TIME-TO-BUILD, GROWTH AND WELFARE (*)

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ABSTRACT

In an AK endogenous growth setup with a time-to-build investment technology, the steady-state growth rate and the level of welfare are shown not to be independent ot the time distribution of the financing of an investment project. We emphasize that welfare effects in the AK model are of opposite sign to those arising in an exogenous growth model when both theorical economies share the same long-run growth rate and the same steady state real interest rate.

RESUMEN

En una modelo AK de crecimiento endógeno con tecnología de inversión *time-to-build*, tanto la tasa de crecimiento de estado estacionario como el nivel de bienestar dependen de la distribución temporal de financiación de un proyecto de inversión. Además, se muestra que los efectos que sobre el bienestar provocan los cambios en la citada distribución temporal en el modelo AK tienen el signo contrario a los efectos que surgen en el modelo de crecimiento exógeno, cuando las economías modelizadas tienen la misma tasa de crecimiento de largo plazo y el mismo tipo de interés real de estado estacionario.

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1 Introduction

Time-to-build technology assumes that multiple periods are required to build new capital goods and only finished capital goods are part of the productive capital stock. Several authors (Kydland and Prescott (1982), Rouwenhorst (1991) and Christiano and Todd (1996), among others) have studied the role of investment gestation lags in shaping business cycles.

However, the way how the time pattern of investment financing affects economic growth and welfare has not been characterized yet. We show that, under a time-to-build technology, in the AK endogenous growth model, long-run growth and welfare are not independent of the time distribution of financing of an investment project. Therefore, a government might be interested in stimulating a specific investment financing structure in preference to others.

In particular, we show that firms always should build more than a half of every investment project in the first period. The specific way how they should build theirs investment projects depends on: i) the initial conditions of the economies and, ii) the intertemporal elasticity of substitution.

In addition, we study the effects of a temporal structure of investment financing on welfare in an endogenous growth model relative to the effects in a standard exogenous growth model. The results are opposite to each modelling setup. This suggest the relevance of the setup chosen to analize a given economic policy question.

The model is presented in section 2. In section 3, the effects of investment financing on long-run growth are studied. In section 4, we calibrate the model and analyze the effects of a temporal structure of investment financing on welfare. Section 5 concludes.

2 Model

The economy consists of many identical households, which have preferences defined on consumption, C_t. Assume that the utility function is:

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} \quad , \quad \sigma > 0 \quad . \tag{1}$$

Consider a representative agent who seeks to maximize the value of his lifetime utility:

$$V = \sum_{t=0}^{\infty} \rho^t U(C_t) \quad , \tag{2}$$

where $0<\rho<1$ is the utility discount factor. Production, Y_t , takes place according to an AK technology:

$$Y_t = AK_t \quad , \tag{3}$$

where A denotes a constant capital factor productivity and K_t is the capital stock at the begining of period t.

The aggregate of investment, I_t, and consumption cannot exceed output:

$$C_t + I_t \leqslant Y_t \quad . \tag{4}$$

It takes more than one period to build new productive capital. Suppose for instance that agents can only invest in projects that take J periods to complete. Denote by $S_{j,t}$ the number of projects at time t, j stages from completion, $1 \leqslant j \leqslant J$. A project that takes $j+1$ periods to complete, if started today, will be ready j periods from tomorrow:

$$S_{j,t+1} = S_{j+1,t} \quad , \qquad j = 1, 2, ...J-1 \quad . \tag{5}$$

As it is standard in time-to-build economies, we assume that the time structure of financing is the same for all investment projects. To complete a project, a representative agent must pay a proportion $\varphi_j \geqslant 0$ of it at stage j. Total investment at a point in time is therefore given by:

$$I_t = \sum_{j=1}^{J} \varphi_j S_{j,t} \quad , \qquad \text{with} \sum_{j=1}^{J} \varphi_j = 1 \quad . \tag{6}$$

Only completed projects add to the stock of productive capital:

$$K_{t+1} = (1-\delta)K_t + S_{1,t} \quad , \tag{7}$$

where $0<\delta<1$ is the rate of depreciation.

