q_o|u)\right) du}{C_{o,c}(q_o, q_c)} = \beta. \quad (3.19)$$

where q_o is given by Equation (3.18). To compared these results with the one obtained without any information about the foreign exchange market, we combine Equation (3.3) and Equation (3.19) to get

$$CoVaR_{s|oe}(\alpha, \beta) = \frac{\int_0^1 C_{s,o|c}\left(C_{s|c}(F_s(CoVaR_{s|oe})|u), C_{o|c}(F_o\left(VaR_{oe}(\alpha) - F_c^{-1}(u)\right)|u)\right) du}{\alpha} = \beta, \quad (3.20)$$

where $VaR_{oe}(\alpha)$ is obtained from the convolution of the exchange rate and the oil in USD following Equation (3.17). Appendix C provides information about how to build the *CoVaR* measure using copulas conditioned to a bullish oil-related scenario.

3.3 Data

I employ weekly data of the European stock market, the *USDEUR* exchange rate and oil prices from 07 January 2000 to 07 September 2018. I obtain weekly returns from the log difference between two consecutive Fridays. The time series includes several crises during this period, e.g. the dot-com crisis, the 2008 financial crisis and the European debt crisis, where both oil prices and exchange rates experienced great oscillations.

Concerning commodity prices, I use the Europe Brent crude oil spot price sourced from the US Energy Information Agency (http://www.eia.doe.gov), which is the main benchmark to settle the price of light crudes. Brent crude oil is denominated in US dollars per barrel. The $USDEUR$ exchange rate is obtained from the European Central Bank Statistical Data Warehouse (https://sdw.ecb.europa.eu). Regarding the European stock market, I employ the EUROSTOXX index from Datastream.

Table 3.3.1 shows some descriptive statistics for the full sample and two subsample that correspond to the pre-crisis and the post-crisis periods. It indicates a clear change in higher moments, i.e. skewness and kurtosis, and in the relationship between variables.

TABLE 3.3.1: Descriptive statistic for the variables

	Full sample			Pre-crisis period			Post-crisis period		
	A	B	C	A	B	C	A	B	C
μ	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
σ	0,05	0,01	0,03	0,05	0,01	0,03	0,05	0,01	0,03
skewness	-0,50	0,08	-0,96	-0,60	-0,01	-0,23	-0,41	0,17	-1,38
kurtosis	5,19	4,10	9,94	3,95	3,03	4,81	6,77	5,04	12,30
q=95%	0,07	0,02	0,04	0,08	0,02	0,04	0,07	0,02	0,04
q=5%	-0,09	-0,02	-0,05	-0,09	-0,02	-0,05	-0,09	-0,02	-0,05
ρ_{USDEUR}	-0.1933	-	-0.0513	-0.1271	-	0.1649	-0.2554	-	-0.2188
ρ_o	-	-0.1933	0.2153	-	-0.1271	-0.0379	-	-0.2554	0.4326
ARCH test	0,0000	0,0007	0,0000	0,0000	0,6604	0,0000	0,0000	0,0031	0,0251
LBQ test	0,4992	0,7454	0,0090	0,4223	0,5941	0,8540	0,1537	0,2942	0,0125

A: Oil in USD, B: USDEUR exchange rate, C: EUROSTOXX. All the series are shown in returns.
LBQ test refers to the p-value of the Ljung-Box Q-test for autocorrelation performed with 20 lags.
ARCH test refers to the p-value of the Engle's ARCH Test for heteroscedasticity performed with 1 lag.
The 15 September 2008 is chosen as breakpoint to define a crisis date.
ρ_{USDEUR} and ρ_o shows the Pearson's linear correlation coefficient of the variables against the $USDEUR$ and the Oil in USD respectively.

3.4 Results

This section presents the results from the estimation of the final model in a first stage. The model selection process and some robustness checks are shown in Appendix E. The backtesting tests provide the model fit for different quantiles of the returns distribution, while the Akaike Information Criterion and the bivariate histogram help us in the model selection process. The implications of the scenario design for the conditional percentile of the Eurostoxx are analysed in a second stage.

3.4.1 Model diagnosis

The estimates of the model and their standard deviations are shown in Table 3.4.1. Oil in US dollars and $USDEUR$ exchange rate returns double the level of variance when we change from state 1 to state 2,as seen in the estimate for the parameter $\kappa_{s_t=2}$. The variance of EUROSTOXX returns under the high-variance regime triples the variance under the calm state. The student copula provides the best fit for the dependence between oil and exchange rate under state 1. Within the high-variance regime, i.e. state 2, the Gaussian copula suit better the co-movement between oil in

USD dollars and the USDEUR exchange rate. The dependence between these two assets is negative with a correlation around −20% regardless the current state. There is no link between the exchange rate and the European stock market under the calm regime, but under the high-variance regime there is a negative association with tail dependence when the euro appreciates against the dollar and European stock market is in the upper tail of its returns distribution. The relationship between oil in USD and the European stock market is positive within both states. Nevertheless, the dependence is weak and without existence of tail dependence under the state 1. The Gaussian copula with a correlation parameter with $\rho < 0.1$ between the oil in USD dollars and the EUROSTOXX and the independence between the foreign exchange rate and the EUROSTOXX indicate the low dependence of the European stock market to movements in the FX and oil markets under the calm regime. However, the dependence between oil in USD dollars and the EUROSTOXX is positive and presents a lower tail dependence under the regime 2. The presence of tail dependence under the high-variance regime could be explained by the investors' herd behaviour (Aloui et al. (2013)). The probability of remaining in the same state for the next week is higher than 98%, indicating a high persistence in both regimes.

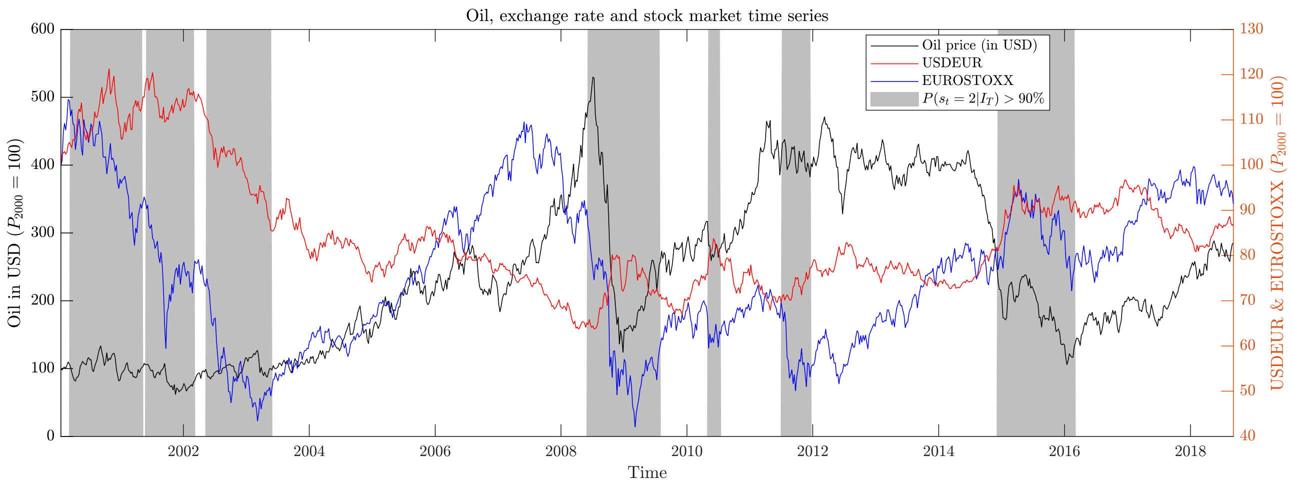
The Figure 3.4.1 shows the time series of oil price in US dollars (left axis) and the USDEUR exchange rate (right axis) jointly with those periods where the probability of being in a high-variance regime where higher than 90% (grey area). It provides evidence about the role of the exchange rate in these periods. A threshold model depending on the level of oil price could not explain the high probability of being in state 2 before 2003, when oil prices were stable but the foreign exchange rate and the European stock market experience great oscillations. The periods after 2008 where the probability of being in the regime 2 is high coincide with turbulences in all the markets.

TABLE 3.4.1: Model with a complete vine structure

	A	B	C
ϕ_0	0.00 **	-0.00	0.00 ***
	(0.00)	(0.00)	(0.00)
ϕ_1	0.06 *	0.04	-0.05 *
	(0.04)	(0.03)	(0.04)
$\kappa_{s_t=2}$	2.16 ***	2.17 ***	3.77 ***
	(0.24)	(0.23)	(0.43)
α_0	0.00 ***	0.00 ***	0.00 ***
	(0.00)	(0.00)	(0.00)
α_1	0.12 ***	0.09 **	0.15 ***
	(0.04)	(0.04)	(0.06)
State 1		State 2	
T_1 (A,B)			
$\rho_{A,B}$	-0.20 ***	$\rho_{A,B}$	-0.18 ***
	(0.05)		(0.05)
$\nu_{A,B}$	12.54 ***		
	(0.41)		
T_1 (B,C)			
		$\theta_{B,C}$	0.06 **
			(0.03)
T_2 (A,C | B)			
$\rho_{A,C}$	0.08 *	$\theta_{A,C}$	0.14 ***
	(0.05)		(0.05)
p_{11}	0.99 ***	p_{22}	0.98 ***
	(0.01)		(0.01)
		LL	-6671.80

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (3.6) and (3.9) and for the parameters of the best copula choice according to the $AICC$ value reported by Table 3..6.
LL is the log-Likelihood value.
A: Oil in USD, B: USDEUR exchange rate, C: EUROSTOXX. $\rho_{A,B}$ and $\eta_{A,B}$ is the correlation and number of degrees of freedom between oil in USD and USDEUR returns. $\theta_{B,C}$ is the estimate of the 90° Rotated Clayton under state 2 between USDEUR and EUROSTOXX. $\rho_{A,C}$ is the correlation between oil in USD and EUROSTOXX under state 1 once the dependence between those variables and USDEUR has been considered. $\rho_{A,C}$ is the estimate of the Clayton copula between oil in USD and EUROSTOXX under state 2 once the dependence between those variables and USDEUR has been considered.
Vine structure: *Oil-USDEUR*- State 1: Student, State 2: Gaussian. *USDEUR-EUROSTOXX*-State 1: Independence, State 2: 90° Clayton. *Oil-EUROSTOXX*$|$*USDEUR*-State 1: Gaussian, State 2: Clayton.

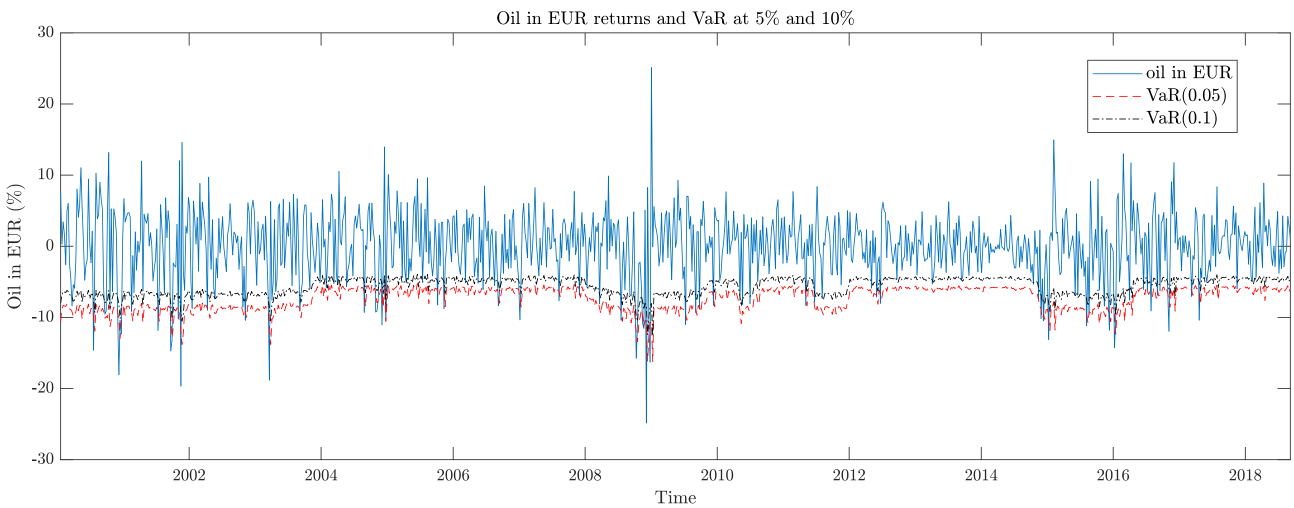
FIGURE 3.4.1: Time series of assets prices and high-volatility periods.



This figure shows the time series of the price of oil in USD dollars (left axis) and the USDEUR exchange rate and EUROSTOXX (right axis), while the grey area indicates those periods where the smoothed probability of being in the high-variance regime is higher than 90%. The price at the beginning of the sample is 100 for the three assets.

To observe how well the distribution of oil returns in euros is fitted by the convolution function in Equation (3.17), Figure 3.4.2 plots the oil returns in euros together with its $5-th$ and $10-th$ percentiles obtained from the convolution. The VaR adapts to the changes in volatility indicating an adequate fit for the empirical data.

FIGURE 3.4.2: Oil returns denominated in euros and its 5-th and 10-th percentiles

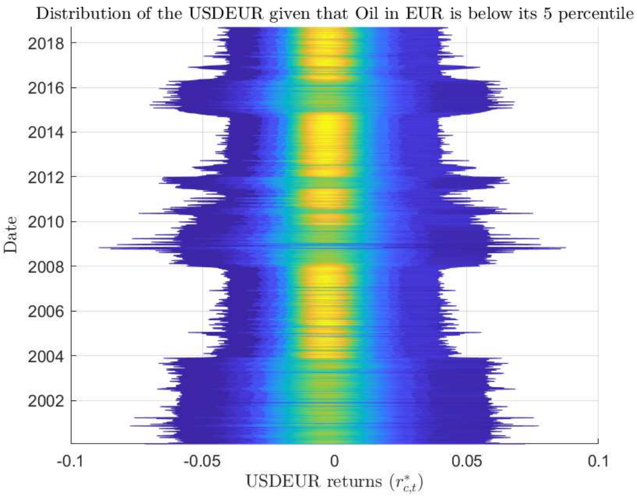


Historical time series of the oil returns denominated in euros and its 5-th and 10-th percentile obtained from the convolution function from Equation (3.5) and the model from Figure 3.4.1.

