The underlying structure in Atanassov's IFS

J. Montero
Facultad de Matemáticas
Universidad Complutense
Madrid 28040, Spain
e-mail: monty@mat.ucm.es

D. Gómez
Escuela de Estadística
Universidad Complutense
Madrid 28040, Spain
e-mail: dagomez@estad.ucm.es

H. Bustince, E. Barrenechea
Departamento de Automática y Computación
Universidad Pública de Navarra
Pamplona 31006, Spain
e-mail: bustince@unavarra.es

Abstract

In this paper we shall stress the key relevance of addressing a specific structure for the valuation space in Atanassov's IFS, which may help to establish certain differences with other related models. In particular, we point out that each one of the valuation states proposed by Atanassov plays a different role, so they should be distinguished from their particular properties with respect to the other valuation states. In the same way, we realize that those three valuation states are not interchangeable and that each state should in principle allow a clear definition, as in any standard classification problem, always subject to potential direct estimation. Considering for example a transition (directed) graph in order to connect states provides in addition a new framework for useful more general models, taking simultaneously into account several logics when needed and avoiding the still binary heritage in Atanassov's model.

Key words: Atanassov's Intuitionistic Fuzzy Sets, Interval Valued Fuzzy Sets, Type-2 Fuzzy Sets, L-Fuzzy sets.

1 Introduction.

Since the introduction of Atanassov's intuitionistic fuzzy sets [4] (see also [5, 6, 7, 10, 28]), quite a number of papers have been published developing his model both from a theoretical and a practical point of view (see [11] for an overview of this an other formal generalizations of fuzzy sets [32] and some of their applications). Nevertheless, the serious scientific criticism received about the term *intuitionistic* [29] and even about the originality of the model itself

[13, 15] might many researchers not pay attention on the key issues Atanassov was addressing with his proposal (see also [8, 9]). Based upon a paper of the authors recently published [20] in this paper we pretend to stress that Atanassov model, if viewed as a classification system in the sense of [1, 2], is offering a contextual framework for fuzzy sets which is extremely close to linguistic modelling needs. As it will be shown below, the most basic unit for knowledge requires three valuation states, as Atanassov is postulating.

Let us briefly remind here that, given a fuzzy property, Atanassov model [4] is defined as a particular type-2 fuzzy set (see [14, 16, 17])

$$\mu : \mathcal{X} \to [0,1]^3$$

where X represents the family of objects under consideration in such a way that the degrees of *membership*, *non-membership* and *indeterminacy* are associated to each object $x \in X$, summing 1 these three values, for all $x \in X$. In this way the Zadeh's fuzzy sets are extended by imposing a simultaneous view into these three fuzzy sets.

In fact, nothing should be said about a fuzzy set without taking into account the whole family of fuzzy sets directly related to our fuzzy set of interest. For example, nothing should be said about *tallness* without taking into account *shortness*. But Atanassov wrote in his model that the standard alternative to a fuzzy set was its negation, keeping in this way an unconsciously binary framework, which leaded as a consequence to a third state (*indeterminacy*) difficult to explain.

Moreover, Atanassov did not realize that his model was not simply made out of three states, but from three states that require certain structure behind. This is being clear from the names he uses for his three classes. His *indeterminacy* is clearly different in nature to *membership* and *non-membership*, but this difference in nature is not being reflected anywhere in his model, apart from the different name they receive. Things are not different because they have a different name, but the other way round: things get different names because they are different.

In this paper we offer a model being consistent to Atanassov's intuition. The introducing of a particular structure (than can become more complex either in the number and relation between valuation states or in the logic allowing their manipulation) will avoid the isomorphism with interval valued fuzzy sets, since each model will require different structures (see [20]).

2 About Atanassov's negation

Our main argument is to view Atanassov's model as a classification model. As pointed out in [21], this decision making aid framework is in general closer to a fuzzy approach, rather than pursuing the crisp decision some models have as output (see also [22]). But in a classification framework it is a standard requirement in practice that classes under consideration allow a direct intuition. In this sense, conceiving a fuzzy set together its negation as the basic classification system should not be considered as appropriate. In fact, it looks quite unnatural to ask somebody to what extent *John is not-tall*.