The solution to the time to build model is given by maximizing (2) subject to (3)-(7), which must hold at all t, and initial conditions K_0 and $S_{j,0,}$ $j = 1, 2, ...J-1$. The decision variables at time t are C_t and $S_{J,t}$. In particular, a balanced growth path is $\{Z_t, \gamma_{k,t+J}\}_{t=0}^{\infty}$ such that satisfied:

$$\begin{aligned} \varphi_J Z_t^{-\sigma} \quad = \quad & \sum_{j=1}^{J-1} \rho^j \left[\varphi_{J-j+1}(1-\delta) - \varphi_{J-j}\right] Z_{t+j}^{-\sigma} \left(\prod_{i=0}^{j-1} \left(\gamma_{k,t+i}\right)^{-\sigma} \right) \\ & + \rho^J \left[\varphi_1(1-\delta) + A\right] Z_{t+J}^{-\sigma} \left(\prod_{i=0}^{J-1} \left(\gamma_{k,t+i}\right)^{-\sigma} \right) \end{aligned} \tag{8}$$

$$Z_t + \sum_{j=1}^{J-1} \left[\varphi_j - \varphi_{j+1}(1-\delta)\right] \left(\prod_{i=0}^{j-1} \gamma_{k,t+i} \right) + \varphi_J \left(\prod_{i=0}^{J-1} \gamma_{k,t+i} \right) = A \tag{9}$$

where $Z_t = \frac{C_t}{K_t}$, $\gamma_{k,t} = K_{t+1}/K_t$ and the equation (8) is the standard Euler condition, and (9) is the aggregate resource constraint obtained from (4)-(7).

This model generates steady-state growth, being straightforward to show that all variables grow in steady state at the same rate, γ^*.

3 Time-to-build and growth

In this section, the growth rate γ^* is shown not to be independent of the time distribution of the financing of an investment project. For simplicity, we assume that just two periods are required to construct each investment project ($J = 2$), so $0 \leqslant \varphi_1, \varphi_2 \leqslant 1$, $\varphi_1 + \varphi_2 = 1$. Then, the conditions defining steady state are:

$$1 - \varphi_1 = [(1 - \varphi_1)(1 - \delta) - \varphi_1]\,\rho\,(\gamma^*)^{-\sigma} + [\varphi_1(1 - \delta) + A]\,\rho^2\,(\gamma^*)^{-2\sigma} \tag{10}$$

$$\left(\frac{C_t}{K_t}\right)^* + [(1 - \varphi_1)(1 - \delta) - \varphi_1]\,\gamma^* + (1 - \varphi_1)(\gamma^*)^2 - \varphi_1(1 - \delta) = A \tag{11}$$

where $\gamma^* = \left(\frac{K_{t+1}}{K_t}\right)^*$, $\forall t$, and an asterisk denotes a steady state value. The equation (10) is (8), and (11) is (9), both evaluated at steady-state under $J = 2$.

We can define the real interest rate on a j-period investment by:

$$1 + r_{j,t} = \frac{\partial U_t / \partial C_t}{\rho^{t+j} \partial U_{t+j} / \partial C_{t+j}}, \quad j = 1, 2. \tag{12}$$

Therefore, the steady state j-period real interest rate is:

$$1 + r_j^* = \frac{1}{(\gamma^*)^{-j\sigma}\,\rho^j}, \quad j = 1, 2. \tag{13}$$

Lemma 1: *Under full depreciation, the steady state growth rate is an increasing (decreasing) function of φ_1 if and only if the steady state real interest rate is positive (negative).* [1]

