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Near field of stacked diffraction gratings

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ABSTRACT

We obtain a general analytical formulation for determining the near field produced by N diffraction gratings disposed in stack using a scalar approximation. Parameters of the gratings such as type-amplitude/phase-, fill factors, periods and relative positions between gratings along the x and y axes are considered. The obtained formulation is useful for analyzing problems which involve several diffraction gratings, such as optical encoders since it is computationally faster than integral formulations. Finally, analytical results are compared with numerical simulations based on the Rayleigh–Sommerfeld equation.

1. Introduction

Diffraction gratings are elements which periodically modulate one or more properties of the impinging light [1]. They can be found in numerous applications and different areas of research, such as spectroscopy, optical metrology, and Moiré interferometry [2–5]. In the near field, systems with 1 and 2 gratings have been well analyzed since they produce Talbot and Moiré effects, respectively [6,2]. Nevertheless, there exist optical devices which use three and even more diffraction gratings in stack as, for example, optical encoders [7–10], or other applications [11–13]. Besides, optical devices in which the beam passes through the same grating several times can also be found [14].

When the optical device has more than two gratings, or the light beam passes through the grating more than twice, the problem of determining the field after them, although simple, becomes tedious. For example, optical linear encoders with 3 diffraction gratings have been analyzed by Crespo et al. [7]. Also, a multi-grating system has been studied for Lau effect. In this case, the intensity distribution for N gratings is described as a joint correlation amongst the functions of gratings [15]. The use of optical elements in stack has also been used for improving the diffractive focusing capabilities of X-rays using stacked Fresnel π-plates [16]. In these two last cases, the integrals involved cannot be solved and the results are given in an integral form. Goodman analyzes how to describe the optical propagation process in a complex optical system [17], although an analytical non-integral solution to the problem is not proposed.

In the present work, we calculate an analytical general expression for the near-field diffracted by a number of diffraction gratings disposed in stack when they are illuminated by a monochromatic plane wave. An scalar approach has been applied, considering Fresnel approximation. The final result, obtained after solving the involved integrals, can be applied, for example, in the analysis of optical encoders. The numerical performances are greatly improved when integrals are not present into the computation.

2. Theoretical description

A sketch of the problem is shown in Fig. 1. A monochromatic plane wave, with wavelength λ impinges on a system composed by N diffraction gratings, separated distances $z_1, z_2, \ldots, z_{N-1}$ to each other. The observation plane is placed at a distance z_N from the last diffraction grating. Considering Thin Element Approximation, their transmittances can be described as Fourier series expansions

$$t_j(x_j) = \sum_{n_j} a_{j,n_j} \exp[i q_j n_j (x_j + \Delta x_j)], \quad j = 1, 2, \ldots, N, \tag{1}$$

where $i = \sqrt{-1}$, a_{j,n_j} are the Fourier coefficients for the grating j-th with n_j integer, $q_j = 2\pi/p_j$, and p_j is the period of the grating j-th.

The purpose of this work is to reach a general non-integral expression to describe the near field produced by a system of N stacked gratings. Firstly, let us show the field produced by one diffraction grating. Considering the Fresnel formalism and plane wave illumination, the field, $U_1(x_1, z_1)$, at a distance z_1 from the grating can be obtained solving [18]

$$U_1(x_1, z_1) = \frac{(1+i)\exp(ikz_1)}{\sqrt{2\lambda z_1}} \int_{-\infty}^{\infty} t_1(x_0) \exp\left[i\frac{k}{2z_1}(x_1 - x_0)^2\right] dx_0, \tag{2}$$

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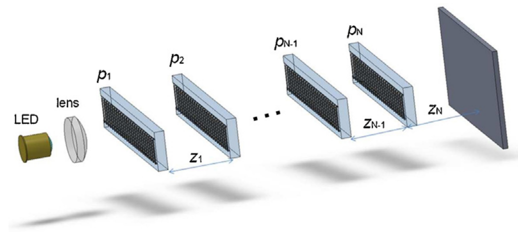

