

# A two steps phase-shifting demodulation method using the VU factorization

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In this paper, we present a two steps phase-shifting fringe pattern demodulation method using the VU factorization. Compared with other methods, this method is robust to non-constant background illuminations and modulating terms (non-normalized fringes). We use the two given fringe patterns, and from these, we synthetically generate another two images that approach the background illumination in order to obtain a total set of four images. Such a set of images is used in a matrix model that can be factored using the VU factorization method to recover the modulating phase. Our proposal is able to recover the phase without: 1) known phase shift, 2) normalized fringe patterns, and 3) any special non-linear pre-processing. Tests and results will be shown to demonstrate the feasibility of this method. © 2021 Optical Society of America

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## 1. INTRODUCTION

A fringe pattern is an interferogram obtained by means of optical interference, however, this is not the only situation where we have fringe patterns to process [1, 2]. We may have two classes of fringe patterns: 1) open fringes and 2) closed fringes. Demodulating open fringes can be made with a single fringe pattern using linear methods such as the Fourier transform method [3]. However, demodulating closed fringes is much more complex because we have a sign ambiguity to be solved [4–6]. This sign ambiguity is solved by the phase-shifting interferometry techniques [1]. In phase shifting interferometry, if we know the phase shift, a formula can be deduced to obtain the modulating phase [7, 8], but, if this is unknown, a non-linear process is involved [9].

The special case in this context is when we have only two fringe patterns. By knowing the phase shift a formula can be deduced, but, it is necessary to pre-process each fringe pattern to a) remove the background illumination and b) normalize the fringes. From these two tasks, normalizing the fringes is the more challenging task [8, 10, 11].

In this work, we show a method to demodulate the phase

given only two fringe patterns with unknown phase shift. The method presented here follows a completely different approach that makes it interesting and an important contribution to the state of the art [9]. This method does not require any pre-filtering process to the fringe patterns in order to remove the background illumination and normalize them. In this method, we use the two given fringe patterns and synthetically generate another two images to end up with a set of four images. This set of images is modeled by a matrix form that can be factored in order to recover the modulating phase. For this purpose, we use the VU factorization method.

In the following sections we are going to show how it is build the set of four images and its matrix model along with the VU factorization. Then, we are going to apply this method to some simulated and non-simulated experiments to show the results of the demodulated phase. These results will be compared with the Lissajous Ellipse Fitting (LEF) method method given that is a well-known state-of-the-art technique and one of the most popular for the demodulation of two step phase shifted patterns [12–14]. At the end, some conclusions and discussion will be given.

## 2. METHOD

In Phase Shifting Interferometry (PSI), we have an interferogram set modeled as follows:

$$g_{m,n} = a_m + b_m [\cos(\phi_m) \cos(\delta_n) - \sin(\phi_m) \sin(\delta_n)], \quad (1)$$

where sub-index  $m \in \{1, 2, \dots, M\}$  enumerates the pixels of an image and  $n \in \{1, 2, \dots, N\}$  enumerates the images of the interferogram set. We can model the interferogram set in matrix form as:

$$G = \begin{pmatrix} \mathbf{a} & \mathbf{c} & -\mathbf{s} \end{pmatrix} \begin{pmatrix} \mathbf{1}^T \\ \mathbf{u}^T \\ \mathbf{v}^T \end{pmatrix}, \quad (2)$$

where  $\mathbf{a}$ ,  $\mathbf{c}$ , and  $\mathbf{s}$  are column vectors described as  $\mathbf{a} = \{a_m\}$ ,  $\mathbf{c} = \{b_m \cos \phi_m\}$  and  $\mathbf{s} = \{b_m \sin \phi_m\}$ , for  $m = 1, 2, \dots, M$ ; corresponding to the background illumination and interference or modulating terms of the fringe patterns, respectively. The terms  $\mathbf{1}$ ,  $\mathbf{u}$  and  $\mathbf{v}$  are other column vectors described as  $\mathbf{1} = \{1\}$ ,  $\mathbf{u} = \{\cos \delta_n\}$  and  $\mathbf{v} = \{\sin \delta_n\}$ , for  $n = 1, 2, \dots, N$ ; corresponding

to a constant term of ones and to the sine and cosine of the phase shifts, respectively. So, matrix  $G$  has dimensions  $M \times N$  with terms  $g_{m,n}$  described as Eq. (1).