Proof. Denote by $x = (\gamma^*)^{-\sigma}\rho$, the inverse of the steady state level of the 1-period interest rate $(1 + r_1^*)$ [see (13)], to write the only positive root of (10), when $\delta = 1$, as:[2]

$$x^+ = \frac{\varphi_1 + \sqrt{\varphi_1^2 + 4A(1 - \varphi_1)}}{2A} \tag{14}$$

So that

$$\frac{\partial x^+}{\partial \varphi_1} = \frac{1}{2A}\left[1 + \frac{\varphi_1 - 2A}{\sqrt{\varphi_1^2 + 4A(1 - \varphi_1)}}\right] = \frac{1}{2A}\left[1 + \frac{\varphi_1 - 2A}{\sqrt{(2A - \varphi_1)^2 - 4A(A - 1)}}\right]. \tag{15}$$

The reader can check that (15) is analogous to:

$$\frac{\partial x^+}{\partial \varphi_1} = \frac{1}{2A}\left[1 + \frac{1}{g(\varphi_1)}\right], \quad \text{if} \quad A \leqslant \varphi_1/2 \tag{16}$$

[1] When depreciation is not full, the result still holds but it can only be shown numerically.

[2] The other root of (10): $x^- = \frac{\varphi_1 - (\varphi_1^2 + 4A(1 - \varphi_1))^{1/2}}{2A}$ is negative. This implies $1 + r_1^* < 0$, which it is not very likely in actual economies. So, attention is not paid to this case.

$$\frac{\partial x^+}{\partial \varphi_1} = \frac{1}{2A} \quad \left[1 - \frac{1}{g(\varphi_1)}\right], \quad \text{if} \quad A \geqslant \varphi_1/2 \tag{17}$$

where $g(\varphi_1) = \sqrt{1 - \frac{4A(A-1)}{(2A-\varphi_1)^2}}$, and that (14) is similar to:

$$x^+ = \frac{\varphi_1 + \sqrt{(2A-\varphi_1)^2 - 4A(A-1)}}{2A}. \tag{18}$$

If $A > 1$, we have:

i) $x^+ < 1$ since in (18): $\sqrt{(2A-\varphi_1)^2 - 4A(A-1)} < 2A - \varphi_1$. So, $r_1^* > 0$.

ii) It is easy to show that $0 < g(\varphi_1) < 1$, for all φ_1. So, from (17), $\frac{\partial x^+}{\partial \varphi_1} < 0$ and from the definition of x, it is obvious that $\frac{\partial \gamma^*}{\partial \varphi_1} > 0$.

If $A \leq 1$:

i) $x^+ \geq 1$ since $\sqrt{(2A-\varphi_1)^2 - 4A(A-1)} \geqslant 2A - \varphi_1$ in (18). So, $r_1^* \leq 0$, and,

ii) it is easy to show that $g(\varphi_1) \geq 1$, for all φ_1. From (16) if $A \leqslant \varphi_1/2$ or else from (17) it can be got $\frac{\partial x^+}{\partial \varphi_1} \geq 0$, and from the definition of x, $\frac{\partial \gamma^*}{\partial \varphi_1} \leq 0$. ■

An intuition about this lemma is given by using the Brock-Mirman neoclassical model with exogenous growth and a time-to-build technology. In this model, the production function is: $Y_t = BK_t^\alpha(\gamma^t L_t)^{1-\alpha}$, where γ is the exogenous growth rate, $L_t = 1$, $0 < \alpha < 1$ and $B > 0$. In this case, under $\delta = 1$, the steady state Euler condition similar to (10) is:

$$\frac{1 - (1-\tilde{\rho})\varphi_1}{\tilde{\rho}} = \tilde{\rho}\ PMg(\varphi_1), \tag{19}$$

where $PMg(\varphi_1)$ is the steady state marginal product of capital under $\varphi_1 \in [0,1]$, and $\tilde{\rho} = \gamma^{-\sigma}\rho$. Comparing (19) under different values of φ_1, it is easy to see that in this exogenous growth model, if $\tilde{\rho} < 1$, implying $r_1^* > 0$, the steady state stock of capital under $\varphi_1 < 1$ is smaller than under $\varphi_1 = 1$. The reason to underaccumulate capital is because it takes longer to the agent to obtain the return on his/her investment.

