

FUZZY MULTICRITERIA DECISION SUPPORT FOR BUDGET ALLOCATION IN THE TRANSPORT SECTOR

Javier Montero
Department of Statistics and OR
Faculty of Mathematics
Complutense University
Madrid 28040, Spain

Alan D. Pearman
School of Business and Economic Studies
University of Leeds
Leeds LS2 9JT, England

Juan Tejada
Department of Statistics and OR
Faculty of Mathematics
Complutense University
Madrid 28040, Spain

ABSTRACT

This paper compares two ways of providing decision support for the allocation of a fixed financial budget among a set of competing highway investment proposals. The first, which is described only in outline, uses a broadly conventional, hierarchically structured linear additive multicriteria model. The technical focus of the paper, however, is on the second, an approach based in fuzzy multicriteria modelling. The thinking which led us to explore this approach is set out, together with the formal structure of the model. The results of a small case study are given and an assessment is made of how decision makers' understanding of the investment options available can be enhanced by using the two models in tandem.

Keywords: linear additive value models; fuzzy multicriteria models; budget allocation; transportation planning

1. BACKGROUND

Transport planners working for individual city or regional authorities are annually faced with the problem of deciding, from a wide range of proposals, which subset of highway investments to undertake within their limited budget. Although the total expenditure each year by any one authority may be quite high, the individual projects are typically small ones. In Great Britain, where the applied work for this paper is based, project costs might range from £50k to £1m. The projects themselves are diverse in character with a correspondingly diverse set of impacts. Many are designed to achieve improvements in safety, environment or access and have impacts which are not readily quantifiable. Particularly for smaller projects, the amount of effort that can be expended on the project evaluation process must be quite limited, if it is not to undermine the potential cost-effectiveness of the projects themselves. At the same time, there are widespread doubts that small-scale but good-value-for-money projects are being set aside in favour of more expensive, higher-profile but less effective ones. Hence a way of comparing the full range of potential schemes in a single assessment model is highly desirable.

To address this problem, a hierarchical linear additive value multicriteria model was developed, together with associated computer software, *COMPASS* [5], [10]. In this paper, the original *COMPASS* model and the thinking behind it serves as the starting point for the development of a fuzzy multicriteria approach. This approach, we argue, offers insights into the relative performance of competing schemes over and above those provided by the linear additive model used in *COMPASS*. It can usefully be employed to complement the scheme rankings emerging from the conventional model. In the first two sections of the paper, we provide some brief background on *COMPASS*, to put the development of the fuzzy multicriteria model in context.

COMPASS uses a set of 32 lowest-level criteria to assess competing schemes. A substantial minority of the criteria are assessed on 0-10 subjective judgmental scales. The 32 criteria can be successively aggregated to 11 or to 4 criteria (Figure 1). At this final most aggregate level, projects are assessed in terms of their contributions (measured on a judgmental scale or by the

selection of a suitable proxy) towards improvements in safety, traffic conditions, environment and, finally, the planning and development of the area. The ability to assess schemes in less detail than is required when working with the complete set of 32 criteria facilitates comparison between schemes of different costs. Expensive schemes can be assessed in full, cheaper ones only in terms of aggregate impacts. Further details are given in [9].

The 32 criteria are intended to capture the effectiveness of each scheme, measured in terms of the difference between the evaluation of the situation with and without the project in place:

$$E_h = \sum_{i=1}^{32} w_i v_i(X_{hi}) \quad (1)$$

where E_h is the aggregate effectiveness of project h ; w_i is the weight given to the i th impact and $v_i(X_{hi})$ is the scaled score of project h on impact i .

Capital cost is assessed separately and is treated by *COMPASS* as the rationed resource, with the final ranking of schemes depending on their effectiveness:capital cost ratios. This basis for ranking derives from the parallel with cost-benefit analysis. In the presence of a binding capital cost constraint, net present value is in general an incorrect basis on which to select from among a group of competing investments. A fully optimal selection of projects requires the solution of a potentially unwieldy integer programming problem, but selecting projects in decreasing order of benefit:capital cost ratio until the budget is exhausted is usually an effective heuristic [8].

COMPASS diverges from ‘textbook’ multicriteria analysis at a number of points, largely in response to important practicalities. It is effectively a ‘production line’ system, which must be amenable to use by non-specialists on a wide range of schemes. The attribute set, measurement scales and initial attribute weights are fixed in advance, without reference to the particular set of projects being analysed. The attribute set is also a large one. This led us, in combination with the practical problems of getting reliable input data, to build a deterministic model of what is in principle a choice problem under uncertainty. We opted instead for a system which emphasised sensitivity testing as a way of exploring doubts about weights and

data precision, one which was intended to support the complex overall process of project selection, rather than purporting to be prescriptive.

2. ASSESSMENT OF THE LINEAR ADDITIVE MODEL

The original *COMPASS* model has now been applied to a number of sets of highway improvement schemes. As a result, it is possible to make an initial assessment of how the model has performed. One of the difficulties of doing so, however, is the lack of a norm - there is no set of projects for which we have a "correct" ranking. Hence assessment is essentially assessment of the functioning of the system and users' reaction to it and not of the results themselves.

