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A NEW METHOD FOR EARTH TIDE DATA ANALYSIS

por

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A New Method for Earth Tide Data Analysis.

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1. Introduction.

Let Y is an Earth tide observation, i.e. an ordinate of a record like those on Fig. 1. The general model for Y is

$$(1) \quad Y = G + D + \varepsilon$$

where G is the pure tidal component of the record, D is the drift and ε is the error of the observation. If the popular slang of the communication technique is used, G is the useful signal of the record, ε is the noise and D - somewhat between signal and noise.

The task of the tidal analysis is to estimate the parameters upon which depends G , or, which is the same, to get the estimate \tilde{G} of G . From this point of view D is a function of the time which describes the zero-line of the record.

The same data Y can be used for non-tidal problems like recent crustal movements, superficial deformations, volcano and earthquake precursors etc. These problems need the determination of the second component of (1), i.e. the estimate \tilde{D} of D . For example as precursors can be considered some accelerations and jumps of \tilde{D} (by the way, we can look for precursors in G as well).

In order to get \tilde{G} we need to separate it from D . This can be done only if we obtain, in one or another way, the estimate \tilde{D} . Thus we see that the tidal analysis of Y has to satisfy both tidal and non-tidal problems.

Evidently, we shall have good estimates \tilde{G} and \tilde{D} if we can reduce the effect of the noise ε on both of them. It is out of question to estimate ε directly. We can only define the residuals,

$$(2) \quad r = Y - \tilde{G} - \tilde{D},$$

Schüller, 1977, Jentzsch, 1977) as well as a more flexible variant of M67 (Venedikov, 1984). However, none of these methods could show significant advantages with respect to M67.

A most interesting modern approach to the problem was made by the method used in Mizusawa (Ishiguro, Akaike, Ooe & Nakai, 1983, Tamura, Sato, Ooe & Ishiguro, 1991). In this method the gaps define natural intervals of the record. In each interval we get a separately estimated drift. The drift model is related with a condition for every sequence of three ordinates.

Here we shall propose a method (here and further M92) which is partly a generalization of M67, partly its alternative. The motives for its creation are the following.

First, there are modern observations of high quality with very stable drift. For them the model used by M67, which is good for an irregular drift, may be not necessary. We can hope that the drift can be approximated in intervals Δ much longer than 48 hours.

Second, for such observations the correlation of the noise may be not so important. We can try to avoid the use of the filtered numbers and return to the hourly ordinates.

Finally, now we have powerful computers which allow the development of very sophisticated methods. We can make very complicated experiments and we are, in a way, obliged to exploit these possibilities.

However, a more sophisticated method does not yet means a better method and we do not suggest necessarily to replace M67 by M92. What seems quite sure, it could be useful to apply in parallel both methods.

2. The tidal model.

If G_t is the value of G at time t , we have the theoretical model

$$(4) \quad G_t = \sum_k \delta H_k \cos (\phi_k + \omega_k t + \kappa)$$

where each term represents a tidal wave with an index k .

The quantities ω_k (angular velocity), H_k (theoretical amplitude) and ϕ_k (initial theoretical phase) are well known. The parameters δ

(amplitude factor) and κ (phase shift) are the unknown parameters of G whose estimates are the object of the analysis. They can be considered as functions of the angular velocity ω ,

$$(5) \quad \delta = \delta(\omega_k) \quad \text{and} \quad \kappa = \kappa(\omega_k).$$

A problem is the great number of the tides in (4) - in the modern tidal potential developments of Tamura (1987) and Xi Quinwen (1985) there are more than 1000 tidal waves.

This is easy to overcome this problem by using that (5) are slowly varying functions. We can shape groups of tides, say g groups, with close ω_k and accept that δ and κ are constants within a group. If $k = k_j$ are the indices of the tides in the j -th group, we can set up

$$(6) \quad \delta(\omega_k) = \delta_j = \text{const.} \quad \text{and} \quad \kappa(\omega_k) = \kappa_j = \text{const.} \\ \text{for } k = k_j, \quad j = 1, 2, \dots, g$$

and introduce the unknowns (Venedikov, 1961, 1966)

$$(7) \quad \xi_j = \delta_j \cos \kappa_j \quad \text{and} \quad \eta_j = -\delta_j \sin \kappa_j$$

With them the expression (4) becomes,

$$(8) \quad G_t = \sum_{j=1}^g (\xi_j c_j(t) + \eta_j s_j(t)) \quad \text{where} \\ c_j(t) = \sum_{k=k_j} H_k \cos(\phi_k + \omega_k t) \quad \text{and} \quad s_j(t) = \sum_{k=k_j} H_k \sin(\phi_k + \omega_k t)$$

The grouping or the separation of the tides should be made in dependence on the total length of the record. It should guarantee a linear independence (close to orthogonality) of the functions $c_j(t)$ as well as $s_j(t)$.

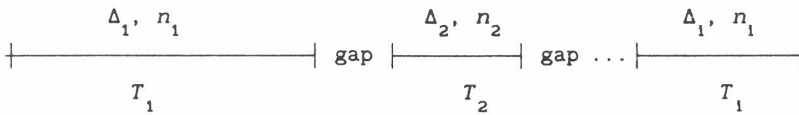
For a more convenient manipulation with (8) we can replace it by

$$(9) \quad G_t = \sum_{j=1}^m a_{tj} x_j,$$

where x_j and a_{tj} , $j = 1, 2, \dots, m = 2g$, replace the unknowns (7) and the functions (8) respectively.

3. Model of the drift.

The model of D in M92 is related with the scheme



Intervals Δ_1 in the method of analysis M92.

The record is subdivided into N intervals denoted by Δ_1 , with a central epoch T_1 and, generally, a variable length n_1 hours. Within any interval Δ_1 are no gaps so that n_1 is also the number of the hourly ordinates in Δ_1 .

In the following the letter i ($i = 1, 2, \dots, N$) will be used exclusively as an index of an interval Δ_1 .

In every Δ_1 we have separate approximation of the drift through

$$(10) \quad D_t = \sum_{k=0}^{v_1} z_{k1} p_{tk} \quad \text{for a given } \Delta_1.$$

At the moment we shall accept that

$$(11) \quad p_{tk} = (t - T_1)^k \quad \text{for } \Delta_1$$

so that (10) is a polynomial of power v_1 . The coefficients z_{k1} of the polynomial are unknowns specific for Δ_1 . The power v_1 is also specific for every Δ_1 . The selection of v_1 will be discussed a little bit later.

In the practical application of M92, Δ_1 are defined by:

(i) in any case by the gaps, like in the method of Mizusawa, and, optionally,

(ii) by the epochs of the calibrations,

(iii) by epochs given as input data, related with places where we suspect changes in the drift's behaviour,

(iv) in dependence on two integers n_{\min} and n_{\max} which $n_{\min} \leq n_i \leq n_{\max}$ or

(v) by choosing a length n and $n_i = n = \text{const}$. In this case we have the scheme of M67 with $n = 48$ replaced by the chosen value of n .

Let us return to the powers v_i in (10).

We must be able to work with v_i depending on i , i.e. with polynomials (10) of different order for every Δ_i . When n_i is variable, we can expect higher v_i for longer Δ_i and lower v_i for shorter Δ_i . We may need different v_i corresponding to the changes in the drift properties and behaviour, independently of whether the length n_i is variable or $n_i = n = \text{const}$.

Evidently, we are not able to choose, before the processing, a set of fixed values v_i , $i = 1, 2, \dots, N$. That is why M92 presumes a variation of v_i separately for every Δ_i and selection of optimum values. This will be discussed in paragraphs 5 and 6. Now, in the next paragraph, we shall consider the use of the models (9) and (10) for getting the estimates of the unknowns only for a given fixed set of powers v_1, v_2, \dots, v_N .