In the endogenous growth model described in this section, the same intuition holds. But now, since capital is the engine of growth, and $\varphi_1 = 1$ generates the highest steady state stock of capital, it also generates the highest long run growth rate steady-state growth, then decreases for lower values of φ_1.

When $\tilde{\rho} \geq 1$ it occurs the opposite in both models.

4 Time-to-build and welfare

The level of welfare of the representative agent in the model in the previous section can not be characterized analytically. Thus, we study the effects of a time-to-build technology on welfare by calibrating and numerically solving the model in a neighborhood of the steady state.

The equations that describe the dynamics of the system under $J = 2$ are:

$$(1-\varphi_1)\, Z_t^{-\sigma} = [(1-\varphi_1)(1-\delta) - \varphi_1]\, \rho\, \gamma_{k,t}^{-\sigma}\, Z_{t+1}^{-\sigma} + [\varphi_1 (1-\delta) + A]\, \rho^2\, \gamma_{k,t}^{-\sigma}\, \gamma_{k,t+1}^{-\sigma}\, Z_{t+2}^{-\sigma}\ , \tag{20}$$

$$Z_t + [(1-\varphi_1)(1-\delta) - \varphi_1]\, \gamma_{k,t+1} + (1-\varphi_1)\, \gamma_{k,t+1}\gamma_{k,t} - \varphi_1 (1-\delta) = A\ , \tag{21}$$

The log-linear approximation to the system made up by equations (20) and (21) is:

$$\Omega_1 \begin{bmatrix} X_{t+2} - X^* \\ X_{t+1} - X^* \\ g_{t+1} - g^* \end{bmatrix} = \Omega_2 \begin{bmatrix} X_{t+1} - X^* \\ X_t - X^* \\ g_t - g^* \end{bmatrix}, \tag{22}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_1(1,1) & \Omega_1(1,2) & \Omega_1(1,3) \\ 0 & 1 & 0 \\ 0 & 0 & (1-\varphi_1)(\gamma^*)^2 \end{bmatrix},$$

$\Omega_1(1,1) = \rho^2 (Z^*)^{-\sigma} (\gamma^*)^{-2\sigma} [\varphi_1 (1-\delta) + A]$,
$\Omega_1(1,2) = \rho (Z^* \gamma^*)^{-\sigma} [(1-\varphi_1)(1-\delta) - \varphi_1]$,
$\Omega_1(1,3) = \rho^2 (Z^*)^{-\sigma} (\gamma^*)^{-2\sigma} [\varphi_1 (1-\delta) + A]$,

$$\Omega_2 = \begin{bmatrix} 0 & (1-\varphi_1)(Z^*)^{-\sigma} & (Z^*\gamma^*)^{-\sigma} \rho \left[(1-\varphi_1)(1-\delta) - \varphi_1 + (\gamma^*)^{-\sigma} \rho (\varphi_1 (1-\delta) + A)\right] \\ 1 & 0 & 0 \\ 0 & -Z^* & -Z^*\gamma^* [\varphi_1 + (\gamma^* - (1-\delta))(1-\varphi_1)] \end{bmatrix},$$

and $X_t = \log Z_t$, $g_t = \log \gamma_{k,t}$, and again an asterisk denotes a steady state value. The first row in Ω_1 and Ω_2 is the log-linearization of (20), the second row is a definition and, finally, the third row is the log-linearization of (21).

Since the variable g_t in the system is predetermined and the others $(X_{t+1},\ X_t)$ are free, the system will have a unique, locally determined equilibrium when the steady state is a saddle point. This requires that matrix $\Omega = \left[\Omega_1^{-1}\Omega_2\right]$ has one characteristic root be less than 1 in absolute value and the others greater than 1. Stability conditions are ortogonality conditions between each unstable left eigenvector of the matrix Ω and the vector of variables in deviations of the steady state of (22), for all t. Stable solutions for $\{X_{t+1},\ g_t\}_{t=0}^{\infty}$ can be computed by adding the two estimated stability conditions to the equation (21). Then, we recover time series for the levels of the endogenous variables using the definitions for X_t, g_t, Z_t and $\gamma_{k,t}$, given the initial conditions (i.e., K_0, K_1).

