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# New black holes in $D = 5$ minimal gauged supergravity: Deformed boundaries and frozen horizons

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A new class of black hole solutions of the five-dimensional minimal gauged supergravity is presented. They are characterized by the mass, the electric charge, two equal magnitude angular momenta and the magnitude of the magnetic potential at infinity. These black holes possess a horizon of spherical topology; however, both the horizon and the sphere at infinity can be arbitrarily squashed, with nonextremal solutions interpolating between black strings and black branes. A particular set of extremal configurations corresponds to a new one-parameter family of supersymmetric black holes. While their conserved charges are determined by the squashing of the sphere at infinity, these supersymmetric solutions possess the same horizon geometry.

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## I. INTRODUCTION

There has recently been considerable interest in solutions of the five-dimensional gauged supergravity models, mainly motivated by the AdS/CFT correspondence [1,2]. The black holes (BHs) play a central role in this context, providing the thermodynamic saddle points of the dual four-dimensional theory.

The Schwarzschild-anti–de Sitter (AdS) BH provides the simplest example, possessing a spherical horizon and a single global charge: the mass. As expected, the inclusion of other charges (e.g. the angular momenta) generically deforms the horizon shape. Despite the existence of a number of partial results, the study of the horizon geometrical properties (in particular, its deformation) in conjunction with other BH properties is a rather poorly explored subject, presumably due to the complexity of the problem. However, this study is greatly simplified by restricting to BHs with a spherical horizon topology which possess two equal-magnitude angular momenta. Then one can use a cohomogeneity-1 ansatz which factorizes the angular dependence of the metric and the gauge potential and leads to a homogeneous squashing of the horizon geometry. Then, without any loss of generality, the induced horizon metric can be written as

$$ds^2_{\mathcal{H}} = \frac{L_H^2}{4}(\sigma_1^2 + \sigma_2^2 + \epsilon_H^2\sigma_3^2), \tag{1}$$

with $L_H > 0$ and the left invariant one-forms $\sigma_1 = \cos\psi d\theta + \sin\psi\sin\theta d\phi$, $\sigma_2 = -\sin\psi d\theta + \cos\psi\sin\theta d\phi$, $\sigma_3 = d\psi + \cos\theta d\phi$ (where coordinates $\theta$, $\phi$, $\psi$ are the Euler angles on $S^3$, with the usual range). The deformation parameter $\epsilon_H$ gives the ratio of the $S^1$ and the round $S^2$ parts of the (squashed $S^3$-) horizon metric, while the horizon area is $A_H = 2\pi^2 L_H^3\epsilon_H$.

A black hole in minimal gauged supergravity with the horizon geometry (1) has been constructed in closed form by Cvetič *et al.* in Ref. [3]. This solution is characterized by three nontrivial parameters, namely the mass $M$, the electric charge $Q$, and a rotation parameter $J$. An extension which possesses an extra parameter $\Phi_m$ associated with a nonzero magnitude of the magnetic potential at infinity has been reported in the recent work [4].

These solutions possess a conformal boundary geometry which is the static Einstein universe. However, a remarkable property of the AdS/CFT correspondence is that it does not constrain the way of approaching the boundary of spacetime, asymptotically *locally* AdS (AlAdS) solutions being also relevant. An interesting case here corresponds to configurations whose conformal boundary metric is the product of time and a squashed sphere,

$$ds^2_B = \frac{L^2}{4}(\sigma_1^2 + \sigma_2^2 + \epsilon_B^2\sigma_3^2), \tag{2}$$

with $\epsilon_B > 0$ being a squashing parameter and $L$ the AdS length scale.

The main purpose of this work is to investigate the correlation between the squashing parameters $\epsilon_B$ and $\epsilon_H$ and, more general, how the BH properties are affected by the deformation of the boundary sphere. A new class of BH solutions is reported in this context. Possessing arbitrary values of the squashing parameters $\epsilon_H$ and $\epsilon_B$, the generic solutions interpolate between black strings and black branes. A particular limit describes a new one-parameter family of supersymmetric (SUSY) BHs, which possess special properties.