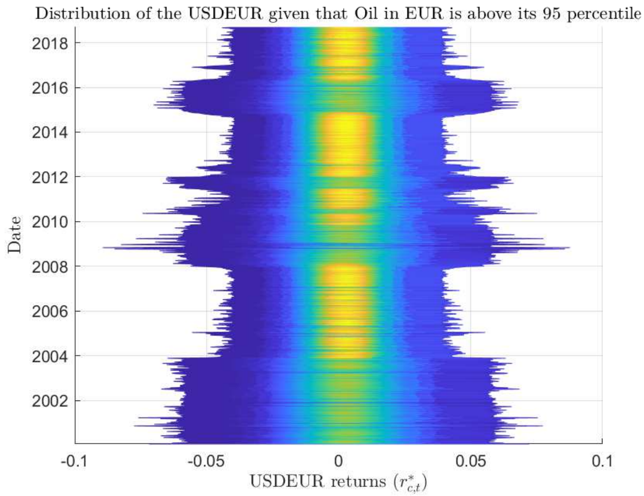
3.4.2 Stress test for the Eurostoxx given a distress scenario for oil returns in euros and the role of the exchange rate.

This subsection starts looking at the conditional distribution of the exchange rate returns under different scenarios for oil in euros. Lighter colours in Figure 3.4.3 indicates a higher probability for those values of the exchange rate returns. If the exchange rate was independent from the scenario for oil prices in euros, the conditional distribution would be identical no matter which oil-related scenario conditions foreign exchange rate. However, the FX distribution exhibits skewness features depending on the scenario, meaning that the literature has been implicitly assuming an expected response of the exchange rate when defining an oil-related scenario in the bivariate analysis.

FIGURE 3.4.3: Distribution of USDEUR returns under different scenarios for Oil in EUR.



(a) Oil in EUR below 5-th percentile

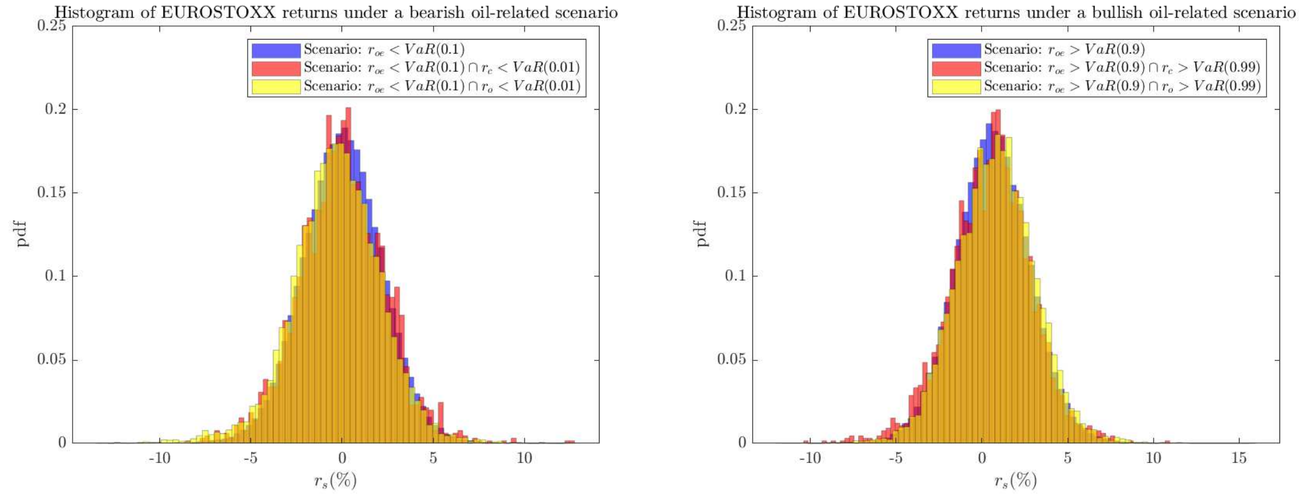


(b) Oil in EUR above 95-th percentile

This figure shows the distribution of the exchange rate returns under different scenarios for oil in euros. The distribution of exchange rate returns exhibits skewness features depending on the scenario of oil in Euros. The lighter colour indicates a higher probability for those values. The conditional distribution of the exchange rate is obtained as $f(r_c|r_{oe} < VaR_{oe}(\alpha)) = C_{o|c}(F_o(VaR_{oe} - r_c)|F_c(r_c))f(r_c)\frac{1}{\alpha}$ where the subscript t is ignored for notational convenience.

An overview of the response in the returns distribution of the European stock market can be obtained by simulation. Following Algorithm A we can generate realizations from the joint distribution to get the properties when a certain event occurs.First, I generate 1000000 simulations from the joint distribution of oil, foreign exchange rate and the European stock market. Then, I choose those observations that meets some criteria, e.g. stock realizations on those simulations where the oil returns in euros is below its 10-th percentile. Figure 3.4.4 shows in the left (right) chart the histogram of the European stock returns when a downward (upward) movement in oil prices denominated in euros occurs. The blue bars presents the conditional distribution of the EUROSTOXX when the oil-related scenario materialises. The red (yellow) bars indicate the conditional distribution of the European stock market on an oil-related scenario triggered mainly by the FX (oil) market. Two main findings should be highlighted looking at Figure 3.4.4. On the one hand, the returns distribution of the European stock market when a bearish oil-related scenario materialises presents higher losses if the exchange risk triggers the scenario. On the other hand, the EUROSTOXX distribution under a bullish oil-related scenario presents higher losses if the oil market triggers the conditioning event. The source of risk that triggers the scenario conditions strongly the conditional distribution of the stock market returns.

FIGURE 3.4.4: Conditional distribution of the EUROSTOXX on the scenario for oil price in euros and the FX.



(a) Bearish oil-related scenario (b) Bullish oil-related scenario

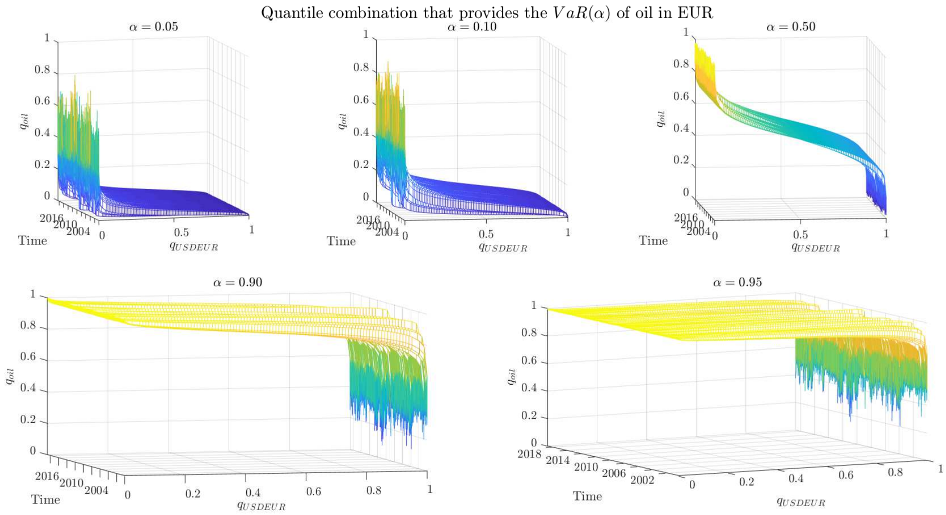
These figures show the histogram of the EUROSTOXX returns when a bearish (a) or bullish (b) scenario for oil prices in euros materialises. The returns are obtained following the simulation process shown in Appendix A. I simulate 1000000 realizations taking the values of the parameters at the end of the sample period. The blue histogram represents the conditional returns distribution for the European stock market when the source of risk that triggers the event scenario is unknown. The red histogram shows the conditional distribution of the stock market when the exchange rate triggers the scenario for oil prices in euros, while the scenario is triggered by the oil market in the yellow histogram. Looking to the left tail of the distribution we can observe that the variable that triggers the conditional scenario could be as important as the scenario for oil in euros. Taking into account the source of risk could enhance the precision in the response of the European stock market to the materialisation of the scenario.

Figure 3.4.5 shows the combination of quantiles (top) / returns (bottom) of oil returns in US dollars and $USDEUR$ that provides the $VaR(\alpha)$ of the oil returns in euros. Note that bottom chart is a straight line, because the oil return in US dollar is a linear function given a $VaR(\alpha)$ of the oil returns in euros and a value for the exchange rate returns (see Equation (3.18)). The changes over time are due to variations in the $VaR(\alpha)$ of the oil in EUR. Note that the relationship is not linear when we are dealing with quantiles (top chart).

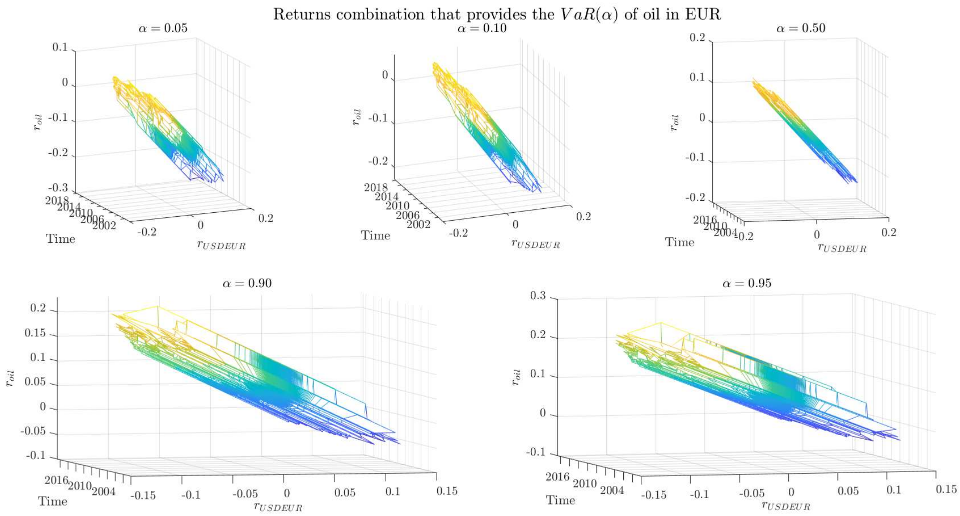
Figure 3.4.6 shows the distribution of the conditional 10-th percentile of the Eurostoxx returns over the sample 2000-2018. Left chart presents a scenario where the oil in euros is below its $10-th$ percentile ($\alpha = 0.1$) while the right chart shows a scenario where oil in euros is above its $90-th$ percentile ($\alpha = 0.9$). X-axis compares the same scenario depending on the upper (left chart) or lower (right chart) threshold for the quantile of the exchange rate (q_c). On the one side, left figure shows a scenario where oil prices denominated in euros experience a downward movement and the $USDEUR$ returns are below its q_c100-th percentile. On the other side, right graph presents a scenario where oil prices denominated in euros experience a upward movement and the $USDEUR$ returns are above its q_c100-th percentile. Label C in the x-axis refers to the convolution of oil and the exchange rate, i.e. without any assumption about the source of risk that triggers the conditioning event following Equation (3.20). The bearish $CoVaR$ of the EUROSTOXX returns presents higher dispersion over time than the bullish $CoVaR$. For both scenarios $CoVaR$ increases for higher quantiles of the exchange rate returns. This implies than bearish $CoVaR$

where the main source of risk is the movement in oil prices denominated in US dollar and bullish *CoVaR* where the source of risk comes from the exchange rate are the most harmful scenarios for the European stock market.

FIGURE 3.4.5: Combination of oil in US dollars and USDEUR such that the sum is the $VaR(\alpha)$ of the oil denominated in euros



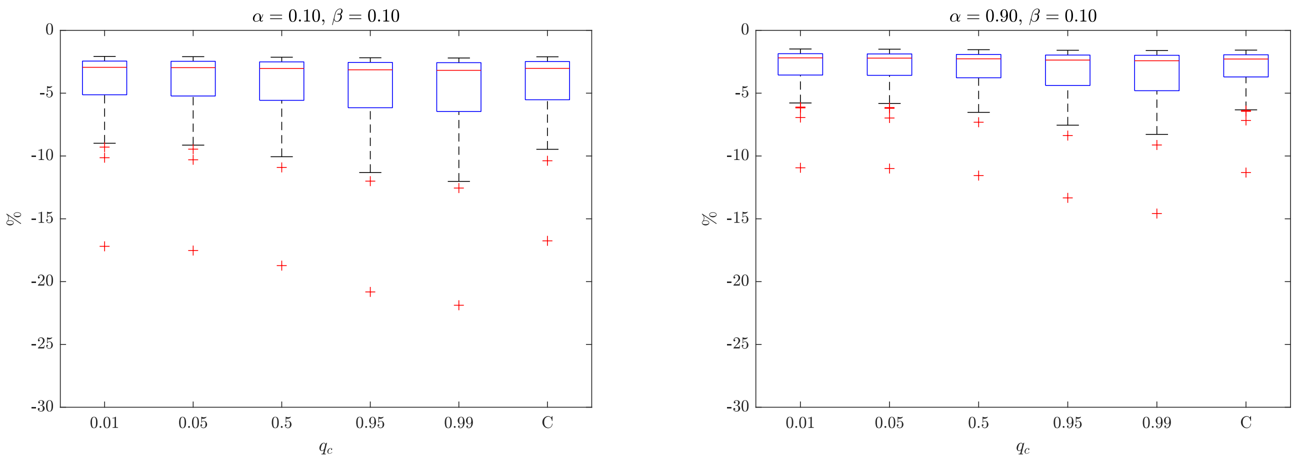
(a) Quantile combination to get $VaR(\alpha)$ of oil returns in euros



(b) Returns combination to get $VaR(\alpha)$ of oil returns in euros

This figure shows the combination of quantiles (top) / returns (bottom) of oil in US dollars and USDEUR that provides the $VaR(\alpha)$ of the oil returns in euros. Note that the bottom figure is a straight line, because oil return in US dollar is a linear function of the $VaR(\alpha)$ of the oil returns in euros and the exchange rate return. The changes over time are due to the changes in the $VaR(\alpha)$ of the oil in euros. Note that when we are dealing with quantiles (top chart) the relationship is not linear.