Using the matrix  $V = (\mathbf{a} \ \mathbf{c} \ -\mathbf{s})$  and matrix  $U = (\mathbf{1} \ \mathbf{u} \ \mathbf{v})$ , we can write matrix  $G$  as the product

$$G = VU^T. \quad (3)$$

So the idea of the VU factorization is to find the matrices  $V$  and  $U$  given the matrix  $G$ . This can be achieved, for example, by assuming we know  $U$  to obtain  $V$  as

$$V = G[U(U^T U)^{-1}] \quad (4)$$

and so on [cite]. The problem here is that matrix  $U^T U$  is invertible only for  $N \geq 3$ , but for  $N = 2$  we have a singular matrix. Hence, we can't obtain a VU factorization when having only two fringe patterns. However, we can overcome this singularity by extending the interferogram set in the following way:

$$\begin{aligned} \hat{g}_{m,1} &= \sum_{k=1}^M g_{k,3} h_\sigma(k-m) \\ \hat{g}_{m,2} &= \sum_{k=1}^M g_{k,4} h_\sigma(k-m), \\ g_{m,3} &= a_{m,1} + b_m \cos(\phi_m) \cos(\delta_1) + b_m \cos(\phi_m) \cos(\delta_1) \\ g_{m,4} &= a_{m,2} + b_m \cos(\phi_m) \cos(\delta_2) + b_m \cos(\phi_m) \cos(\delta_2), \end{aligned} \quad (5)$$

Note here that images  $\hat{g}_{m,1}$  and  $\hat{g}_{m,2}$  are synthetically generated from the given fringe patterns  $g_{m,3}$ ,  $g_{m,4}$ . The operator  $h_\sigma(\cdot)$  is a convolution kernel corresponding to a low-pass filter with wide-band  $\sigma$ . For clarity, in Eq. (5) the convolution is written as a 1D convolution, however, in practice we must use 2D convolution operation. Thus, this set of four images can be modeled in matrix form as:

$$G = \begin{pmatrix} \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{c} & -\mathbf{s} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & \cos \delta_1 & \cos \delta_2 \\ 0 & 0 & \sin \delta_1 & \sin \delta_2 \end{pmatrix}, \quad (6)$$

where  $\mathbf{g}_1 = \{g_{m,1}\}$  and  $\mathbf{g}_2 = \{g_{m,2}\}$ , for  $m = 1, 2, \dots, M$ , are the two synthetically generated images as column vectors, which approach the background illumination of the fringes patterns given in images 3 and 4 of the new set Eq. (5).

Now, by taking  $V = (\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{c} \ -\mathbf{s})$  and

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & \cos \delta_1 & \sin \delta_1 \\ 0 & 1 & \cos \delta_2 & \sin \delta_2 \end{pmatrix}, \quad (7)$$

we can write the matrix  $G$  as in Eq. (3). In this case the matrix  $U^T U$  is invertible as it is not singular. Therefore, using this set of 4 images and its matrix model, we can proceed with the VU factorization as follows in algorithm 1.

In this way, we have a method to demodulate a phase-shifting interferogram sequence having only two fringe patterns with unknown phase shifts.