4. Observational equations and their solution after the MLS.

If we joint (9) and (10) into (1) we get the observational equations

$$(12) \quad Y_t = \sum_{j=1}^m a_{tj} x_j + \sum_{k=0}^{v_i} p_{tk} z_{ki} \quad \text{for } \Delta_i, i = 1, 2, \dots, N$$

where Y_t is an ordinate at time t . Here the error ε is ignored but we keep it in mind.

According to (11) p_{tk} is related with T_i and Δ_i and it would be more correct to write p_{itk} . In order to avoid too many indices we shall

drop away the index i .

In a given Δ_1 we have n_1 hourly ordinates Y_t . For them the argument time t takes the values

$$(13) \quad t = T_1 - \nu_1, T_1 - \nu_1 + 1, \dots, T_1 + \nu_1, \quad \text{where } \nu_1 = (n_1 - 1)/2.$$

We shall deal first with the equations (12) for a given Δ_1 and we shall introduce the following matrices related with the values (13) of t :

$$(14) \quad (n_1 \times 1) \text{ dimensional column vector}$$

$$Y_1 = \|Y_t\|, \quad t = T_1 - \nu_1, T_1 - \nu_1 + 1, \dots, T_1 + \nu_1,$$

$$(15) \quad (n_1 \times m) \text{ dimensional matrix}$$

$$A_1 = \|a_{tj}\|, \quad j = 1, 2, \dots, m, \\ t = T_1 - \nu_1, T_1 - \nu_1 + 1, \dots, T_1 + \nu_1 \text{ and}$$

$$(16) \quad (n_1 \times \nu_1 + 1) \text{ dimensional matrix}$$

$$P_1 = \|p_{tk}\|, \quad k = 0, 1, \dots, \nu_1 \\ t = T_1 - \nu_1, T_1 - \nu_1 + 1, \dots, T_1 + \nu_1.$$

Further we shall arrange the unknowns in the column vectors:

$$(17) \quad z_1 = \|z_{k1}\|, \quad k = 0, 1, \dots, \nu_1, \quad \text{dimension } (\nu_1 + 1 \times 1),$$

$$x = \|x_j\|, \quad j = 1, 2, \dots, m, \quad \text{dimension } (m \times 1),$$

where x represents the tidal unknowns (ξ and η) which are common for all Δ_1 while z_1 represents the drift unknowns, which are related with Δ_1 , i.e. which depend on i . The number of the elements of z_1 , $\nu_1 + 1$, is also depending on i .

By using these matrices we can rewrite (12) in the matrix form

$$(18) \quad Y_1 = A_1 x + P_1 z_1 \quad \text{for } \Delta_1, \quad i = 1, \dots, N.$$

In principle, (18) can be directly solved by the MLS. However, this is difficult even for a powerful computer because the number of the drift unknowns in all z_1 can be very high. For example, if we use fixed length intervals with $n_1 = 48$ and polynomials of fixed power $v_1 = 2$, a record of length one year will provide 546 drift unknowns.

This inconvenience can be avoided if we separate the unknowns z_1 from x . Namely, we can orthogonalize P_1 , then A_1 towards P_1 , in the following classical way.

We replace P_1 by a matrix

$$(19) \quad Q_1 = P_1 S_1, \quad \text{with} \quad S_1 S_1^* = (P_1^* P_1)^{-1}$$

where S_1 is an upper triangular matrix of dimensions $(v_1+1 \times v_1+1)$, Q_1 has the same dimensions as P_1 and $*$ means the transpose of a matrix.

We replace also A_1 by the matrix

$$(20) \quad B_1 = A_1 - Q_1 (Q_1^* A_1)$$

which has the dimensions of A_1 .

It is easy to see that Q_1 is orthonormal and B_1 is orthogonal to both Q_1 and P_1 , i.e.

$$(21) \quad Q_1^* Q_1 = I \quad \text{and} \quad B_1^* Q_1 = B_1^* P_1 = 0$$

where I is a unit matrix and 0 is a zero matrix. The properties (21) are observed separately for every Δ_1 .

Now with Q_1 and B_1 we transform the equations (18) into

$$(22) \quad Y_1 = A_1 x + P_1 z_1 = B_1 X + Q_1 Z_1, \quad i = 1, 2, \dots, N,$$

where Z_1 and X are new unknowns which replace z_1 and x . From here and the expressions (19) and (20) about Q_1 and B_1 we get

$$(23) \quad X = x \quad \text{and} \quad Z_1 = S_1^{-1} (z_1 + (Q_1^* A_1) X)$$

i.e. the tidal unknowns \mathbf{x} are not changed, but the drift unknowns \mathbf{z}_1 are transformed into $\tilde{\mathbf{z}}_1$.

Now we can apply the MLS by taking into account (21). Then we get the following systems of normal equations for the estimates $\tilde{\mathbf{z}}_1$ of \mathbf{z}_1 and $\tilde{\mathbf{X}}$ of \mathbf{X} :

First N matrix equations for the drift unknowns

$$(24) \quad (\mathbf{Q}_1^* \mathbf{Q}_1) \tilde{\mathbf{z}}_1 = \mathbf{Q}_1^* \mathbf{Y}_1, \quad i = 1, 2, \dots, N,$$

then a matrix equation for the tidal unknowns

$$(25) \quad (\mathbf{B}^* \mathbf{B}) \tilde{\mathbf{X}} = \mathbf{B}^* \mathbf{Y}, \quad \mathbf{B} = \|\mathbf{B}_1\|, \quad \mathbf{Y} = \|\mathbf{Y}_1\|, \quad i = 1, \dots, N$$

Here \mathbf{B} is obtained through the arrangement of \mathbf{B}_1 ($i = 1, \dots, N$) into a single matrix and \mathbf{Y} - through the arrangement of \mathbf{Y}_1 into a single column vector. If \tilde{n} is the sum of n_1 , i.e. the total number of the data, \mathbf{B} has the dimensions ($\tilde{n} \times m$) and \mathbf{Y} - ($\tilde{n} \times 1$).

From the solution of (24) and (25) we can get the estimates

$$(26) \quad \tilde{\mathbf{z}}_1 = (\mathbf{Q}_1^* \mathbf{Q}_1)^{-1} (\mathbf{Q}_1^* \mathbf{Y}) = \mathbf{Q}_1^* \mathbf{Y}$$

$$(27) \quad \tilde{\mathbf{X}} = (\mathbf{B}^* \mathbf{B})^{-1} (\mathbf{B}^* \mathbf{Y}),$$

Thus \mathbf{z}_1 are obtained in a most simple way. However, if we want to get the original drift unknowns, i.e. the coefficients z_{ki} of the polynomials (10), we have to go back from \mathbf{z}_1 to \mathbf{z}_i through the computation of

$$(28) \quad \tilde{\mathbf{z}}_1 = \mathbf{S}_1 (\tilde{\mathbf{z}}_1 - (\mathbf{Q}_1^* \mathbf{A}_1) \tilde{\mathbf{X}})$$

For the computations it is important that we can get directly $\mathbf{B}^* \mathbf{B}$ and $\mathbf{B}^* \mathbf{Y}$ through \mathbf{A}_1 and \mathbf{Q}_1 after

$$(29) \quad B^*B = \sum_{i=1}^N B_i^*B_i = \sum_{i=1}^N \left(A_i^*A_i - (Q_i^*A_i)^*(Q_i^*A_i) \right)$$

$$B^*Y = \sum_{i=1}^N B_i^*Y_i = \sum_{i=1}^N \left(A_i^*Y_i - (Q_i^*A_i)^*(Q_i^*Y_i) \right).$$

5. Variation of the drift model.

Practically this is done by choosing two parameters v_{\min} and v_{\max} . Then v_1 , for every Δ_1 , is let taking all values

$$(30) \quad v_{\min} \leq v_1 \leq v_{\max}.$$

A most natural value of v_{\min} seems to be $v_{\min} = 1$. This is a default value of M92 but it can be changed optionally.