FIGURE 3.4.6: Boxplot of the CoVaR distribution of the EUROSTOXX over the full sample

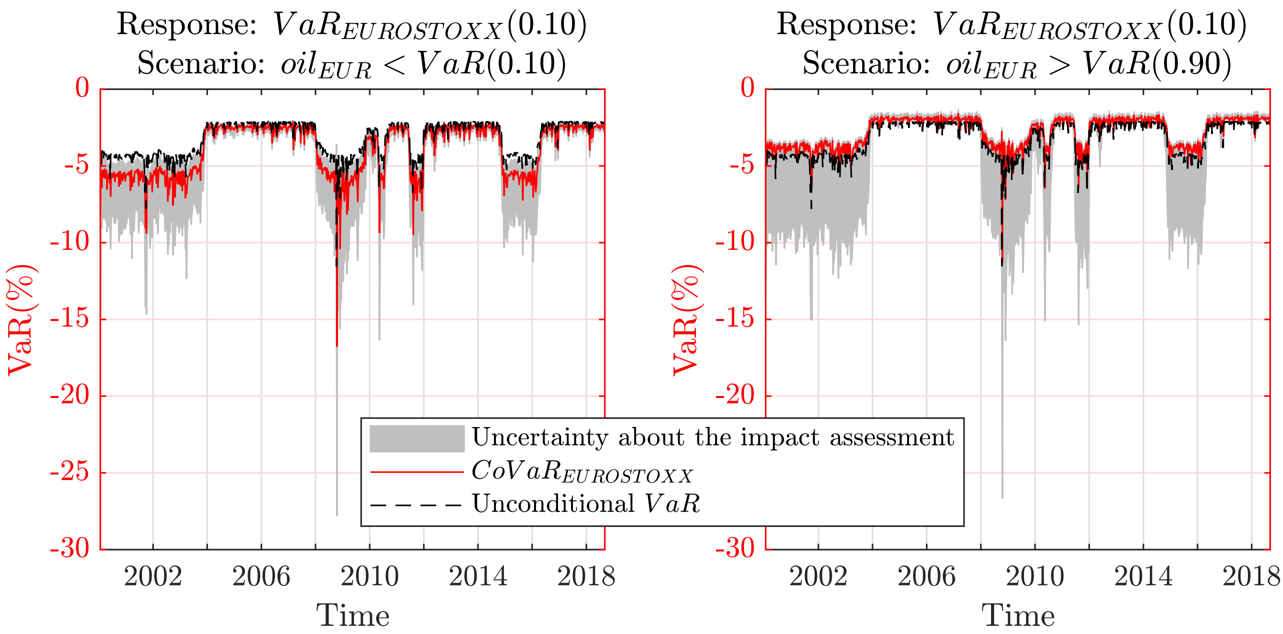


This figure shows the distribution of the CoVaR over the sample 2000-2018. Left chart presents a scenario where the oil returns in euros is below its $10 - th$ percentile ($\alpha = 0.1$) while right chart shows a scenario where oil returns in euros is above its $90 - th$ percentile ($\alpha = 0.9$). X-axis compares the same scenario depending on the quantile of the exchange rate (q_c). Left figure shows a bearish scenario for oil returns in euros and USDEUR is below its q_c100-th percentile, while right graph presents a bullish scenario for oil returns in euros where the USDEUR is above its its q_c100-th percentile. Label C in the x-axis refers to the convolution of oil returns and the exchange rate, i.e. without doing any assumption regarding the stress in the exchange rate.

Figure 3.4.7 presents the Value at Risk of EUROSTOXX (black dashed line), the Value at Risk of the EUROSTOXX under a distress scenario for the oil price in euros (red solid line) and its range of potential responses depending on the source of risk that triggers the distress scenario (grey area). Left figure shows a bearish scenario for oil denominated in euros, i.e. below its 10-th percentile, while the right chart indicates a bullish scenario, i.e. oil returns in euros above its 90-th percentile. The response of the EUROSTOXX VaR might be different depending on the source of risk that triggers the event, i.e. the foreign exchange market or the oil market. The grey area indicates how the source of risk could change the response of the European stock market when a oil-related scenario materialises. This are provides a magnitude regarding the uncertainty of the conditional behaviour of the stock market due to the trigger of the conditioning event.[12] On the one side, the bearish $CoVaR$ is lower than the VaR of EUROSTOXX returns no matter which is the source of risk, although the magnitude of the difference between $CoVaR$ and VaR might vary. On the other side, bullish $CoVaR$ returns are higher than the VaR returns of EUROSTOXX, but this could change depending on the source of risk that triggers the scenario.

[12]To build the range of *uncertainty* I choose a set of quantiles of the exchange rate returns (q_c) from 10^{-8} to $1 - 10^{-8}$.

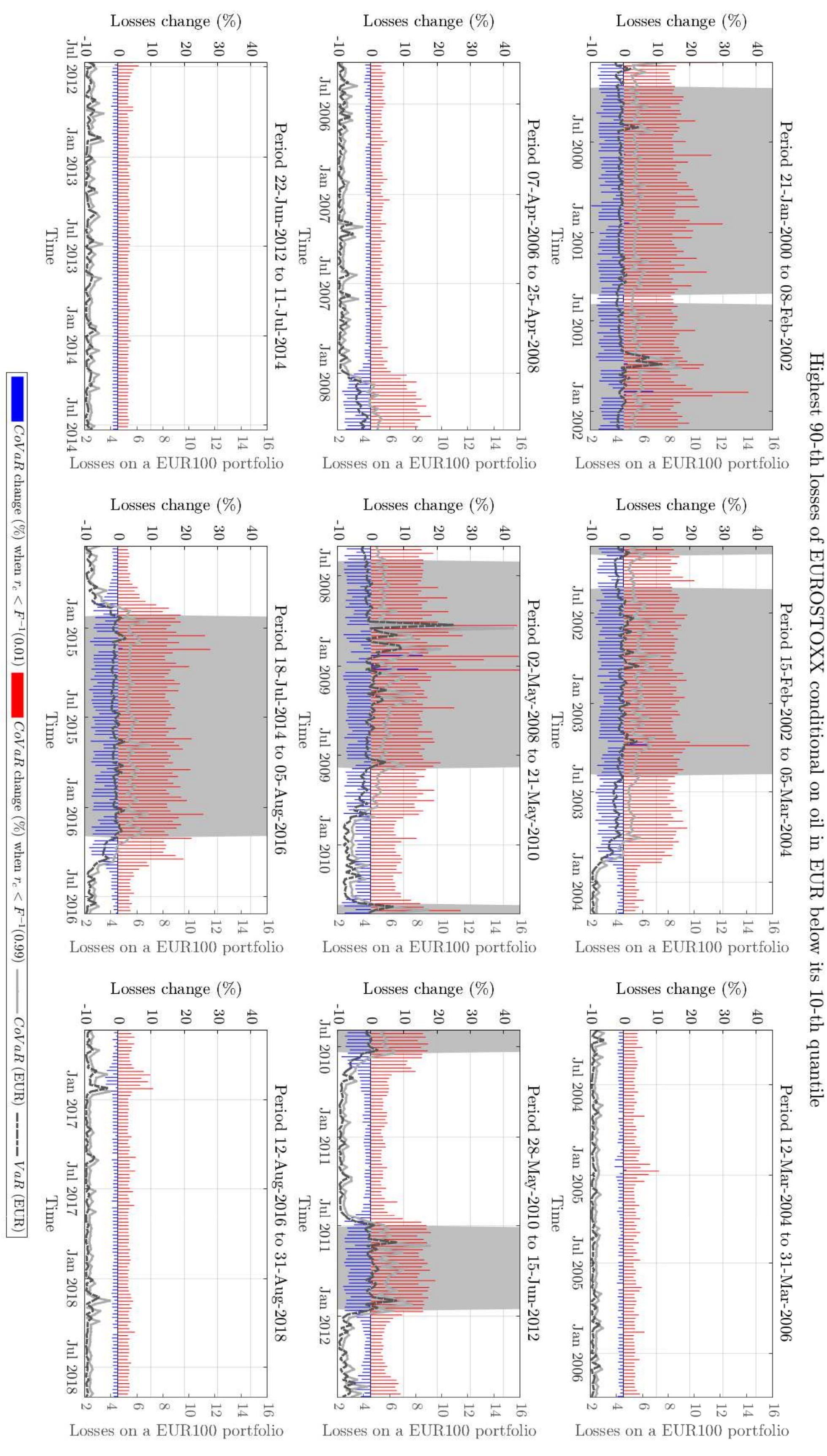
FIGURE 3.4.7: Value at Risk of the EUROSTOXX under different oil-related scenarios



This figure shows the Value at Risk of EUROSTOXX (black dashed line), the Value at Risk of the EUROSTOXX under a distress scenario for oil in euros (red solid line) and its range of potential values depending on the source of risk that triggers the distress scenario for oil prices in euros (grey area). Left figure shows a bearish scenario for oil in euros, i.e. below its 10-th percentile, while the right chart indicates a bullish scenario, i.e. oil returns in euros above its 90-th percentile. The response of the EUROSTOXX VaR might be different depending on the source of the shock, i.e. arising from the exchange rate or from the oil trade. Grey areas show how the response of EUROSTOXX could vary under the same scenario for oil in euros depending on the source of the scenario. This allows us to build a range of *uncertainty* regarding the impact of the scenario.

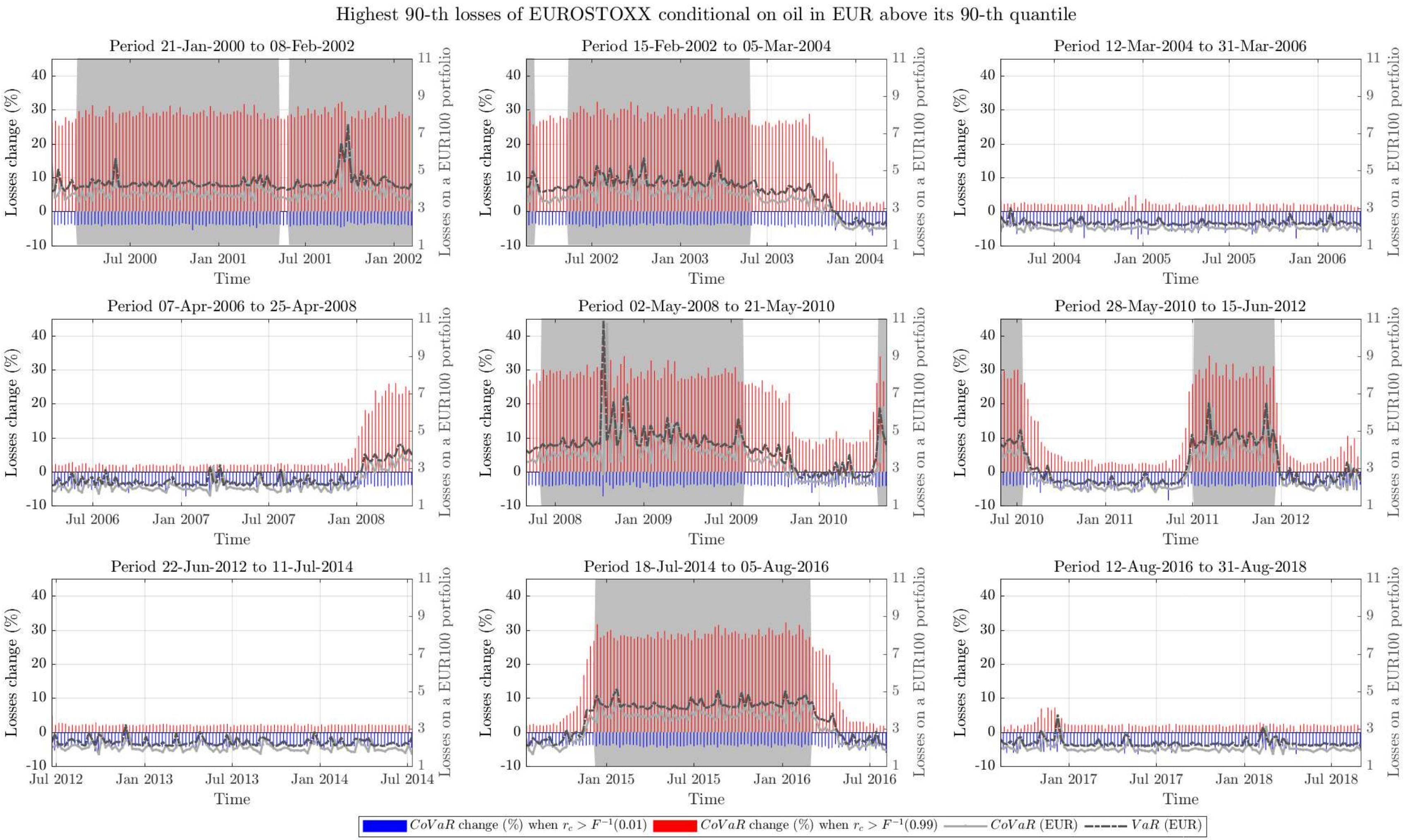
To assess how the source of risk could condition the losses in a stock portfolio, let us assume that the VaR and the $CoVaR$ losses of the EUROSTOXX occur. Then, given that $r_{s,t} = log(P_{s,t}) - log(P_{s,t-1})$, the losses in a $EUR100$ portfolio would be $100(1 - \exp(VaR))$ and $100(1 - \exp(CoVaR))$ respectively. Figures 3.4.8 and 3.4.9 show in the right axis the losses on a $EUR100$ portfolio when the distress scenario materialises. Grey line indicates the losses in the portfolio when the $CoVaR$ scenario occurs. The $CoVaR$ losses come from a downward movement in oil prices denominated in euros (Figure 3.4.8) or from an upward movement (Figure 3.4.9). The $CoVaR$ is obtained setting an undefined the source of risk using Equation (3.20). Black dashed line indicates the losses that comes from the VaR of the EUROSTOXX returns. Grey areas indicate periods where the smoothed probabilities of being at the high-variance state are higher than 90%. This regime is identified in three main periods: before 2003, coinciding with the dot-com crisis; between 2008 to 2011, when the financial crisis and the European sovereign debt crisis occur; and between 2014 to 2016, matching with the oil glut period.