#### Algorithm 1. Two-frames VU factorization

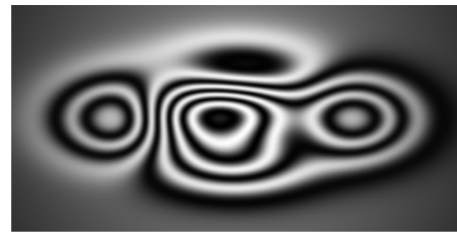
```

1: procedure FACTORIZATION( $G, tolerance$ )  $\triangleright G$  is extended as in Eq. (5)
2:    $\delta_1 \leftarrow \text{random} \in [-\pi, \pi]$ 
3:    $\delta_2 \leftarrow \text{random} \in [-\pi, \pi]$ 
4:    $U \leftarrow \text{evaluate as in Eq. (7)}$ 
5:    $error \leftarrow \|\delta_1 + \delta_2\|$ 
6:   while  $error > tolerance$  do
7:      $V \leftarrow GU(U^T U)^{-1}$ 
8:      $\hat{\phi} \leftarrow \arctan(-V_{:,3}, V_{:,2})$ 
9:      $V_{:,2} \leftarrow \cos(V_{:,2}), \quad V_{:,3} \leftarrow -\sin(V_{:,2})$ 
10:     $U^T \leftarrow (VV^T)^{-1} V^T G$ 
11:     $\hat{\delta}_1 \leftarrow \arctan(U_{3,4}, U_{3,3}), \quad \hat{\delta}_2 \leftarrow \arctan(U_{4,4}, U_{4,3})$ 
12:     $U \leftarrow \text{evaluate as in Eq. (7) using } \hat{\delta}_1, \hat{\delta}_2$ 
13:     $error \leftarrow \|(\delta_1 - \hat{\delta}_1) - (\delta_2 - \hat{\delta}_2)\|$ 
14:     $\delta_1, \delta_2 = \hat{\delta}_1, \hat{\delta}_2$ 
15:  return  $\hat{\phi} \quad \triangleright \hat{\phi}$  is the estimated phase

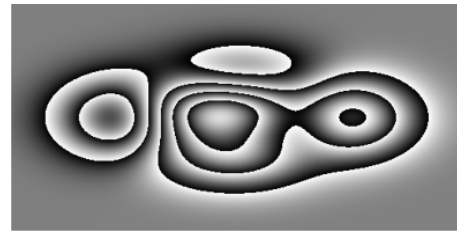
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(a)  $g_{m,1}$

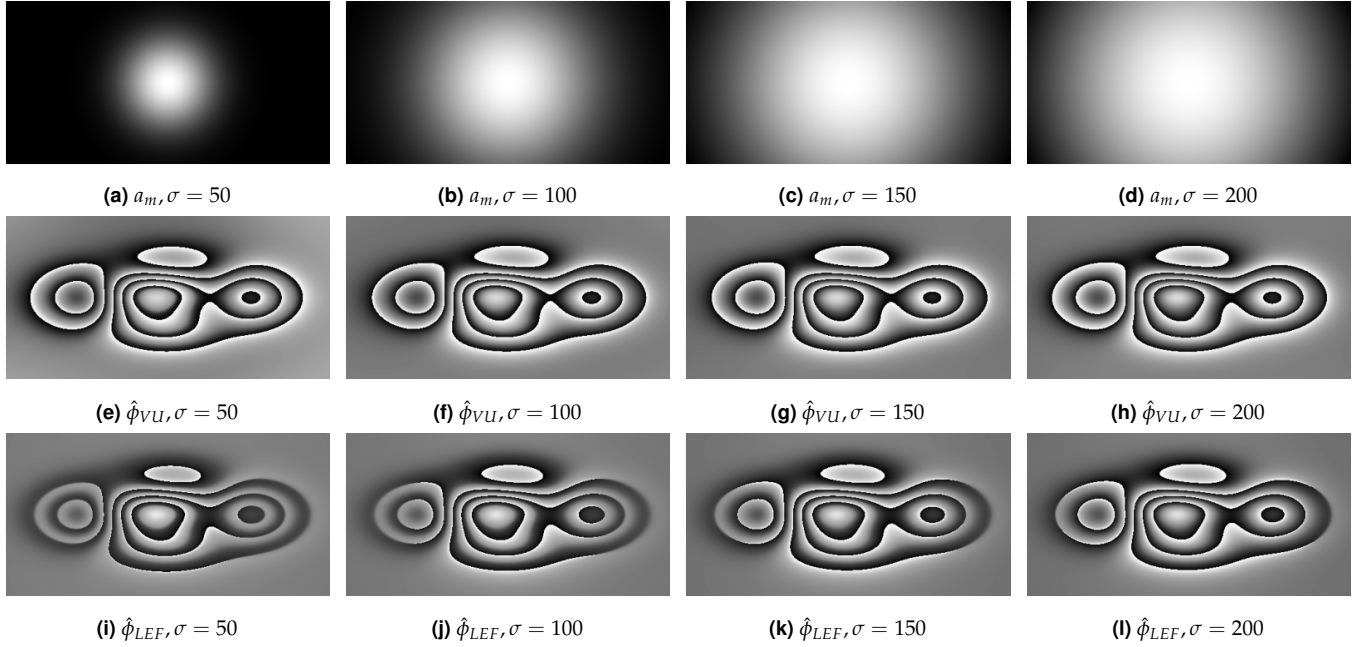


(b)  $g_{m,2}$



(c)  $\phi$

**Fig. 1.** Synthetic fringe patterns with variable background illumination.



**Fig. 2.** (a) - (d) Background illumination functions added to the signal, (e) - (h) Phase estimations using VU factorization, (i) - (l) Phase estimations using the LEF method.

### 3. EXPERIMENTS AND RESULTS

In order to prove the feasibility of our method, we present the results obtained with synthetic and experimental patterns. For comparative purposes, we also present the results obtained with the Lissajous Ellipse Fitting (LEF) method given that is a well-known state-of-the-art technique for the demodulation of two step phase shifted patterns [12–14].

#### A. Synthetic patterns

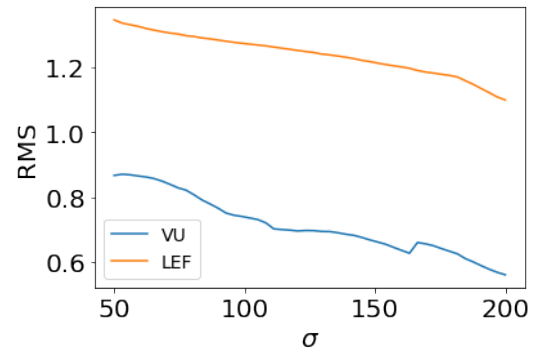
As mentioned in Section 2, our main contribution is that our method is able to estimate the phase  $\phi_m$  from the patterns presented in Eq. (1) without retrieving the background illumination  $a_m$ . To demonstrate the capabilities of our method, we performed a series of phase estimations of synthetic patterns with variable background illuminations, which in this case are 2D Gaussian signals with different variance ( $\sigma$ ). In Figures 1a and 1b are presented a pair of synthetic patterns with resolution  $512 \times 256$  with a Gaussian background illumination of  $\sigma = 100$  (shown in Figure 2b) and Figure 1c present the ideal phase  $\phi$  of the synthetic pattern.