Fig. 1. Sketch of the system with N stacked gratings

being $k = 2\pi/\lambda$ and x_0 the x-coordinate at the grating plane. This integral is easily solved[19]

$$U_1(x_1, z_1) = \frac{\exp(ikz_1)}{\sqrt{\lambda z_1}} \sum_{n_1} a_{1,n_1} \exp\left[\frac{2\pi i n_1 (x_1 + \Delta x_1)}{p_1}\right] \times \exp\left(\frac{-2\pi i n_1^2 z_1}{z_T}\right), \tag{3}$$

where $z_T = 2p_1^2/\lambda$ is the so-called Talbot distance. The intensity after the grating is obtained from the field (Eq. (3)) just performing $I_1(x_1, z_1) = U_1(x_1, z_1) U_1^*(x_1, z_1)$

$$I_1(x_1, z_1) = \frac{1}{\lambda z_1} \sum_{n_1, n'_1}^{\infty} a_{1,n_1} a^*_{1,n'_1} \exp\left[\frac{2\pi i (n_1 - n'_1)(x_1 + \Delta x_1)}{p_1}\right] \times \exp\left[\frac{-2\pi i (n_1^2 - n'^2_1) z_1}{z_T}\right]. \tag{4}$$

Eq. (4) reveals the well known Talbot effect [6], which consists of the apparition of replications of the grating intensity pattern at integer multiples of the Talbot distance from the grating.

Now, let us calculate the field diffracted by two gratings in tandem. The second grating is placed at a distance z_1 from the first one. The field after the second grating, $U_2(x_2; z_1, z_2)$, is obtained by multiplying Eq. (3) by the transmittance of the second grating and propagating the field to a distance z_2 from the second grating. Solving the integral, it results

$$U_2(x_2; z_1, z_2) = \frac{\exp[ik(z_1 + z_2)]}{\lambda\sqrt{z_1 z_2}} \sum_{n_1, n_2} a_{1,n_1} a_{2,n_2} \times \exp\{i[n_1 q_1 (x_2 + \Delta x_1) + n_2 q_2 (x_2 + \Delta x_2)]\} \times \exp\left(-i\frac{q_1^2 n_1^2 z_1}{2k}\right) \exp\left[-i\frac{(n_1 q_1 + n_2 q_2)^2 z_2}{2k}\right]. \tag{5}$$

In the same fashion as the previous case, the intensity after the second diffraction grating, $I_2(x_2; z_1, z_2) = U_2(x_2; z_1, z_2) U_2^*(x_2; z_1, z_2)$, results in

$$I_2(x_2; z_1, z_2) = \frac{1}{\lambda^2 z_1 z_2} \sum_{n_1, n'_1, n_2, n'_2} a_{1,n_1} a^*_{1,n'_1} a_{2,n_2} a^*_{2,n'_2} \times \exp[i(n_1 - n'_1) q_1 (x_2 + \Delta x_1) + i(n_2 - n'_2) q_2 (x_2 + \Delta x_2)] \times \exp\left[\frac{-i(n_1^2 - n'^2_1) q_1^2 z_1}{2k}\right] \times \exp\left\{\frac{-i[(n_1 - n'_1) q_1 + (n_2 - n'_2) q_2][(n_1 + n'_1) q_1 + (n_2 + n'_2) q_2] z_2}{2k}\right\}. \tag{6}$$

Classical Moire effect can be explained based on the previous equation. It occurs when the period of both gratings is equal, $q_2 = q_1$, $z_1 = m z_T$ with m integer, and the observation plane is placed just after the second grating, $z_2 = 0$ [19].

In a similar way, we can obtain the field, $U_3(x_2; z_1, z_2, z_3)$, for the case of three gratings

$$U_3(x_2; z_1, z_2, z_3) = \frac{\exp[ik(z_1 + z_2 + z_3)]}{\lambda^{3/2}\sqrt{z_1 z_2 z_3}} \sum_{n_1, n_2, n_3} a_{n_1,1} a_{n_2,2} a_{n_3,3} \times \exp\{i[n_1 q_1 (x_3 + \Delta x_1) + n_2 q_2 (x_3 + \Delta x_2) + n_3 q_3 (x_3 + \Delta x_3)]\} \times \exp\left(-i\frac{q_1^2 n_1^2 z_1}{2k}\right) \times \exp\left[-i\frac{(n_1 q_1 + n_2 q_2)^2 z_2}{2k}\right] \times \exp\left[-i\frac{(n_1 q_1 + n_2 q_2 + n_3 q_3)^2 z_3}{2k}\right]. \tag{7}$$

Analyzing the results obtained for one, two, and three gratings, it is possible to extract a general analytical solution for the case of N stacked gratings at the near field. The complex field after the N-th grating, at a distance z_N from it, is given by

$$U_N(x_N; z_1, \ldots, z_N) = \frac{1}{\lambda^{N/2}} \prod_{j=1}^{N} \frac{1}{\sqrt{z_j}} \sum_{n_j} a_{j,n_j} \exp(ikz_j) \exp[i n_j q_j (x_N + \Delta x_j)] \times \exp\left[-\frac{i}{2k}\left(\sum_{h=1}^{j} n_h q_h\right)^2 z_j\right]. \tag{8}$$