One area where difficulties have been encountered concerns units of measurement for the attributes and the weighting of attributes. It is clearly important that users fully understand the units in which the different dimensions of impact are assessed, but there have been cases where they have not. This is particularly important with regard deriving new or amended sets of attribute weights. The number of attributes alone makes the weighting problem a difficult one and this is compounded by the judgmental scales on which a number of the attributes are assessed. Vagueness about measurement scales serves further to compound these problems.

A related difficulty is that of sensitivity testing in multiattribute models. There are major problems here which have not been satisfactorily resolved [13]. Although *COMPASS* permits a full range of one-variable-at-a-time sensitivity testing, it does no more. This must be a matter of concern in a model with a potential 32 variables.

It is this basic concern with potential inaccuracy in input data and weights and the limited possibilities for sensitivity testing which has led us to investigate an alternative model of the decision process, one based on fuzzy multicriteria analysis.

3. THE FUZZY MULTICRITERIA MODEL

Fuzzy multicriteria decision making has found a range of applications. Of those which have been concerned with choice within a discrete set of alternatives, most have had as their focus the identification of a single, most preferred alternative. They have tended to be concerned with relatively small sets of alternatives. The highway project priority assessment problem, however, generates potentially large sets to be evaluated and is concerned with ranking, rather than with simply identifying a single preferred option.

The administrative context is also important. Highway project selection is not a problem where it is realistic to expect active involvement on a regular basis from the elected representatives who will finally determine the list of projects to be implemented. Their wish is to set guidelines that reflect their political judgements and then to delegate responsibility for identifying appropriate projects to the authority's transport planners. Only immediately prior to the finalisation of the project list will the politicians become involved again, to adjust priority rankings and perhaps to veto some proposals.

The procedure described in the present paper is based on a family of three related programs, each of which uses a fuzzy multicriteria assessment procedure adapted from Siskos *et al.* [17], which in turn is based on the *ELECTRE III* method developed by Roy [15]. The adaptations improve the functioning of the model in this particular application and expand the information available to the decision maker, along the lines proposed in [6]. From the point of view of highway project priority assessment, the model has the following important characteristics:

- it can be formally specified in advance of the ranking process itself and uses a set of weights to reflect the relative importance attached by decision makers to different dimensions of impact;
- it can accommodate substantial numbers of projects;

- it is based on pairwise comparison of projects, thus complementing the value function model in a manner which is in keeping with the way in which the fine-tuning of the priority list is often undertaken;
- it acknowledges potential inaccuracy in measuring different schemes' impacts so that small differences in a given dimension need not contribute to the assessment of whether one project is preferable to another;
- it acknowledges possible incomparabilities between schemes, so that projects which are qualitatively very different from each other are not automatically processed into a single undifferentiated ranking list.

The Siskos model is based on the integration of fuzzy outranking relations into a domination structure. First, fuzzy outranking relations are established using partial fuzzy relations on one criterion at a time. Secondly, the fuzzy outranking relation is used to define the fuzzy set of non-dominated alternatives. Siskos in his applied work then looked to select for implementation the single alternative whose degree of non-domination by the other alternatives was highest and close to one.

In this paper, building on the Siskos approach, we consider a finite set of investment alternatives, $A = \{a, b, c, \dots\}$, each evaluated on n criteria by a real number, g_i ($i = 1, \dots, n$). $g_i(a)$ represents the performance of alternative a on the i th. criterion and is assumed to be such that higher values correspond to preferred levels of performance. Any alternative, a , is assessed by an n -vector $g(a)$ with entries corresponding to its performance on each criterion. Each criterion is given a weight, w_i , where the sum of the weights is normalised to one. The weights reflect the relative importance which the decision maker gives to each criterion and in our model are those initially derived for *COMPASS*, see [5]. The weights are thus the relative importance attached to unit changes in project scores on each of the 32 criteria. In the Siskos model, what are weighted are partial fuzzy outranking relations. Although it is intuitively defensible to weight contribution to overall outranking in the same way as contribution to overall value, a more specific, axiomatically-based justification would be desirable. This is

more the case in our larger-scale, relatively mechanistic application than it was in Siskos' original work (see [17], pp. 264/5).

The basic components of the Siskos evaluation model are n pairs of partial fuzzy relations, $d_i(a, b)$ and $D_i(a, b)$. The first is the partial fuzzy outranking relation of b by a , defined as follows for the i th. criterion:

$$d_i(a, b) = \begin{cases} 1 & \text{if } g_i(b) - g_i(a) \leq 0 \\ 0 & \text{if } g_i(b) - g_i(a) \geq s_i \\ \text{between 0 and 1} & \text{otherwise} \end{cases} \quad (2)$$

The second is the fuzzy discordance relation:

$$D_i(a, b) = \begin{cases} 1 & \text{if } g_i(b) - g_i(a) \geq v_i \\ 0 & \text{if } g_i(b) - g_i(a) \leq s_i \\ \text{between 0 and 1} & \text{otherwise} \end{cases} \quad (3)$$

(2) and (3) implement a view of preference judgement which is close in spirit to the pseudo-criterion concept introduced in [16]. Similar ideas are further developed in [3] and [12].