The concrete variation of v_1 , for $v_{\min} = 1$ and $v_{\max} = 4$, is demonstrated through the following scheme, where v'_1 means an optimum value of v_1 for Δ_1 :

$v_{\min} = 1, v_{\max} = 4, v'_1$ means an optimum value.

variation of v_1 in Δ_1				variation of v_2 in Δ_2				variation of v_N in Δ_N			
v_1	v_2	...	v_N	v_1	v_2	...	v_N	v_1	v_2	...	v_N
1	1	...	1	v'_1	1	...	1	v'_1	v'_2	...	1
2	1	...	1	v'_1	2	...	1	v'_1	v'_2	...	2
3	1	...	1	v'_1	3	...	1	v'_1	v'_2	...	3
4	1	...	1	v'_1	4	...	1	v'_1	v'_2	...	4
v'_1				v'_2							v'_N

- We make the analysis of all data, i.e. all Δ_1 , by choosing $v_1 = 1, 2, \dots, v_{\max}$, while $v_2 = v_3 = \dots = v_N = 1$
- Through statistical criteria we choose

the optimum value v'_1 of v_1

- We make the analysis of all data, i.e. all Δ_1 , by choosing

$$v_1 = v'_1, v_2 = 1, 2, \dots, v_{\max}, v_3 = \dots = v_N = 1$$

- Through statistical criteria we choose

the optimum value v'_2 of v_2

and so on till we choose

the optimum value v'_N of v_N .

The whole procedure can be repeated by getting as initial values, instead of $v_1 = v_{\min} = 1$, the selected optimum values v'_1 and look for new optimum values.

The analysis, i.e. the creation and the solution of the normal equations (24) and (25), is made for all indicated values of v_1 . Thus, when the initial value is $v_{\min} = 1$, we have to process a number of Nv_{\max} analyses.

Such a quantity of computations is too big and needs as many as possible facilitations. A most important facilitation is the following. It consists in using the matrices created for a given variant of v_1 for the next variant.

Let us suppose that we have made the computation for a given set of values of v_1, v_2, \dots, v_N , in particular for $v_1 = v$ for a given interval Δ_1 . Then the matrices P_1 and Q_1 related with Δ_1 have $v + 1$ columns, i.e. they can be represented as

$$(31) \quad P_1 = [p_0 \ p_1 \ \dots \ p_v], \quad Q_1 = [q_0 \ q_1 \ \dots \ q_v],$$

where p_k and q_k are columns of dimension $(n_1 \times 1)$. Here, as far as S_1 is triangular, q_k depends on p_0, \dots, p_k only and it does not depend on p_{k+1}, \dots, p_v ($k < v$).

Let us now increase $v_1 = v$ to $v_1 = v + 1$. Then P_1 and Q_1 are to be replaced by

$$(32) \quad P'_1 = [p_0 \ p_1 \ \dots \ p_v \ p_{v+1}], \quad Q'_1 = [q_0 \ q_1 \ \dots \ q_v \ q_{v+1}].$$

Due to the above considerations, the first $v+1$ columns of Q_1 remain the

same and we need only to compute the additional q_{v+1} .

For the new value $v_1 = v + 1$ the estimates of the drift unknowns \tilde{Z}_1 are to be replaced by

$$(33) \quad \tilde{Z}'_1 = Q'_1 * Y_1$$

where \tilde{Z}'_1 has $v + 2$ elements, while \tilde{Z}_1 has $v + 1$ elements. The first $v + 1$ elements of \tilde{Z}'_1 remain the same as in \tilde{Z}_1 , while the new element of \tilde{Z}'_1 is

$$(34) \quad \tilde{Z}_{1v+1} = q_{v+1} * Y_1.$$

We shall have a new matrix B'_1 instead of B_1 , a new matrix B' instead of B and, instead of (25), new normal equations

$$(35) \quad (B' * B') \tilde{X}' = B' * Y,$$

for new estimates \tilde{X}' of the same tidal unknowns.

For the practical application it is important that

$$(36) \quad \begin{aligned} B' * B' &= B * B - (q_{v+1} * A_1) * (q_{v+1} * A_1) \\ B' * Y &= B * Y - (q_{v+1} * A_1) (q_{v+1} * Y_1) . \end{aligned}$$

For a given variant with normal equations (24) and (25) the sum of squares of the residuals can be obtained, according to the theory of the MLS, after

$$(37) \quad S_o^2 = Y * Y - \sum_{i=1}^N \tilde{Z}_i * \tilde{Z}_i - \tilde{X} * (B * Y), \quad \text{d.f. } f = \sum_{i=1}^N (n_i - v_i - 1) - m$$

where d.f. means degrees of freedom.

When $v_1 = v$ is increased to $v_1 = v + 1$, as above, the corresponding sum of squares becomes

$$(38) \quad S_1^2 = \mathbf{Y}^* \mathbf{Y} - \sum_{i=1}^N \tilde{Z}_i^* \tilde{Z}_i - \tilde{Z}_{1v+1}^2 - \tilde{\mathbf{X}}' * (\mathbf{B}' * \mathbf{Y}), \quad \text{d.f. } f_1 = f - 1$$

where $\tilde{\mathbf{X}}'$ are the new tidal estimates, related with $v_1 = v + 1$.

The estimates of the variances of the observations are respectively

$$(39) \quad \sigma_0^2 = S_0^2/f \quad \text{and} \quad \sigma_1^2 = S_1^2/f_1$$

where σ_0 and σ_1 are the corresponding standard deviations or mean square errors. The estimation of the precision of $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{X}}'$ can be obtained, according to the MLS, through σ_0 and $(\mathbf{B} * \mathbf{B})^{-1}$ and σ_1 and $(\mathbf{B}' * \mathbf{B}')^{-1}$ respectively.

6. Selection of an optimum variant.

We shall denote by V_0 the variant with $v_1 = v$ and by V_1 the variant with $v_1 = v+1$ for a fixed Δ_1 . The difference in V_0 and V_1 is in the models of the drift for Δ_1 which are

$$(40) \quad V_0: \quad D_t = z_0 p_{t0} + z_1 p_{t1} + \dots + z_v p_{tv}$$

$$V_1: \quad D_t = z_0 p_{t0} + z_1 p_{t1} + \dots + z_v p_{tv} + z_{v+1} p_{tv+1} \quad \text{for } \Delta_1$$

where the index i at z_{k1} is dropped away for convenience.

We see that V_0 can be obtained from V_1 under the "zero hypothesis"

$$(41) \quad H_0: \quad z_{v+1} = 0$$

Then the problem to select a better variant among V_0 and V_1 is equivalent to the testing of H_0 . If H_0 is rejected then V_1 is to be chosen and, vice versa, if H_0 cannot be rejected then V_0 can be preferred.

This can be done by the method of analysis of variances. We have to compute the famous ratio F of Fisher, in this case

$$(42) \quad F = (S_0^2 - S_1^2) / \sigma_0^2.$$

When H_0 is true and V_0 is to be chosen F should be a small number. When H_0 is not true and V_1 is to be chosen, F should be a big number. We can decide what is a big or a small number according to the following.

