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# Light scalar susceptibilities and isospin breaking

R.Torres Andrés and A.Gómez Nicola

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## Abstract.

Making a thermal analysis in the context of NLO  $SU(3)$  Chiral Perturbation Theory we see that isospin breaking (IB) corrections (both electromagnetic and QCD ones) to quark condensates are of order  $\mathcal{O}(e^2)$  and  $\mathcal{O}(\varepsilon)$ , with  $\varepsilon$  the  $\pi^0 - \eta$  mixing angle. However the combination  $\chi_{uu} - \chi_{ud}$  of flavour breaking susceptibilities, which vanishes in the isospin limit and can be identified essentially with the connected susceptibility, has an order  $\mathcal{O}(1)$  contribution enhanced with  $T$  because of the  $\pi^0 - \eta$  mixing. Finally we present a thermal sum rule that relates quark condensate ratios and the light scalar susceptibility without IB,  $\chi(T) - \chi(0)$ .

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## 1. INTRODUCTION

The low-energy sector of QCD has been successfully described over recent years within the chiral lagrangian framework. Chiral Perturbation Theory (ChPT) is based on the spontaneous breaking of chiral symmetry and provides a consistent and systematic scheme to calculate low-energy observables [1, 2, 3]. This formalism has been also extended to include finite temperature effects, in order to describe meson gases and their evolution towards chiral symmetry restoration [4, 5]. The effective ChPT lagrangian is constructed as an expansion of the form  $\mathcal{L} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \dots$  where  $p$  denotes a meson momentum or mass compared to the chiral scale  $\Lambda_\chi \sim 1$  GeV.

ChPT can take into account both QCD (due to the light quark masses difference  $m_u - m_d \neq 0$ ) and electromagnetic IB by means of new terms that implement the chiral symmetry breaking pattern. The former generates a  $\pi^0 - \eta$  mixing in the  $SU(3)$  lagrangian which introduces corrections of order  $(m_d - m_u)/m_s$  that, as we will see in section 3, will be important when considering some combinations of the light scalar susceptibilities at finite temperature. On the other hand, the presence of electromagnetic interactions induces mass differences for the light mesons through the presence of virtual photons. These corrections have been included in the ChPT effective lagrangian [6, 7, 8, 9] by means of terms like  $\mathcal{L}_{e^2}$ ,  $\mathcal{L}_{e^2 p^2}$  and so on, with  $e$  the electric charge. These terms are easily incorporated in the ChPT power counting scheme by considering formally  $e^2 = \mathcal{O}(p^2)$ .

The aim of this work is to explore within the thermal ChPT formalism the IB corrections to quark condensates (section 2) and their corresponding light scalar susceptibilities (section 3) to NLO, both physical objects directly related to chiral symmetry restoration. More details are given in [10].

## 2. QUARK CONDENSATES AND THERMAL SUM RULE

Quark condensates are defined as the order parameters of the chiral transition. For their  $\mathcal{O}(p^4)$  calculation we need the terms  $\mathcal{L}_{p^2}$  (which generates the meson loops),  $\mathcal{L}_{e^2}$  (that induces the charged mesons mass difference to LO); and, in order to renormalize the results, the next order ones  $\mathcal{L}_{p^4}$ ,  $\mathcal{L}_{p^2e^2}$  and  $\mathcal{L}_{e^4}$  to tree level. The relevant terms and details of the renormalization are given in [16].

For the sum of up and down quark condensates, and their difference, in  $SU(2)$  we get to one loop :

$$\langle \bar{u}u + \bar{d}d \rangle_T = \langle \bar{u}u + \bar{d}d \rangle_0 + 2F^2 B_0 [g_{\pi^0}(T) + 2g_{\pi^\pm}(T)] \quad (1)$$

$$\langle \bar{u}u - \bar{d}d \rangle_T = 4B_0^2(m_d - m_u)h_3 - \frac{8}{3}F^2 B_0 e^2 k_7 \quad (2)$$

where  $g_i(T) = \frac{g_1(M_i, T)}{2F^2}$ , and  $g_1(M, T) = \frac{1}{2\pi^2} \int_0^\infty dp \frac{p^2}{E_p} \frac{1}{e^{\beta E_p} - 1}$ , with  $E_p^2 = p^2 + M^2$  and  $\beta = 1/T$ .