## II. GENERAL SOLUTIONS

In the minimal case, the bosonic sector of the $d=5$ gauged supergravity consists of the graviton and an Abelian vector only, with action

$$I = \frac{1}{16\pi}\int_{\mathcal{M}} d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - F_{\mu\nu}F^{\mu\nu} - \frac{2}{3\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}A_\mu F_{\nu\alpha}F_{\beta\gamma}\right], \tag{3}$$

where $R$ is the curvature scalar, $A$ is the gauge potential and $F = dA$ is the field strength tensor.

The appropriate ansatz for the metric and the gauge potential is given by [4,5],

$$\begin{aligned} ds^2 &= F_1(r)dr^2 + \frac{1}{4}F_2(r)(\sigma_1^2+\sigma_2^2) \\ &\quad + \frac{1}{4}F_3(r)(\sigma_3 - 2W(r)dt)^2 - F_0(r)dt^2, \\ A &= a_0(r)dt + a_k(r)\frac{1}{2}\sigma_3. \end{aligned} \tag{4}$$

The BHs in [3,4] can be written in this form and have $\epsilon_B = 1$, while $\epsilon_H$ presents a complicated dependence on the global parameters (e.g. with $\epsilon_H > 1$ for $Q = \Phi_m = 0$).

We have found that these BHs possess a generalization with a squashed Einstein universe in the boundary metric. As such, apart from $\{M, J, Q, \Phi_m\}$ the new BHs have an additional free geometric parameter, the boundary squashing $\epsilon_B$. These asymptotics are compatible with the following approximate expression of the metric functions at infinity [6] (which fixes the boundary conditions imposed in the numerics),

$$\begin{aligned} F_0 &\sim \left(\frac{r}{L}\right)^2 + \frac{1}{9}(13 - 4\epsilon_B^2) + \cdots, \\ F_1^{-1} &\sim \left(\frac{L}{r}\right)^2 - \frac{1}{9}(14 - 5\epsilon_B^2)\left(\frac{L}{r}\right)^4 + \cdots, \\ F_2 &\sim r^2 - \frac{5L^2}{4}(1-\epsilon_B^2) + \cdots, \\ F_3 &\sim \epsilon_B^2 r^2 + \frac{13L^2\epsilon_B^2}{9}(1-\epsilon_B^2) + \cdots, \\ W &\sim -\frac{\hat{j}}{r^4} + \cdots, \end{aligned} \tag{5}$$

while the U(1) potential behaves as

$$\begin{aligned} a_k &\sim -2\Phi_m + \left[\mu - 4\Phi_m L^2\epsilon_B^2 \log\left(\frac{L}{r}\right)\right]\frac{1}{r^2} + \cdots, \\ a_0 &\sim V_0 + q/r^2 + \cdots. \end{aligned} \tag{6}$$

The asymptotically flat solutions necessarily have $\Phi_m = 0$; however, an A(l)AdS spacetime effectively acts like a box [7–10]. This allows for the existence of a nonvanishing asymptotic magnetic field, $F_{\theta\phi} \to \Phi_m \sin\theta$, such that the parameter $\Phi_m$ can be identified with the magnetic flux at infinity through the base space $S^2$ of the $S^1$ fibration [4],

$$\Phi_m = \frac{1}{4\pi}\int_{S^2_\infty} F. \tag{7}$$

The global charges of the solutions are encoded in a set of free coefficients which enter their asymptotic expansion, being computed by using the standard holographic renormalization procedure [11–13]. One finds e.g. the angular momentum and the (holographic) electric charge,

$$J = \frac{\pi\hat{j}\epsilon_B^3}{4}, \qquad Q = -\pi\left(q\epsilon_B + \frac{16}{3\sqrt{3}}\Phi_m^2\right). \tag{8}$$

The solutions possess a horizon located at $r = r_H > 0$, where, restricting to the nonextremal case, $F_0 = f_0(r - r_H)^2 + \cdots$, while the remaining functions are nonzero. The Hawking temperature is $T_H = \frac{1}{2\pi}\sqrt{f_0/F_1(r_H)}$, while the horizon metric is given by (1), with $L_H^2 = F_2(r_H)$ and $\epsilon_H^2 = F_3(r_H)/F_2(r_H)$.

The BHs are obtained numerically, by solving the field equations subject to the boundary conditions described above [14], the results being displayed in units with $L = 1$. All configurations reported here (including the SUSY ones) are regular on and outside the horizon. Also, they do not present other pathologies [such as closed timelike curves (CTCs)].

