AFFECT, MATHEMATICAL THINKING AND INTERCULTURAL LEARNING

A STUDY ON EDUCATIONAL PRACTICE

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Studies dealing with affect and mathematical learning in multicultural contexts are rare. This paper explores the interaction between mathematical thinking and affect in these contexts. A study made in 2003, in Belgium, carried out with secondary level students of Portuguese origin is presented. The methodology and the schema of analysis involved have been previously used by this author in earlier investigations in Spain. It is a framework with parameters which disclose the origin and the evaluation of the emotional responses in the subjects.

The reconceptualisation on affect and mathematical thinking in the present decade is marked by two essential features. One is the attempt to consolidate a satisfactory theoretical framework for its interpretation, the other is the development of measurement instruments and other methodological tools for researcher within the social context in which learning takes place (see e.g, Evans, Hannula, Philippou & Zan (2003), Hannula et.al. (2004)). In accordance with this aim our paper seeks to increase the coherence between the observing instruments and the theory itself.

In a previous published paper I presented a model for the study of the interactions between cognitions and affects in the learning of Mathematics (Gómez-Chacón, 2000a and 2000b). This model is used here for describing emotional responses, their origin as well as for surveying their evolution in the subjects under consideration. Three dimensions related to affects and cognitions are specifically focused, namely, affect itself, meta-affects and belief systems. Attention is drawn to the importance of taking into consideration these dimensions in investigations of this nature, particularly in the case of school-failing students and in multicultural contexts.

Two groups of students, both of secondary level, have been involved in the study, one from Spain the other from Belgium. The study in Spain was accomplished first. Here I present the study carried out in Belgium, from February to June 2003, with students of Portuguese origin living in Brussels, in their 7^{th} to 12^{th} grades of education.

1.- Theoretical Framework

Working from a holistic perspective, which requires to consider the particular situation of the individual person, forces us to approach the theoretical framework of our work from a variety of cognitive fields:

- The appropriation of insights from a socio-cultural and socio-constructive perspective;
- The affective dimension in mathematics education, applying theoretical frameworks from psychology and sociology into the context of mathematics education.

- The perspective of social and cultural identity.

Dimensions related to affects and cognitions, such as affect itself, meta-affects and belief systems have been already used and defined by other authors (Goldin, 1998). At the beginning of my research (Gomez-Chacon, 1995, 1997) I considered them as a departure point, but moved toward a more precise and operational definitions of the constructs. Since then, myself and others, have distanced ourselves from this approach in order to seek a more dynamic viewpoint, which is required for dealing with the complexity of affect-cognition interplay in social learning situations. Taking into account the embeddedness of students' knowledge as well as beliefs in the social context (see e.g. de Abreu, Bishop, and Pompeu, 1997) the interpretation and appraisal processes that ground students' emotions in the classroom are constituted by the social-historical context in which they are situated. The emotions clearly have a rationale with respect to the local social order, giving rise to a particular notion of identity, all of which play a central role in the Mathematics classrooms (Cobb & Hodge (press))

With others, I hold the general view that cognitive theories and socio-cultural theories can be brought together in an effort to create a comprehensive theory of human activity (Eisenhart, 1988:10). My specific contribution in this respect is to integrate the emotional dimension in mathematics learning into this general view. I consider that the group, with its culture, its communication system and its institutional structure (accepted to be basic social and anthropological phenomena in mathematical education), is as important as the personal dimension, with its intra-individual aspects of cognition and of psychological relationships. I claim to have verified that a critical integration of these perspectives assists our understanding of the complex interaction of the affective, cognitive and cultural factors which play a role in learning mathematics.

In this perspective emotions are not a mere result of automatic responses or consequences of physiological impulses but rather are a complex result of learning, of social influences and of interpretation. In social interaction, emotions play a basic role in establishing relationships of social belonging and of social status. Social identity is considered to be the central organising principle which mobilises the totality of emotional responses of each individual towards mathematics and its learning. The central tenet of this perspective is that the individual's social identity shapes the social and global structure of affectivity in mathematics.

The adoption of this holistic perspective requires of us to look for an understanding of the research questions at three levels: the individual level; the micro level of classroom and workshop interaction; and the level of the social and cultural contexts.

As I have already indicated, emotional responses show their qualitative character if they are placed in the social context which originates them. Consequently the analysis of emotion has not been limited to simple laboratory scenarios (specific phases of

problem-solving, mistakes, etc.) but I include more complex scenarios[1] which regard the students in their social context and take account of students' self-concept as learners of mathematics, and the values and feelings that the students have as members of a social group.

2.- Instruments for identifying (diagnosing) the cognition-affect interaction

The investigation is qualitative in character, combining methods proper to ethnography and case-studies. (Clearly, a form of research involving a multi-methodological dimension cannot easily be described in detail in the limited space at our disposal in a paper. Here the essentials are presented and we direct the interested reader to consult Gomez-Chacon, 1997 and 2000).