The significance threshold for criterion i , s_i , provides a way of responding to doubts about data reliability and to some extent to doubts about the weights as well. The pair of alternatives (a, b) belongs to the set of pairs in which a is at least as good as b with membership degree 1 only if a outscores b according to the i th. criterion. But even if b outscores a , the possibility exists that criterion i might contribute to the general conclusion that a is at least as good as b , for example, because of misperceptions or measurement errors. Hence up to a prespecified sensitivity level, s_i , $g_i(b) - g_i(a) > 0$ is still taken as potential support for (a, b) belonging to the set in which a is at least as good as b according to criterion i , but with a membership degree that monotonically decreases from one to zero as the difference tends towards s_i . Once the difference is above s_i , (a, b) no longer belongs to the set.

$D_i(a, b)$ reflects incomparability between a and b on account of the alternatives' relative performance on criterion i . The basic idea is that once the difference $g_i(b) - g_i(a)$ goes above

a certain veto level, v_p , alternative b is so much better under criterion i than alternative a that this last alternative cannot outrank alternative b in the final outranking relation; (a, b) belongs with degree one to the set of pairs of alternatives for which incomparability against a exists on criterion i . In cases where the difference is greater than the significance threshold but less than the veto level, the pair (a, b) belongs to the same set, but with membership degree less than one. $D_i(a, b)$ is monotonically decreasing as $g_i(b) - g_i(a)$ decreases; once this difference falls below s_p , no case for incomparability can be established.

In our initial application of the Siskos framework, we have used linear interpolation for both $D_i(a, b)$ and $d_i(a, b)$ and have set v_i high (Figure 2). The choice of linear interpolation was motivated largely by simplicity. The decision to set the veto threshold high derives partly from the wish to avoid unnecessary incomparabilities in circumstances where the true values for the v_i may be hard to establish and partly by the fact that highway priority assessment techniques (PATs) are intended to analyse a wide range of project types and hence to facilitate the comparison of schemes with diverse outcomes. It would only be in extreme circumstances that we would wish the method to use the veto threshold.

With $d_i(a, b)$ and $D_i(a, b)$ as basic inputs, the Siskos method now proceeds to the first phase of the ranking process for alternatives considered in their entirety. In this phase, the n partial fuzzy outranking relations are aggregated into a single fuzzy concordance relation:

$$C(a, b) = \sum_{i=1}^{32} w_i d_i(a, b) \quad (4)$$

Next, the fuzzy concordance relation $C(a, b)$ and the n fuzzy discordance relations $D_i(a, b)$ are linked to obtain a fuzzy outranking relation, $d(a, b)$, defined in the first instance as:

$$d(a, b) = \sum_{i=1}^{32} d_i(a, b) \times \text{Min}_i \{1 - D_i(a, b)\} \quad (5)$$

In respect of (5), we diverge from Siskos, who used Roy's original *ELECTRE III* definition [14]. We do so to ensure that, in computing the degree of outranking, the basic concordance measure is modified by the existence of discordance to a lesser extent than Roy's formulation

would do. The motivation is to allow a high level of discordance on any one criterion to downgrade the standing of an alternative but not to induce the major downgrading that can occur as the cumulative consequence of a number of minor discordances if a multiplicative adjustment factor is used when there are a large number of criteria for which discordance may potentially arise. An alternative approach would be to replace the latter element of (5) by a weighted product (geometric average) with some appropriate, previously defined importance coefficients, as suggested in [12] (p. 51). While using (5) has the disadvantage of using only the worst case, a weighted product makes use of all cases and also overcomes the undesirable cumulation effect.

Once the fuzzy outranking relation (5) has been computed as described above, the Siskos model moves into a second phase aimed at providing rankings of alternatives. For this it is necessary to define a fuzzy domination relation that, for every pair of alternatives, measures the outranking intensity between them:

$$d^v(a,b) = \begin{cases} d(a,b) - d(b,a) & \text{if } d(a,b) \geq d(b,a) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

From (6) may be computed directly the fuzzy set of non-dominated alternatives with membership degree defined in Orlovsky [7] by:

$$\mu^{ND}(a) = 1 - \max_{b \in A} \{d(b,a) - d(a,b), 0\} \quad (7)$$

At this stage, because our concern is with ranking alternatives rather than selecting a single preferred option, we now depart from the Siskos procedure. Individual alternatives are ranked using a successive discarding technique due to Montero and Tejada [6], in which the worst alternatives are successively dropped, taking into account at each iteration only those alternatives which have not previously been rejected:

- STEP 1: Set $Y = A$ and evaluate $\mu^{ND}: Y \rightarrow [0, 1]$
- STEP 2: Discard $D = [a \in Y: \mu^{ND}(a) = \text{Min}_{a \in Y} \{\mu^{ND}(a)\}]$ (8)
- STEP 3: Replace Y by $Y - D$ and repeat STEP 2 until $Y = \emptyset$

In this way a first ranking of alternatives is obtained. The type of choice mechanism represented by (8) is potentially non-monotonic, in the sense that altering the valuation of only one alternative can, in principle, change the whole ranking. Nonetheless, the analysis of relative ranking of alternatives from iteration to iteration certainly complements the information available from the basic *COMPASS* and *ELECTRE III* outputs, also included in our *PAT* set of programs. In practical applications attempted so far, this has been found to be a useful tool for getting better insight and understanding of each particular preference structure.