The most remarkable feature of these BHs is that they interpolate between two classes of solutions with different topologies: black strings and black branes. Starting with the vacuum static case [15], we exhibit in Fig. 1 the domain of existence of the BHs in the $(A_H, T_H)$-plane, which shows that the $\epsilon_B = 1$ pattern is generic. Their horizon deformation is also displayed, and one can see that the parameters $\epsilon_H$ and $\epsilon_B$ are correlated, with $\epsilon_H/\epsilon_B \to 1$ for large horizon size (for instance this happens in Fig. 1 also in the $\epsilon_B = 5$ curve for very large values of the area, which are not displayed in the range shown). However, the small BHs are always close to sphericity, with a well-defined $L_H \to 0$ limit. This is a smooth, horizonless geometry, which can be viewed as a deformation of the globally AdS spacetime, providing a natural *background* for a model with $\epsilon_B \neq 1$. Moreover, the ratio $\epsilon_H/\epsilon_B$ is well defined as $\epsilon_B \to 0$, depending only on the value of $L_H$. Then, after the rescaling $\psi \to \bar{\psi}/\epsilon_B$, one finds that in the $\epsilon_B \to 0$ limit,

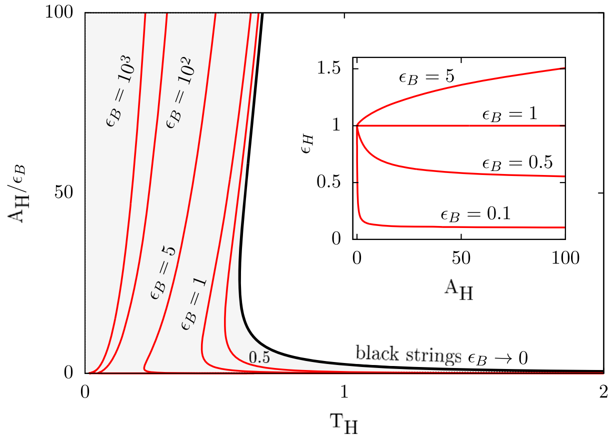


FIG. 1. The $(T_H, A_H)$-domain of existence of static vacuum black holes with a deformed sphere at infinity. The inset shows the horizon deformation $\epsilon_H$ for several values of the boundary squashing $\epsilon_B$.

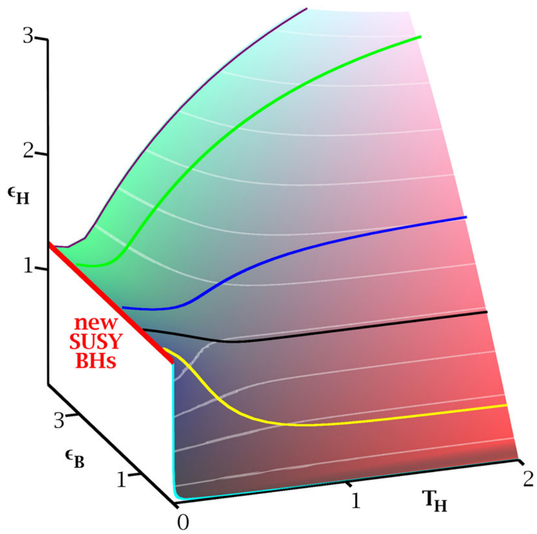


FIG. 2. The $(T_H, \epsilon_B, \epsilon_H)$-domain of existence is shown for a particular set of black holes. The $(Q, J, \Phi_m)$-dependence of the boundary squashing parameter $\epsilon_B$ is imposed such that the extremal limit corresponds to supersymmetric solutions [e.g. $\Phi_m = L(\epsilon_B^2 - 1)/2\sqrt{3}$]. Color curves are isolines of constant $\epsilon_B$, and grey curves are isolines of constant $\epsilon_H$.

the solutions become AdS black strings. For such configurations, both the boundary and the horizon metric are the direct product $S^2 \times S^1$, with an arbitrary periodicity for $S^1$ as parametrized by $\bar{\psi}$ [e.g. $ds_B^2 = \frac{L^2}{4}(d\theta^2 + \sin^2\theta d\phi^2 + d\bar{\psi}^2)$]. These solutions were originally found in [17] (see also [18,19]), and provide natural AlAdS generalizations of the (uniform) black strings in $d = 5$ Kaluza-Klein theory. We also notice that $\epsilon_B^2$ can be continued to negative values, which results in a different set of solutions with CTCs.