Considered as a random variable, F has the F -distribution of Snedecor with d.f. 1: f (practically 1: ∞). For a given confidential probability P or level of significance $\alpha = 1-P$, like $P = 95\%$ or $\alpha = 5\%$, we can find a critical value F_{crit} . Then we can accept that the computed F is big or small if

$$(43) \quad F \geq F_{crit} \quad \text{or} \quad F < F_{crit}$$

respectively.

A practical inconvenience is that through F we can easily compare only two variants. When we have to use many variations in the model function it becomes rather complicated the comparison of all variants, two by two.

Much more comfortable way to choose the optimum variant is the Akaike's Information Criterion (AIC) (Sakamoto, Ishiguro & Kitagawa, 1986) which has also a serious theoretical background. It can be computed after

$$(44) \quad AIC = \tilde{n} \log 2\pi + \tilde{n} \log \sigma^2 + \tilde{n} + 2(\tilde{m} + 2)$$

where \tilde{n} is the total number of the data, \tilde{m} is the total number of all unknowns (including the drift ones) and σ^2 is the estimate of the variance. As an optimum variant can be accepted the case with the lowest computed (observed) AIC.

In the practical application of M92 the program search automatically the minimum of AIC for every Δ_1 . If the minimum is found for the variant $v_1 = v'$, then the quantity F is computed for the variants

$$(45) \quad F_1 \text{ for } (v'-1, v') \quad \text{and} \quad F_2 \text{ for } (v', v'+1).$$

We have a confirmation of the solution taken on the basis of AIC if F_1 is a big number and F_2 - a small number, in the sense discussed above.

7. Separation of the main tidal groups.

In M67, as well as in the earlier methods of Lecolazet, Doodson-Lennon and Pertsev, there is a separation of the D and SD tides at the same stage as the elimination of the drift. More concretely, in M67 this means that in every interval of 48 hours the filters for the SD tides eliminate the D tides and the filters for the D tides eliminate the SD tides.

This is reasonable as far as there are variable meteorological waves, S_1 with period 24^h and S_2 with period 12^h .

In M92 a similar option is included in the following way for the separation of the SD tides from the S_1 D tide in every Δ_1 .

Optionally, we can set up

$$(46) \quad p_{t1} = c_{S_1}(t) \quad \text{and} \quad p_{t2} = s_{S_1}(t)$$

where c and s are the functions (8) computed for the S_1 tidal group. Then p_{t0} is kept as before, after (11), but the other polynomial terms are shifted as

$$(47) \quad p_{3k} = (t-T_1)^1, \quad p_{4k} = (t-T_1)^2, \quad \dots$$

Then, following just the same algorithm, together with the drift elimination, we get an elimination of the S_1 tide when SD tides are determined. Just in the same way, by replacing S_1 by S_2 in (46), we can get an elimination of S_2 when the D tides are determined.

Thus, in addition to the variants with different power for the drift, we get the variants with and without elimination of the S_1 or S_2 tide.

8. Practical application and some results.

M92 and the corresponding computer program has been applied on two series of data:

LAN - gravity observations by an LCR gravimeter installed in the volcanic geodynamic station Cueva de los Verdes in Lanzarote and

BRU - gravity observations with a superconducting gravimeter in Brussels (Observatoire Royal de Belgique).

It has to be immediately declared that the "first" hypothesis, made in paragraph 1 has not been justified. It appeared practically impossible, i.e. absolutely inefficient, the approximation of the drift in very large time intervals. Generally, when the drift was approximated in longer intervals we used to get higher variances (mean square errors).

This is a conclusion made through the analyses of the whole series. For a graphic demonstration we have used two pieces of monthly series without gaps:

LANM: 1000 hourly ordinates of LAN, Figures 3.a, 4.a and 5.a

BRUM: 770 hourly ordinates of BRU, Figures 3.b, 4.b and 5.b.

The units used on the plots are : abscissa - time in hours, ordinate - gravity in $0.1 \mu\text{gal}$.

On Fig. 3 the drift D is approximated through a single polynomial of a fixed power $v = 4$, without the variation discussed in Paragraph 6. We see important residuals r which are evidently auto-correlated. There is, possibly, a deterministic signal in the r .

On Fig. 4 we see the approximation of D in LANM and BRUM again by a single polynomial but this time the power v is allowed to vary. The optimum power v' is selected automatically through the AIC criterion. The resulted v' is: for LANM $v' = 17$ and for BRUM $v' = 8$.

Here is a more complicated estimated \tilde{D} which is charged by a part of the residuals from the case $v = 4$ on Fig.3. The residuals have a lower level but still important correlation, better seen for BRUM.

The comparison between Fig. 3 and 4 shows the importance of the variation of the power v as well as that the statistical criterion runs well, at least not too badly.

At the last Fig. 5, we have an approximation of D in intervals of 48 hours, in a way like in M67, with a variation of the power v for every interval. An interesting numerical result is that for the whole series LAN and BRU we have got the following mean values of the optimum powers

LAN: mean $v' = 1.71$, BRU: mean $v' = 2.44$.

Thus, somewhat unexpectedly, D in BRU has a more complicated behaviour than in LAN. This can be seen from the plotted curves of the estimated \tilde{D} .

Now we have a much lower level of the r with a much less evident correlation, for both LANM and BRUM.

Nevertheless, from the study of r for the whole series LAN and BRU, we have stated that the auto-correlation does not disappear totally.

On Fig. 6 we have the estimated auto-correlation of R for BRU when the intervals of the approximation are of length $n = 36 = \text{const}$. The picture is quite similar for $n = 48$ and much less favorable for higher values of n .

Here we see that we have an improvement of the auto correlation if the S_1 tide is removed (when SD are determined). This is an indication that the auto correlated noise has really a meteorological origin. Nevertheless, with or without the elimination of S_1 , we have a non-negligible auto-correlation. This may affect considerably the estimates of the precision. Namely, we can expect lower estimated variances (mean square errors σ) than the actual ones.

Generally, one way to avoid a correlated noise of meteorological origin, is to incorporate meteorological data in the analysis. This problem needs deeper consideration and we shall not discuss it here.

Tables 1 and 2 represent the results for LAN and BRU respectively from the application of M67 and M92.

In all cases we have lower σ for M92 compared to M67. The ratio between σ for the two methods, grossly estimated, is

$$\begin{aligned} \sigma(\text{M67})/\sigma(\text{M92}) &\cong 3:2 \text{ for LAN, SD tides} \\ &\quad \text{LAN, D tides} \\ &\quad \text{BRU, SD tides} \\ &\cong 4:1 \text{ for BRU, D tides} \end{aligned}$$

even higher than 4:1 in the last case.

The ratio 3:2 is not too big and, with some optimism, we can believe that we have a real raise of the precision due to a new powerful technique of the analysis. However, 4:1 is a too big ratio, too beautiful, to be true.

When these results are interpreted it is necessary to take into account that in M92 we have not a separate estimation of the precision as in M67. For the computation of σ and its interpretation a very important condition is the independence of the errors ϵ , i.e. the assumption that ϵ is a white noise. If actually this condition is not observed, it is possible M92 to provide lower σ , in particular for the D tides.

One should think carefully what is better - to have lower or higher estimates of the precision. When the computed σ are lower than the actual errors we can be misled to make interpretations and conclusions going too far. When the computed σ are higher we are prevented from this but some details in the results can escape from our attention.

Anyway, our final conclusion is that M92 can be a good numerical and mathematical tool for the analysis of the Earth tide data. However, it should be used carefully, in parallel with other methods, e.g. M67. What seems quite certain, M92 can procure a rather good and detailed description of the drift function. The latter can be useful for studying problems of the non-tidal Earth deformations.