The ranking just derived is based on project effectiveness alone; an important further consideration in the *PAT* application, however, is the treatment of capital cost, as set out in section 2. With this in mind, we next define a fuzzy set of good and cheap projects with a membership function given by:

$$\mu^{FC}(a) = \mu^{ND}(a) \times \left\{ \frac{K}{\text{Cost}(a)} \right\}$$

where (9)

$$K = \text{Min}_{a \in A} \{\text{Cost}(a)\}$$

Thus good and cheap alternatives are defined by the product form of the fuzzy intersection operation (effectively a multiplication of membership levels) and degree of cheapness is defined relative to the lowest cost alternative being considered. The previous non-domination and successive discarding operations are then repeated to provide a second project ranking. This adjustment endeavours to compensate for the possibility that the relatively poor performance of one alternative may be a direct consequence of its being a low-cost one. With, in practice, differences in capital cost as large as 50:1 when comparing individual projects, it is clearly important that such relativities be acknowledged. While this could be done simply by

introducing capital cost as an appropriately weighted 33rd. criterion, we prefer to maintain a degree of separateness in the treatment of this rationed resource.

Next the ranking process is repeated a third time, on this occasion using (10) to extend the previous outranking relation (5):

$$d'(a,b) = d(a,b) \times \left\{ \text{Min}\left[1, \frac{\text{Cost}(a)}{\text{Cost}(b)}\right] \right\} \quad (10)$$

Thus outranking is influenced by alternatives' relative cost, which in turn ensures that capital cost influences the nondomination and discarding operations (7) and (8).

Finally, a straightforward effectiveness-cost ratio is computed on individual alternatives' scores, as in the original *COMPASS*.

This basic structure forms the core of the trio of evaluation programs. The first of these, *PAT1*, receives as data input scores for each of the projects for all 32 impacts, together with criteria weights and the significance and veto thresholds, details of each project's capital costs and a target aggregate cost range for the set of projects to be implemented. It then uses (2) through to (10) to analyse and rank the individual projects under consideration (weights were taken from the original *COMPASS*). Having done so, the program then moves into a second, separate mode of operation, in which the alternatives being compared are not individual projects, but sets of projects. The rationale for this is that the final decision required is a choice about a set of projects and in some respects the set has characteristics which are not straightforwardly assessable by considering its individual members in isolation. For example, the inclusion of a project with certain undesirable environmental impacts may be unacceptable on a project by project basis of comparison, but, if it is known that there are a number of other projects to be included in the same investment package which are environmentally desirable, the inclusion of one poor project may be acceptable, in order to gain, say, the safety benefits

that the project concerned may offer. In this way, a new angle is given to the decision making process, over and above what is offered by the effectiveness-cost rankings in *COMPASS*.

PAT1 takes the information on what is an acceptable cost range (typically an upper bound which may slightly exceed the stated budget ceiling and a lower bound which falls a little below it) and uses this information to create a set of what are termed maximal subsets of projects. These are sets of projects whose total cost falls within the given capital cost range and are such that no other project can be added to the set without exceeding the range's upper bound (maximal subsets of projects are obtained here by means of a particular algorithm included in *PAT1*, but any other analogously efficient algorithm could have been applied). Once the set of maximal subsets has been established, the score for each one of these maximal subsets is obtained by adding the single scores of the projects it contains, criterion by criterion (since the scores of projects with overlapping consequences cannot be simply added, independence between projects should be checked previously). The program then analyses those subsets in precisely the same way that the individual projects were treated. In particular, the analysis offers guidance as to which feasible subsets appear to perform well both in terms of a conventional effectiveness-cost ratio and using the unadjusted and cost-adjusted fuzzy multicriteria ranking techniques based on non-domination.

One potential problem with this procedure is that for large sets of projects and/or for wide cost ranges, the number of maximal subsets can become unwieldy. For this reason *PAT2*, the second of the trio of programs, works not on the full set of originally specified projects, but on any subset that the user chooses to define, e.g., on the basis of the ranking of individual projects which emerges from *PAT1*. *PAT2* simply analyses this smaller set of projects in the same way as *PAT1* handles the full set, except that it no longer restricts itself to maximal subsets but examines all subsets that fall within the cost range. In this way, the cost-effectiveness of small marginal projects can be addressed, with the possibility that the decision maker may schedule such a project late in the financial year or even choose deliberately to underspend if circumstances outside the orbit of the model suggest this may be desirable.

Lastly, *PAT3* works only with preselected groups of individual projects. Its role is to produce a final ranking of project sets using the same four bases for ranking as did *PAT1*, but with no seriously sub-optimal sets involved. *PAT3* also provides – as do the previous *PAT1* and *PAT2* – information about the similarity and non-comparability relations between the sets. In practice, it might be this group of sets and the ranking of its members that is the principal focus of discussion with decision makers in terms of finally selecting a set of projects to implement (see [11], for additional details on the output of the *PAT1*, *PAT2* and *PAT3* programs).