From now on the superscript  $R$  in the low energy constants will mean that the results have been already renormalized in the  $\overline{MS}$  scheme, and the subscript 0 will refer to the zero temperature results, which can be found in [16]. As a nontrivial check of our calculation, one can see that the condensates in (1)-(2) are finite and scale-independent.

These results are compatible with the expressions for  $e = 0$  and  $T = 0$  in [2], for  $T \neq 0$ ,  $m_u = m_d$  and  $e = 0$  in [4], and for the condensate difference (2) with  $e \neq 0$  and  $T = 0$ , in [8].

In the  $SU(3)$  case, we have calculated, also to one loop, the light  $u, d$  and strange  $s$  condensates, taking into account both sources of IB. The main distinctive feature in this case, as commented above, is the appearance of  $\pi^0 - \eta$  mixing term through the tree-level mixing angle  $\varepsilon$  defined by  $\tan 2\varepsilon = \frac{\sqrt{3}}{2} \frac{m_d - m_u}{m_s - \bar{m}}$ . The condensates are

$$\begin{aligned} \langle \bar{u}u + \bar{d}d \rangle_T^{(3)} &= \langle \bar{u}u + \bar{d}d \rangle_0^{(3)} + 2F^2 B_0 \left( \frac{1}{3} (3 - \sin^2 \varepsilon) g_{\pi^0}(T) + 2g_{\pi^\pm}(T) + g_{K^0}(T) \right. \\ &\quad \left. + g_{K^\pm}(T) + \frac{1}{3} (1 + \sin^2 \varepsilon) g_\eta(T) \right) \\ \langle \bar{u}u - \bar{d}d \rangle_T^{(3)} &= \langle \bar{u}u - \bar{d}d \rangle_0^{(3)} + 2F^2 B_0 \left( \frac{\sin 2\varepsilon}{\sqrt{3}} [g_{\pi^0}(T) - g_\eta(T)] + g_{K^\pm}(T) - g_{K^0}(T) \right) \end{aligned} \quad (3)$$

$$\langle \bar{s}s \rangle_T = \langle \bar{s}s \rangle_0 + F^2 B_0 \left( \frac{4}{3} [g_{\pi^0}(T) \sin^2 \varepsilon + g_\eta(T) \cos^2 \varepsilon] + 2 [g_{K^\pm}(T) + g_{K^0}(T)] \right). \quad (4)$$

These results are finite and scale-independent with the renormalization of the LEC, including the EM ones, given in [3, 7]. On the other hand, the condensates agree with those of [3] for  $e = 0$  and  $T = 0$ .

An interesting application of the above calculations is a thermal sum rule relating the ratio  $\frac{\langle \bar{q}_i q_i \rangle_{e \neq 0}}{\langle \bar{q}_i q_i \rangle_{e=0}}$  and the divergence-free light scalar susceptibility without IB,  $\chi(T) - \chi(0)$ ,

which dominates the thermal behaviour of  $\frac{\langle \bar{q}_i q_i \rangle^{e \neq 0}}{\langle \bar{q}_i q_i \rangle^{e=0}}(T) - \frac{\langle \bar{q}_i q_i \rangle^{e \neq 0}}{\langle \bar{q}_i q_i \rangle^{e=0}}(0)$  in both two and three flavour ChPT. One can see that, up to order  $e^2$  we have

$$\frac{\langle \bar{q}_i q_i \rangle^{e \neq 0}}{\langle \bar{q}_i q_i \rangle^{e=0}}(T) - \frac{\langle \bar{q}_i q_i \rangle^{e \neq 0}}{\langle \bar{q}_i q_i \rangle^{e=0}}(0) = \frac{M_{\pi^\pm}^2 - M_{\pi^0}^2}{F^2} g_2(M_{\pi^0}, T) = \frac{4Ce^2}{3F^4} \frac{\hat{m}^2}{M_{\pi^0}^4} [\chi(T) - \chi(0)] \quad (5)$$

with  $g_2(M, T) = -\frac{\partial g_1(M, T)}{\partial M^2}$  and  $\chi$  being defined as  $\chi = \partial \langle \bar{q} q \rangle / \partial m$ , where  $m = m_u = m_d$  is the light quark mass in the isospin limit. As mentioned, we do not distinguish here between different flavour-breaking susceptibilities  $\chi_u, \chi_d$ , as we will do in section 3.