Using Eq. (8), a general analytical solution for the intensity can also be obtained, $I_N(x_N; z_1, \ldots, z_N) = U_N(x_N; z_1, \ldots, z_N) U_N^*(x_N; z_1, \ldots, z_N)$, resulting in

$$I_N(x_N; z_1, \ldots, z_N) = \frac{1}{\lambda^N} \prod_{j=1}^{N} \frac{1}{z_j} \sum_{n_j, n'_j} a_{n_j,j} a^*_{n'_j,j} \exp[i q_j (n_j - n'_j)(x_N + \Delta x_j)] \times \exp\left\{-\frac{i}{2k}\left[\sum_{h=1}^{j} (n_h - n'_h) q_h\right]\left[\sum_{l=1}^{j} (n_l + n'_l) q_l\right] z_j\right\}. \tag{9}$$

3. Numerical comparison

As an example, the intensity computed with Eq. (8) after a system with 1–5 Ronchi amplitude gratings in stack is shown in Fig. 2a. The period for the gratings is $p_j = 20\ \mu$m, they are separated $z_j = 50\ \mu$m along the z axis and $\Delta x_j = 0$ along the x axis. The figure shows the intensity after the last grating for each system of gratings. The wavelength is $\lambda = 0.5\ \mu$m. For the computation, diffraction orders $n_j = -3, -1, 0, 1, 3$ have been considered, since they present ≈95% of the total energy for this kind of gratings in transmission configuration. These results are compared to those obtained with a numerical integration of the Rayleigh–Sommerfeld formula [20]. As it can be observed in Fig. 2b, the results are quite similar. We can see that light concentrates as the number of gratings increases, producing a kind of array illuminator, as it is shown in [21]. The differences between analytical and numerical results are due to truncations of terms in the analytical equation.

4. Conclusions

In this work, we obtain an analytical non-integral solution for the diffracted optical field after N diffraction gratings in stack. The result is valid for the near field in scalar approach, that is, when the period of the grating is much larger than wavelength, which can be useful for the analysis of optical encoders and other optical devices where light passes through many diffraction gratings. Finally, we compute an example and compare the analytical results

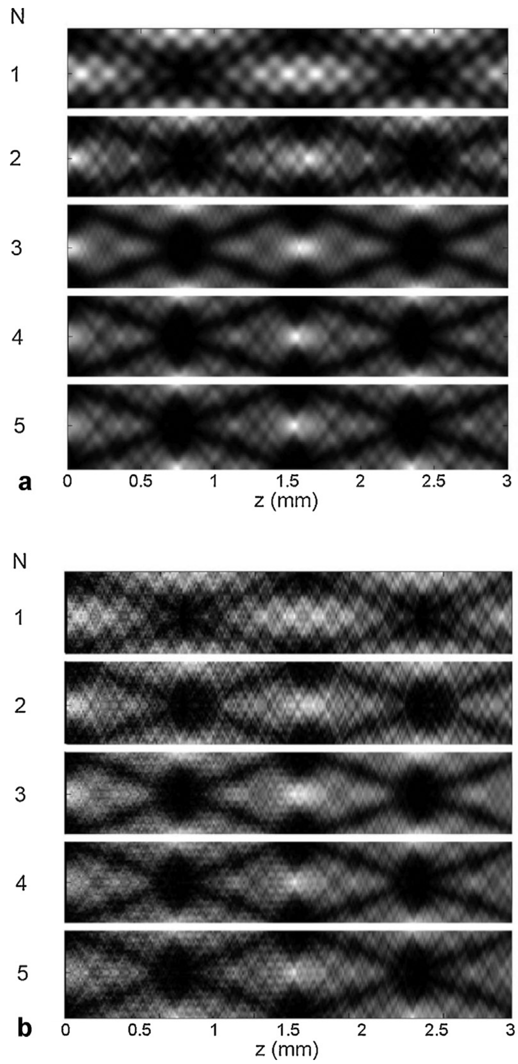

Fig. 2. (a) Analytical and (b) numerical intensity distribution at a distance z from the last grating for the case of 1–5 Ronchi gratings in stack. The period for all the gratings is $p_j = 20\ \mu m$, and they are separated $z_j = 50\ \mu m$ along the z axis and $\Delta x_j = 0$ along the x axis. The wavelength is $\lambda = 0.5\ \mu m$. The intensity after the last grating is shown.

with those obtained by using a numerical implementation of the Rayleigh–Sommerfeld formula, showing the similarities between them.

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