# Analytical method to measure bending deformations in prismatic optical films 

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#### Abstract

The aim of this work is to provide an analytical method based on experimental measurements in order to obtain the prismatic film deformation for different curvatures of Hollow Cylindrical Prismatic Light Guides (CPLG). To conform cylindrical guides is necessary bend the film to guide the light, changes induced by curving the film give rise to deformation shifts. Light losses affected by deformation has been experimentally evaluated and numerically analyzed. The effect of deformation in prism angle is specially increased for CPLG of curvatures higher than $20 \mathrm{~m}^{-1}$. An experimental method for accurate transmittance measurements related to bending is presented.


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The guiding of light is a valuable challenge for the industry. Many studies have been done according to many parameters like efficiency, health ambient and design $[1,2,3]$. The Hollow Cylindrical Prismatic Light Guide (CPLG) has the shape of a transparent light guide whose walls are composed of extruded prism ${ }^{[4]}$, they offer a technical alternative as light transport in the standard tubular reflective guidance systems. The hollow light guide is internally covered with prismatic film whose outer face is composed of $90^{\circ}$ microprismatic structure longitudinal to the axis of the guide. Light suffer total internal reflection (TIR) in the prismatic structure which has been developed and manufactured using dielectric materials. During the last years, diverse geometrical and photometrical analysis were proposed due to the need of optimization and manipulation of the light output and its distribution in several applications ${ }^{[5,6]}$. Different studies had been shown in order to prove experimentally that the standards of quality of prismatic guides are high enough to compete favorably with other available types of light guides like aluminum ones, in which light suffers from intrinsic loss of energy at each surface reflection. In light guidance structures like fiber optic guides, the optical radiation can waste its energy within a dielectric structure ${ }^{[7,8]}$. In contrast, the CPLG is an optically ideal device from the point of view of the propagation of light ${ }^{[4]}$.

Prismatic films are composed of microprismatic structures whose width is usually of the order of several hundred micrometers and are usually made of acrylic resin materials or clear grade optic polycarbonate.

Considerations related to light losses of prism film are very important from the point of view of long distance transmission of light. Although such light guides can be fairly efficient, losses in prism light guides are influenced by absorption, scattering, attenuation
coefficients and optical imperfections ${ }^{[9,10]}$. Hence, it is important to consider the imperfections of the surfaces of the prismatic microstructure: surfaces which are not optically flat or which deviate from the expected angle, optical inhomogeneities in the material, and the existence of surface irregularities on the prism peaks which could modify the optical behavior of the prismatic film. Furthermore, there are mechanical deformations on the prismatic surfaces due to the induced bend in the prismatic film. In this letter, it is provided a characterization method based on experimental measurements in order to obtain the deformation rate in the geometry for different bending of the prismatic film used in Hollow Cylindrical Prismatic Light Guides (CPLG). The influence of the light losses affected by deformation has been evaluated.

This letter is organized as follows. Firstly, an experimental recognition procedure algorithm has been developed in order to measure the deformation in the geometry of the prisms structure of a prismatic film sheet for different curvatures by image processing techniques. The curvature $k$ of a circle of radius $R$ is defined to be the reciprocal of the radius $(k=1 / R)$. We apply the Hough Transform (HT) ${ }^{[11]}$ to a line-enhanced binary image to generate a Hough domain image for detecting lines and angles associated to curvature defects. After that, experimental procedures and software simulations have been carried out to quantify luminous flux losses due to mechanical deformations. Finally, conclusions and future developments are presented.