4. RESULTS FROM THE FUZZY MODEL

The programs *PAT1*, *PAT2* and *PAT3* have been applied on an exploratory basis to the analysis of twelve schemes which have been under consideration by a local government highway authority in south-east England (see Table 1).

TABLE 1 ABOUT HERE

These twelve projects are reasonably typical of the range which has to be considered, with the exceptions that:

- the schemes divide clearly into high-cost and low-cost groups, with few in between;
- the high-cost schemes are substantially higher in number than would be the case if the sample were a truly representative one.

Central to the performance of the fuzzy multicriteria evaluation method is the basic calculation of non-domination [equation (7)]. As can be seen from the table, for the most part ND parallels effectiveness as calculated in *COMPASS* [equation (1)], but with some noteworthy differences. Project 5, which in fact scores very badly on certain important criteria and hence

overall in terms of effectiveness, ranks better in terms of ND. Project 8, which has no particularly weak points (although also few strong ones) is raised in the ranking. Project 1 drops, because its poor scores on certain environmental impacts are no longer, with ND, compensated so much by good scores on the traffic criteria. Generally, all-round adequate schemes do better and very good or very poor performances are rated less extreme. This is essentially because extreme performances are not so likely to be distinguished from good or poor ones by an ND measure with the veto threshold set high, so that, provided a project has some good aspects, it can record a reasonably good ND score.

In order to test the working of the method more fully, it was hypothesised that the authority concerned had an investment budget of £8000K, which was treated for the purpose of applying the *PAT* programs as a range from £7750K, to £8000K. On this basis and using simply effectiveness:cost ratios *COMPASS* would have selected to undertake in order of preference projects 11, 8, 9, 12, 10, 7, 1, 6, and 4 [Table 1]. The fuzzy multicriteria model's ND:cost ratio suggests the same except that 6 would be selected before 1 and 4 would only be included after 2 and 5 had been rejected as exceeding the remaining budget.

The *PAT* trio of programs in fact rank projects using successive discarding, as explained earlier. Successive discarding applied to ND and to each of the two cost-adjusted versions gives the results in Table 2.

TABLE 2 ABOUT HERE

Although there are potentially differences in ranking using discarding, since ND is recomputed after each discarding, in fact the discarding ranks are very similar in this application to those given by the initial ND calculation. This is especially apparent in comparing ranking in terms of μ^{FE} [equation (9)], where the effect of the adjustment to ND is equivalent to taking the original ND:cost ratio [Table 1]. However the assessment in terms of relative cost adjusted

non-domination [equation (10)] is somewhat different, notably in advancing project 5 (a poor 12th. in EFF:C ratio terms) to rank fourth. This seems to be a consequence of the neutralising influence of the ND calculation on poor scores (referred to earlier) combined with a tendency of this form of cost relative to have only limited effect and so to advance in the ranking projects with a high ND even if they are relatively expensive.

Moving into the aggregative phase of *PAT1*, the program identifies in this example 15 maximal subsets with aggregate costs in the prescribed range. Applying in turn each of the three bases for successive discarding yields the same ranking for the top four subsets:

TABLE 3 ABOUT HERE

The reason the three methods provide identical rankings is that the cost range is very low in relative terms (£250K. in £8000K.) so the potential influence of either cost adjustment is limited. However, it is worth noting that the EFF:C ratio ranking of the subsets is somewhat different, scoring subsets B and I in the top ranked positions, but relegating subsets C and H to rank 9th. and 10th. respectively. Subset A ranks 3rd. and is the only other one of the fifteen that does not contain project 5. Thus the difference seems to arise from the nature of the ND definition discussed earlier.

On this basis, it was decided to take each of the subsets from Table 3 and apply *PAT2* to them in turn. *PAT2* examines all subsets within the cost range, not just maximal subsets. In each case, the maximal subset continued to rank first on all three successive discarding calculations. Thus on this occasion, in no case did removing one or more small projects improve overall performance. Given that it is the small projects that are cost-effective in this particular set, this is scarcely surprising. It is also worth noting that dropping project 5 was impossible since, with a cost of £3300K., it would have taken total cost below the lower bound of the stated cost range.

With this confirmation that the four leading subsets identified by *PAT1* seemed to be the most promising, they were then input to *PAT3* for a final comparison against each other, without other subsets having any potential influence. Each of the three discarding methods ranked the four the same as in *PAT2* (Table 3). The effectiveness:cost ranking was B, I, C, H, as expected, since it is not affected by the presence of other comparators and this was the order derived in *PAT1*.

5. COMPARISON AND ASSESSMENT OF THE TWO APPROACHES

At present, the fuzzy multicriteria model has only been used experimentally. An important issue is now to start to assess the extent to which it is providing a better or different set of insights about projects than does the original *COMPASS* program.

As stated earlier, we have no absolutely correct set of rankings against which to assess the model's performance. Our judgement must be based on our own and users' responses to the way the model operates. The fact that both *COMPASS* and the new model are producing broadly similar rankings and selections is comforting. The fact that they are not identical is not surprising, but is potentially important. It suggests at least tentatively that the multicriteria model, as applied to this set of competing road investment projects, is capturing in its evaluation some characteristics of projects and sets of projects different from the linear additive model, as discussed in section 4.