The infinitely squashed limit is also well defined. After taking $\epsilon_B \to \lambda N$ together with rescaling the coordinates, $r \to \lambda r$, $\theta \to \Theta/\lambda$, $\psi \to -\Psi/(N\lambda^2) - \phi$, $t \to t/\lambda$, one finds that $\lambda \to \infty$ results in an AlAdS twisted black brane [20], with a conformal boundary metric which is the product of time and

$$ds_B^2 = \frac{L^2}{4}\left(d\Theta^2 + \Theta^2 d\phi^2 + \left(d\Psi + N\frac{\Theta^2}{2}d\phi\right)^2\right). \tag{9}$$

The same type of line element is found for the horizon metric (although with different factors for the two distinct parts),

$$ds_{\mathcal{H}}^2 = \frac{L_H^2}{4}\left(d\Theta^2 + \Theta^2 d\phi^2 + m_H\left(d\Psi + N\frac{\Theta^2}{2}d\phi\right)^2\right). \tag{10}$$

In the generic spinning magnetized case, the relation between the horizon and boundary deformations is more intricate. Depending on the values of $M$, $J$, $Q$ and $\Phi_m$, one finds e.g. solutions with a large $\epsilon_B$ and arbitrarily small $\epsilon_H$. Similarly, there are spinning magnetized BHs whose horizon is a round sphere, $\epsilon_H = 1$, while the value of $\epsilon_B$ is very large (or very small). Some of these features can be seen in Fig. 2, where the $(\epsilon_B, \epsilon_H)$-diagram is shown vs $T_H$ for a particular set of solutions. In this figure it can be seen that the black holes reach a finite value of $\epsilon_H$ as we approach extremality ($T_H \to 0$). This is always the case as long as $\epsilon_B$ is different from 0. On the other hand, in this figure we can see that for nonextremal black holes (with $T_H \neq 0$), decreasing the value of $\epsilon_B$ to 0 also makes $\epsilon_H$ go to 0. This indicates the presence of a regular string limit, with $\epsilon_H/\epsilon_B$ being finite (note e.g. the linear relation between $\epsilon_B$ and $\epsilon_H$ for large enough values of the temperature). In Fig. 3 we show the area $A_H$ as a function of $\epsilon_H$ and $T_H$. In this figure we can see that the $\epsilon_H \to 0$ limit of nonextremal solutions causes the area to go up to infinity (see how the surface bends up on the right side of the figure). Actually this limit has a finite $A_H/\epsilon_B$ limit, indicating that the density $A_H/\epsilon_B$ of the limiting string is finite. We have verified numerically these features by directly constructing the string configurations and comparing the corresponding charge densities (such as $M/\epsilon_B$, etc.).

Therefore the black string limit is well defined for a part of the parameter space only (for example, close to extremality the correlation between $\epsilon_H$ and $\epsilon_B$ is lost, without a smooth black string limit in the extremal case). Nevertheless, the infinite squashing limit is well behaved, resulting in a family of charged and magnetized black branes with the same conformal boundary metric as in the static vacuum case [21].

Let us mention that, unsurprisingly, the squashed spinning and magnetized BHs share some common features with the $\epsilon_B = 1$ solutions in [3,4]. For example, their thermodynamics is qualitatively similar to that case, the BHs with a large enough boundary magnetic field

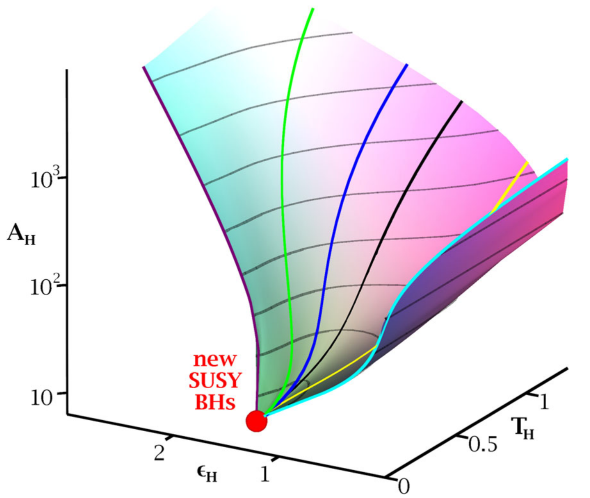


FIG. 3. The $(T_H, \epsilon_H, A_H)$-diagram for the same solutions as in Fig. 2. The full set of SUSY solutions is mapped here to a single point, possessing the same horizon area and deformation. Isolines of constant $\epsilon_B$ and $A_H$ are marked with color and grey lines, respectively.