CPLG can have diameters ranging from 3 to 50 cm and the length varies from 1 meter to more than 20 meters for larger diameter guides. They can be produced in a variety of sizes which may be required for specific applications. To predict the reproducibility in the output
flux and then optimize the design, we define the angle deformation range of a prismatic structure with diameter changes. By considering different stages of one experimental deformation process, it is possible to compute the incremental changes associated to $k$. In order to investigate the defects of prismatic structures in the system, computer analysis such as changes in prism angle, defined as the angle between flat surfaces, and plane shape were carried out in different curvatures $k$ of prismatic film: 0 (plane), 6.67, 10.00, 20.00, 40.00, 57.14 and $66.67 \mathrm{~m}^{-1}$. For this purpose, the profile of a prismatic film sample with the desired curvature was analyzed by digital imaging processing (Fig.1). The Optical Lighting Film has a base of 356 wide and its peak height is 178 $\mu \mathrm{m}$.

A precision cutting was performed on transverse section in film to avoid damaging the material, after that, the sample was covered by evaporating magnesium oxide onto the surface in order to improve the image contrast and to avoid the error caused by the lighting in the changes of curvature.


Figure 1. Image of a cross-section of the prism film used to analyze the contour (57X).

By processing the image of the prism profile and using morphological operations, measurement of the inclination angle of the profile with high accuracy can be achieved. The optical microscope used to obtain the prism image is a Motic SMZ-143 equipped with a digital camera (Moticam 2000).

The HT is an effective technique for detecting and finding the images within noise in the straight-line detection. We applied the HT for prism angle detection. First, a threshold is applied to the input image in order to make it binary. The threshold value of a binary image is determined from a grey-level histogram of the image and later the edge is separated from the background. The profile edge description is obtained from the operator Canny ${ }^{[12]}$, this operator is considered as an optimal edge objects and hard edges. The Canny method finds edges by looking for the local maxima of the gradient of the image; the gradient is calculated using the derivative of a Gaussian filter. The method uses two thresholds to detect strong and weak edges, and includes the weak edges in the output only if they are connected to strong edges. Later, we used the HT to detect the parameters that control the accuracy of the right angle at the vertex of the prism. The HT is used to identify the parameters of the line and it uses the parametric representation of a straight line $y=m x+b$, where $m$ is the slope or gradient of the line, $b$ is the $y$ intercept of the line, the point where the graph of the line crosses the $y$-axis, and $x$ is the independent variable of the function $y=\mathrm{f}(x)$. This parameter is fitted to a set of given edge points (see Fig.2). Fig. 1 shows the plot of the prismatic film profile with Hough lines identified through the
digital processing for prism film curvature of $20 \mathrm{~m}^{-1}$ in red color.


Figure 2. Edge map (black color) with Hough lines identified (red color) in prismatic film profile with curvature of $20 \mathrm{~m}^{-1}$.

After that, the HT uses a polar representation $\rho=$ $x \cos (\theta)+y \sin (\theta)$, where $\rho$ indicates the perpendicular distance from the origin, located at $(1,1)$, to the line and $\theta$ the angle in which it is shown the sum of intensities in the image peaks. It takes as input the grey scaled image, and produces as output, an image showing the positions of tracked intensity discontinuities. The result of the HT is stored in a matrix that can be considered an accumulator (Fig. 3). One dimension of this matrix shows the angles $\theta$ and the other dimension shows the distances $\rho$. Moreover, each element has a value of the pixels which are positioned on the line with parameters ( $\rho, \theta$ ). Thus, the element with the highest value shows the line most represented in the input image. The resolution of the digitization will determine how well we can estimate the gradient of the curve. Therefore, optimum detection features parameters as rhoresolution and thetaresolution are selected to object representation. Rhoresolution was set on 0.4 to build the accumulation matrix with jump between two consecutive pixels, and thetaresolution was set on 0.05 , it represents the spacing of the Hough transform bins along the $\theta$ axis. The HT determines what the features are and how many of them exist in the image. The maximum value in the $\rho$ plane corresponds to the parameters of the straight line with the most amounts of points. The highest peaks located in the HT matrix of prism film curvature of $20 \mathrm{~m}^{-1}$ are shown in Fig. 3. The red square shows the peaks of data in the Hough matrix of the three prisms image used to obtain a mean value. Every peak corresponds to the slope of a prism, the negative angle corresponds to the right slopes of the image and the positive angle corresponds to the left slopes.