The most terrible heritage of binary logic is perhaps the constant temptation to use *simple* models in order to escape complexity instead of approaching such a complexity. For example, we can view the question *is John tall?* as a classification problem between two possible answers, *yes* and *no*. This is because we realize that any question requires the possibility of more than one possible answer, at least *something* and its *opposite*. But the opposite of a concept within a classification framework is not its negation, but its *dual* or *antonym* concept (see, e.g., [23, 24]). And it has experimentally shown that people confuse duality and negation only within a very particular contexts where concepts are truly crisp [25] (note that from a classification point of view, *"no"* is the dual concept of *"yes"*, not its negation).

If we accept that the basic piece in our approach should be a classification unit, this unit should be at least formed by a concept and its antonym (see also [30]). But of course we should open the possibility to more complex non-binary classification units. For example, and coming back to the question *is John tall?*, most people will ask for additional intermediate possible answers apart from the extremes *yes* and *no*.

Many standard classification units are based upon a finite family of linearly ordered valuation states.

3 About Atanassov's indeterminacy

As already pointed out, choosing a concept together with its negation as the basic classification unit produces as a consequence a difficulty in explaining the role of *determinacy* in Atanassov's model. From a logical point of view it is difficult to justify that something is left outside a concept and its negation. But once negation is changed into antonym, of course there is room (and necessity) for a third state in order to capture what is not captured between a concept and its dual or antonym. For example, *ignorance* should appear in a quite general context, since *ignorance* use to be the previous status to knowledge process. The introduction of this extra *ignorance* valuation state, outside the standard classification valuation possibilities, seems to us a relevant originality in Atanassov's proposal. In fact, it is quite surprising, even in a crisp framework, that when there is no information some mathematical models still force to choose between *truth* and *falsehood*, or define a probability distribution between both states, when the only clear thing is that we know nothing (i.e., the degree of membership to *ignorance* is 1).

Atanassov's two-states plus *ignorance* classification unit is obviously related to Ruspini's partition [26], but it must be clearly stated that we can not assume that our classification should be a fuzzy partition in the sense of [26], which can perhaps be associated to ideal classification units in some particular context (its existence is neither assured in general, see [2]).

If a classification unit is based upon a finite family of linearly ordered valuation states, adding an *ignorance* state, the new classification unit will be consistent to Atanassov's main arguments.

4 About the underlying structure

As already pointed out, states can not be properly understood without a structure justifying names and expected properties of each name. Notice that, for example, classical 5-states valuation unit (given by *none*, *Poor*, *Average*, *Good* and *Very good*) is not just a bunch of states, but they define a very particular structure: those 5 valuation states define a linear order.

When we consider only two valuation states, the question of the existence of a particular structure explaining roles of each state seems no relevant at all. But as pointed out in [20], the underlying structure of Interval Values Fuzzy Sets [27, 31] and the underlying structure of Atanassov's IFS are not the same, as they both are different from Lukasiewicz's model (*true*, *possible*, *false*) which has also three valuation states. As already pointed out in [20], these three models have three valuation states, but depending on the model each *middle* state has a different relation with the remaining two states.

Valuation spaces require the definition of their particular structure, most probably a directed graph, which will hopefully show the transition learning procedure.

5 About the manipulation of states and objects

Once a directed graph had being fixed, we may wonder about combinations of states, so a logic must be defined between states, but of course unconnected classes can not be combined. The same applies to objects, which may require a different logic (see [18, 19]).

6 The model

Let us then assume

1.- A well-defined (crisp) finite set of *objects*, $\mathcal{X}$.

2.- A finite valuation space $\mathcal{C}$ of *valuation states*, fuzzy in nature but well defined from a representation point of view (perhaps a finite family of linguistic labels representing a concept).

Following [20], we stress the relevance of certain type-2 fuzzy sets [14, 16, 17] given by a mapping

$$\mu : \mathcal{X} \to [0, 1]^{\mathcal{C}}$$

once they are provided with a particular structure (including the *ignorance* state) in such a way that

A1 $\mathcal{X}$ is a well-defined non-empty, but finite, set of *objects* such that

- A1.1 There exists a crisp directed graph $(\mathcal{X}, \mathcal{P})$ showing physical immediacy between two distinct objects $x, y \in \mathcal{X}$ ($p_{xy} = 1$ in case there is immediacy between $x, y \in \mathcal{X}$ and $p_{xy} = 0$ otherwise).
- A1.2 There exists a logic on $\mathcal{X}$ allowing a consistent evaluation of questions about objects.