becoming thermodynamically stable for the full range of $T_H$; see Fig. 3. Also, for any $\epsilon_B > 0$, the zero horizon size limit of the solutions with $\Phi_m \neq 0$ is nontrivial and describes a one-parameter family of spinning charged (nontopological) solitons. Such solutions possess no horizon, with the size of both parts of the horizon metric vanishing as $r_H \to 0$ (while $g_{rr}$ and $g_{tt}$ remain nonzero). Interestingly, for a given $\epsilon_B$, the solitons form a one-parameter family of solutions, most of their properties being determined by $\Phi_m$. For example, the following relation holds,

$$J = \Phi_m Q, \quad \text{with} \quad Q = -\frac{8\pi\Phi_m^2}{3\sqrt{3}}, \tag{11}$$

as implied by the existence of two first integrals of the system [21]. Although no similar expression exists for $M$, the relation

$$M = \frac{3\pi L^2}{32}\left(1 + \frac{32}{\sqrt{3}}\frac{\Phi_m}{L}(\epsilon_B^2 - 1) - \frac{64}{3}\frac{\Phi_m^2}{L^2}\epsilon_B^2\right) \tag{12}$$

provides a good fit for the mass of the solutions with $\epsilon_B$ close to 1 and a small boundary magnetic field.

## III. SUPERSYMMETRIC BLACK HOLES

As found in Ref. [22], a particular set of $\epsilon_B = 1$ BHs [3] preserves one quarter of the supersymmetry. Then it is natural to inquire if these special configurations survive when deforming the boundary geometry according to (2). To address this issue, we use the framework proposed in [22] and consider a line element,

$$ds^2 = -f^2(\rho)(dt + \Psi(\rho)\sigma_3)^2 + \frac{1}{f(\rho)}ds_B^2,$$
$$\text{with } ds_B^2 = d\rho^2 + a^2(\rho)(\sigma_1^2 + \sigma_2^2) + b^2(\rho)\sigma_3^2, \tag{13}$$

and a U(1) potential,

$$A = \frac{\sqrt{3}}{2}\left[f(\rho)dt + \left(f(\rho)\Psi(\rho) + \frac{L}{3}p(\rho)\right)\sigma_3\right]. \tag{14}$$

All functions which enter the above ansatz are determined by $a(\rho)$ and its derivatives [22], $a(\rho)$ being the solution of a sixth order equation,

$$\left(\nabla^2 f^{-1} + 8L^{-2}f^{-2} - \frac{L^2 g^2}{18} + f^{-1}g\right)' + \frac{4a'g}{af} = 0, \tag{15}$$

where $\nabla^2$ is the Laplacian for $ds_B^2$, $g = -\frac{a'''}{a'} - \frac{3a''}{a} - \frac{1}{a^2} + \frac{(4a')^2}{a^2}$, and $f^{-1} = \frac{L^2}{12a^2a'}(4(a')^3 + 7aa'a'' - a' + a^2a''')$. Any solution of this equation corresponds to a configuration which preserves (at least) one quarter of the supersymmetry.

Without any loss of generality, the horizon is located at $\rho = 0$, with a Taylor series expansion of the solution $a(\rho) = L\sum_{k\geq 1}\alpha_k(\frac{\rho}{L})^k$. Combining this expansion with the sixth order Eq. (15), one finds $\alpha_1 \neq 0$, $\alpha_2 = 0$, together with the constraint

$$(11\alpha_1^2 - 8)\alpha_4 = 0. \tag{16}$$

The choice $\alpha_4 = 0$ corresponds to the exact solution found by Gutowski and Reall in Ref. [22], with $a(\rho) = \alpha L\sinh(\rho/L)$, where $\alpha_1 = \alpha > 1/2$. These BHs possess a horizon with $L_H = L\sqrt{(4\alpha^2 - 1)/3}$ and $\epsilon_H = \sqrt{\alpha^2 + 3/4} > 1$, while $\epsilon_B = 1$ and $\Phi_m = 0$.