The experiment consists on generating different background distributions by adding a 2D Gaussian function and changing the variance  $\sigma$  from 50 to 200. Then, we estimate the phase using the VU factorization and the LEF method. Finally, we calculate the Root Mean Square (RMS) error between the ideal pattern,  $\phi$  (Figure 1c), and the estimated ones,  $\hat{\phi}$ . In Figure 2 we present some representative results of the experiment in order to show the influence of the background in the phase extraction process. Figures 2a, 2b, 2c, 2d show some of the background illuminations added to the signal, figures 2e, 2f, 2g, 2h present phase estimations using VU factorization and figures 2i, 2j, 2k, 2l the estimations using the LEF method.

As it can be seen in figures 2i to 2l, the accuracy of the estimation using the LEF method increases as the background illumination is less variable; this is due to the fact that most of

the techniques for two step phase estimation using randomly shifted patterns assume that the background term is constant spatially and temporally [9]. In exchange, our method present consistent results along the different background signals.

In Figure 3 we present the comparison of the RMS between the VU factorization and the LEF method along different levels of variance in the Gaussian background. As mentioned before, as the  $\sigma$  term gets bigger, the background is less variable and both methods increase their accuracy; nevertheless, even with highly variable background, the VU factorization presents a smaller error.



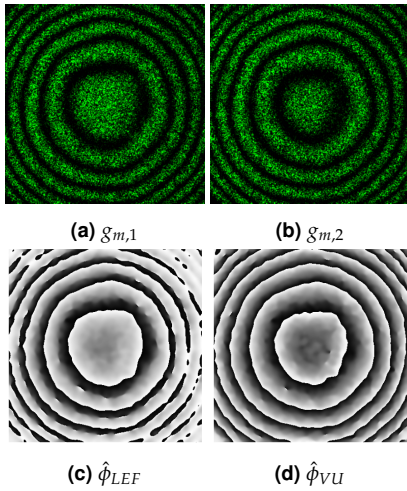
**Fig. 3.** Estimated RMS between the phases obtained with the VU factorization and the LEF method respect to the ideal pattern.

#### B. Experimental patterns

To demonstrate the capabilities of our method on experimental results, we implemented our method on fringe patterns obtained from a Parallel Radial Shear Interferometer (PRSI) as the one proposed in [15]. We used a  $\lambda = 532nm$  laser to illuminate the sample, the shifts are generated by a polarizer array which

allows us to capture two steps simultaneously. The patterns were captured by a  $2048 \times 1536$  CCD camera and the estimated phase step is define by the rotation of the polarizers (for more details of the arrangement please refer to [15]).

In figures 4a and 4b we present typical fringe patterns from the PRSI, in this case the induced phase shift is  $\delta = \pi/3$ . Prior to the phase estimation, we only applied a Gaussian pre-filtering to reduce the speckle produced by the laser. Then, we performed the phase recovery using the LEF method and the VU factorization. As it can be seen, the phase recovered by the LEF method, shown in Figure 4c, loses information as it gets farther from the center of the image; this is due to the Gaussian nature of the wavefront, generating a non constant background illumination so normalization would be required. On the other hand, the results obtained with the VU factorization, shown in Figure 4d, present consistency on all the regions of the phase.

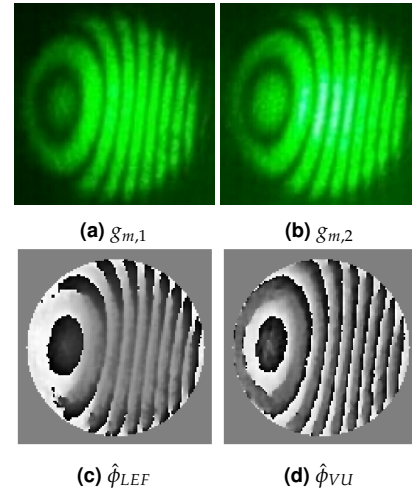