FIGURE 3.4.8: Losses on a EUR100 portfolio when the distress scenario (*VaR* or *CoVaR*) materialises and the percentage change of *CoVaR* losses depending on the source of the scenario. Bearish scenario for oil in euros.



This figure shows the losses of a EUR100 portfolio when the distress scenario defined by the risk measures materialises (right axis). The oil price denominated in euros presents a downward movement under the CoVaR scenario. Left axis shows the percentage change of the *CoVaR* losses depending on the source of the shock. Grey areas indicate periods where the smoothed probabilities of being at a high-volatility state are higher than 90%.

FIGURE 3.4.9: Losses on a EUR100 portfolio when the distress scenario (VaR or CoVaR) materialises and the percentage change of CoVaR losses depending on the source of the scenario. Bullish scenario for oil in euros.



This figure shows the losses of a EUR100 portfolio when the distress scenario measured by the risk measures materialises (right axis). The oil price denominated in euros presents a upward movement under the CoVaR scenario. Left axis shows the percentage change of the *CoVaR* losses depending on the source of the shock. Grey areas indicate periods where the smoothed probabilities of being at a high-volatility state are higher than 90%.

Left axis presents the changes in percentage of the nominal losses on the $EUR100$ portfolio depending on the source of risk, compared to the $CoVaR$ losses with an undefined source of risk. The losses decrease between $1\% - 9\%$ compared to the bearish $CoVaR$ if the exchange rate triggers the downward movement in oil prices. Indeed, the 10% highest EUROSTOXX losses under a bearish oil-related scenario alleviate if the appreciation of the euro generates the decrease in oil prices. On the other side, $CoVaR$ losses increase between $4\% - 20\%$ when oil movements are generating the downward trend in oil price in euros. The bearish $CoVaR$ triggered by the exchange rate depicts a scenario where the appreciation of the euro indicates a high foreign demand of European goods, which appreciates the domestic currency. The bearish $CoVaR$ triggered by the oil market might be related to an economic slump scenario where the oil demand decreases, coinciding with higher losses in thestock market. Regarding the bullish scenario for oil returns in euros, $CoVaR$ losses decrease around 4% when the event is led by the oil returns in US dollars. This could be explained by the fact that economies in the expansion phase of the economic cycle present a high demand of energy products (Fernández Casillas et al. 2012). Losses increase between $3\% - 30\%$ when the depreciation of the euro explains the bullish trend in oil prices. The depreciation of the euro could be indicative of an economic crisis in the euro area and the existence of trade imbalances.

The findings in this section prove that the same scenario for oil in euros might describe very different economic frameworks depending on the source of risk. Hence, identifying the trigger that leads the distress scenario is relevant to build tailor-made stress tests and to get a better understanding about the relationship between variables in extremes scenarios.

3.5 Conclusion

The academic literature has not distinguished the trigger of a distress scenario in international markets when analysing the response of a domestic economy. Leaving the source of risk undefined may affect the consistency of estimates of the response of a given market because it may be strongly conditioned by the trigger that led to the distress scenario. On the contrary, having more detailed information on the scenario will generally lead to more accurate estimates of the response of the domestic economy. This article suggests combining the vine copula approach with the convolution concept, getting the most out of financial data to design stress test scenarios where the global markets and the exchange rate interact to define the distress event. The convolution approach allows us to take into account not only the degree of distress in the conditioning event but also the trigger that generates such event. The vine copula approach allows for modelling complex multivariate distributions while the convolution copula can capture the interaction between oil prices and the exchange rate. This framework allows for considering tailor-made scenarios, reducing the uncertainty regarding the role that the foreign exchange rate plays in the distress scenario.

I perform an empirical exercise using weekly returns of EUROSTOXX, Brent oil in US dollars and the foreign exchange rate for the period 2000-2018 to analyse the dependence of the European stock market on the foreign exchange rate under an oil-related scenario. A given event for oil prices in euros is consistent with different combinations of scenarios for the euro dollar exchange rate and oil markets. Whether it is exchange rate risk or commodity risk that triggers the conditioning event should be expected to have an impact on the response of the European stock market to an energy-related scenario. I employ a SWARCH model where the copula and the variance switch jointly across regimes to reflect the structural change observed in the data. Indeed, empirical evidence shows periods of increased volatility in global markets jointly with a higher degree of co-movement and tail dependence across financial variables. These structural changes have been identified before 2003, between 2008-2011 and between 2014-2016, in coincidence with the early 2000s recession, the financial crisis with the consequent European sovereign debt crisis, and the 2010s oil glut. The EM algorithm provides the estimates of the model following an iterative process, reducing the complexity of the optimization problem.

The results indicate that when an upward movement in oil prices in euros materialises, the magnitude of the 10% highest losses in the European stock market depends on the source of risk triggering the scenario. Such extreme losses increase when a downward movement in oil prices in euros materialises, with independence of the trigger. However, the size of the increase depends on the source of risk. Stock responses to oil-related scenarios present a higher dispersion under downward swings than under upward movements. On the one hand, the dominant role of commodity risk in scenarios where the oil prices in euros experience a downward movement can sharply increase the losses of the European stock market. On the other hand, the exchange rate risk might exacerbate stock losses if it triggers an extreme event where oil prices in euros increases. A simulation exercise shows that the conditional distribution of stock returns in a scenario where oil prices sharply decrease is more left-skewed when the oil market triggers the conditioning event. The distribution of stock returns also presents a left-skewed feature in scenarios where a upward swing in oil prices occur due to extreme movements in the foreign exchange market. The decrease of oil demand in economic crises and the depreciation of the domestic currency, due to political uncertainty and weak economic fundamentals, may explain these results.

The proposed approach can improve our understanding of exchange rate movements might affect stress test exercises in global markets. Possible extensions of the methodology could study the interactions between the European stock market and the international markets where the foreign exchange rate plays a role regardless of whether the effects emerge contemporaneously or with some lags. Combining the convolution and the copula methodology (Cherubini et al. 2016) we could build a flexible VAR model that allows for non-linearities. The copula approach establishes the link between current and past returns, while the convolution provides the distribution of the current returns as the sum of past returns and an innovation. The VAR model would be enhanced by the possibility of analysing the mean effect of an oil-related shock on the tail of the European stock market distribution. Additional studies could deal with interactions of exchange rate to foreign economies where there is a significant exposure. For instance, Spanish financial institutions have a large exposure to Latin American countries. Analysing the response of the financial firms to extreme events in these countries depending on the source of the shock, i.e. foreign stock markets or exchange rates, will be useful to design better hedging strategies, increasing the resilience of the financial sector to instabilities in the region.

Thus, these findings have consequences, firstly, for risk managers, investors and traders, who wish to control the exposure of its stock positions to commodity and exchange rate risk; secondly, for regulatory authorities and supervisors, who look for tailor-made stress test scenarios that consider the role of the foreign exchange rate; thirdly, for monetary authorities, who are interested to quantify stock market losses if scenarios of unstable energy prices materialise; lastly, for policy makers, who wish to understand the interactions between the main variables that drive the economy.

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Appendix

A Algorithm for the simulation process

Algorithm 4 Simulation of dependence under a Vine in dimension N=3 over a time period τ and a copula structure that follows a two-state Markov switching.

procedure SIM-DEPENDENCE($\theta, P(s_{t-1} = 1|I_T), P(s_t = 1|I_T), p_{11}, p_{22}$)
for $\omega \leftarrow 1, \dots, W$ **do**
if $rand < P(s_{t-1} = 1|I_T)$ **then**
$state_{1,\omega} = 1$
else
$state_{2,\omega} = 2$
end if
if $rand < P(s_t = 1|I_T)$ **then**
$state_{2,\omega} = 1$
else
$state_{2,\omega} = 2$
end if
for $t \leftarrow 1, \dots, \tau$ **do**
if $state_{t+1,\omega} = 1$ **then**
if $rand < p_{11}$ **then**
$state_{t+2,\omega} = 1$
else
$state_{t+2,\omega} = 2$
end if
else
if $rand < p_{22}$ **then**
$state_{t+2,\omega} = 2$
else
$state_{t+2,\omega} = 1$
end if
end if
$u_{t,\omega,1} = rand$
$u_{t,\omega,2} = C^{-1}_{2|1}\left(rand|u_{t,\omega,1}; \theta_{state_{t+2,\omega}}\right)$
for $n \leftarrow 3, \dots N$ **do**
$u_{t,\omega,n} = rand$
for $k \leftarrow 1, \dots, n-1$ **do**
$u_{t,\omega,n} = C^{-1}_{n|k}\left(u_{t,\omega,n}|u_{t,\omega,k}; \theta_{state_{t+2,\omega}}\right)$
end for k
end for n
end for t
end for ω
Return u and $state$
end procedure

θ_s are the set of parameters for the copula structure under regime s. $P(s_{t-1} = 1|I_T)$ and $P(s_t = 1|I_T)$ are the smoothed probabilities of being in state 1 at $t-1$ and t.
p_{11} and p_{22} are the diagonal values from the transition matrix (see Equation (3.25)).
rand refers to an uniform-distributed random realization.
The *OUTPUT* u is a uniform-distributed matrix that has the joint dependence presented in the model. The *OUTPUT* *state* is a matrix that indicates in which regime is the model at each time within each simulation.

Algorithm 5 Simulation from a AR(1)-SWARCH(2,1) over a time period τ and Gaussian distribution assumption for the innovation process.

procedure SIM-PATH($u, state, \phi_0, \phi_1, \alpha_0, \alpha_1, \kappa_2, r_{T-1:T}$)
  for $n \leftarrow 1, \ldots, N$ **do**
    for $w \leftarrow 1, \ldots, W$ **do**
      for $t \leftarrow 1, \ldots, \tau$ **do**
        if $t = 1$ **then**
          $\varepsilon = r_{n,T} - \phi_{n,0} - \phi_{n,1} r_{n,T-1}$
        end if
        if $state_{t,\omega} = 1$ **then**
          $h_{t,\omega,n} = \alpha_{n,0} + \alpha_{n,1}\varepsilon^2$
        else
          $h_{t,\omega,n} = \alpha_{n,0} + \alpha_{n,1}\frac{\varepsilon^2}{\kappa_{n,2}}$
        end if
        if $state_{t+1,\omega} = 1$ **then**
          $\sigma_{t,\omega,n} = \sqrt{h_{t,\omega,n}}$
        else
          $\sigma_{t,\omega,n} = \sqrt{\kappa_{n,2} h_{t,\omega,n}}$
        end if
        $\varepsilon = \Phi^{-1}(u_{t,\omega,n})\sigma_{t,\omega,n}$
        if $t = 1$ **then**
          $r_{t,\omega,n} = \phi_{n,0} + \phi_{n,1} r_{n,T} + \varepsilon$
        else
          $r_{t,\omega,n} = \phi_{n,0} + \phi_{n,1} r_{t-1,\omega,n} + \varepsilon$
        end if
      end for t
    end for w
  end for n
  Return r
end procedure

u is a N-dimension matrix ($TxWxN$) obtained from Algorithm 4.
ϕ_0 and ϕ_1 are vectors of parameters of length N that drive the dynamic in Equation (3.6).
α_0, α_1, κ_2 are vectors of parameters of length N that drive the dynamic in Equation (3.9).
The *OUTPUT* r is a N-dimension matrix ($\tau xWxN$) of W simulated paths of length τ for the N returns.

B Copula set for modelling joint distribution

Gaussian and Student copula are elliptical copulas, i.e., the bivariate joint density under these copulas has elliptic isodensities.
Gumbel and Clayton are Archimedean copulas, which implies that can be expressed as a function of the generate function ϕ and its inverse ϕ^{-1}, i.e. $C(u_1, u_2, \theta) = \phi^{-1}[\phi(u_1;\theta) + \phi(u_2;\theta);\theta]$ where θ is the copula parameter.
To enhance the features of copulas that only allow for positive dependence, they are rotated to capture negative tail dependence. The next table shows the tail dependence for the 90° rotated copulas. The 90° rotated copulas are built modifying

slightly the standard copula, i.e.

$$C_{90}(u_1, u_2) = u_2 - C\,(1 - u_1, u_2)$$

TABLE 3..1: Tail dependence for the 90° rotated copulas

	$\tau_{L\|U}$	$\tau_{U\|L}$
90°R Clayton	$2^{-1/\theta}$	-
90°R Gumbel	-	$2 - 2^{1/\theta}$

θ is the parameter from the original copula. Further information about the rotated copula can be found in Brechmann and Schepsmeier (2013), Cech (2006), Georges et al. (2001) and Luo (2010).

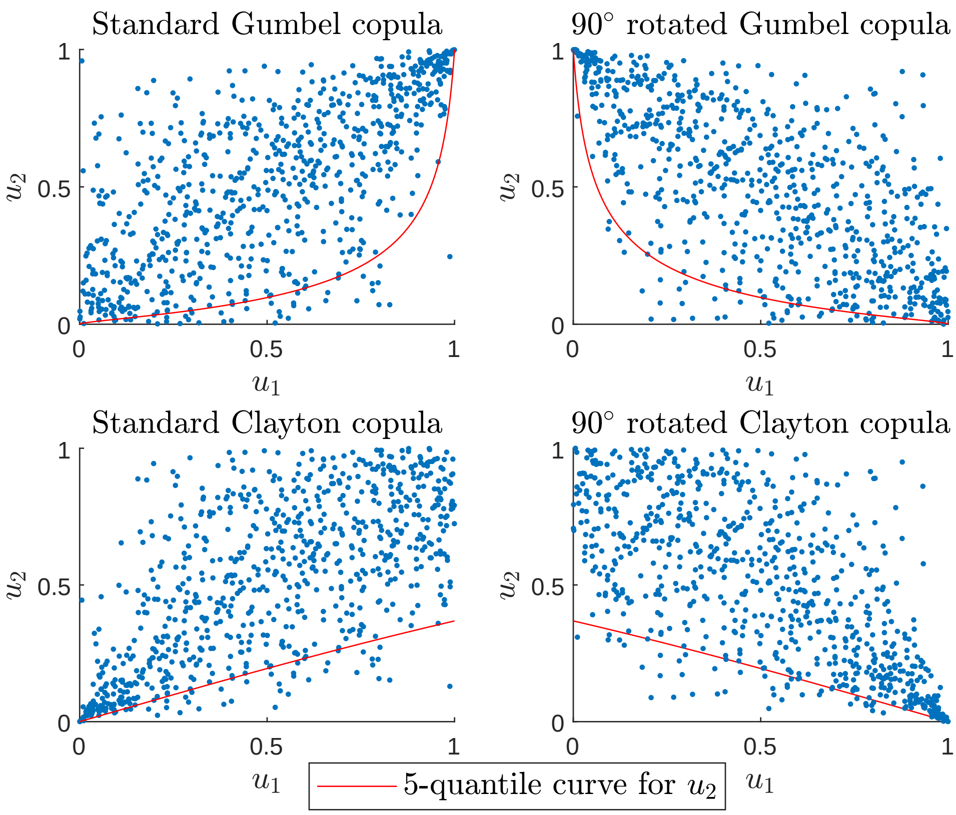
Let u_1 and u_2 denote two variables uniformly distributed across (0,1).