A2 $\mathcal{C}$ is a finite valuation space, with at least three elements, such that

- A2.1 There is a crisp directed graph $(\mathcal{C}, \mathcal{R})$, in such a way that $r_{ij} = 1$ in case there is immediacy between $i, j \in \mathcal{C}$ and $r_{ij} = 0$ otherwise.

A2.2 There exists a logic in $\mathcal{C}$, allowing a consistent evaluation of questions about valuation states.

A3 There exists an *ignorance* state $I \in \mathcal{C}$ such that

A3.1 For every $i \in \mathcal{C}$ there exist a path connecting i from I.

A3.2 $\mu_x(I) = 1, \forall x \in \mathcal{X}$, and $\mu_x(i) = 0, \forall x \in \mathcal{X}, \forall i \neq I$, when there is no available information (complete ignorance).

Additional conditions can be introduced. For example, we may impose the following condition that may be assumed by default, when there is no information about the structure of objects:

B1 $\mathcal{X}$ is a well-defined non-empty, but finite, set of objects such that its associated crisp directed graph $(\mathcal{X}, \mathcal{P})$ verifies that $p_{xy} = 0, \forall x \neq y$, so reference to the structure of objects can be avoided (no immediacy between two objects).

Analogously, we can also impose the following condition that may be assumed by default, when there is no information about the structure of the valuation space:

B2 $\mathcal{C}$ is a finite valuation space, with at least three elements, such that its associated crisp directed graph $(\mathcal{C}, \mathcal{R})$ verifies that for all $r_{ij} = 1, \forall i \neq j$, so reference to the structure of states can be partially avoided (immediacy between every two valuation states).

So we can conclude with the following definition as a model that in our opinion fits main Atanassov's requirements, but allowing an explanation of the dispute on Atanassov's original model, once the underlying classification structure is fixed (see [20] for details).

Definition 6.1 *Let*

$$\mu_A : \mathcal{X} \longrightarrow [0,1]^{\mathcal{C}}$$

be a function where $\mathcal{X}$ represents a finite family of objects and $\mathcal{C}$ represents a finite family of classes *or* states*. We shall say that A is a* Meta Fuzzy Set *if a directed graph $(\mathcal{C}, \mathcal{R})$ verifying all the above conditions B1,A2 and A3 has been defined, together with a logic $(\bigvee, \bigwedge, \neg)$ on $(\mathcal{C}, \mathcal{R})$.*

This logic should be defined taking into account the existence of a structure (see, e.g., [3]).

7 Final comments

Now, the next step in our research should be to check how key properties can be translated from Atanassov's original model into this model, taking into account alternative valuation spaces, alternative structures and alternative logics (putting together the results obtained in [3] and [12]).

Nevertheless, even if we acknowledge that Atanassov's model is not properly intuitionistic, and that it is mathematically equivalent to another pre-existent model, researchers should look ahead and realize from Atanassov's arguments that Zadeh's original proposal is still heavily dependent on a binary view of reality. Representing knowledge requires more complex models, starting with a proper representation of ignorance. Moreover, Atanassov's arguments point out the necessity of a global view to any question. In this way, some Atanassov's modelling deficiencies have been bypassed by including the existence of a structure in potentially very different valuation states where *ignorance* plays a significative role.

Acknowledgements: This paper has been partially supported by the National Science Foundation of Spain, grants TIN2006-06290 and TIN2007-65981.

References

[1] A. Amo, D. Gómez, J. Montero and G. Biging (2001): *Relevance and redundancy in fuzzy classification systems.* Mathware and Soft Computing 8:203–216.

[2] A. Amo, J. Montero, G. Biging and V. Cutello (2004): *Fuzzy classification systems.* European Journal of Operational Research 156:459–507.

[3] A. Amo, J. Montero and E. Molina (2001): *Representation of consistent recursive rules.* European Journal of Operational Research 130:29–53.

[4] K.T. Atanassov (1983): *Intuitionistic fuzzy sets.* In: V. Sgurev, ed., VII ITKR's Session, Sofia, June 1983 (deposed in Central Science and Technical Library, Bulgarian Academy of Sciences, 1697/84, in Bulgarian).

[5] K.T. Atanassov (1986): *Intuitionistic fuzzy sets.* Fuzzy Sets and Systems 20:87–96.

[6] K.T. Atanassov (1989): *More on intuitionistic fuzzy sets.* Fuzzy Sets and Systems 33:37–45.

[7] K.T. Atanassov (1999): *Intuitionistic Fuzzy sets*, Physica-Verlag, Heidelberg, New York.