Acknowledgments

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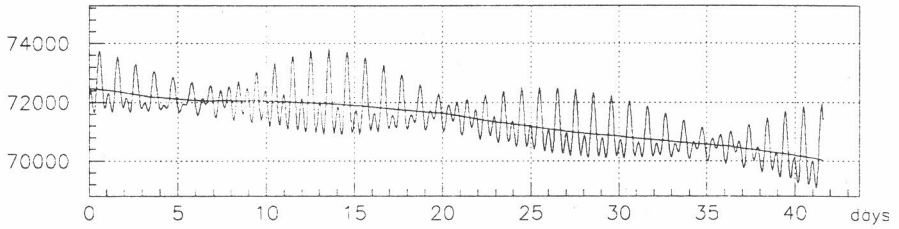
The data from the superconductive gravimeter has been kindly offered by Prof. P.Melchior and Dr. B.Ducarme from the Belgian Royal Observatory.

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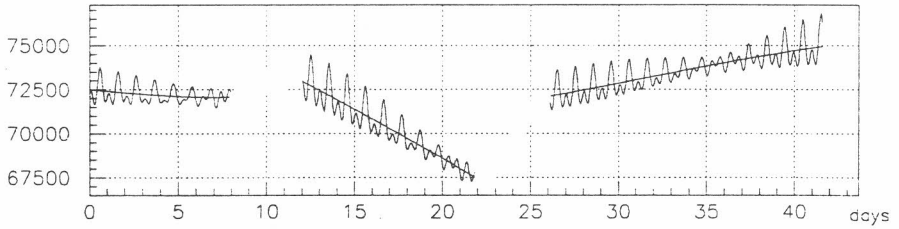
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Fig. 1.a



Observed tidal variations of the gravity with a drift

Fig. 1.b



Observed tidal variations of the gravity with gaps and irregular drift
unit = 0.1 microgal

Figure 1

Fig. 2.

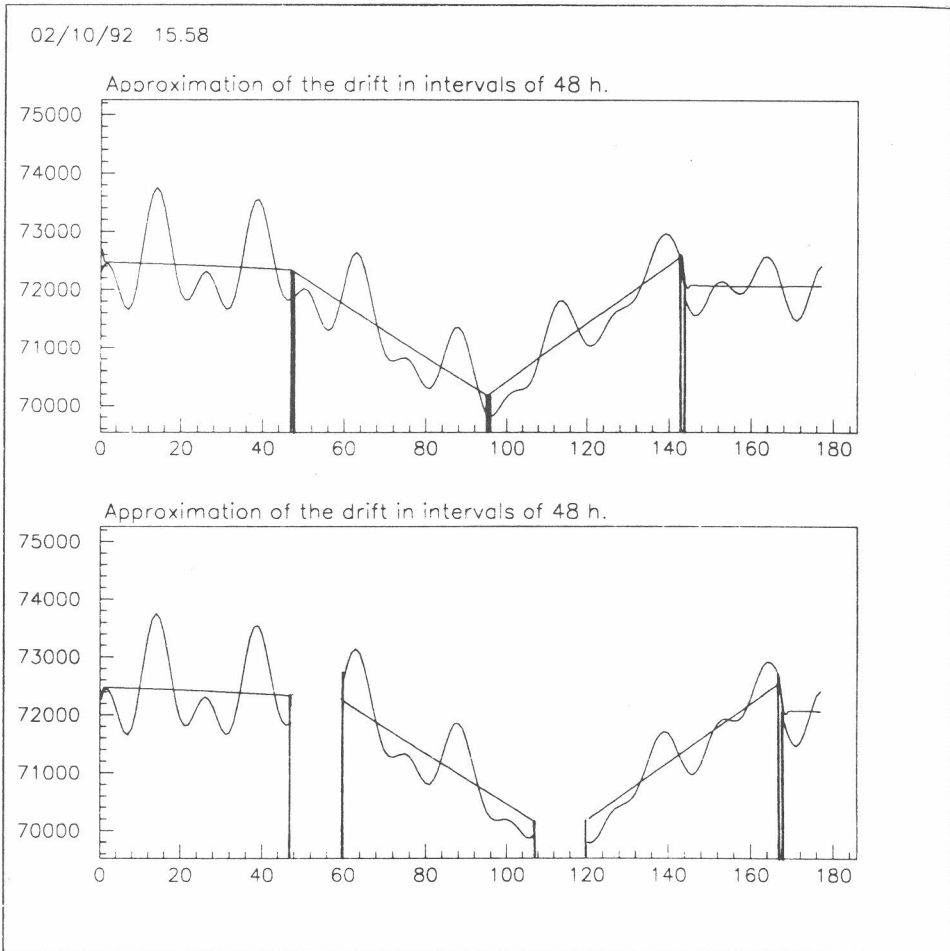


Fig. 3.a

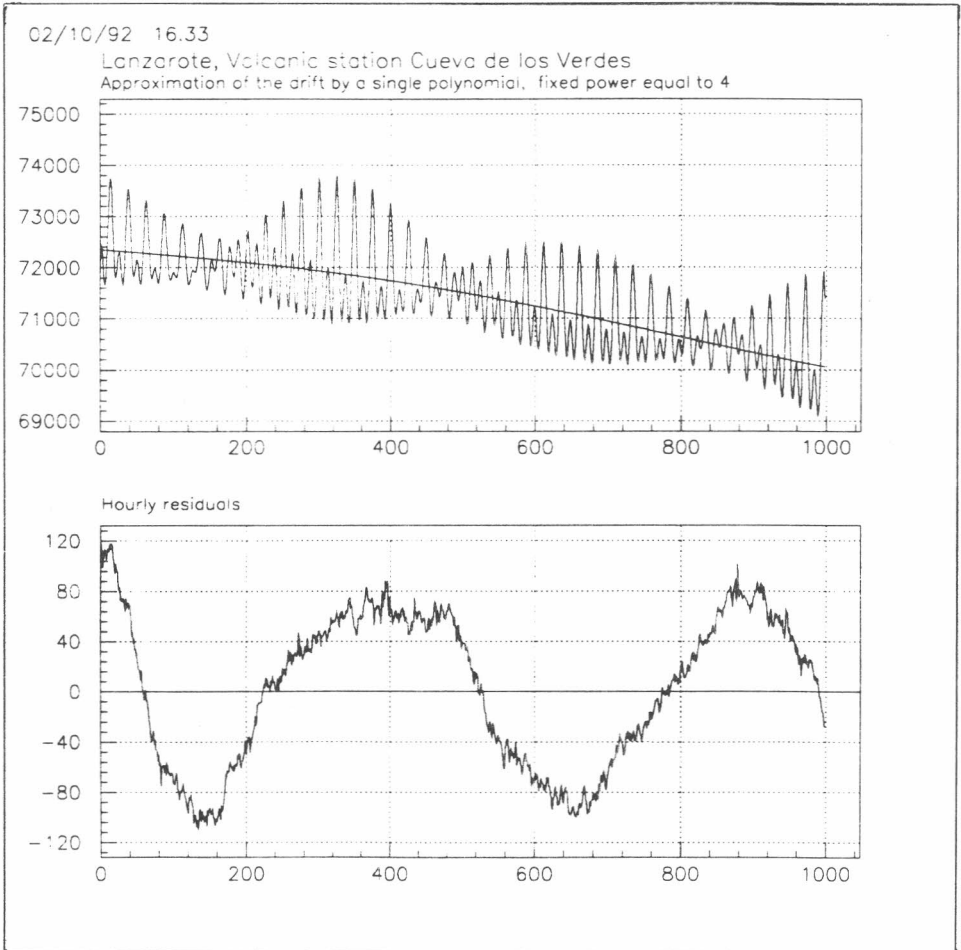
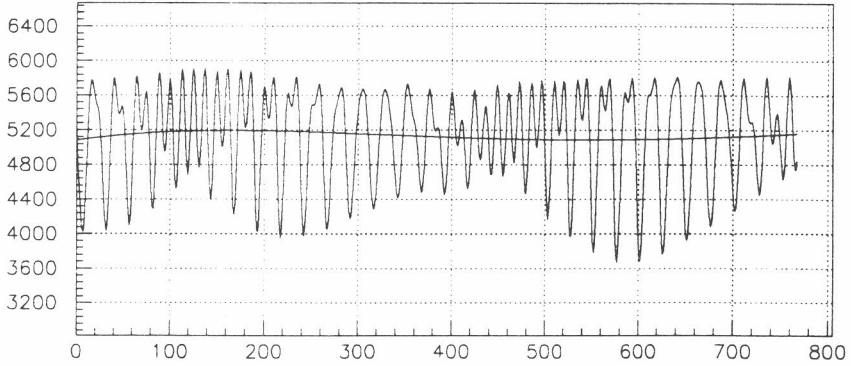