The negative lower tail dependence, $\tau_{L|U}$, is defined as $\tau_{L|U} = lim_{q\to 0} P(u_2 < q | u_1 > 1 - q)$.

The negative upper tail dependence, $\tau_{U|L}$ is defined as $\tau_{U|L} = lim_{q\to 1} P(u_2 > q | u_1 < 1 - q)$.

Figure 3..1 shows an example of how change the distribution and the tail joint behaviour when the 90° rotated copula is employed. See Zhang (2008) for further details about negative tail dependence.

FIGURE 3..1: Rotated copulas employed to capture negative tail dependence



This figure shows 800 simulations from the same seed but under different copula assumptions. Rotating 90 degrees allows us to capture negative upper tail dependence (90° rotated Gumbel), negative lower tail dependence (90° rotated Clayton). The red line indicates the threshold below which the 5% of the u_2 are found given the values taken by u_1. Gumbel and Clayton copula has a copula parameter $\theta = 2$.

Gaussian copula. This copula has a parameter ρ that gathers linear correlation. When $\rho = 1$ the tail dependence is 1, otherwise this copula does not present tail dependence. There is not a closed form expression due to the fact that Gaussian copula is an implicit copula. Meyer (2013) takes a in-depth look at this copula.
The copula probability density function is

$$c(u_1, u_2; \rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left\{ -\frac{\rho^2\Phi^{-1}(u_1)^2 - 2\rho\Phi^{-1}(u_1)\Phi^{-1}(u_2) + \rho^2\Phi^{-1}(u_2)^2}{2(1-\rho^2)} \right\},$$

where Φ^{-1} stands for the Gaussian inverse cumulative distribution function.
The conditional copula $C_{2|1}(u_2|u_1;\rho)$ is

$$\Phi\left(\frac{\Phi^{-1}(u_2) - \rho\Phi^{-1}(u_1)}{\sqrt{1-\rho^2}} \right).$$

Student copula. This copula allows for positive and negative symmetric tail dependence. The parameter ρ measures correlation and the parameter η, the number of degrees of freedom, controls the probability mass assigned to extreme joint co-movements of risk factors changes.[13] When $\eta \to \infty$ corresponds to the Gaussian

[13] For more information about the properties of the t-Student copula see Demarta and McNeil (2005)

copula.[14] Student copula has not a closed form because it is aN implicit copula. The copula probability density function is

$$\begin{aligned} c(u_1, u_2; \eta, \rho) &= K \frac{1}{\sqrt{1-\rho^2}} \\ &\left[1 + \frac{T_\eta^{-1}(u_1)^2 - 2\rho T_\eta^{-1}(u_1) T_\eta^{-1}(u_2) + T_\eta^{-1}(u_2)^2}{\eta(1-\rho^2)}\right]^{-\frac{\eta+2}{2}} \\ &\left[(1+\eta^{-1}T_\eta^{-1}(u_1)^2)(1+\eta^{-1}T_\eta^{-1}(u_2)^2)\right]^{\frac{\eta+1}{2}}, \end{aligned}$$

where $K = \Gamma(\frac{\eta}{2})\Gamma(\frac{\eta+1}{2})^{-2}\Gamma(\frac{\eta+2}{2})$.
The conditional copula $C_{2|1}(u_2|u_1; \rho, \eta)$ is

$$T_{\eta+1}\left(\sqrt{\frac{\eta+1}{\eta+(T_\eta^{-1}(u_1))^2}}\frac{T_\eta^{-1}(u_2) - \rho T_\eta^{-1}(u_1)}{\sqrt{1-\rho^2}}\right)$$

where T_η is the cdf of a t-Student with the numbers of degrees of freedom equal to η and T_η^{-1} represents its inverse [15]

Clayton copula. This copula allows positive dependence and asymmetric lower tail dependence. The Clayton copula has a dependence parameter $\theta \in (0, +\infty)$. When $\theta \to 0$ implies independence and when $\theta \to \infty$ implies perfect dependence. The Clayton copula is

$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta},$$

and the copula probability density function is

$$c(u_1, u_2; \theta) = (\theta+1)\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-2-\frac{1}{\theta}}(u_1 u_2)^{-\theta-1}.$$

The conditional copula $C_{2|1}(u_2|u_1; \theta)$ is

$$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1+\theta}{\theta}} u_1^{-\theta-1}$$

Gumbel copula. This copula allows for positive dependence and asymmetric upper tail dependence. The Gumbel copula has a dependence parameter $\theta \in [1, +\infty)$. When $\theta = 1$ implies independence and when $\theta \to \infty$ implies perfect dependence. The Gumbel copula is

$$C(u_1, u_2; \theta) = \exp\left(-\left\{(-\log u_1)^\theta + (-\log u_2)^\theta\right\}^{1/\theta}\right),$$

[14] The Gaussian copula underestimates the probability of joint extreme co-movements in high volatility and correlation scenarios (see Aussenegg and Cech (2011))

[15] See for instance Cech (2006)

and the copula probability density function is

$$\begin{aligned} c(u_1, u_2; \theta) &= (A + \theta - 1)\, A^{1-2\theta} \exp(-A) \\ &\quad (u_1 u_2)^{-1} (-\log u_1)^{\theta-1} (-\log u_2)^{\theta-1}, \end{aligned}$$

where $A = \left[(-\log u_1)^{\theta} + (-\log u_2)^{\theta}\right]^{\frac{1}{\theta}}$.
The conditional copula $C_{2|1}(u_2|u_1; \theta)$ is

$$\exp\left(- \left\{ (-\log u_1)^{\theta} + (-\log u_2)^{\theta} \right\}^{1/\theta} \right) \left\{ (-\log u_1)^{\theta} + (-\log u_2)^{\theta} \right\}^{1/\theta - 1} (-\log u_1)^{\theta-1} \frac{1}{u_1}$$

C Considering the role of the exchange rate in a bullish scenario for oil returns in euros

Following Ojea Ferreiro (2019), I define the bullish $CoVaR_{s|oe}(\alpha, \beta)$ as the $\beta 100\%$ lowest stock returns given that oil returns in euros are above its α quantile, i.e.

$$\begin{aligned} P\left(r_s < CoVaR_{s|oe} | r_{oe} > VaR_{oe}(\alpha)\right) &= \frac{P\left(r_m < CoVaR_{s|oe}, r_{oe} > VaR_{oe}(\alpha)\right)}{P(r_{oe} > VaR_{oe}(\alpha))} \\ &= \beta, \end{aligned}$$

where $P(r_{oe} > VaR_{oe}(\alpha)) = 1 - \alpha$.
Following the same reasoning that in Subsection 3.2.3 for a given lower bound q_c for the quantile of the exchange rate returns we get

$$r^*_{oe}(\alpha) \leq F_c^{-1}(q_c) + r_o,$$

where $r^*_{oe} = VaR_{oe}(\alpha)$. Consequently, oil returns denominated in US dollars should be greater

$$r_o \geq r^*_{oe} - F_c^{-1}(q_c)$$

which in terms of quantiles would be

$$\begin{aligned} P\left(r_o \geq r^*_{oe} - F_c^{-1}(q_c)\right) &= 1 - F_o(r^*_{oe} - F_c^{-1}(q_c)) \\ &= 1 - q_o. \end{aligned} \tag{3.21}$$

Hence, the bullish $CoVaR(\alpha, \beta)$ when the exchange rate returns are above its q_c100-th quantile would be obtained implicitly from

$$\begin{aligned} P\left(r_s < CoVaR_{s|oe} | r_{oe} > VaR_{oe}(\alpha), r_c > VaR_c(q_c)\right) &= \frac{P\left(r_s < CoVaR_{s|oe,c}, r_{oe} > VaR_{oe}(\alpha), r_c > VaR_c(q_c)\right)}{P(r_{oe} > VaR_{oe}(\alpha), r_c > VaR_c(q_c))} \\ &= \beta. \end{aligned}$$

Taking into account the chosen vine copula structure, where the first link between the variables arises from a common exposure to the exchange rate while the direct relationship between oil and stock returns is modelled once this connection through the exchange rate has been considered, we get the following expression

$$\frac{\int_{q_c}^{1} C_{s|c}(F_s(CoVaR_{s|oe,c})|u) - C_{s,o|c}\left(C_{s|c}(F_s(CoVaR_{s|oe,c})|u), C_{o|c}(q_o|u)\right) du}{1 - q_o - q_c + C_{o,c}(q_o, q_c)} = \beta, \tag{3.22}$$

where the probabilities of being above the threshold are obtained considering the rotation of copulas.[16]

D Markov switching specification

Let us define Ψ as a vector 2x2 that gathers the conditional joint density function of $r_{o,t}$, $r_{c,t}$, $r_{s,t}$ given by a low-volatile or high-volatile regime at t and $t-1$, where the relationship across variables might change, i.e.

$$\Psi = \begin{bmatrix} f\left(r_{o,t}, r_{c,t}, r_{s,t}; \Theta_{s_t=1,s_{t-1}=1}\right) & f\left(r_{o,t}, r_{c,t}, r_{s,t}; \Theta_{s_t=1,s_{t-1}=2}\right) \\ f\left(r_{o,t}, r_{c,t}, r_{s,t}; \Theta_{s_t=2,s_{t-1}=1}\right) & f\left(r_{o,t}, r_{c,t}, r_{s,t}; \Theta_{s_t=2,s_{t-1}=2}\right) \end{bmatrix}, \tag{3.23}$$

where $\Theta_{s_t,s_{t-1}}$ is the vector of parameters under the regime s_t at time t and regime s_{t-1} at time $t-1$. Note that s_{t-1} is only considered for the variance given by the $SWARCH(2,1)$, while the dependence across variables only depends on the current state s_t.
The conditional densities depend only on the current regime s_t and the previous regime s_{t-1}, i.e.

$$f\left(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, s_t = j, s_{t-1} = i; \Theta_{s_t,s_{t-1}}\right) = f\left(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, s_t = j, s_{t-1} = i, s_{t-2} = k...; \Theta_{s_t,s_{t-1}}\right),$$

for $i, j = 1, 2$ and I_{t-1} refers to the information set at $t-1$. I assume that the evolution of s_t follows a first order Markov chain independent from past observations, i.e.

$$p_{ij} = P(s_t = j | s_{t-1} = i) = P(s_t = i | s_{t-1} = j, s_{t-2} = k, I_{t-1}), \tag{3.24}$$

for $i, j, k = 1, 2$.
The transition matrix defined by the Markov Chain is

$$P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \tag{3.25}$$

where each column i indicates the probability of remaining on the state i (p_{ii}) or moving to state j (p_{ij}) conditioned to the fact that we are currently at state i for $i, j = 1, 2$ and $i \neq j$. Obviously, $p_{ii} + p_{ij} = 1$ because only two states exist. That is the reason why p_{ij} is presented as $1 - p_{ii}$.
Let us assume that the set of parameters Θ are known. Let us gather the probability assigned to the observation at time t of being the result of regime j, i.e. $P(s_t = j | I_t; \Theta)$, in a vector $\hat{\xi}_{t|t}$,

$$\hat{\xi}_{t|t} = \left[P(s_t = 1 | I_t; \Theta), P(s_t = 2 | I_t; \Theta)\right]'.$$

$\hat{\xi}_{t|t}$ comprises the inference about the regime at time t given the information available at that period. The probability assigned to the observation at time $t+1$ of being the result of regime j given the information at time t is collected in vector $\hat{\xi}_{t+1|t}$,

$$\hat{\xi}_{t+1|t} = \left[P(s_{t+1} = 1 | I_t; \theta), P(s_{t+1} = 2 | I_t; \theta)\right]'.$$

$\hat{\xi}_{t+1|t}$ is the probability forecast of being in the next period $t+1$ at each regime given the information available at t. The forecast probability for the next period is obtained

[16] See Ojea Ferreiro (Ojea Ferreiro) as a reference on this topic.

as the product of the inference probability by the transition matrix, i.e.

$$\hat{\xi}_{t+1|t} = P\hat{\xi}_{t|t}.$$

The link between $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ is obtained by the updated probabilities, including the new available information through Bayes' theorem, i.e.

$$P(s_t = j|I_t;\Theta) = \frac{\sum_{i=1}^{2} P(s_t = j, s_{t-1} = i|I_{t-1};\Theta) f(r_{o,t}, r_{c,t}, r_{s,t}|I_{t-1};\Theta_{s_t=j,s_{t-1}=i})}{L_t(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, \Theta)}, \quad (3.26)$$

where $P(s_t = j, s_{t-1} = i|I_{t-1};\Theta) = P(s_{t-1} = i|I_{t-1};\Theta)p_{ij}$ and $L_t(r_{o,t}, r_{c,t}, r_{s,t}; I_{t-1}, \Theta)$ is the likelihood function at time t. To get the likelihood at time t we have to assess the sum of the product of the joint density conditioned to the occurrence of each possible set of states at t and $t-1$ by their probability given the information set at $t-1$, i.e.

$$L_t(r_{o,t}, r_{s,t}, r_{c,t}; I_{t-1}, \Theta_t) = \sum_{j=1}^{2}\sum_{i=1}^{2} f(r_{o,t}, r_{s,t}, r_{c,t}|\Theta_{s_t=j,s_{t-1}=i}, I_{t-1}) P(s_t = j, s_{t-1} = i|I_{t-1}), \quad (3.27)$$

where $\Theta_{s_t=j,s_{t-1}=i}$ stands for the set of parameters of the joint distribution at regime j at time t and regime i at time $t-1$. Rewriting Equation (3.26), that connects $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$, in a matrix form

$$\hat{\xi}_{t|t} = \frac{(P \odot [\hat{\xi}_{t-1|t-1}, \hat{\xi}_{t-1|t-1}] \odot \Psi)' 1_2}{1_2'\{(P \odot [\hat{\xi}_{t-1|t-1}, \hat{\xi}_{t-1|t-1}] \odot \Psi)' 1_2\}},$$

where Ψ was defined in Equation (3.23) while $\odot$ represent the element-wise product. To start the iteration we need a value for $\hat{\xi}_{1|0}$, for which I use the unconditional probabilities of each state that can be expressed in a matrix form as

$$\hat{\xi}_{1|0} = \pi = (A'A)^{-1} A'(0,0,1)'$$

where

$$A = \begin{bmatrix} I_2 - P \\ 1_2' \end{bmatrix} = \begin{bmatrix} 1 - p_{11} & p_{22} - 1 \\ p_{11} - 1 & 1 - p_{22} \\ 1 & 1 \end{bmatrix}.$$

and I_N is the identity matrix of size NxN and 1_N is a $(Nx1)$ vector of ones. To finish this subsection I present the Kim (1994)'s algorithm for smoothed inferences, which are used to present the probabilities of being in each state at each time t given the complete information of the sample T, i.e.