The aims of the study were:

- To establish and describe significant relationships between cognition and affect (local and global affect, two constructs defined in the study);

- To trace the origin of the affective responses and identify the undergone evolutions in the subjects (differences, changes, etc.) after their participation in a programme of active learning which integrates the affective dimension (meta-affect);

- To analyse if it is possible to interpret the emotional responses of the young from a perspective of social identity and of cultural identity;

- To promote school instruction (teaching-and-learning) as a *process of socialization and counter-socialization* in the students beliefs, and the need to revise these methods of instruction.

Description of the study:

The study in Belgium was done with Portuguese students living in Brussels (displaced or emigrants) (Figueiral and Gómez-Chacón, 2003) of grades 7^{th} to 12^{th}, between Feb. and June 2003. During a five-month period, forty pupils of grades 10^{th}, 11^{th} and 12^{th} were closely monitored[2]. Two of the groups were followed in their classrooms, in the Belgium schools, and three groups in classes of Portuguese language and culture (each group followed once a week). The researcher was as an action researcher who collaborated as teacher with their official teacher. The students were integrated in multicultural classes. There were students from Portugal, Belgium,

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[1] The terms *complex scenarios* are used here with the sociological meaning of *scenario*. Therefore to speak of scenario is to speak of what makes a scene to be organised and how it is organised. It is, particularly, to speak of what is being acted in a concrete space and time, with specific resources. Whenever this scenario is acted in similar circumstances, the persons participating will behave, more or less, in the same way because their individual and social learning predisposes them to act so. For readers interested in this aspect see Gómez Chacón (2002). Here we have exemplified some of the various *scenarios* in which students behaviours originate and are recognised by their teachers and by a number of other persons. By means of this typology we have aimed at showing the influence of various processes, (cognitive, meta-cognitive, social, cultural), making explicit the causes and consequences of emotional interaction in learning.

[2] It is relevant to note that 64% of the students in the group researched were born in Portugal, and more than half of these, 56%, arrived in Belgium when they were over 5 years old.

Cameroon, Turkey, Albania, Italy, Morocco and children of mix-marriages, Belgium-Italian, Turkish- Albanian, Italian- American and so on. The common characteristic of the groups was that the largest number were Portuguese.

In the study we[3] explored and described the possible "tensions, conflicts and resistances" that could occur in learning mathematics in multicultural contexts, and that could be related to the stance the students take in relation to how their social and cultural group is represented.

The data was collected through field notes, interviews, questionnaires and other biographical and family data of aspects related to their experiences and learning situations as a "displaced group". The research sought to find out their experience in the school in Belgium, and their academic development in both countries, how competent they were in the languages (French / Portuguese), their identity and their expectations for the future. Data was also gathered on their affects, --attitudes, beliefs and emotions-- in relation to Mathematics.

Summary of data collection tools:

1. Interviews about situations. A tool to collect data about the belief systems and values associated with mathematical knowledge at school level and about applied knowledge, in an educationally disadvantaged context (level 1 and 2, global emotion, complex scenario).
2. Mood map of the problems. A tool to diagnose emotional and self-regulatory responses (level 2 and 3, local emotion, simple scenario)
3. Questionnaire on beliefs about mathematics and learning mathematics (levels 2 and 3, local emotion, simple and complex scenarios)ç
4. Field notes. Field notes written by the (author as) researcher, either during the class or afterwards. This exercise completed the transcription made in classroom session. The researcher took the role of teacher by developing learning modules, designed for classroom practice. Also the researcher registered the notes to describe the conduct of students and teacher outside the classroom (level 1, 2 and 3, local and global emotion, and simple and complex scenarios*).*

3.- Analysis and results

The different categories of beliefs about mathematics learning and problem solving is not only determined by the mental context of the student and the classroom. Other influential factors are, the way classes are presented, the activities in which the student takes part, the family culture, parents beliefs about mathematics, the social ideas about mathematics, etc. The stimulation to learning, beliefs, interpretations, instant emotions, and the construction of meanings they all take place in this interface experience (Volet, 2001).

[3] I use "we" because several researchers are collaborating in this project.

All these aspects are not always easy to analyse or interpret. We found difficulty in analysing and interpreting the intersubjective perceptions when there was a cultural displacement, as was the case with the study done in Belgium.

The results of the investigation on the emotional dimension of the students regarding mathematics showed that of all the levels mentioned above the one most clearly visible was the level of appraisal of the experience which includes the immediate classroom context.

The classes of mathematics, in spite of the different and diverse styles of teachers, were all based on photocopied cards with a summary of the topic and mechanical and routine exercises to do or solve by the students. Therefore, the same methods were repeated: to do the card work, to correct it by the students or by the teacher on the blackboard. The method followed conditioned their emotional reaction: routine exercises without emotion.