Fig. 3.b

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Brussels, superconducting gravimeter

Approximation of the drift by a single polynomial, fixed power equal to 4



Hourly residuals

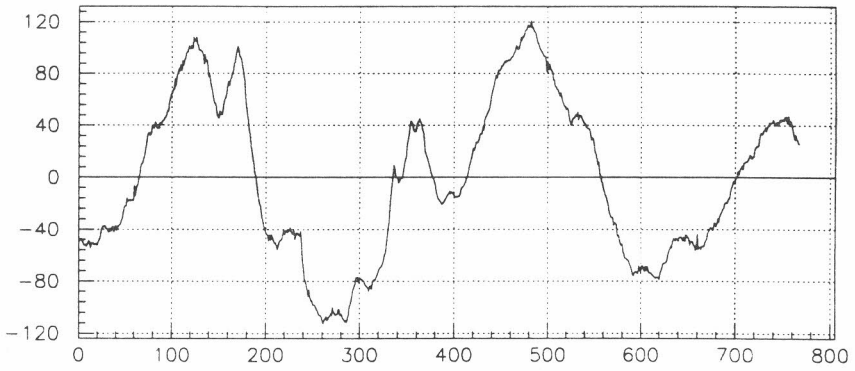


Fig. 4.a

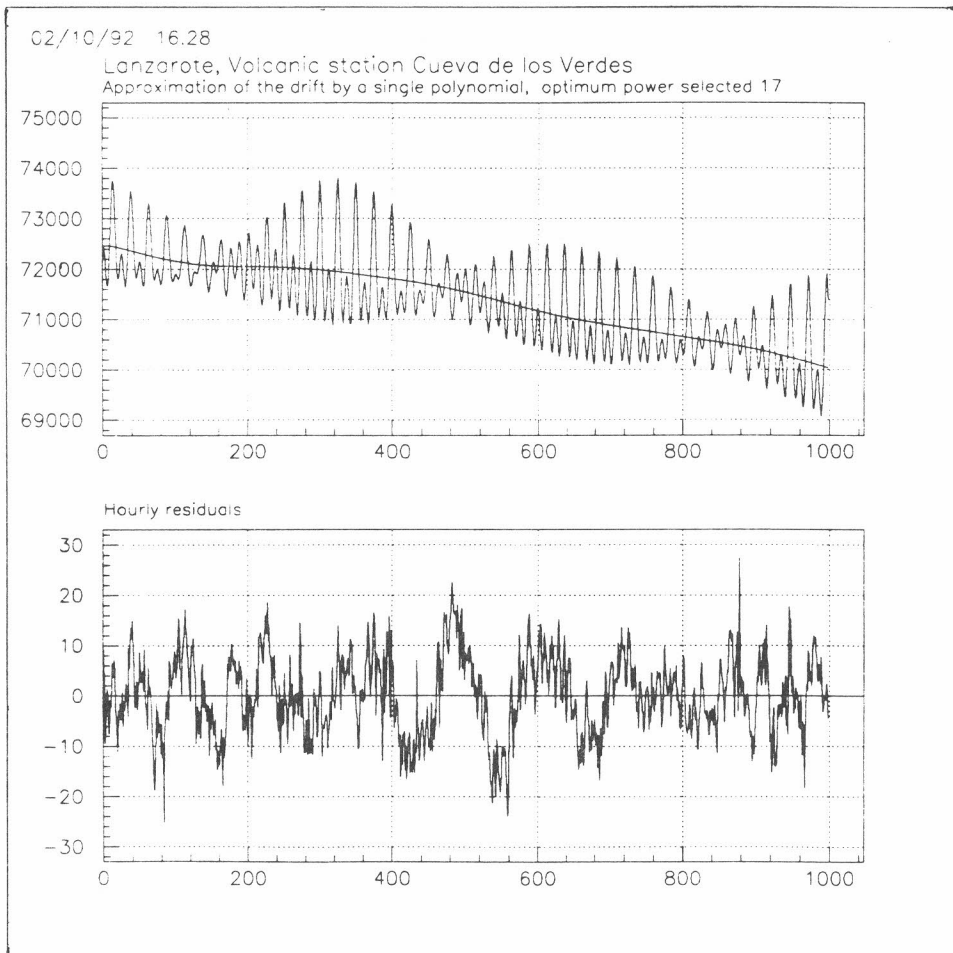


Fig. 4.b

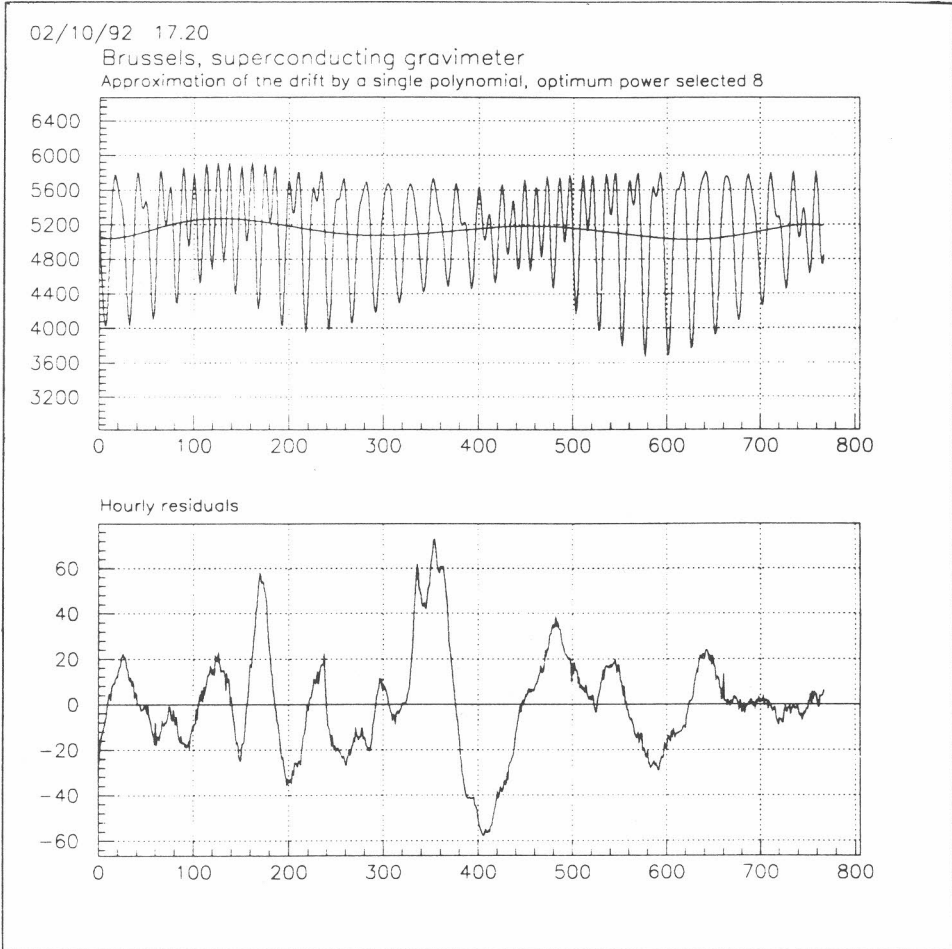


Fig. 5.a

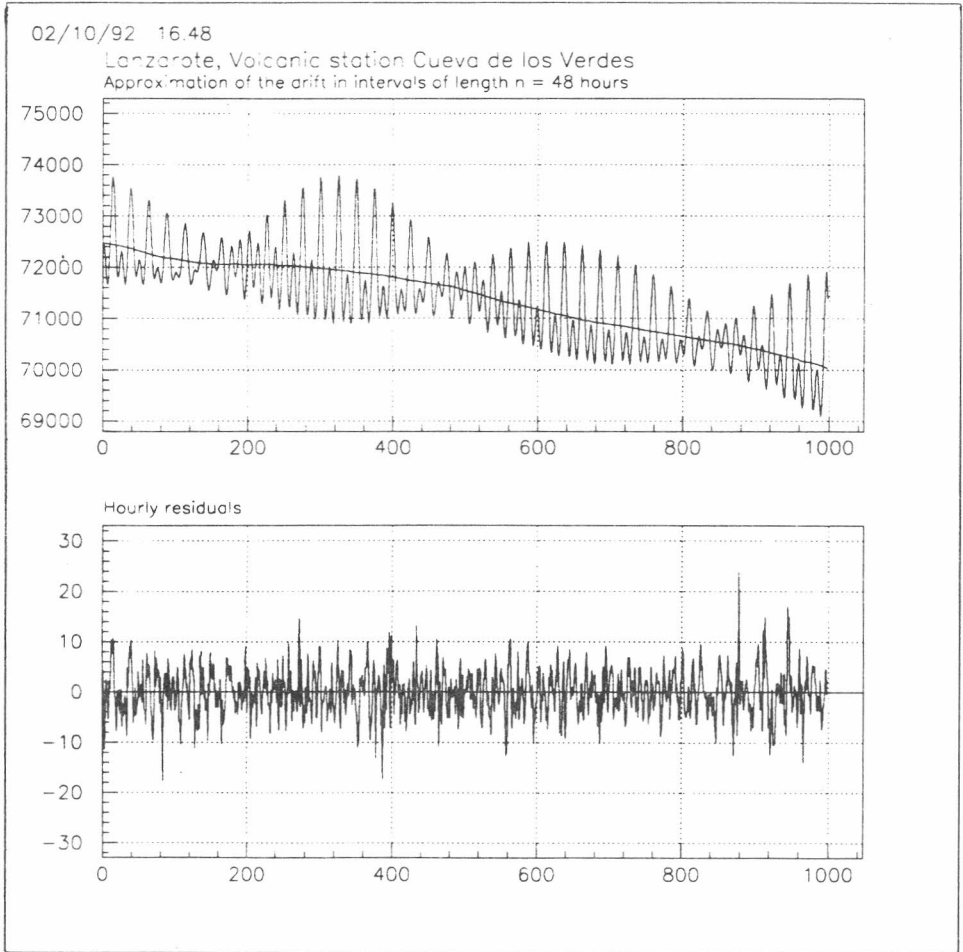


Fig. 5.b

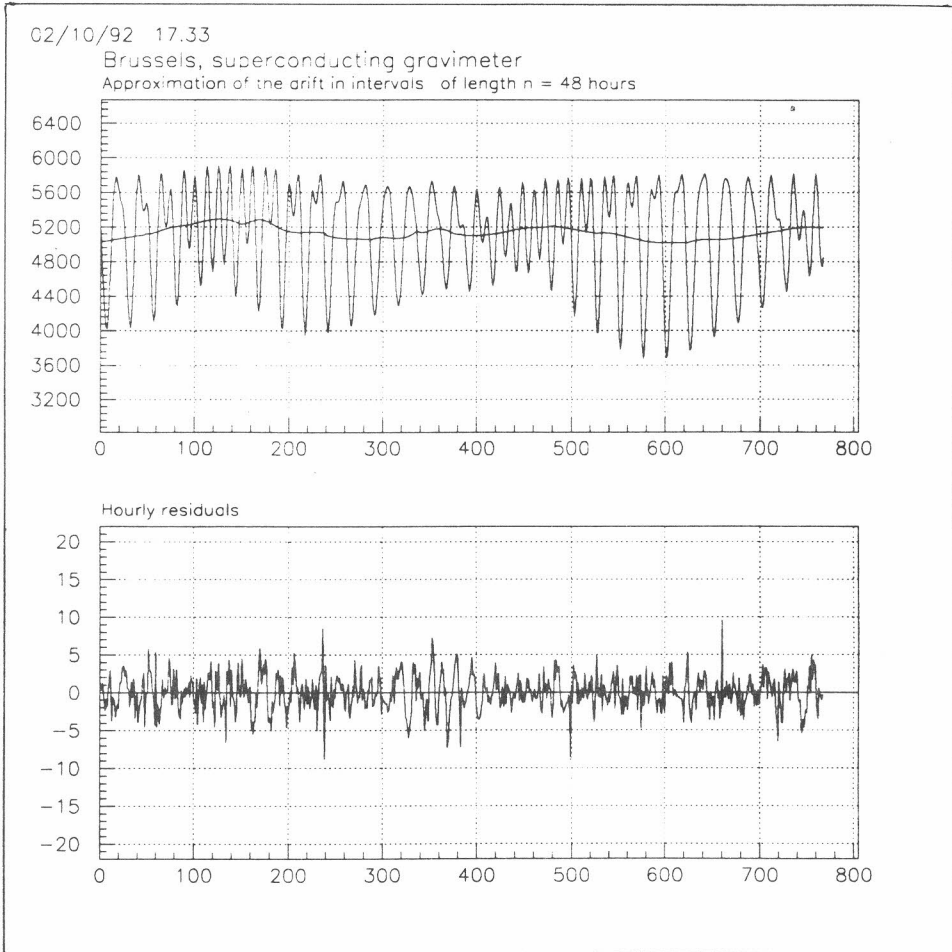


Fig. 6.