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \left\{ P' \left[\hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t} \right] \right\},$$

where $\odot$ and $(\div)$ represent the element-wise product and division respectively. Taking into account that current set of parameters depends on the state at t and $t-1$, we can rewrite previous equation as

$$\hat{\xi}_{t|T} = 1_2' \underbrace{\left\{ [\hat{\xi}_{t|t}, \hat{\xi}_{t|t}]' \odot P \odot \left[\hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t}, \hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t} \right] \right\}}_{\xi_{t+1,t|T}},$$

where $\xi_{t+1,t|T} = \begin{bmatrix} P(s_{t+1}=1, s_t=1|I_T;\Theta) & P(s_{t+1}=1, s_t=2|I_T;\Theta) \\ P(s_{t+1}=2, s_t=1|I_T;\Theta) & P(s_{t+1}=2, s_t=2|I_T;\Theta) \end{bmatrix}$.
The smoothed probability of being in state j at t and in state i at $t-1$ is

$$P(s_t = j, s_{t-1} = i|I_T;\Theta) = \frac{P(s_t = j|I_T;\Theta)}{P(s_t = j|I_{t-1};\Theta)} p_{ij} P(s_{t-1} = i|I_{t-1};\Theta), \tag{3.28}$$

for $t > 1$.

E Robustness check

Regarding the model choice, this appendix goes from the simplest model to the most sophisticated one. Simplest model, i.e. a truncated vine structure using Gaussian copulas, provides useful information concerning the data fit to implement further improvements. Performing a likelihood ratio test against the Student copula provides essential information concerning the significance of tail dependence in the model structure. The analysis using graphical tools help us to infer the actual dependence between the percentiles of the variables given by their estimated marginal distributions. The analysis obtained from the simplest model would point to a truncated model where the dependence could be different between states. This intermediate model, where the truncated vine structure could be non-elliptic and different between states, is the cornerstone to build more complex structures. Indeed, following Figure 3.2.1, the copula choice in step T_2 depends on the copula choice in step T_1, i.e. the truncated vine. The analysis of the conditional distribution of oil and stock returns given the exchange rate returns would give us an idea about the dependence between oil and stock returns once considered a common exposure to the exchange rate. This analysis would lead to the last model, the most complete one, to get a comprehensive idea about the links between these three key variables in the economy.

Simplest model: truncated vine structure using elliptical copulas

I present first the results for the elliptical models. Table 3..2 reports the estimate of the model, where the exchange rate is linked to oil in USD and EUROSTOXX, but EUROSTOXX and oil in USD are not directly connected, i.e. a truncated vine structure. Left table presents the results under Gaussian assumptions while right table shows the estimates under Student copula. The link between EUROSTOXX and the $USDEUR$ exchange rate is quite weak, $|\rho_{B,C}| < 0.1$, while the relationship between Oil in USD and $USDEUR$ is statistically significant and negative in both regimes. Hence, there is a link between the increases in oil prices and the appreciation of the euro against the US dollar. The table also shows the likelihood ratio statistic between the Student model and the Gaussian model. Its p-value is lower than 5%, indicating the significance of the tail dependence to explain the relationship between the set of variables.

Figure 3..2 presents the histogram and the likelihood under the Gaussian distribution where the variance within each state might differ following the $SWARCH$ model. The excess of kurtosis in the Gaussian distribution could be explained by a realization from a Gaussian distribution with higher variance. This feature of $SWARCH$ models was already underscored by Leon Li and Lin (2004).

Figure 3..3 presents the unconditional coverage backtesting test proposed by Kupiec (1995). The x-axis shows different quantiles of the marginal distribution chosen as threshold to count exceedances. The right axis presents the p-value where the null hypothesis is that $\alpha 100\%$ of the sample is below the threshold shown by the $VaR(\alpha)$. This analysis provides a useful robustness check regarding the fitting of the model for several quantiles. Left axis indicates the number of exceedances. Black line presents the current number of exceedances while the red lines are the bounds at 10%, 5% and 1% under the null hypothesis. These charts help us to check how well the model suits the data. The subgraph related to oil returns indicates that our model presents less outliers than expected in the data for quantiles between 0.45 and 0.15, but the model fits well the tail below quantile 0.15. On the other side, the model fits well EUROSTOXX distribution above quantile 0.05. The $USDEUR$ returns is fitted well by our model, even for extreme quantiles the p-value is higher than 0.05.

Figure 3..4 presents the conditional coverage backtesting test proposed by Christoffersen (1998), where the null hypothesis is that VaR violations are independent while the alternative hypothesis is that VaR violations follows a first order Markov Chain. Right axis shows the p-value of the Christoffersen (1998)'s test while left axis presents the number of exceedances. Left axis presents the number of observation. Red solid line presents the number of observations without exceedances at t and $t-1$. Red dashed line shows the number of pairwise observations where we have an exceedance at t but not at $t-1$ while the black dotted line shows the opposite. Red dotted line shows the number of pairwise observations with two consecutive exceedances. The p-value is higher than 0.10 for most of the quantiles. Hence, there is no evidence of a clustering of exceedances.

Finally, Figure 3..5 presents the bivariate histogram between oil in USD and $USDEUR$ returns (top figures) and the bivariate histogram between $USDEUR$-EUROSTOXX (bottom figures). The probability integral transform is chosen from state j if the smoothed probability of being at regime j is higher than 90% where $j = 1, 2$. The oil in USD - $USDEUR$ relationship presents a cluster of data in high quantiles of oil returns and to a lesser extent in the opposite tail. These features could be explained by a Student or a 90° rotated Clayton. The oil in USD-$USDEUR$ link shows some degree of higher dependence in high quantiles of exchange rate and low quantiles of oil returns under state 2. Gaussian copula or a 90° rotated Gumbel might fit well the data as potential copulas. The $USDEUR$-EUROSTOXX link is quite homogeneously distributed under state 1, so a Gaussian or independent copula could match the data, while there is a higher dependence in high quantiles of exchange rates returns and low quantiles of EUROSTOXX returns under state 2, which could be consistent with a 90° Gumbel copula. These potential copulas are analysed and compared in the next subsection.

TABLE 3..2: Gaussian and Student t models

	Gaussian model			Student model		
	A	B	C	A	B	C
ϕ_0	0.00 **	-0.00	0.00 **	0.00 *	-0.00	0.00 ***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
ϕ_1	0.06 **	0.04	-0.05	0.06 **	0.04 *	-0.06 *
	(0.04)	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)
$\kappa_{s_t=2}$	2.27 ***	2.25 ***	3.74 ***	2.21 ***	2.21 ***	3.64 ***
	(0.32)	(0.26)	(0.32)	(0.26)	(0.29)	(0.08)
α_0	0.00 ***	0.00 ***	0.00 ***	0.00 ***	0.00 ***	0.00 ***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α_1	0.12 ***	0.08 **	0.15 ***	0.12 ***	0.09 **	0.16 ***
	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)	(0.05)

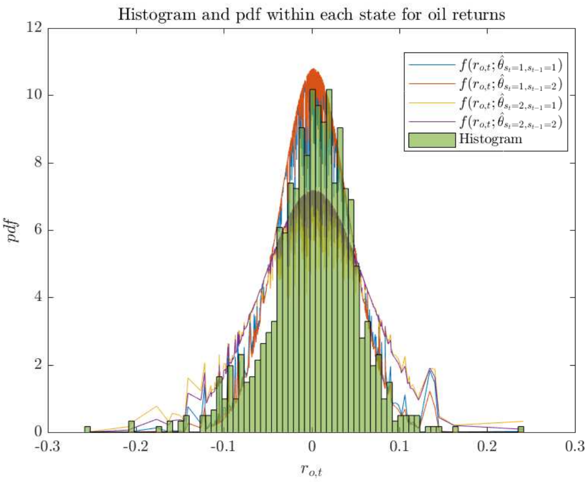
Gaussian			Student		
State 1	$\rho_{A,B}$	-0.23 ***	State 1	$\rho_{A,B}$	-0.22 ***
		(0.05)			(0.05)
	$\rho_{B,C}$	-0.09 *		$\eta_{A,B}$	13.35 ***
		(0.05)			(0.24)
State 2	$\rho_{A,B}$	-0.17 ***		$\rho_{B,C}$	-0.08 *
		(0.05)			(0.05)
	$\rho_{B,C}$	-0.03		$\eta_{B,C}$	24.80 ***
		(0.05)			(0.24)
	p_{11}	0.99 ***	State 2	$\rho_{A,B}$	-0.17 ***
		(0.00)			(0.05)
	p_{22}	0.98 ***		$\eta_{A,B}$	100.00 ***
		(0.01)			(1.17)
	LL	6668.53		$\rho_{B,C}$	0.07
					(0.07)
				$\eta_{B,C}$	7.35 ***
					(0.52)
				p_{11}	0.99 ***
					(0.00)
				p_{22}	0.98 ***
	RL	5,51			(0.01)
	RL p-value	0,0263		LL	6674.04

The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (3.6) and (3.9) and for the parameters of the Gaussian and Student t copula.
LL is the log-Likelihood value. RL is the logarithm of the likelihood ratio between the Student (unrestricted model) and the Gaussian (restricted model).RL p-value is the probability a results at least as extreme as the one obtained under the null hypothesis. The likelihood ratio is distributed under the null hypothesis as

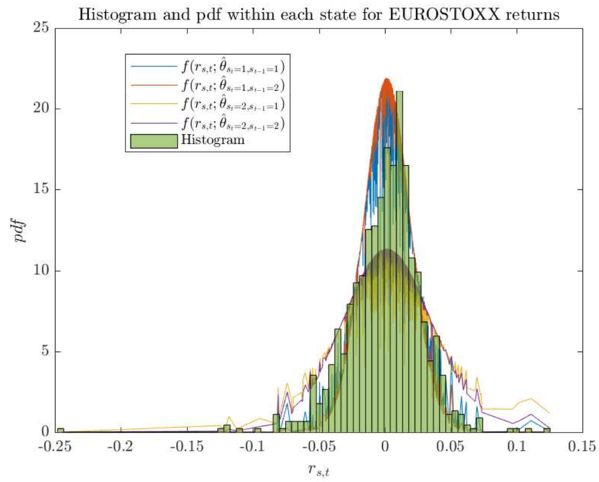
$$-2(log(Likel_R) - log(Likel_{UR})) \sim X_{k_{UR}-k_R}$$

A: Oil in USD, B: USDEUR exchange rate, C: EUROSTOXX.
$\rho_{A,B}$ is the correlation between Oil in USD and USDEUR returns. $\rho_{B,C}$ is the correlation between USDEUR exchange rate and EUROSTOXX returns.
$***/**/*$ indicates statistical significance at 1/5/10%

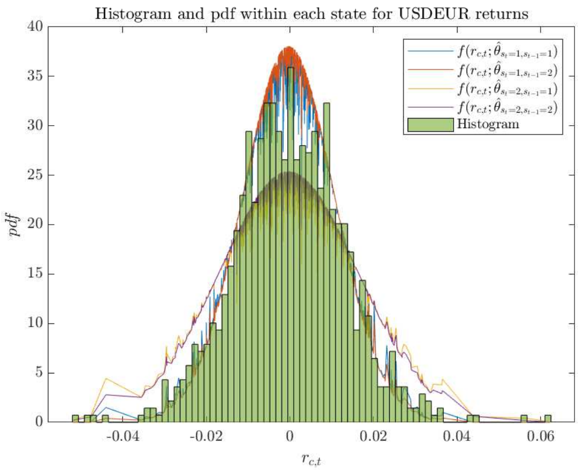
FIGURE 3..2: Histogram and Marginal distribution within each state



(a) Oil returns



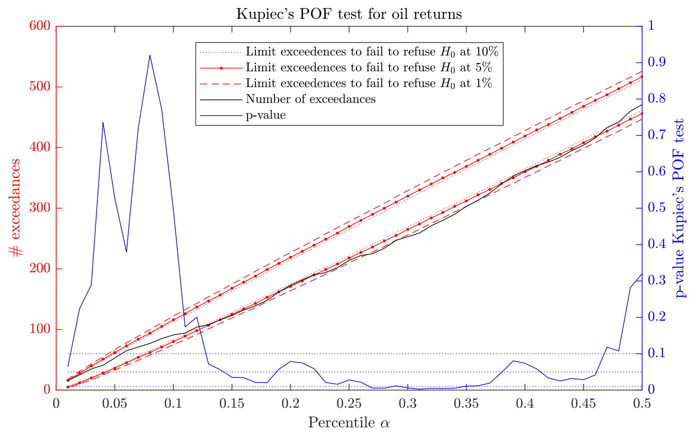
(b) EUROSTOXX returns



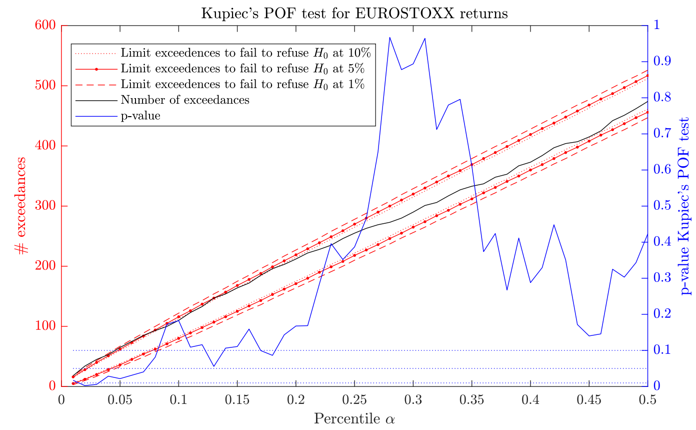
(c) USDEUR returns

The histogram (green bars) is scaled to be equivalent to the probability distribution function within each state. Although at time t we have only 2 states we have four pdf because the current variance according to the SWARCH model in the equation (3.9) depends on the state at t and the state at $t-1$. Note that higher moments can be obtained given higher probability to the distribution with higher dispersion for extreme values.