The interviews with the teachers showed that, among the teachers themselves, there was very little acknowledgement of the process of change and development of a cultural identity that these students were experiencing as a result of their moving between European areas with different cultures. The adaptation of these students to school is seen mainly in cognitive and linguistic terms, no other dimensions were considered, nor were there any questions or informed debate as to the role the schools may play in forming their cultural identities.

To the question: *Have you observed any difference in the behaviour of X and in the rapport of his/her family with the school when you compare him/her to students of other countries?* The answers were : *No. I don't belief that the differences are traceable to the country of origin.*

And in relation to the question, *Have you had contacts with the parents of X?* The answers indicated that belonging to a particular cultural group was not the cause of their behaviour: "*yes*", "*no*", "*the same as for other students*". However, it was considered obvious and normal, in Belgium, that these students will be one year behind because of their language, but they do not recognise that it is in fact a difficulty[4].

In relation to the teaching and learning of mathematics and its relationship to the students' cultural and social identity we observed that:

- None of the activities of solving problems or exercises made any reference to the history of mathematics nor to mathematics as a constructed and cultural body of knowledge.
- There were no activities for investigation and solution of problems.
- There were no applications or didactic or dynamic resources.

[4] This scenario is not unique to the schools that received Portuguese immigrants in Belgium (Cline Abreu, Gray, Lambert and Neale, 2002).

This type of activities would have facilitated their integration and would have allowed them to tackle mathematics from a more cultural perspective and as part of social and cultural identity.

The students outlined as difficulties some of the elements of what we have called "interface" experience. Such experience has to do with the climate generated in the classroom, with those invisible elements that give quality and stimulate learning and that, from our point of view, are linked to a characteristic of Portuguese culture and their way of interacting.

They missed a warmer kind of communication on the part of the Belgians. The students valued[5] positively the multicultural society in Belgium, because of the co-existence of many displaced groups from different countries. They considered negative the intrinsic double linguistic culture of Belgium. We also observed a tendency to attribute positive aspects to non-Belgians and negative ones to Belgians : "they are cold". However, it is interesting to point out how these students saw themselves. Although they considered themselves Portuguese their place of belonging was not so clear and at times this realisation became painful.

In describing the most difficult part of their experiences a student wrote: "to have your arse between two stools" and another expressed it with no room for doubt: "having the label of foreigner in Belgium and in Portugal".

In view of all these data we claim that teaching-learning is fundamentally a communication process based on social interaction.

For the learner to take part in this process a double task is called for: The first is centred in the conceptual content to be learned, and the other in the form of interaction-communication in which the content is inserted.

When we compared these data with other studies done in Spain, a new example emerges of what we called *complex scenario and expression of the global affect of the person*. In our investigation, the data specifically showed that the reaction to mathematics in the classroom of the first group of students was manifested basically in scenarios of re-adjustment, self-righteousness, demand for interdependence and answers to messages or distinctions (resistances). Here, in the investigation with Portuguese students in Belgium, it appears again the demand for interdependence – obviously with different nuances-- and a new scenario that may be called *scenario of interaction-communication* that refers to the characteristics of the interchanges between student and teacher.

In the data collected about this scenario of interaction-communication, we noticed that interaction-communication between student and teacher was not frequent and did not go beyond questions and answers about the exercises.

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[5] Questions such as the following were asked in the interview: What do you like best of Belgium society? What do you like least? What do you value most of what you consider to belong to your Portuguese culture? What do you appreciate least of your Portuguese culture? In relation to your experience of coexistence with different cultural realities: what do you personally consider most positive?

In the data emerging from the students about the question of 'what they would like in the classes of mathematics' we gathered three types of suggestions:

- *Ones related to resources and methodology,* for example: " I would like to be able to use a computer to see the drawings and graphs"; "to have less theory and make the classes more lively", "to give more practical examples"
- *Ones that considered that more time given to explaining the concepts would help the understanding:* "more time to get the sense of it, that the teachers give more time to these subjects", "there should be the minimum number of students possible, so that the teacher could explain better to each one", "to have the classes in the afternoon to have more time for mathematics", "the teachers know a lot, but they should have a lighter programme to give them more time to explain it to us",
- *Ones related to the mathematics motivation that the students think that the teachers should foster in the classroom:* "I would like to change the teacher because mine explains things very badly", "I think that the teachers should explain things for all the pupils and not only for the good ones", "the teacher should explain things better for all and not think that it is simple and that all should follow his ideas", " that they don't start from the premise that is all logical", "I would advise that the teachers stimulate love for mathematics because often they are the cause of us not liking it", "a teacher should be near the students and be patient", " I believe that it is necessary to have a teacher that likes teaching to awaken the interest of the students", " I believe that they explain things well but they are too serious".