Figure 3. The HT of the three prisms image of prismatic film curvature of $20 \mathrm{~m}^{-1}$ with Hough peak identified. The red square shows the peaks of data in the Hough matrix indicates $\rho$ the perpendicular distance from the origin to
the line and $\theta$ the angle at which the sum of intensities in the image peaks.
Fig. 4(a) shows the relationship between analyzed curvatures and the Hough peak obtained after HT represented as prism angle obtained by the sum of the corresponding positive and negative slope ( $\theta$ ) of the measure. The highest peak (Hough peak) of the structure analyzed becomes quite stable from non curved film or plane (referenced in figure as cero curvature) to $20 \mathrm{~m}^{-1}$ of curvature, after that, there is an increase of $2.5^{\circ}$ (from $89.5^{\circ}$ to $92^{\circ}$ ) in the curvature interval from 20.00 to 57.14 $\mathrm{m}^{-1}$. A stabilization of $92^{\circ}$ is obtained in the last curvature period (from 57.14 to 66.66 ). Although the increasing of points that belong to different angles in the contour, Hough peak remains stable. It is possible to appreciate this phenomenon in the angular deviation rate analysis illustrated in Fig. 4 (b). Polycarbonates are thermoplastic polymers that have a linear stress strain behavior at low values of pressure, described by Hook's Laws. However, mechanical response is sensitive to strain rates, especially high strain rate. The pressure applied to the prismatic geometry at curvatures higher than $20 \mathrm{~m}-1$ could increase the deformation in the prism geometry causing the gradient change in figure 4 a .


Figure 4. Angle $\theta$ obtained by HT related with CPLG curvature. The graph legend shows curvatures evaluated in $\mathrm{m}^{-1}$. Global changes in prismatic angle related with Hough peaks are shown in (a) and local changes in prism half-angle in (b).

To determine accurately changes in prism angle, a detailed analysis of the prismatic apex angle is presented in Fig. 4(b), which shows local changes in prism apex half-angles of several curvatures with regard to the plane prismatic film. A bigger angular distance rate around the main half-angles $\left(-45^{\circ}, 45^{\circ}\right.$ and $\left.0^{\circ}\right)$ is detected for higher values of curvature 20 due to the contour
deformation by the increasing number of points that belong to different angles in the edge due to the changes in film curvature. This effect correspond with the higher increase observed in Fig. 4(a). Deformation is minimal in the optimal slope (positive $45^{\circ}$ and negative $-45^{\circ}$ ) which correspond with the optimum prism half-angle apex in figure. In contrast, several peaks appear in nearby values which determine the total prism angle reproduce in Fig. 4 (a). Deformation does not increase proportionally with increasing curvature, because internal stresses of the material produced asymmetric changes in slope. Although the increasing of several points that belong to different angles in the contour, Hough peak remains stable by the prevalence of the angle shown. Hence, we can use HT as a way to classify the deformation of the prism profile.

In this section we report light losses of experimental and ray-tracing simulations of CPLG to check curvature influence. After that, measurement data are compared to evaluate the performance associated of prismatic light guides curvature.

The light losses of prismatic film used in CPLG, with a range of curvatures from plane to $57.14 \mathrm{~m}^{-1}$ were experimentally measured using physical prototypes on a real size scale to investigate the influence of curvature of prismatic film in flux transmittance (see Fig.5). Light output losses of the light guides prototypes, for several curvatures was experimentally measured using a calibrated laser. The output light beam from a $\mathrm{He}-\mathrm{Ne}$ laser (JDS Uniphase 1508 Helium Neon Gas Laser) with a wavelength of 632.8 nm was optically expanded ten times by a beam expander to be collimated to the desired beam diameter of 4.8 mm . Even though the prismatic film provides minimal spectral changes, by using light of a single wavelength the measurements are independent of the spectral responses of the CPLG. Light escaped is recorded by a Photometer (Gamma Scientific's flexOptometer) with a Si-based detector placed together with the prismatic surface. Light supplied by the $\mathrm{He}-\mathrm{Ne}$ laser is injected into the plane surface of the prismatic film for incidence angles ( $\varphi$ ) over the range of $15^{\circ}$ to $45^{\circ}$ ( $5^{\circ}$ step size) large enough to evaluate acceptance angle influence $\left(\sim 30^{\circ}\right)$ in transmittance measurements ${ }^{[15]}$.