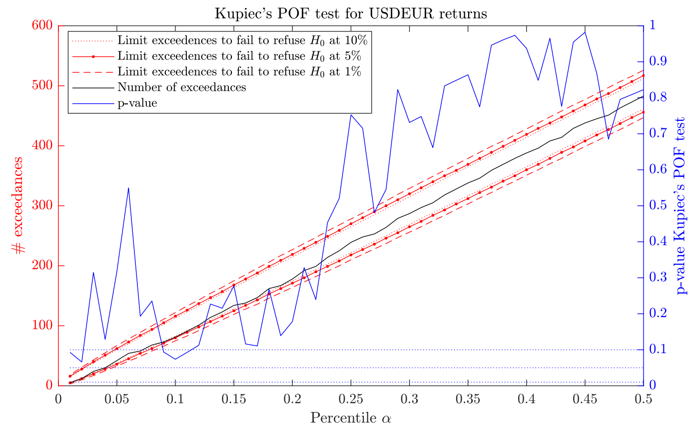
FIGURE 3..3: Kupiec's POF test



(a) Oil returns



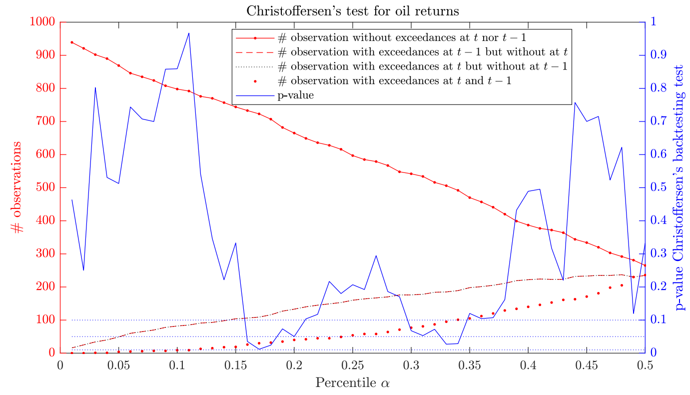
(b) EUROSTOXX returns



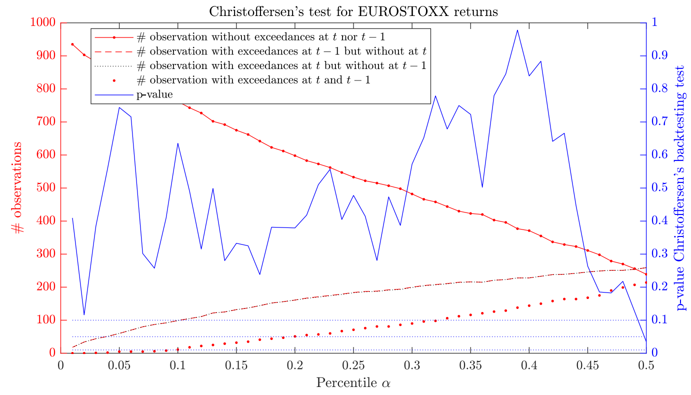
(c) USDEUR returns

These figures present the unconditional coverage backtesting test proposed by Kupiec (1995) to check the number of exceedances of a VaR with a α% significance level (x-axis). Right axis shows the p-value of the Kupiec (1995)'s test while left axis presents the number of exceedances. Confidence intervals for the null hypothesis are presented in the red lines for the 1%,5% and 10% significance level. Black line presents the current number of exceedances.

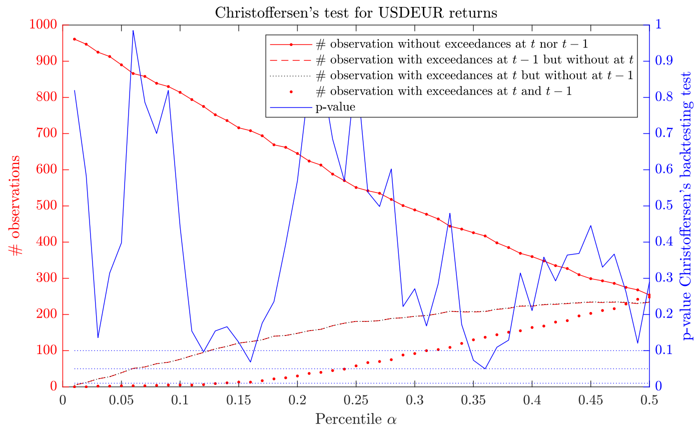
FIGURE 3..4: Christoffersen test



(a) Oil returns



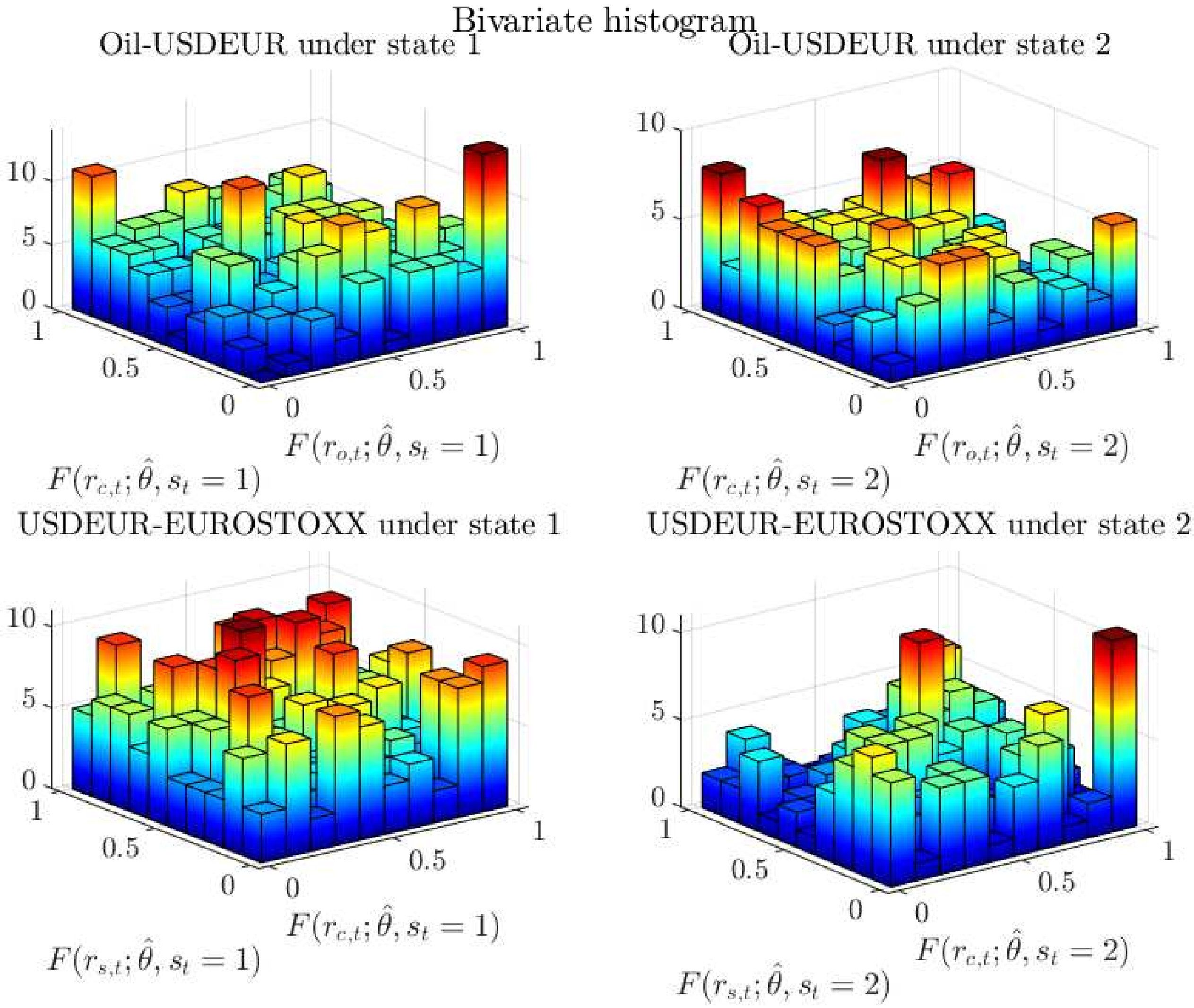
(b) EUROSTOXX returns



(c) USDEUR returns

Conditional coverage backtesting test proposed by proposed by Christoffersen (1998) are used for testing the number of exceedances of a VaR with a α% significance level (x-axis). Right axis shows the p-value of the Christoffersen (1998)'s test while left axis presents the number of exceedances. Left axis presents the number of observation. Red solid line present the number of observations without exceedances at t and $t-1$. Red dashed line shows the number of pairwise observations where we have an exceedance at t but not at t while the black dotted line shows the opposite case. Red dotted line shows the number of pairwise observations with two consecutive exceedances.

FIGURE 3..5: Bivariate Histogram



This figure shows the bivariate histogram of the probability integral transforms of oil returns in US dollars and USDEUR returns (top figures) or USDEUR returns and EUROSTOXX returns (bottom figures). We suppose that observation at time t beyond to a regime j if the smoothed probability of being at t in state j is higher than 90%. These figures give us an idea about the type of relationship that we could expect from each set of two variables within each regime.

Intermediate model: truncated vine structure

Table 3..3 shows the AICC values for the potential copulas indicated by Figure 3..5. Lowest value indicates the best copula fit for the truncated vine structure. According to $AICC$ results, the best fit is provided by the Student copula (state 1) and the Gaussian copula (state 2) for the Oil-$USDEUR$ link and the independence copula (state 1) and the 90° rotated Clayton (state 2) for the EUROSTOXX-$USDEUR$ dependence. Table 3..4 indicates the estimates for the best copula model within the truncated vine structure. Figures 3..6 and 3..7 present the uncoverage and coverage backtesting test for the $CoVaR(\alpha,\beta)$ of oil returns (top figure) and EUROSTOXX (bottom figure) given that the exchange rate returns are below its $VaR(\alpha)$.[17] X-axis shows the joint probability of observing an exceedance, i.e. $\alpha\beta$, where $\alpha=\beta$. Figure 3..6 and 3..7 indicate that the copula choice meets the criteria in terms of number of exceedances

[17]Further information on how to build backtesting tests for the $CoVaR$ can be found in Appendix A of Ojea Ferreiro (2018).

and the independence of these VaR violations.

Table 3..5 presents the results of the independence test based on the empirical Kendall's τ. The conditional distribution of EUROSTOXX and Oil in USD given exchange rate returns are assumed to be independent by the truncated vine structure. This hypothesis is rejected at 1% significance level. Hence, the vine structure should include a direct link between oil and EUROSTOXX returns, even once the exchange rate connection is taken into account. The copula choice for this conditional dependence between oil in USD and EUROSTOXX is studied in the next subsection.

TABLE 3..3: AICC values to choose the best model fit

A	B	C	D	E
-13294.01	-13296.70	-13298.53	-13292.29	-13293.90
F	G	H	I	J
-13287.44	**-13298.66**	-13293.60	-13293.43	-13288.06

Notes: $AICC$ denotes Akaike Information Criterion corrected for small sample bias.

$AICC = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$ where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum $AICC$ value (in bold letters) indicates the best copula fit.

A: *Oil-USDEUR*- State 1: Gaussian, State 2: Gaussian
USDEUR-EUROSTOXX- State 1: Gaussian, State 2: Gaussian.

B: *Oil-USDEUR*- State 1: Student, State 2: Student
USDEUR-EUROSTOXX- State 1: Student, State 2: Student.

C: *Oil-USDEUR*- State 1: Student, State 2: Gaussian
USDEUR-EUROSTOXX-State 1: Gaussian, State 2: 90° Clayton.

D: *Oil-USDEUR*-State 1: 90° Clayton, State 2: Gaussian
USDEUR-EUROSTOXX-State 1: Gaussian, State 2: 90° Clayton.

E: *Oil-USDEUR*-State 1: Student, State 2: 90° Gumbel.
USDEUR-EUROSTOXX-State 1: Gaussian, State 2: 90° Clayton.

F: *Oil-USDEUR*-State 1: 90° Clayton, State 2: 90° Gumbel.
USDEUR-EUROSTOXX-State 1: Gaussian, State 2: 90° Clayton.

G: *Oil-USDEUR*- State 1: Student, State 2: Gaussian.
USDEUR-EUROSTOXX-State 1: Independence, State 2: 90° Clayton.

H: *Oil-USDEUR*-State 1: 90° Clayton, State 2: Gaussian.
USDEUR-EUROSTOXX-State 1: Independence, State 2: 90° Clayton.

I: *Oil-USDEUR*-State 1: Student, State 2: 90° Gumbel.
USDEUR-EUROSTOXX-State 1: Independence, State 2: 90° Clayton.