**Fig. 4.** Experimental results of a Parallel Radial Shear Interferometer (PRSI).

Another interesting result is the one presented on Figure 5. Here, the captured fringe patterns are shifted a  $\delta = \pi/2$  and present saturation with variable spatial distribution, so they do not have the same visibility (Figures 5a and 5b). Using other two-steps demodulation methods, as with the LEF, normalization of fringes is required to obtain the phase as expected. However, as fringes are not normalized, the LEF method, shown in Figure 5c, is not able to estimate the phase. On the other hand, the VU factorization, presented in Figure 5d, proved to be more robust to variations in the background between the interferograms; also, this results shows that the VU can estimate the phase when the fringes present levels of saturation in the fringes.

#### 4. CONCLUSIONS

We presented a two steps phase-shifting demodulation method using the VU factorization. Our proposal is based on generating a set of four images: the two original fringe patterns and two synthetically generated. Then we propose a matrix model that fits with these four images and given this, we perform the VU factorization. The method proved to be robust to non-constant background illuminations, modulation terms (non-normalized fringe patterns) and random phase shift. Also, as shown in the experimental results, it tolerates variations of the background illumination among the interferograms. The feasibility of this method has been proved by performing a series of experiments



**Fig. 5.** Experimental results of a Parallel Radial Shear Interferometer (PRSI). Saturated fringe patterns.

with synthetic and experimental fringe patterns and comparing it with the two steps LEF method, which has derived other works and algorithms for phase demodulation. The ideas behind the method presented here are completely new. Based on the results, we found our method as a fast algorithm able to obtain the phase from challenging fringe patterns, without any normalization process.

#### 5. DISCUSSION

As in any other phase-shifting interferometry (PSI) technique, note that the mathematical model of the interferogram fringes, shown in Eq. (1), does not impose any restriction to the modulating phase so this one can be monotonically increasing, decreasing (open fringes, no rings) or have local maximums or minimums (closed fringes or having rings). Even the phase may have discontinuities. In any of these conditions, PSI algorithms are able to recover the phase correctly because they process the images in the temporal domain where the images were taken. The method presented here, is not an exception because as any other PSI algorithm, the processing happens in the temporal domain of the sequence. In the spatial domain, if images complain with the interference model of Eq. (1) the interferograms may have incomplete or any number of fringes. The numerical results presented here supports this.