Table 3, along with the results from the two previous tables, implies that subsets I and B are probably the main contenders for implementation, since the other two involve project 5 which is arguably over-valued by the fuzzy approach and retained in the subset analysis in part as a consequence of the choice of a narrow pair of budget limits. Thus the discussion begins to focus on three issues:

- whether to implement project 2 or projects 1 and 4 together, the latter being the more expensive option;

- the extent to which groups of projects may have characteristics over and above the characteristics of their individual members;
- the treatment of projects like project 5, that have either very good or very poor dimensions of performance.

In this last case, there is also a more general question on veto thresholds and whether they have been set inappropriately high.

Overall, it seems that the fuzzy multicriteria model is capable of providing a set of project subsets to form the agenda for the final, politically driven decision making process, together with indications of issues to bear in mind when discussing the different aspects of the shortlist. To some extent against this must be set the relative complexity of the model and the possibility that concepts such as sensitivity and veto thresholds and possibly non-linear membership functions may prove too complex for engineers, politicians or both. Whether or not this is a serious problem may be capable of being mitigated by the way in which it is presented; users do not have to be closely involved with these relatively technical ideas to get the insights that the trio of programs offers.

Clearly, these are matters for further research/development work, along with further consideration of the setting of the threshold levels and the shape of the interpolations in equations (2) and (3). Over and above this, perhaps the key area for more fundamental research is one which is not directly addressed in this paper, the treatment of small projects. In *COMPASS*, smaller projects can be evaluated in a cost-effective way by retreating up the value tree hierarchy to a more aggregate level, see [10]. In fuzzy multicriteria models, evaluation and aggregation within fuzzy hierarchies seems to have been given relatively little attention. If the fuzzy model is to have the range of application that the conventional linear additive model does, a fuller understanding is needed of how comparable evaluations at high and low levels of aggregation can be made.

6. FINAL COMMENTS

The question of choosing a set of alternatives subject to some constraints - e.g., total cost - has been already addressed by the *PROMETHEE V* software [1], with generalised criteria similar to those proposed in this paper, and considering alternatives grouped in clusters (for example, based on their geographical area). However, the treatment of the cost variable differs between the two approaches. Additionally, there are non-trivial differences in the way the comparisons between sets of projects are undertaken.

First, cost in our model is not considered as a standard criterion. It is quite common in multicriteria analysis to keep capital or similar costs separate, e.g., [4]. What are reasonable aggregation rules to apply for other criteria often do not transfer readily to permit inclusion of a cost dimension. Since cost can require special treatment, we propose to incorporate the cost dimension separately, once other criteria have been already aggregated into one. We do this by means of a separate rule - particular proposals are expressions (9) and (10). Decision makers do want to optimise performance criteria, but in many situations spending as much as possible of the available budget and showing high cost effectiveness are two further objectives.

Secondly, global evaluation of a subset of alternatives is obtained in *PROMETHEE V* by adding final dominance evaluations for each alternative within such a subset, while we derive dominance values for each group of alternatives by summing basic criteria evaluations. In this way, we maintain part of the *COMPASS* approach. Scores for each subset of alternatives and each criterion are defined as the sum of the scores of those alternatives for that criterion, allowing subsets of projects to be treated as a standard alternative. The dominance evaluation process is then developed every time from these group scores. Adding the basic evaluations - our input data - seems to us more intuitive than adding dominance evaluations - the output of a quite complex process. Evaluations of alternatives with overlapping consequences should not be by summation - in either approach - but at this point some of the *PROMETHEE V* features can be adapted into our model in order to consider more appropriate sets of non-

overlapping alternatives. Restrictions relative to compatibility or geographical area can also be considered in our search for maximal subsets of alternatives within the cost range.

Finally, the role of the decision maker in the *PROMETHEE V* procedure is different to that in our proposal, since we need the decision maker to act several times in the middle of the process. Instead of having one procedure elaborated from the basic input data alone, to be checked at the end of the process, we propose three simpler procedures, to be structured by the decision maker. This provides an extra opportunity to respond to the decision maker's insights about the particular problem being analysed, although, clearly, all the features already included in recent versions of the *PROMETHEE* software - including the *GAIA* graphical analysis - will certainly allow *PROMETHEE* to be used as a highly evolved and flexible tool, available as a commercial package, see [2].

ACKNOWLEDGEMENTS

The preparation of this paper has been supported by Direccion General de Investigacion Cientifica y Technica (national grant PB91-0389) and under a British-Spanish Accion Integrada (project 180/07) through the British Council. The authors would like to express their appreciation to an anonymous referee for valuable suggestions and comments.

REFERENCES

- [1] J.P. Brans and B. Mareschal, 'PROMETHEE V; MCDM problems with segmentation constraints'; *INFOR*, vol. 30, 1992, pp.85-96
- [2] J.P. Brans and B. Mareschal , 'The PROMETHEE-GAIA decision support system for multicriteria investigations'; *Decision Support Systems* (to appear)
- [3] J.P. Brans and Ph. Vincke, "A preference ranking organisation method", *Management Science*, vol. 31, 1985, pp. 647-56.
- [4] W. Edwards and J.R. Newman, *Multiaattribute Evaluation*, Sage Publications, Beverly Hills, 1982.
- [5] P.J. Mackie, A.D. May, A.D. Pearman and D. Simon, "A computer-aided priority assessment technique for local authority transport projects", pp.161-9 in Highway Appraisal and Design, Proceedings of the PTRC Summer Annual Meeting Seminar E, PTRC, London, 1988.
- [6] J. Montero and J. Tejada, 'Fuzzy preferences in decision making', in B. Bouchon and R.R. Yager (eds.) *Uncertainty in Knowledge-Based Systems*, Springer Verlag, Berlin, 1986.
- [7] S.A. Orlovsky, 'Decision-making with a fuzzy preference relation', *Fuzzy sets and Systems*, vol. 1, 1978, pp. 155-67.