However, the condition (16) can also be satisfied by taking $\alpha_1 = 2\sqrt{2/11}$, $\alpha_4 \neq 0$. This leads to a new set of BHs with the following near-horizon expansion [23]:

$$\frac{a(\rho)}{L} = 2\sqrt{\frac{2}{11}}\frac{\rho}{L} + \alpha_3\left(\frac{\rho}{L}\right)^3 + \alpha_4\left(\frac{\rho}{L}\right)^4 + \cdots. \tag{17}$$

These asymptotics can be smoothly matched to a large-$\rho$ expansion of $a(\rho)$ with the following leading order terms,

$$\frac{a(\rho)}{L} = a_0 e^{\frac{\rho}{L}} + \left(a_2 + c\frac{\rho}{L}\right)\frac{e^{-\frac{\rho}{L}}}{a_0} + \left(a_4 + \frac{2 - 16a_2 - 5c}{12}c\frac{\rho}{L} - \frac{2}{3}c^2\left(\frac{\rho}{L}\right)^2\right)\frac{e^{-\frac{3\rho}{L}}}{a_0^3} + \cdots, \tag{18}$$

$\{\alpha_3, \alpha_4; a_0, a_2, a_4\}$ in the above relations being free parameters (with $a_0 \neq 0$) and $c = (1 - \epsilon_B^2)/4$.

This results in a family of AlAdS BHs, which, after moving to a nonrotating frame at infinity, can also be viewed as a special class within the ansatz (4). Their spatial boundary is a squashed sphere, with $F_3/F_2 \to \epsilon_B^2$. The global charges are determined by the squashing parameter $\epsilon_B$, with [21]

$$M = \pi L^2\left(\frac{7913}{34848} + \frac{33280}{35937}\frac{1}{\epsilon_B^2} - \frac{7}{36}\epsilon_B^2 + \frac{89}{864}\epsilon_B^4\right),$$

$$J = -\pi L^3\left(\frac{16640}{35937} - \frac{2795}{8712}\epsilon_B^2 + \frac{1}{9}\epsilon_B^4 - \frac{1}{27}\epsilon_B^6\right),$$

$$Q = -\pi\sqrt{3}L^2\frac{1}{13068}(6449 - 1936{\epsilon_B}^2 + 968{\epsilon_B}^4). \qquad (19)$$

They necessarily possess a boundary magnetic field, with $\Phi_m = \frac{2}{\sqrt{3}}(\epsilon_B^2 - 1)$, while the horizon angular velocity is $\Omega_H = 2/(L\epsilon_B^2)$. The most unusual feature of the new BHs is that although $\epsilon_B$ is arbitrary, their horizon geometry (1) is frozen, with $L_H = L\sqrt{7/11}$, $\epsilon_H = \sqrt{65/44}$ and

$$A_H = 7\pi^2 L^3 \frac{\sqrt{455}}{121}. \qquad (20)$$

As $\epsilon_B \to 1$, the solutions bifurcate from a critical Gutowski-Reall BH with $\alpha = 2\sqrt{2/11}$. Also, as seen in Figs. 2 and 3, they are approached smoothly as a particular limit of the general solutions. However, different from the nonextremal case, these BHs do not possess a solitonic limit. Nevertheless, SUSY solitons with $\epsilon_B \neq 1$ exist as well [24], satisfying a different set of boundary conditions at $\rho = 0$ and bifurcating from the globally AdS background. Again, most of their physical properties are determined by the boundary squashing parameter $\epsilon_B$. A diagram summarizing the picture for these three different types of SUSY solutions is shown in Fig. 4.

The frozen horizon geometry prevents the SUSY solutions from approaching a black string limit as $\epsilon_B \to 0$. Instead, a BH with nonasymptotically flat, non-AlAdS asymptotics is approached. The limit $\epsilon_B \to \infty$ is also nonstandard. The same scaling as in the non-SUSY case leads to an exact plane-fronted wave solution which is not asymptotically AdS [25].

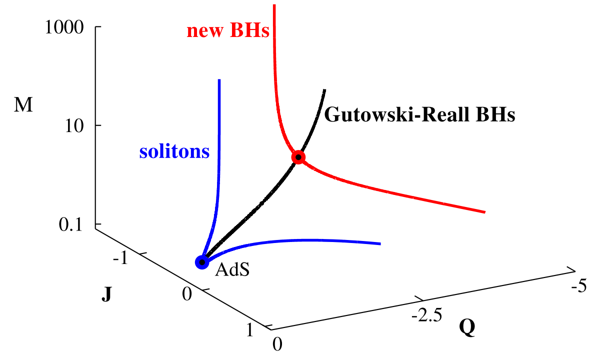


FIG. 4. The $(Q, J, M)$-diagram is shown for three families of supersymmetric AdS solutions.