In relation to this scenario of interaction-communication we recorded the following dialogue in an interview of group debate about particular "cases". It shows clearly the deficit in learning on the part of the students.

- Researcher: "and the classes of mathematics?"
- Diana: " The teacher is a pain. All teachers are a pain but this one is the worst"
- Researcher: "The class is a bit unruly and you, although not indisciplined, don't do much work. You begin work late, don't have the materials… calculators…"
- Bruno: "today I have everything, but we are tired"
- Diana: "With her (the teacher) you cannot rest. She makes us work too much. Always go, go; that is why, she sometimes gets on our nerves"
- Bruno: " we lack motivation"
- Diana: It is not the study, it is understanding and if I miss a class it becomes much more difficult"
- Researcher: "You can take advantage of the extra recuperation classes"
- They react with laughter: " that's just what we need. With all the hours of mathematics that we have now one more…" (Extract from an Interview of a Group Debate School Ma Ch.) (there is a contradiction between what they say

here and what they asked for before, that more time and less pupils would help their understanding).

The same *protocol of Interviews about situations* was used in Belgium and in Spain (Gómez –Chacón, 2000 or 1998). The objective was to compare the beliefs of both populations (beliefs and social context).

As already indicated we constructed an in-depth semi-structured interview on situations. The situations were presented through photographs. For each photograph a number of questions were prepared in relation (a) to the use and success of school mathematics, (b) to the beliefs about mathematics as an object of knowledge. The questions directed the interviewee to describe what he/she saw and to explain –from his own point of view—what he/she believed about the manner in which the action was taking place.

As the interview progressed the beliefs and responses concerning mathematics were ascertained. They spontaneously emerged in the conversation with the student, or they were provoked by the interviewer's deliberate intervention at a given moment, thus manifesting the local and global affect of the subject.

The protocol of this interview had questions which could not be answered except as responses about the true beliefs the subject had. For example, the students were asked, when the photographs were shown, to suggest possible scenarios while thinking about their friends, or to give advice relevant to situations in which other young people, like themselves, are participants. The student projected himself in the choices he makes.

Ultimately what we seeked to know was not merely what these young people thought and how they lived, but rather how they manifested their beliefs and emotional responses concerning their educational and cultural world.

In the results gathered from the interviews concerning beliefs about mathematics we ascertained a clear tendency to identify mathematics with three large areas: school mathematics; mathematics as a body of knowledge needed for some professions (mainly the more intellectual type; for the others only some "basic mathematics" were needed), and mathematics as a means to develop certain types of reasoning (logic and strategy). Some of the answers present a diffuse idea of mathematics as something that is present everywhere and explains the world in a complicated manner.

Levels of inclination and tendencies were more difficult to ascertain in this study due to the individual personal expressions of the students in mathematics classes. However, the global data collected, clearly manifested that the adaptation of Portuguese students to Belgium made a significant impact in their sense of self and demanded from them to reconsider their cultural identity.

The four main characteristics in the way they related their new identity were:

- Commitment to family customs and the historical patrimony of their country.

- A reinforced feeling of wanting to perfect cultural instruments associated to their Portuguese heritage (for example: to become fluent in Portuguese) and to speak French very well to be well integrated.
- A reinforced feeling (because Belgium is itself a multicultural society) of wanting to co-exist with many displaced groups, although they denounce the tensions generated by biculturalism and by their own bicultural situation.
- A feeble feeling when the differences associated to their Portuguese origin were not accepted / recognised or taken as lack of competence.

Conclusion

In summary this study illustrates the significance of the multidimensional and dynamic aspects of learning and identity in context. The studies done in different contexts, and in the interface experience, have shown that what constitutes the adaptative and appropriation elements of learning, is not only a subjective perception, but it is also objectively situated in communities of practices.

This is of extreme importance for educational practice in intercultural contexts. The contexts should not be considered as static but rather as dynamic units.

There is a tendency --identified in our investigation--to conceptualise the cultural contexts and the culture of origin as something fixed, even in those high schools that mention the intercultural dimension in their school plan. They manifest a static and respectful vision towards the other but without interaction or estimation of the value of what is proper of each culture.

From our own perspective, we claim that in order to build a good framework and to interpret the emotional dimension of the person in their context, it is necessary to conceptualise the macro-micro relations and the role that communication and face-to-face encounters play in the building of these relations. This is a *complex matter*, and many authors have written extensively about it. They denounce that face-to-face meetings are for many researchers, an element (and a minor one at that) among many others in society. However, for the micro-ethnograph authors, these encounters are the central constitutive element of the social world.

These issues raise new questions for investigation: Can the identity, cognition, and emotion be conceptualised and *treated empirically* at three or more levels of specification? What complex emotional scenarios can be typified in multicultural contexts? What beliefs are stable and which ones are linked to context? What are the differences inter and intra individuals in the different levels of the context?

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