- [8] D.W. Pearce and C.A. Nash, *The Social Appraisal of Projects*, MacMillan, Basingstoke, 1981.
- [9] A.D. Pearman, *Developing a Multicriteria Model for Use as a Highway Investment Priority Assessment Technique*, Working Paper 289, Institute for Transport Studies, University of Leeds, 1989.
- [10] A.D. Pearman, P.J. Mackie, A.D. May and D. Simon, "The use of multicriteria techniques to rank highway investment proposals", pp. 157-165 in A.G. Lockett and G. Islei (eds.) *Improving Decision Making in Organisations*, Springer Verlag, Berlin, 1989.
- [11] A.D. Pearman, J. Montero and J. Tejada, "Fuzzy multicriteria techniques: an application to transport planning", pp.510-9 in: B. Bouchon-Meunier, R.R. Yager and L.A. Zadeh (eds.), *Uncertainty in Knowledge Bases*, Springer-Verlag, Berlin, 1991.
- [12] P. Perny and B. Roy, "The use of fuzzy outranking relations in preference modelling", *Fuzzy Sets and Systems*, vol. 49, 1992, pp. 33-53.
- [13] D. Rios Insua and S. French, "*A Framework for Sensitivity Analysis in Discrete Multi-Objective Decision-Making*", School of Computer Studies, University of Leeds, Report 89.1, 1989.
- [14] B. Roy, "Partial preference analysis and decision aid: the fuzzy outranking relation concept", pp. 40-75 in D.E. Bell, R.L. Keeney and H. Raiffa (eds.) *Conflicting Objectives in Decisions*, Wiley, New York, 1977.
- [15] B. Roy, "*ELECTRE III: un algorithme de classements fondé sur une représentation floue des préférences en présence de critères multiples*", *Cahiers du Centre d'Etudes de Recherche Opérationnelle*, vol. 20, 1978, pp. 3-24.
- [16] B. Roy and Ph. Vincke, "Relational systems of preference with one or more pseudo-criteria: some new concepts and results", *Management Science*, vol. 30, 1984, pp. 1323-35.

- [17] J. Siskos, J. Lochard and J. Lombard, "A multicriteria decision making methodology under fuzziness: application to the evaluation of radiological protection in nuclear power plants", pp. 261-83 in H.J. Zimmermann, L.A. Zadeh and B.R. Gaines (eds.) *Fuzzy Sets and Decision Analysis*, North Holland, Amsterdam, 1984.

Table 1 Details of the Twelve Projects

| PROJECT | COST (£K) | EFF | EFF RANK | ND | ND RANK | EFF:C RATIO | EFF:C RANK | ND:C RATIO | ND:C RANK |
|---------|-----------|--------|----------|--------|---------|-------------|------------|------------|-----------|
| 1 | 2870 | 0.4305 | 2 | 0.4464 | 6 | 0.1500 | 7 | 0.1555 | 8 |
| 2 | 3710 | 0.3018 | 3 | 0.4473 | 5 | 0.0813 | 10 | 0.1205 | 9 |
| 3 | 6030 | 0.1437 | 7 | 0.4630 | 4 | 0.0238 | 11 | 0.0767 | 12 |
| 4 | 850 | 0.0965 | 9 | 0.0908 | 12 | 0.1136 | 9 | 0.1069 | 11 |
| 5 | 3300 | -0.125 | 12 | 0.3884 | 7 | -0.037 | 12 | 0.1177 | 10 |
| 6 | 3600 | 0.5305 | 1 | 1 | 1= | 0.1473 | 8 | 0.2777 | 7 |
| 7 | 201 | 0.1532 | 6 | 0.2471 | 9 | 0.7625 | 6 | 1.2294 | 6 |
| 8 | 50 | 0.1782 | 5 | 1 | 1= | 3.5650 | 2 | 20 | 2 |
| 9 | 30 | 0.0740 | 11 | 0.1851 | 11 | 2.4697 | 3 | 6.1712 | 3 |
| 10 | 105 | 0.0949 | 10 | 0.2698 | 8 | 0.9041 | 5 | 2.5700 | 5 |
| 11 | 25 | 0.1795 | 4 | 0.5449 | 3 | 7.1837 | 1 | 21.798 | 1 |
| 12 | 87 | 0.1344 | 8 | 0.2124 | 10 | 1.5457 | 4 | 2.4423 | 4 |

Key: EFF: Effectiveness [equation (1)]

ND: Non-domination [equation (7)]