We assume semiannual periods and every investment project is completed in two periods. In this section, parameter values are: $\delta = 0.05$, $\sigma = 1.5$, $\rho = 0.992$, $A = 0.0698$,

$\varphi_1 = 1$, $\varphi_2 = 1 - \varphi_1 = 0$. These parameter values imply an annual depreciation rate of 10%, an annual steady state growth rate of 1.5% and an annual steady state real interest rate of 4% from (10) and (13), respectively.

The stability analysis for these parameter values shows that there is one stable eigenvalue in the log-linear approximation to the model. So, although the standard AK model lacks transitional dynamics, the model described in section 2 with $J = 2$ exhibits a non-trivial transition between steady states. Besides, the transitional dynamics is oscillatory, because the only stable eigenvalue is negative. This is due to the time-to-build technology.[3]

To compare the level of welfare achieved in economies with different time patterns of project financing, we compute the percent change ϕ in consumption that an individual would require each period to be as well off as under the investment financing policy defined by: $\varphi_1 = 1$. We will present $\Delta C = \phi C_t$ as a percentage of output. Positive (negative) values of this measure mean lower (higher) welfare under $\varphi_1 \in [0, 1)$ than under $\varphi_1 = 1$.

4.1 Numerical experiments

Two experiments are made to study the time-to-build techonology effects on welfare. In each one, economies sharing a given feature are compared. The common feature is specific to each experiment.

First experiment

In the first experiment we compare economies that share the same parameter values above, except for $\varphi_{1.}$ Thus, although these economies start with the same initial conditions K_0 y K_1, each one converges to its own steady state. Figure 1 shows how long-run growth and welfare depend on the time distribution of project financing. The analysis is repeated for several values of the intertemporal elasticity of substitution $(1/\sigma)$ usually considered in RBC models. Graph A extends lemma 1 to a case with $\delta < 1$ and a positive steady state real interest rate. The lower graph shows the value of φ_1 for which welfare is maximum as a function of the initial conditions ratio (K_1/K_0). This graph only shows the results that have been obtained under $K_1/K_0 > \gamma$ since the highest level of welfare is always achieved with $\varphi_1 = 1$ under $K_1/K_0 \leq \gamma$.

Graph B shows three facts: i) initial conditions have a significant role in welfare analysis, ii) the highest welfare is never achieved with $\varphi_1 \leq 0.5$. So, the government interested in welfare never have to allow firms to use most of resources to build its investment projects in the second period. iii) The lower the intertemporal elasticity of substitution, the more the rank of the initial conditions ratio (K_1/K_0) for which maximum welfare is attained for a value of φ_1 lower than 1.

Second experiment

In the second experiment we compare economies that share the same long-run growth (γ =$1.015^{0.5}$) and the same parameter values for δ, ρ, σ (described at the beginning of this section). So, they share the same steady state interest rate (r_1^* =$1.04^{0.5}$-1), but their values of φ_1 and A differ. Besides, we assume that $K_0 = 10$ and $K_1 = 10.08$,[4] so the economy is placed outside the balanced growth path for all φ_1 ∈.[0,1].

[3] Computer programs are available on request.

[4] The results of this experiment are independent of initial conditions for K_0 y K_1.

This analysis is carried out in two different setups: the neoclassical growth model (Brock-Mirman (1972)) with exogenous growth and a time-to-build technology, and the endogenous growth model described in section 2.

Endogenous growth model calibration: For each φ_1, the value of A is chosen so that annual long-run growth is 1.5% (γ^* =1.015$^{0.5}$). From (10) and (13) we get:[5]

$$A = (1-\varphi_1)(1+r_1^*)^2 - [(1-\varphi_1)(1-\delta) - \varphi_1](1+r_1^*) - \varphi_1(1-\delta). \tag{23}$$

So,

$$\frac{\partial A}{\partial \varphi_1} = -r_1^*(r_1^*+\delta) < 0. \tag{24}$$

Therefore, the value of A chosen for an economy is lower for high values of φ_1.

