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Born-Infeld effects in the electromagnetic mass of an extended Dirac particle

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The electromagnetic mass of an extended particle with spin is studied in Born-Infeld theory.

I. INTRODUCTION

In the last few years several models of extended fermions have been shown to provide a fair account of the main properties of the nucleons.[1-6] In particular they give a good description of the electromagnetic radii and the form factors at low momentum transfer. All these models are based on a Dirac field with a fourth-order self-coupling term which allows the existence of finite-energy, localized solitary waves, or particlelike solutions, which are used to describe elementary fermions. In the spirit of this line of work it is natural to ask about the effect of nonlinearities in the electromagnetic Lagrangian density. Fortunately, there is a very interesting nonlinear theory, the Born-Infeld electrodynamics,[7,8] which has not been studied much, most probably because of the great difficulties of its quantum development.

Born and Infeld obtained a very interesting solution corresponding to a point charge without a magnetic moment, other solutions being obtained later on.[9] It is natural to ask about a similar solution coupled to an extended charge density. In 1973 Soler[10] studied the particlelike solutions of this model of elementary fermions[1] in interaction with its own electromagnetic field. He found some interesting results, for instance, a maximum value for a parameter which characterizes the ratio between the electromagnetic interaction and the spinor self-coupling (to be called ϵ in Sec. II) above which there are no particlelike solutions, the electrostatic self-repulsion being too large. As this is the basis of the nucleon models previously mentioned it is natural to ask about the modifications that the Born-Infeld theory would impose on Soler's results. There is an expected decrease of the electromagnetic mass without a change in the Coulomb field outside the particle.

Let us finally point out that the Born-Infeld theory has been shown to possess very exceptional properties concerning the propagation of its solutions and the nonappearance of shock waves.[11] Among the nonlinear theories whose linearization is the Maxwell theory it has a very special position.

II. DESCRIPTION OF THE MODEL

The model is based on the Lagrangian density

$$L = L_D + L_{\text{em}} + L_I \,, \tag{1}$$

where

$$L_D = \tfrac{1}{2} i[\bar\psi\gamma^\mu \partial_\mu\psi - (\partial_\mu\bar\psi)\gamma^\mu\psi] - m\bar\psi\psi + \lambda(\bar\psi\psi)^2, \quad \lambda > 0 \tag{2}$$

$$L_{\text{em}} = b^2\left\{1 - \left[1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16b^4}(\tilde F_{\mu\nu}F^{\mu\nu})^2\right]^{1/2}\right\}, \tag{3}$$

$$L_I = -e\bar\psi\gamma^\mu\psi A_\mu \,. \tag{4}$$

The field equations are

$$i\gamma^\mu\partial_\mu\psi - m\psi - e\gamma^\mu\psi A_\mu + 2\lambda(\bar\psi\psi)\psi = 0 \,, \tag{5a}$$

$$\partial_\lambda\left\{\frac{F^{\lambda\mu} - \frac{1}{2b^2}(\tilde F_{\alpha\beta}F^{\alpha\beta})\tilde F^{\lambda\mu}}{\left[1 + \frac{1}{2b^2}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{16b^4}(\tilde F_{\alpha\beta}F^{\alpha\beta})^2\right]^{1/2}}\right\} = e\bar\psi\gamma^\mu\psi \,, \tag{5b}$$

where $\tilde F_{\alpha\beta}$ is the tensor dual to $F_{\alpha\beta}$.

The solution to these equations is a very difficult problem because they do not factorize in spherical coordinates. A variational approximation is not adequate because of the square root. The reason is that the expression for the integral over the angles of the Lagrangian evaluated with some trial functions depends on the value of the radial functions. The procedure would be therefore very difficult. Nevertheless, the effect of the magnetic field must be small in the physical situations. We can therefore neglect the magnetic effects as a first approximation. Fortunately enough the radial equations for a particlelike solution are easily obtained in this case.

Let us take the following form of the fields:

$$\psi = e^{-i\Omega m t}(m/2\lambda)^{1/2}\begin{pmatrix} G\begin{pmatrix}1\\0\end{pmatrix} \\ iF\begin{pmatrix}\cos\theta\\ e^{i\varphi}\sin\theta\end{pmatrix}\end{pmatrix}, \quad A^\mu = \left(\frac{m}{e}V, \vec 0\right), \tag{6}$$

where G, F, V are dimensionless radial functions and Ω is the frequency of the spinor in units of the mass parameter m.

The exact radial equations are

$$F' + \frac{2}{\rho}F - (1 - \Omega + V + F^2 - G^2)G = 0\,,$$
$$G' + (1 + \Omega - V + F^2 - G^2)F = 0\,, \tag{7}$$
$$V'' + \frac{2}{\rho}V'(1 - \eta V'^2) = -\epsilon(F^2 + G^2)(1 - \eta V'^2)^{3/2}\,,$$

where $\rho = mr$, $\epsilon = e^2/2\lambda m^2$, and $\eta = m^4/e^2b^2$. The symmetric energy-momentum tensor is

$$T^{\alpha\beta} = \frac{i}{4}[\bar{\psi}\gamma^\alpha\partial^\beta\psi - (\partial^\beta\bar{\psi})\gamma^\alpha\psi - (\partial^\alpha\bar{\psi})\gamma^\beta\psi + \bar{\psi}\gamma^\beta\partial^\alpha\psi] + \frac{F^{\lambda\alpha}F^{\beta}_{\lambda}}{\left(1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu}\right)^{1/2}} - \frac{e}{2}(\bar{\psi}\gamma^\alpha\psi A^\beta + \bar{\psi}\gamma^\beta\psi A^\alpha) + g^{\alpha\beta}\left[\lambda(\bar{\psi}\psi)^2 + b^2\left(1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu}\right)^{1/2} - b^2\right]\,, \tag{8}$$

where we have omitted the terms which depend on the magnetic field. From (8) the following expression of the energy is obtained:

$$E = \frac{2\pi}{\lambda m}\left(\Omega I_2 + \frac{1}{2}I_2 + \frac{1}{\epsilon\eta}I_3\right)\,, \tag{9}$$

where

$$I_1 = \int_0^\infty (F^2 + G^2)\rho^2 d\rho\,,$$
$$I_2 = \int_0^\infty (F^2 - G^2)^2\rho^2 d\rho\,,$$
$$I_3 = \int_0^\infty [(1 - \eta V'^2)^{1/2} - 1]\rho^2 d\rho\,.$$