J: *Oil-USDEUR*-State 1: 90° Clayton, State 2: 90° Gumbel.
USDEUR-EUROSTOXX-State 1: Independence, State 2: 90° Clayton.

TABLE 3..4: Model with a truncated vine structure

	A	B	C
ϕ_0	0.00 *	-0.00	0.00 **
	(0.00)	(0.00)	(0.00)
ϕ_1	0.07 **	0.04	-0.05 *
	(0.04)	(0.03)	(0.04)
$\kappa_{s_t=2}$	2.17 ***	2.13 ***	3.78 ***
	(0.31)	(0.33)	(1.54)
α_0	0.00 ***	0.00 ***	0.00 ***
	(0.00)	(0.00)	(0.00)
α_1	0.13 ***	0.08 **	0.15 **
	(0.04)	(0.04)	(0.07)

State 1		State 2	
$\rho_{A,B}$	-0.20 ***	$\rho_{A,B}$	-0.18 ***
	(0.06)		(0.05)
$\eta_{A,B}$	12.22 ***	$\theta_{B,C}$	0.07 **
	(1.40)		(0.04)
p_{11}	0.99 ***	p_{22}	0.98 ***
	(0.00)		(0.01)
LL	-6670.82		

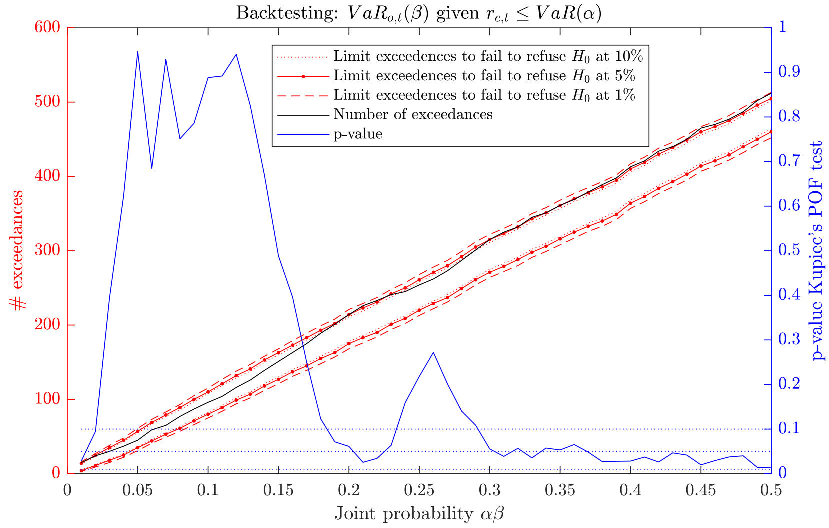
The table reports the estimates and the standard deviation (in parenthesis) for the parameters of the marginal model in Equations (3.6) and (3.9) and for the parameters of the best copula choice according to the $AICC$ value reported by Table 3..3.
LL is the log-Likelihood value.
A: Oil in USD, B: USDEUR exchange rate, C: EUROSTOXX. $\rho_{1,2}$ and $\eta_{A,B}$ is the correlation and number of degrees of freedom between Oil in USD and USDEUR returns. $\theta_{B,C}$ is the estimate of the 90° Rotated Clayton under state 2.
Vine structure: *Oil-USDEUR*- State 1: Student, State 2: Gaussian. *USDEUR-EUROSTOXX*- State 1: Independence, State 2: 90° Clayton.

TABLE 3..5: Conditional independence test result

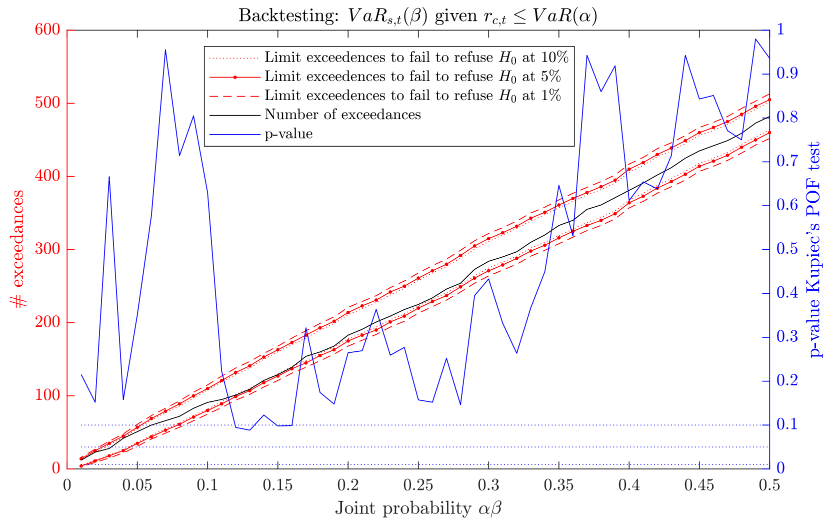
	$s_t = 1, s_{t-1} = 1$	$s_t = 1, s_{t-1} = 2$	$s_t = 2, s_{t-1} = 1$	$s_t = 2, s_{t-1} = 2$
$\hat{\tau}$	0.1062	0.1080	0.1103	0.1130
a	4.9582	5.0422	5.1518	5.2778
p-value	0.0000	0.0000	0.0000	0.0000

The p-values of the the independence test is built as $p - value = 2(1 - \Phi(a))$ where Φ is the Gaussian c.d.f. and $a = \sqrt{\frac{9T(T-1)}{2(2T+5)}}|\hat{\tau}|$ where T is the sample size, and $\hat{\tau}$ is the empirical Kendall's τ of the conditional distribution of oil and EUROSTOXX returns given a certain quantile of the returns of USDEUR exchange rate (see Brechmann and Schepsmeier (2013)). The conditional distribution is obtained given the best copula fit according to the AICC criterion from Table 3..3. The conditional independence is rejected for the four regimes.

FIGURE 3..6: Kupiec's POF test



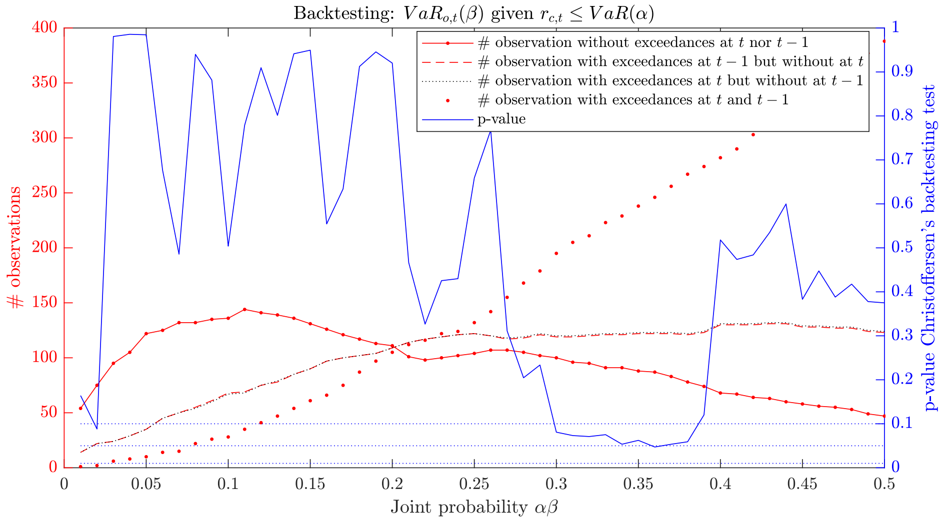
(a) Oil returns



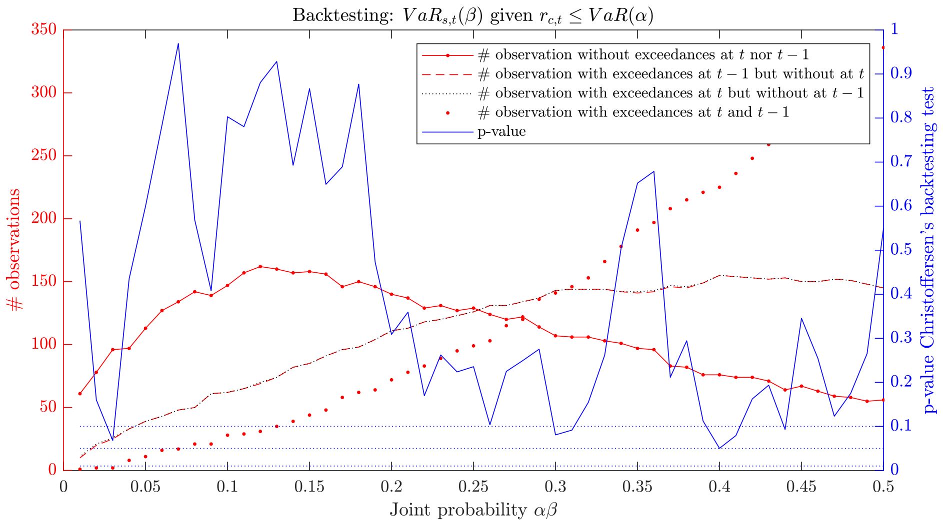
(b) EUROSTOXX returns

These figures present the unconditional coverage backtesting test proposed by Kupiec (1995) to check the number of exceedances of a $CoVaR(\alpha, \beta)$ with a β% significance level given than exchange rate returns are below $VaR(\alpha)$. This figures sets $\alpha = \beta$ while x-axis shows the joint probability, i.e. $\alpha\beta$.
Right axis shows the p-value of the Kupiec (1995)'s test while left axis presents the number of exceedances. Confidence intervals for the null hypothesis are presented in the red lines for the 1%,5% and 10% significance level. Black line presents the current number of exceedances.

FIGURE 3..7: Christoffersen test



(a) Oil returns



(b) EUROSTOXX returns

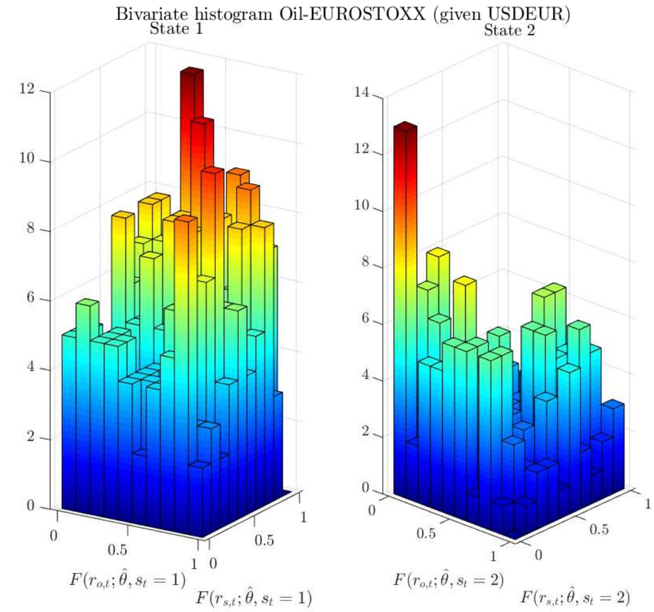
Conditional coverage backtesting test proposed by proposed by Christoffersen (1998) are used for testing the number of exceedances of a $CoVaR(\alpha, \beta)$ with a β% significance level given than exchange rate returns are below $VaR(\alpha)$. This figures sets $\alpha = \beta$ while x-axis shows the joint probability, i.e. $\alpha\beta$.
Right axis shows the p-value of the Christoffersen (1998)'s test while left axis presents the number of exceedances. Left axis presents the number of observation. Red solid line present the number of observations without exceedances at t and $t-1$. Red dashed line shows the number of pairwise observations where we have an exceedance at t but not at t while the black dotted line shows the opposite case. Red dotted line shows the number of pairwise observations with two consecutive exceedances.

Advanced model: vine structure

Figure 3..8 shows the conditional bivariate histogram given the exchange rate returns under the truncated vine structure. The conditional copula is set to be obtained from state j if the probability of being at state j is higher than 90%. There is a higher dependence between low quantiles of oil returns and high quantiles of EUROSTOXX returns under state 1. A Clayton copula could fit the lower tail dependence presented under state 2. Table 3..6 presents the values of the Akaike Information Criterion with a correction for small sample size (AICC) for a set of models where the

Clayton copula defines the dependence between oil in USD and EUROSTOXX conditional on the exchange rate under state 2, while under state 1 we consider the 90° rotated Clayton copula , the Gaussian copula and the independent copula, i.e. the product of the copula inputs. The best fit according to the AICC value is given by the Gaussian copula under state 1 and the Clayton copula under state 2.

FIGURE 3..8: Bivariate histogram conditioned to the exchange rate returns



This figure shows the bivariate histogram of the probability integral transforms of oil returns in US dollars and EUROSTOXX returns given the realization of USDEUR returns, i.e. conditional histogram. We suppose that the realization at time t beyond to a regime j if the smoothed probability of being at t in state j is higher than 90%. These figures give us an idea about the type of relationship that we could expect from each set of two variables within each regime, once the dependence through the exchange link is taken into account.

TABLE 3..6: AICC to choose the best model fit for the stage 2 within the vine structure (T_2)

Model		
A	B	C
-13316,45	-13316,39	-13313,64

Notes: *AICC* denotes Akaike Information Criterion corrected for small sample bias.

$AICC = 2k\frac{T}{T-k-1} - 2\log(\hat{L})$ where T is the sample size, k is the number of estimated parameters and $\hat{L}$ is the Log-likelihood value. Minimum *AICC* value (in bold letters) indicates the best copula fit.

A: *Oil-USDEUR*- State 1: Student, State 2: Gaussian
USDEUR-EUROSTOXX- State 1: Independence, State 2: 90° Clayton .
Oil-EUROSTOXX|USDEUR- State 1: Gaussian, State 2: Clayton.

B: *Oil-USDEUR*- State 1: Student, State 2: Gaussian
USDEUR-EUROSTOXX- State 1: Independence, State 2: 90° Clayton .
Oil-EUROSTOXX|USDEUR- State 1: Independence, State 2: Clayton.

C: *Oil-USDEUR*- State 1: Student, State 2: Gaussian
USDEUR-EUROSTOXX- State 1: Independence, State 2: 90° Clayton .
Oil-EUROSTOXX|USDEUR- State 1: 90° Gumbel, State 2: Clayton.