Exogenous growth model calibration: We assume a production function $Y_t = BK_t^\alpha(\gamma^t L_t)^{1-\alpha}$, where γ is a parameter and $L_t = 1, \forall t$. We rewrite the production function as $y_t = Bk_t^\alpha$, where $y_t = \gamma^{-t}Y_t$ and $k_t = \gamma^{-t}K_t$ are scaled variables that do not grow at the steady state. We considerer an annual exogenous growth rate of 1.5% (γ =1.015$^{0.5}$). Given $\alpha \in (0,1)$, the parameter B is chosen such that $\alpha B(k_0)^{\alpha-1} - \delta = r_1^*$ where r_1^* is $\frac{1}{\gamma^{-\sigma}\rho} - 1$, where ρ and k_0 are the same values as in the endogenous growth economy.[6]

The reader can check that for each value of $\varphi_{1,}$ both setups share the same steady state marginal product of capital, $\alpha B(k^*)^{\alpha-1} = A$, long run growth rate and steady state levels of the real interest rates, r_1^* and r_2^*. So, both imply quite similar long run characteristics.

Figure 2 shows that the changes on welfare due to variations in φ_1 in the exogenous growth model are of opposite sign to those arising in the AK model, and this result does not depend on the value of α. Lower values of φ_1 decreases welfare, relative to financing the full project in its first period in exogenous growth model. In our AK endogenous growth economy, any φ_1 below 1 produces a higher level of welfare than $\varphi_1 = 1$. What lies beneath the result shown in figure 2 is the different behavior of steady state level of output to changes in φ_1 in each setup: (1) In the AK model, since the marginal productivity of capital, A, is chosen to decrease with φ_1, so does output. (2) In the exogenous growth model, it can be shown that the level of output increases with φ_1. For that purpose, the equation analogous to (10) is:

$$1-\varphi_1 = [(1-\varphi_1)(1-\delta) - \varphi_1](\gamma^*)^{-\sigma}\rho + \left[\varphi_1(1-\delta) + \alpha B(k^*)^{\alpha-1}\right](\gamma^*)^{-2\sigma}\rho^2$$

Comparing (10) with the equation above it is easy to show that $\alpha B(k^*)^{\alpha-1} = A, \forall\varphi_1$ (since we are assuming that $\gamma = \gamma^*$). Using this result and (24), it is easy to see that $\frac{\partial k^*}{\partial \varphi_1}$ >0 and $\frac{\partial y^*}{\partial \varphi_1}$ >0 (note: B is a parameter which does not changes with φ_1).

In figure 2 it can be observed a discontinuity between the results for $\alpha = 1$ (AK model) and $\alpha \in (0,1)$. Thus, this latter experiment points out to the relevance of the setup chosen to analize a given economic policy question.

[5]It is easy to see that: $1 + r_2^* = (1+r_1^*)^2$.

[6]We solve numerically this model too. The solution method is the same that we have described above.

5 Conclusion

We show that in the AK endogenous growth model with a time-to-build investment technology the time pattern of investment financing has nontrivial effects on welfare and long-run growth. Growth effects arise because the asset subject to the investment technology is the engine of growth.[7] Once the model is calibrated to economies that grow at a positive rate and being this one a function of the time pattern of investment financing, the welfare effects suggest that firms always should build more than a half of every investment project in the first period. The specific way how they should build theirs investment projects depends on: i) the initial conditions of the economies and, ii) the intertemporal elasticity of substitution.

In addition, we emphasize that welfare effects in the AK model are of opposite sign to those arising in an exogenous growth model when both theorical economies share the same long-run growth rate and the same steady state real interest rate. The investment financing structure that the government would stimulate would be very different if it followed the suggestions of the AK model rather than the exogenous growth model. This result points out to the relevance of the setup chosen to analize a given economic policy question.

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[7] In fact, it could also be shown that in a two-sector endogenous growth model with human capital accumulation, the growth rate depends on physical capital investment gestation lags only if it is an input in the human capital accumulation process.