III. NUMERICAL RESULTS

The field equations (7) were solved by the same procedure described in the paper by Soler.[10]

The importance of the nonlinear Born-Infeld effects is characterized by what is called the Born-Infeld radius $r_0 = (e/4\pi b)^{1/2}$, which is the radius at which the Coulomb electric field takes the value b. The smaller r_0 is, the weaker are the nonlinear effects. The relation between r_0 and η is

$$\eta = \frac{4\pi(mr_0)^4}{e^4}. \tag{10}$$

We have obtained the solution of (7) for the values $\eta = 10^2$, 10^6, 10^7. For the case where the nonlinear Dirac field is used to represent the nucleons, the mass parameter is of the order of the nucleon mass and these values of η correspond to about 0.10 fm, 1.1 fm, and 1.9 fm. It is not interesting to consider higher values of η because if r_0 is greater than the radius of the nuclei the nonlinear effect would be easily detected in atoms. If, however, $r_0 \lesssim 1$ fm, they could be relevant in the structure of the nucleons but very weak in the atoms.

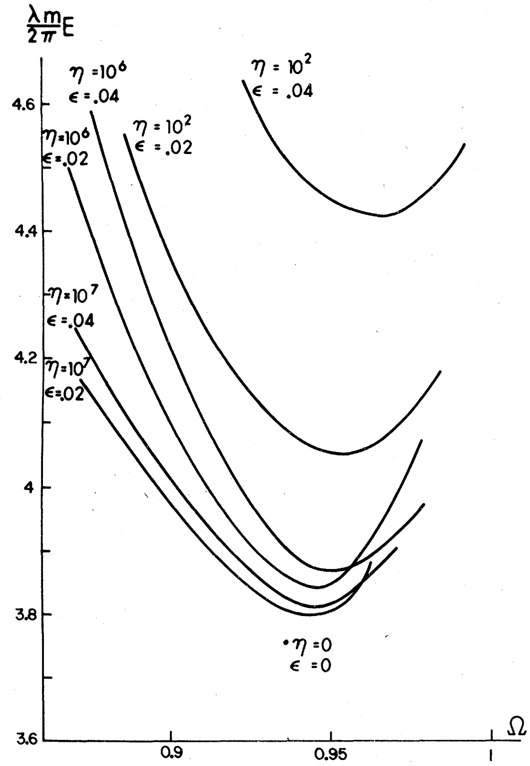

FIG. 1. Energy dependence with different values of ϵ and η.

As was already pointed out by Born and Infeld themselves, the nonlinearity tends to weaken the electromagnetic interaction. This is in agreement with our results. In Fig. 1 we have represented the curve $E(\Omega)$ for the three mentioned values of η. If $\eta = 10^2$ we find almost the same results as for $\eta = 0$ so that the difference cannot be appreciated in the drawing. However, it is clear that for $\eta = 10^6$ and 10^7, when the nonlinearity is relevant, the electromagnetic mass or self-energy is much weaker. If $\eta = 0$ Soler obtained that the minimum of the energy, the frequency, and the parameter ϵ were related by

$$E - E_0 = a\epsilon + b\epsilon^2 = \sigma(\Omega - \Omega_0)\,, \tag{11}$$

where

$$a = 13.0\frac{2\pi}{\lambda m},\quad b = 86.6\frac{2\pi}{\lambda m},\quad \sigma = 21.46\frac{2\pi}{\lambda m}.$$

We have found the following behavior:

$$E - E_0 = a\epsilon + b\epsilon^2 = \sigma_1(\Omega - \Omega_0) + \sigma_2(\Omega - \Omega_0)^2$$

with the values

$$\eta = 10^6,\quad a = 6.05\frac{2\pi}{\lambda m},\quad b = -82.05\frac{2\pi}{\lambda m},$$

$$\sigma_1 = 13.51\frac{2\pi}{\lambda m},\quad \sigma_2 = 414.7\frac{2\pi}{\lambda m},$$

$$\eta = 10^7,\quad a = 3.67\frac{2\pi}{\lambda m},\quad b = -66.25\frac{2\pi}{\lambda m},$$

$$\sigma_1 = 14.27\frac{2\pi}{\lambda m},\quad \sigma_2 = 1079\frac{2\pi}{\lambda m}.$$

IV. SUMMARY

Born and Infeld obtained a solution for their field equations which corresponds to a point charge without a magnetic moment. In this work we have studied the same type of solution in the case of an extended charge density given by a particlelike solution of a nonlinear Dirac field whose structure depends on the same electromagnetic field. This solution was compared with a similar solution obtained with the Maxwell linear theory. An important decrease in the electromagnetic mass was obtained. However, the exterior Coulomb field is not affected. This suggests the possibility that Born-Infeld electrodynamics may be useful in the study of the internal structure of extended particles. In this connection we should be reminded that one of the most difficult puzzles in theoretical physics today is the problem of the electromagnetic mass difference of the nucleons. The results of the present work indicate that the proton would be lighter in Born-Infeld than in Maxwell electrodynamics. This problem will be investigated in a forthcoming work.

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