Figure 5. Experimental setup for the transmittance measurements (upper view).

The angular incidence $\varphi$ is determined with regard to the planar surface and the alignment is related to the axis of the prisms which is in agreement with the axis of the guide. The input flux is controlled by using a 5 mm diameter diaphragm located on the flat surface of prismatic film.

The flux losses induced by curvature of each CPLG as a function of the angle of incidence of the collimated beam are shown in Fig. 6. The light flux lost $\left(\Phi_{k}\right)$ was estimated by dividing the transmission measurements through the film obtained for each particular curvature of CPLG ( $\Phi_{k i}$ ) by the transmission obtained with the corresponding incident flux ( $\Phi_{t}$ ), i.e. light flux obtained with no guide present, $\Phi_{k=} \Phi_{k i} \Phi_{t}$.


Figure 6. Light flux rate (rate of flux lost) scaled from 0 to 1.0 as a function of the curvature of a CPLG for several incidence angles. The graph legend shows incidence angles with regard to the prismatic film flat surface.

The light flux rate escaped through the film is minimum in plane structure. This escaped light will not be longer guided. After curvature deformation, there is a high increase in flux lost, resulting in a mean loss from 1.5 to 3.2 \% in flux loss of CPLG with curvatures higher to 20. Losses increase with the increasing incidence angles. For example, for the incidence angle of $30^{\circ}$, which correspond with the upper incidence angle close to the angle limit for an optimum light guiding, there is an increase in losses of $2.3 \%$ with regard to plane film. Changes in the direction of the light are intensified with curvatures higher than 33, more complex patterns of optical paths appears related with changes in slopes (see Fig. 4(b)) and thus, a slight decrease for the lower incidence angles appears.

Tridimensional simulations were carried out in a non-sequential optical ray-tracing software, TracePro 7.6 ${ }^{[13]}$ which reproduce the experimental setup previously described. Software simulations allow us to quantify losses due to deformations of prismatic film surface by comparing with experimental assembly. Escaped light affected by irregularities in prism corners and changes in the outer angular portion of prismatic structure relates to adjusting the curvature are reproduce in software simulations.

Cylindrical light guides were 3D computer modelled as hollow cylindrical piece of polycarbonate prism film material. For this simulation, a sheet with constructive parameters adapted to the prismatic material commercially called OLF (Optical Lighting Film) has been simulated in 3D CAD software. The prism base is $356 \mu \mathrm{~m}$ wide and its height is $178 \mu \mathrm{~m}$, according to
company datasheet. To check the influence of curvature in CPLG we have designed 3D guides with diverse curvatures from 8.00 to 57.14 . In order to wean light losses caused by changes in prismatic film curvature, we have situated one detector plane situated above to the peak of the prisms, which record the light flux transmitted through the film. The perfect prism was reproduced, as a perfect prismatic structure, in addition, a corner prism with $3 \mu \mathrm{~m}$ radius has been set to compute losses due corner defects. In order to generate a suitable 3D model for a ray-tracing evaluation is necessary to make some approximation in the geometry profile, in this case we consider the corners that include the higher width of defects in cross section as a radius of curvature $r$. This approach is made taking to account the relation of circle with the radii linearly proportional to trigonometric function: $x=2 \operatorname{rsen}(\pi / 2)$, where $x$ is considered the chord length, $\alpha$ is the angle subtended at the center by the chord $\left(90^{\circ}\right)$ and $r$ is the radius set by the enrolled circle. The wavelength was set at 632.8 nm in calculations. The refractive index material of the guide is considered 1.59 determined by using a polycarbonate polymer with a linear absorption coefficient of $1 \cdot 10^{-3} \mathrm{~mm}^{-1}$ according with datasheet. The diffraction loss has not been considered [11]. For the simulation the 3 D modelled system is illuminated with a collimated emission pattern of 4.8 mm diameter spot size. The light source is an emitter which emits above the plane surface of the prismatic film incidence above the approximately critical angle $\theta$ of $30^{\circ}$. This acceptance angle limit is determined by the refractive index of the prismatic film.