## IV. FURTHER REMARKS

The new rotating magnetized BH solutions in this work provide new backgrounds whose AdS/CFT duals describe four-dimensional field theories in a squashed Einstein universe. Also, they can be uplifted either to type IIB or to eleven-dimensional supergravity [26–29].

Their existence raises many questions. In particular, it would be interesting to provide a microscopic interpretation from the boundary CFT for the entropy of the SUSY BHs. Generalizations of these solutions with an arbitrary multipolar structure of the U(1) field at infinity and two independent rotation parameters are also likely to exist, in particular, SUSY configurations.

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[1] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999); Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[3] M. Cvetic, H. Lu, and C. N. Pope, Phys. Lett. B **598**, 273 (2004).
[4] J. L. Blázquez-Salcedo, J. Kunz, F. Navarro-Lérida, and E. Radu, Phys. Lett. B **771**, 52 (2017).
[5] J. L. Blázquez-Salcedo, J. Kunz, F. Navarro-Lérida, and E. Radu, Phys. Rev. D **95**, 064018 (2017).
[6] The gauge freedom in the line element (4) is fixed by taking $F_2/F_1 = r^2(1 + r^2/L^2)$.
[7] C. Herdeiro and E. Radu, Phys. Lett. B **749**, 393 (2015).
[8] M. S. Costa, L. Greenspan, M. Oliveira, J. Penedones, and J. E. Santos, Classical Quantum Gravity **33**, 115011 (2016).

[9] J. L. Blázquez-Salcedo, J. Kunz, F. Navarro-Lérida, and E. Radu, Entropy **18**, 438 (2016).
[10] C. A. R. Herdeiro and E. Radu, Phys. Rev. Lett. **117**, 221102 (2016).
[11] M. Henningson and K. Skenderis, J. High Energy Phys. 07 (1998) 023.
[12] V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).
[13] B. Sahoo and H. U. Yee, J. High Energy Phys. 11 (2010) 095.
[14] We employ a collocation method for boundary-value ordinary differential equations, equipped with an adaptive mesh selection procedure. Typical mesh sizes include around $10^4$, the typical relative accuracy of the solutions being around $10^{-10}$.
[15] Some properties of the $\epsilon_B \neq 1$ static vacuum black holes were discussed in a different context in [16].
[16] K. Murata, T. Nishioka, and N. Tanahashi, Prog. Theor. Phys. **121**, 941 (2009).
[17] K. Copsey and G. T. Horowitz, J. High Energy Phys. 06 (2006) 021.
[18] R. B. Mann, E. Radu, and C. Stelea, J. High Energy Phys. 09 (2006) 073.
[19] A. Bernamonti, M. M. Caldarelli, D. Klemm, R. Olea, C. Sieg, and E. Zorzan, J. High Energy Phys. 01 (2008) 061.
[20] E. D'Hoker and P. Kraus, J. High Energy Phys. 03 (2010) 095.
[21] J. L. Blázquez-Salcedo, J. Kunz, F. Navarro-Lérida, and E. Radu, J. High Energy Phys. 02 (2018) 061.
[22] J. B. Gutowski and H. S. Reall, J. High Energy Phys. 02 (2004) 006.
[23] The existence of this near-horizon expansion was already noticed in Ref. [24].
[24] D. Cassani and D. Martelli, J. High Energy Phys. 08 (2014) 044.
[25] The black brane limiting configuration is also recovered as the $\alpha \to \infty$ limit of the Gutowski-Reall BHs [22].
[26] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Phys. Rev. D **60**, 064018 (1999).
[27] J. P. Gauntlett, D. Martelli, J. Sparks, and D. Waldram, Classical Quantum Gravity **21**, 4335 (2004).
[28] J. P. Gauntlett, E. O. Colgain, and O. Varela, J. High Energy Phys. 02 (2007) 049.
[29] J. P. Gauntlett and O. Varela, Phys. Rev. D **76**, 126007 (2007).