C: Cost

Columns 7 and 9 are multiplied by 1000

Table 2 Ranking of Projects using Successive Discarding

| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---------|----|----|----|----|----|---|---|----|----|----|----|----|
| ND (7) | 6* | 8* | 11 | 3 | 2 | 1 | 5 | 10 | 7 | 12 | 9 | 4 |
| ND (9) | 11 | 8 | 9 | 12 | 10 | 7 | 6 | 1 | 2 | 5 | 4 | 3 |
| ND (10) | 6 | 8 | 11 | 5 | 1 | 2 | 9 | 7 | 12 | 3 | 10 | 4 |

[Numbers in brackets refer to the defining equations

* = tied ranking, equal first]

Table 3 The Best Four Subsets and their Costs

| | | Set Members | | | | | | | | | | |
|--------|----------|-------------|---|---|----|----|----|----|----|----|--|---------|
| Rank 1 | Subset I | 2 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | | £7808K. |
| Rank 2 | Subset C | 2 | 4 | 5 | 8 | 9 | 11 | | | | | £7818K. |
| Rank 3 | Subset B | 1 | 4 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | £7965K. |
| Rank 4 | Subset H | 2 | 4 | 5 | 11 | 12 | | | | | | £7972K. |

Figure 1 The COMPASS Value Tree

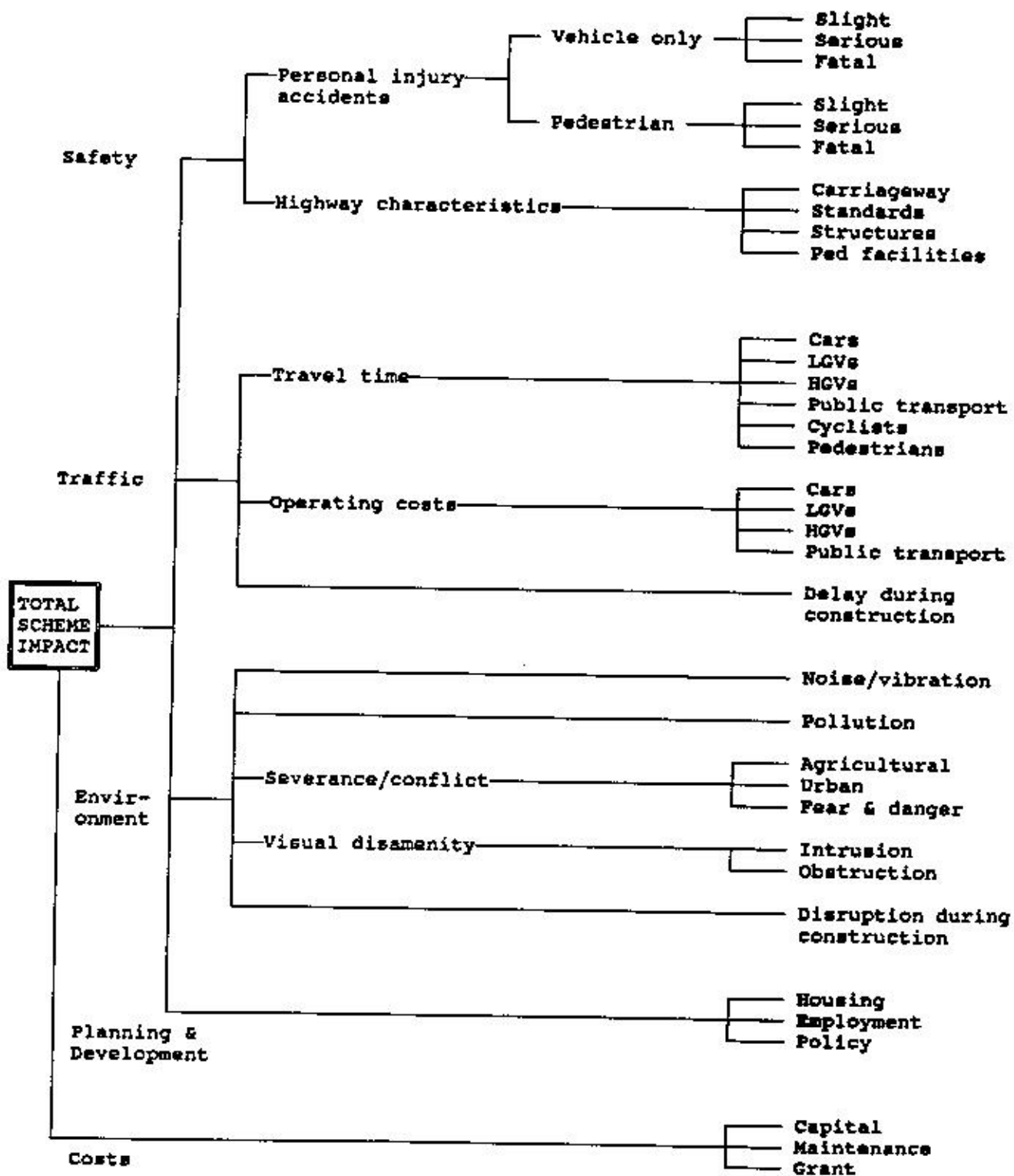


Figure 2 The Partial Fuzzy Outranking and Fuzzy Discordance Relations