The interest of the relation (5) relies on the fact that chiral susceptibility is expected to grow when the system reaches the chiral restoration. Although our results are obtained in a low- $T$  ChPT approach and we cannot reproduce any peak in the susceptibility (as would correspond in the case of a 2+1 flavour case smooth crossover [11, 12, 13]), the growing behaviour predicted for  $\chi(T)$ , which numerically remains below the lattice values, would indicate that the EM mass corrections to condensates may be important near the transition, a fact that should be taken into account for lattice and phenomenological analysis.

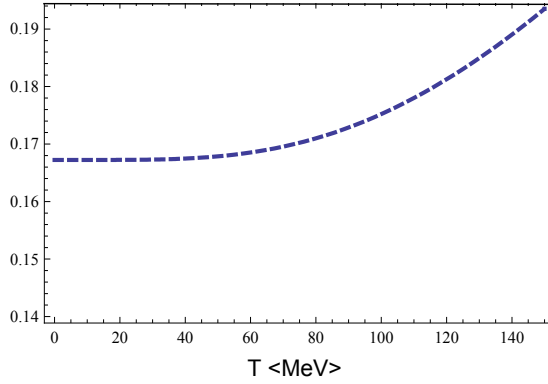
### 3. LIGHT SCALAR SUSCEPTIBILITIES AND THE ROLE OF THE $\pi^0 - \eta$ MIXING

From now on, we will restrict ourselves to the case of SU(3) ChPT with two active flavours  $u$  and  $d$ . Different light quark masses allow to consider three independent light scalar susceptibilities defined as

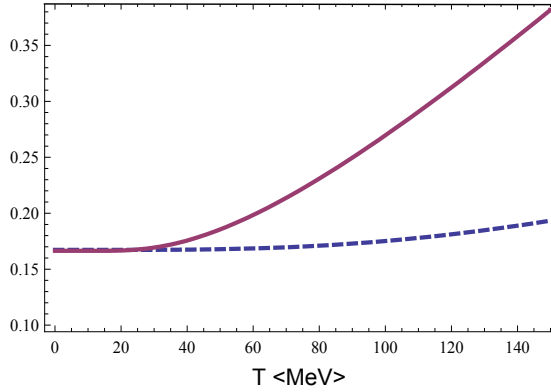
$$\chi_{ij} = -\frac{\partial}{\partial m_i} \langle \bar{q}_j q_j \rangle = \frac{\partial^2}{\partial m_i \partial m_j} \log Z(m_u \neq m_d). \quad (6)$$

To leading order in the mixing angle, the contribution of the  $\pi^0 - \eta$  mixing in the quark condensate difference (2) is of order  $\varepsilon^2$  whereas for (3) it goes like  $\varepsilon$ . When considering finite temperature, the thermal functions  $g_i(T, M_i)$ ,  $i = \pi^0, \eta$  are suppressed by those coefficients and the quark condensates do not receive important corrections. The important point is that differentiating with respect to a light quark mass is essentially the same as differentiating with respect to  $\varepsilon \sim \frac{m_d - m_u}{m_s}$ , so the suppression of the thermal functions is smaller in the case of the susceptibilities than in the quark condensate, even at low and intermediate temperatures.

Because of the linearity in  $\varepsilon$  of (3), the combinations  $\chi_{uu} - \chi_{ud}$  and  $\chi_{dd} - \chi_{du}$  receive an  $\mathcal{O}(1)$  IB correction due to  $\pi^0 - \eta$  mixing, even though the IB effects in the quark condensate are small. These objects are zero in the isospin limit, so all the difference between  $\chi_{uu}$  or  $\chi_{dd}$  and  $\chi_{ud}$  has an IB origin and they are more sensitive to flavour breaking than other combinations such as  $\chi_{uu} - \chi_{dd}$ , which also vanishes without IB, since from the analysis of the  $\varepsilon$  dependence of the quark condensate difference we get  $\chi_{uu} \simeq \chi_{dd}$ .