In the experimental results, we use Gaussian filters for two purposes: 1) for removing the fringes as much as possible and 2) for removing additive noise or smoothing the fringes without removing them. The equation of this Gaussian filter is  $h(x) = \exp[-x^2/(2\sigma^2)]$ . In either case the filter has the same shape but different parameter  $\sigma$  which controls its wide-band. For smoothing the fringes we use  $\sigma = 0.5$  and for removing the fringe, we recommend  $\sigma =$

#### 6. FUNDING

#### 7. ACKNOWLEDGMENTS

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#### REFERENCES

1. D. Malacara, *Optical shop testing*, vol. 59 (John Wiley & Sons, 2007).

2. Z. Wang, D. A. Nguyen, and J. C. Barnes, "Some practical considerations in fringe projection profilometry," *Opt. Lasers Eng.* **48**, 218–225 (2010). Fringe Projection Techniques.
3. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry," *JosA* **72**, 156–160 (1982).
4. J. C. Estrada, M. Servín, J. A. Quiroga, and J. L. Marroquín, "Path independent demodulation method for single image interferograms with closed fringes within the function space  $C^2$ ," *Opt. Express* **14**, 9687 (2006).
5. J. C. Estrada, M. Servín, and J. L. Marroquín, "Local adaptable quadrature filters to demodulate single fringe patterns with closed fringes," *Opt. Express* **15**, 2288 (2007).
6. M. Servín, J. Estrada, and A. Quiroga, "Single-Image Interferogram Demodulation," in *Advances in Speckle Metrology and Related Techniques*, (Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim, Germany, 2011), pp. 105–146.
7. M. Servín, J. C. Estrada, and J. A. Quiroga, "The general theory of phase shifting algorithms," *Opt. Express* **17**, 21867 (2009).
8. M. Servín, J. A. Quiroga, and M. Padilla, *Fringe Pattern Analysis for Optical Metrology* (Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim, Germany, 2014).
9. V. H. Flores, A. Reyes-Figueroa, C. Carrillo-Delgado, and M. Rivera, "Two-step phase shifting algorithms: where are we?" *Opt. & Laser Technol.* **126**, 106105 (2020).
10. J. Antonio Quiroga and M. Servín, "Isotropic n-dimensional fringe pattern normalization," *Opt. Commun.* **224**, 221–227 (2003).
11. K. Staszek and M. Bogusz, "Simple Fringe Pattern Normalization Algorithm," *IFAC Proc. Vol.* **45**, 353–356 (2012).
12. C. Farrell and M. Player, "Phase step measurement and variable step algorithms in phase-shifting interferometry," *Meas. Sci. Technol.* **3**, 953 (1992).
13. Y. Zhang, X. Tian, and R. Liang, "Random two-step phase shifting interferometry based on Lissajous ellipse fitting and least squares technologies," *Opt. express* **26**, 15059–15071 (2018).
14. V. H. Flores and M. Rivera, "Robust two-step phase estimation using the simplified Lissajous ellipse fitting method with gabor filters bank preprocessing," *Opt. Commun.* **461**, 125286 (2020).
15. L. García-Lechuga, P. Pérez-Luna, V. H. Flores, A. Montes-Pérez, A. Quiroz-Rodríguez, J. M. Islas-Islas, and N.-I. Toto-Arellano, "Parallel phase shifting radial shear interferometry with complex fringes and unknown phase shift," *Appl. optics* **59**, 2128–2134 (2020).