[8] K.T. Atanassov (2005): *Answer to D. Dubois, S. Gottwald, P. Hajek, J. Kacprzyk and H. Prade's paper, Terminological difficulties in fuzzy set theory - the case of intuitionistic fuzzy sets.* Fuzzy Sets and Systems 156:496–499.

[9] K.T. Atanassov (2007): *A personal view on intuitionistic fuzzy sets.* In H. Bustince, F. Herrera and J. Montero, eds.: *Fuzzy Sets and Their Extensions: Representation, Aggregation and Models*, Springer Verlag, Berlin.

[10] H. Bustince and P. Burillo (1996): *Structures on intuitionistic fuzzy relations.* Fuzzy Sets and Systems 78:293–303.

[11] H. Bustince, F. Herrera and J. Montero, eds. (2007): *Fuzzy Sets and Their Extensions: Representation, Aggregation and Models*, Springer Verlag, Berlin.

[12] H. Bustince, J. Montero, E. Barrenechea and M. Pagola (2007): *Semiautoduality in a restricted family of aggregation operators*. Fuzzy Sets and Systems 158:1360–1377.

[13] G. Cattaneo and D. Ciucci (2006): *Basic intuitionistic principles in fuzzy set theories and its extensions (a terminological debate on Atanassov IFS).* Fuzzy sets and Systems 157:3198–3219.

[14] S. Coupland and R. John (2007): *Type-2 fuzzy logic and the modelling of uncertainty*. In H. Bustince, F. Herrera and J. Montero, eds.: *Fuzzy Sets and Their Extensions: Representation, Aggregation and Models*, Springer Verlag, Berlin.

[15] D. Dubois, S. Gottwald, P. Hajek, J. Kacprzyk and H. Prade (2005): *Terminological difficulties in fuzzy set theory - the case of intuitionistic fuzzy sets*. Fuzzy Sets and Systems 156:485–491.

[16] J.M. Mendel and R.I. John (2002): *Type-2 fuzzy sets made simple.* IEEE Transactions on Fuzzy Systems 10:117–127.

[17] J.M. Mendel (2007): *Advances in type-2 fuzzy sets and systems.* Information Sciences 117:84–110.

[18] J. Montero (1986): *Comprehensive fuzziness*. Fuzzy Sets and Systems 20:89–86.

[19] J. Montero (1987): *Extensive fuzziness*. Fuzzy Sets and Systems 21:201–209.

[20] J. Montero, D. Gómez and H. Bustince (2007): *On the relevance of some families of fuzzy sets*. Fuzzy Sets and Systems 158:2429–2442.

[21] J. Montero, D. Gómez and V. López (2007): *The role of fuzziness in decision making*. Studies in Fuzziness and Soft Computing 215:337–349.

[22] J. Montero and M. Mendel (1998): *Crisp acts, fuzzy decisions.* In S. Barro *et al.*, eds.: *Advances in Fuzzy Logic*, Universidad de Santiago de Compostela, Santiago de Compostela; pp. 219–238.

[23] A.M. Norwich and L.B. Turksen (1984): *A model for the measurement of membership and the consequences of its empirical implementation.* Fuzzy Sets and Systems 12:1–25.

[24] V. Novak (2001): *Antonyms and linguistic quantifiers in fuzzy logic.* Fuzzy Sets and Systems 124:335–351.

[25] C. Paradis and C. Willners (2006): *Antonymy and negation-The boundness hypothesis*. Journal of Pragmatics 38:1051–1080.

[26] E.H. Ruspini (1969): *A new approach to clustering*. Information and Control 15:22–32.

[27] R. Sambuc (1975): *Function Φ-flous, application a l'aide au diagnostic en pathologie thyroidienne.* These de doctorat en Medicine, Marseille.

[28] E. Szmidt and J. Kacprzyk (2000): *Distances between intuitionistic fuzzy sets.* Fuzzy Sets and Systems 114:505–518.

[29] G. Takeuti and S. Titani (1984): *Intuitionistic fuzzy logic and intuitionistic fuzzy set theory.* Journal of Symblic Logic 49:851–866.

[30] E. Trillas (2006): *On the use of words and fuzzy sets.* Information Sciences 176:1463–1487.

[31] I.B. Türksen (1986): *Interval valued fuzzy sets based on normal forms.* Fuzzy Sets and Systems 20:191–210.

[32] L.A. Zadeh (1965): *Fuzzy sets.* Information and Control 8:338–353.