**FIGURE 1.**  $\chi_{uu}(T) - \chi_{ud}(T)$  normalized to the total susceptibility  $\chi_{tot}(0) = \chi_{uu} + \chi_{dd} + 2\chi_{ud}$  at zero temperature.



**FIGURE 2.**  $\chi_{con}(T)$  (blue dashed line) and  $\chi_{dis}(T)$  (magenta line) normalized to the total susceptibility  $\chi_{tot}(0) = \chi_{uu} + \chi_{dd} + 2\chi_{ud}$  at zero temperature.

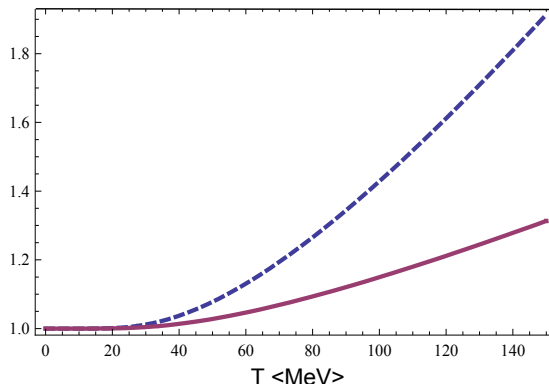
Following the definitions given in [14, 15] one can also relate these flavour breaking susceptibilities with the connected and disconnected susceptibilities (figure 2)

$$\chi_{dis} = \chi_{ud}, \quad (7)$$

$$\chi_{con} = \frac{1}{2}(\chi_{uu} + \chi_{dd} - 2\chi_{ud}). \quad (8)$$

Taking into account, as we have already mentioned above, that  $\chi_{uu} \simeq \chi_{dd}$ , the connected piece, which vanishes in the isospin limit (as noticed in [14]), is essentially the same as  $\chi_{uu} - \chi_{ud}$ , so it will be more sensitive to flavour breaking than the disconnected one. This fact would be interesting for lattice because it shows that, in our model independent scheme, the non-zero character of the connected susceptibility is dominated by IB effects.

Although the total susceptibility with IB is different from that calculated with the quark condensate in the isospin limit and differentiating with respect to  $m = m_u = m_d$  (see figure 3), the restoration temperature seems not to be affected, as can be seen in [10] parametrizing the quark condensate by means of suitable functions that mimic the behaviour at low- $T$  and reproduce the shape of a crossover at large temperatures.



**FIGURE 3.**  $\chi_{tot}(T)$  (blue dashed line) and total susceptibility in the isospin limit,  $\chi_{tot}^{IL}(T)$  (magenta line), normalized to the total susceptibility  $\chi_{tot}(0) = \chi_{uu} + \chi_{dd} + 2\chi_{ud}$  at zero temperature.

## ACKNOWLEDGMENTS

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## REFERENCES

1. S. Weinberg, *Physica* **A96**, 327 (1979).
2. J. Gasser and H. Leutwyler, *Annals Phys.* **158**, (1984) 142.
3. J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).
4. J. Gasser and H. Leutwyler, *Phys. Lett. B* **184**, 83 (1987).
5. P. Gerber and H. Leutwyler, *Nucl. Phys. B* **321**, 387 (1989).
6. G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys. B* **321**, 311 (1989).
7. R. Urech, *Nucl. Phys. B* **433**, 234 (1995).
8. M. Knecht and R. Urech, *Nucl. Phys. B* **519**, 329 (1998).
9. U. G. Meissner, G. Muller and S. Steininger, *Phys. Lett. B* **406**, 154 (1997) [Erratum-ibid. **B 407**, 454 (1997)].
10. A. G. Nicola and R. T. Andres, in preparation.
11. C. Bernard *et al.* [MILC Collaboration], *Phys. Rev. D* **71**, 034504 (2005).
12. Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, *JHEP* **0906**, 088 (2009).
13. M. Cheng *et al.*, *Phys. Rev. D* **74**, 054507 (2006).
14. A. V. Smilga and J. J. M. Verbaarschot, *Phys. Rev. D* **54**, 1087 (1996).
15. F. Karsch [RBC-Bielefeld collaboration], *Nucl. Phys. A* **820** (2009) 99C.
16. A. G. Nicola and R. T. Andres, arXiv:1009.2170 [hep-ph].