Figure 7. Flux loss rate obtained in tridimensional simulations of CPLG guides with curvatures of 8.00 (a) and $57.14(\mathrm{~b})$.

Changes in curvature are reproduced by software tools to detect by comparison with the experimental model, changes in flux related to surface deformations of prisms. The main results of simulations with curvatures of 8.00 and 57.14 are presented in Figure 7, which correspond with a sample of extreme values experimentally analyzed. Flux loss rate in film of curvature 8.00 is $1.3 \%$, in case of 57.14 the change perceived is minimally important being of the order of $1.5 \%$. Flux losses related to corner defects corresponds to $1.3 \%$, the difference between two curvatures shows is minimal ( $0.2 \%$ ) and correspond with an induced increase of $1.13^{\circ}$ in the outer angular portion of the prismatic structure related to adjusting the curvature of CPLG.

In terms of comparison with the results of the experimental prototype we can conclude based on
differences between the theoretical and experimentals (Fig.6) that there is a percentage of light losses estimated in $\sim 1 \%$ due to scratches, cracks, powder, material inhomogeneity and measurement errors. The theoretical values are smaller than the experimental values and show influence by surface deformations. In simulations, curvature of $57.14 \mathrm{~m}^{-1}$ show an increase of lateral losses due to the presence of angles greater than the acceptance angle of the prismatic film (Fig.7 (b)). This behavior explains results for curvatures from 44 to 57 (see Fig. 6), in those cases a fraction of light scape uncontrollably and consequently errors increase. Several rays suffer additional TIRs on the inner prismatic surfaces and then, some rays undergo multiple reflexions on the inner material. In addition, a fraction of flux is guided inside the guide and escape outward in subsequent reflections. Additional research has been development to analyze these specific behaviors of ray lights and will be explained in detail in future works.

In conclusion, a new method to assess the influence of the bending-induced mechanical stress of flexible prismatic film used in light guides is proposed. Through a HT algorithm we evaluate the experimental deformation angle in prisms for pattern detection applied to a sequence of images of prismatic film with different curvature. According to optical analysis, it is concluded that bending deformation has a raised influence in light guides having diameters from 0.1 m to 0.03 m (curvatures from 20 to $66.66 \mathrm{~m}^{-1}$ ). The description of the local and global changes in prism apex half-angle angle helps us to evaluate the prism deformation through experimental measurements.

Such geometry changes induced by bending give rise to deformation shifts. We analyzed how light light behaves in response to curvature. Experimental measurements shows that light transmission changes in response to curvature providing losses of about 2.4 times in a medium diameter evaluated ( $22 \mathrm{~m}^{-1}$ ) for an acceptance angle of $30^{\circ}$. We concluded based on theoretical considerations that there is an amount of losses due to defects in prism corners and small structural changes of prismatic structure. Experimental comparison allow us to estimate the percentage of light losses due to scratches, cracks, powder, material inhomogeneity and measurement errors.

It is important to analyze the existence of bending defects in the surface on the prism film which can modify the behavior of light beam and as a consequence the rays are directed to other directions instead of undergoing TIR.

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