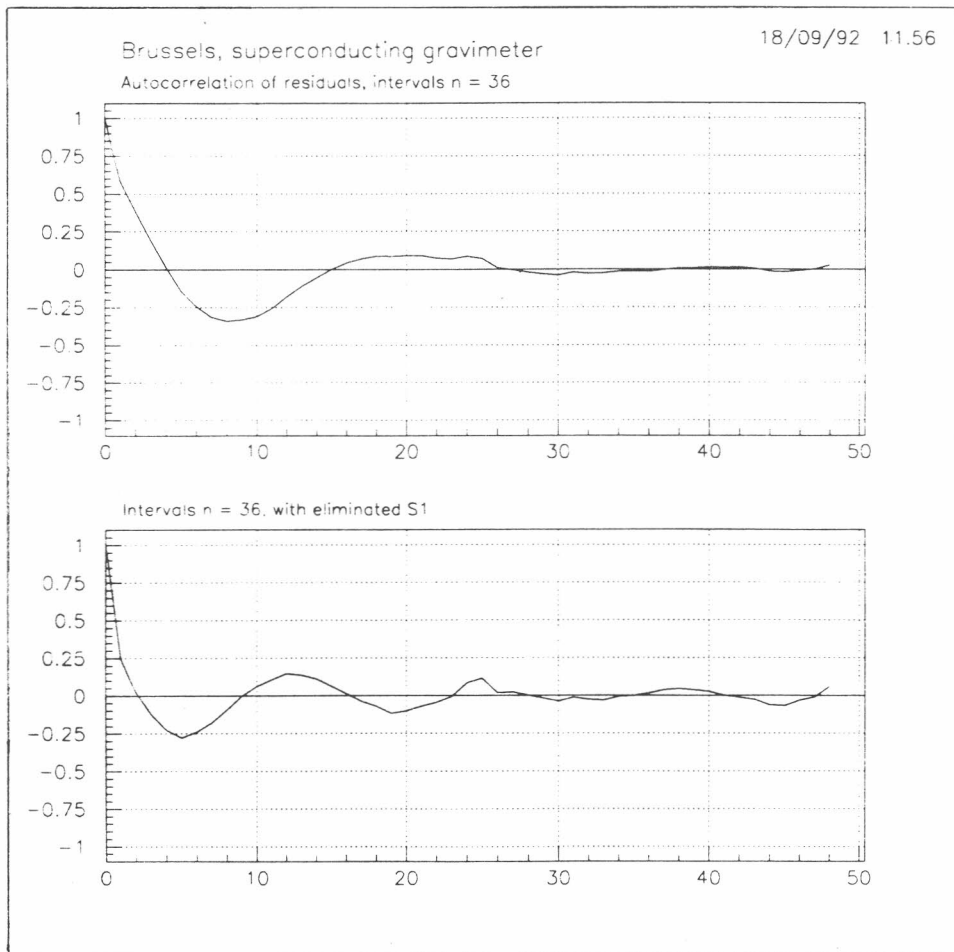


TABLE 1

Lanzarote, volcanic station Cueva de los Verdes, gravimeter LCR

Time interval: 87.05.14./91.06.12.

Comparison between the methods of analysis M67 and M92.

University Complutense of Madrid, computer VAX 9000

Length of the intervals for the approximation of the drift $n = 48^h$

D tides	amplit.factor δ		phase shift κ		SD tides	amplit.factor δ		phase shift κ	
	M92	M67	M92	M67		M92	M67	M92	M67
σ_1	1.2059 ± 413	1.3036 ± 772	6°283 ± 1.954	0°552 ± 3.389	ϵ_2	1.0529 ± 210	1.0792 ± 390	-8°862 ± 1.142	-6°847 ± 2.069
$2Q_1$	1.2528 132	1.2637 237	-1.410 605	-1.660 1.074	$2N_2$	0.9630 67	0.9888 118	-2.000 397	-2.022 684
σ_1	1.2097 110	1.2433 195	1.090 519	0.602 898	μ_2	0.9958 57	0.9836 100	-4.360 327	-3.782 582
Q_1	1.1823 18	1.1831 30	-2.136 84	-2.202 146	N_2	0.9960 9	0.9970 15	0.780 51	0.855 88
ρ_1	1.1527 93	1.1600 158	-2.709 460	-1.021 781	ν_2	0.9963 48	1.0008 81	1.472 276	1.251 462
O_1	1.1500 3	1.1500 6	-1.684 16	-1.661 38	M_2	1.0179 1a	1.0173 3	2.167 9	2.160 16
τ_1	1.2057 345	1.0330 574	-6.299 1.643	-1.009 3.184	λ_2	1.0571 239	1.0765 381	3.832 1.295	1.997 2.029
NO_1	1.1535 40	1.1488 66	-0.699 198	-0.355 327	L_2	1.0666 61	1.0677 96	3.368 325	4.114 518
χ_1	1.1585 220	1.0993 360	1.648 1.088	0.577 1.873	T_2	1.0368 63	1.0424 98	5.838 346	4.863 541
π_1	1.1519 143	1.0995 231	2.809 709	0.834 1.205	S_2	1.0689 4	1.0677 6	4.001 19	3.991 31
P_1	1.1321 8	1.1320 14	0.215 42	0.191 69	K_2	1.0594 11	1.0583 17	3.795 58	3.862 91
S_1	0.7495 509	0.9180 843	24.29 4.087	11.00 5.259	η_2	1.0947 167	1.0712 261	5.606 871	4.800 1.393
K_1	1.1172 3	1.1166 4	0.205 12	0.201 21	$2K_2$	1.1023 413	1.0810 651	0.499 2.147	2.447 3.451
ψ_1	0.9763 340	1.0261 552	-18.11 1.998	-9.869 3.081	M_3	1.1018 103	1.0958 76	1.697 537	1.757 396
φ_1	1.1162 200	1.0877 325	5.329 1.023	5.413 1.711					
θ_1	1.2279 222	1.1430 363	-3.738 1.038	0.190 1.818					
J_1	1.1486 42	1.1494 69	1.251 212	0.467 346					
SO_1	1.2429 245	1.1122 405	0.800 1.131	-0.626 2.084					
OO_1	1.1681 53	1.1507 88	0.367 261	0.786 440					
ν_1	1.1686 265	1.1166 451	2.550 1.303	0.655 2.317					

TABLE 2

Brussels, superconducting gravimeter, P. Melchior, B. Ducarme.

82.04.21/82.05.24 82.06.02/86.10.14 86.11.15/87.11.25

Comparison between the methods of analysis M67 and M92.

Unversity Complutense of Madrid, computer VAX 9000

Length of the intervals for the approximation of the drift $n = 48^h$

D tides	amplit.factor δ		phase shift κ		SD tides	amplit.factor δ		phase shift κ	
	M92	M67	M92	M67		M92	M67	M92	M67
σQ_1	1.2745 ± 105	1.2575 ± 487	-0°262 ± 470	-0°332 ± 2.219	ϵ_2	1.1307 ± 98	1.1463 ± 180	3°420 ± 494	3°341 ± 900
$2Q_1$	1.1611 33	1.1642 150	-1.749 162	-1.885 737	$2N_2$	1.1473 32	1.1494 55	2.324 157	2.444 276
σ_1	1.1540 27	1.1524 122	-1.181 134	-1.152 608	μ_2	1.1946 26	1.1981 46	4.365 126	4.150 219
Q_1	1.1605 4	1.1603 19	-0.469 20	-0.473 093	N_2	1.1767 4	1.1770 7	2.807 20	2.821 34
ρ_1	1.1644 22	1.1769 98	-0.647 108	-0.780 477	ν_2	1.1776 22	1.1760 37	2.672 107	2.807 180
O_1	1.1630 08	1.1634 3	-0.088 3	-0.086 17	M_2	1.1950 08	1.1949 1	2.482 3	2.481 6
τ_1	1.1415 73	1.1026 319	-1.948 366	-5.174 1.660	λ_2	1.1492 109	1.1432 174	3.062 544	3.571 870
NO_1	1.1668 10	1.1682 42	0.844 47	0.717 208	L_2	1.1952 22	1.1946 34	1.565 104	1.592 165
χ_1	1.1779 51	1.1710 225	0.481 250	-0.292 1.102	T_2	1.2081 29	1.2091 45	1.224 138	1.134 215
π_1	1.1830 32	1.1856 138	0.503 151	0.169 668	S_2	1.2141 17	1.2143 3	1.123 8	1.133 12
P_1	1.1643 19	1.1648 8	0.150 9	0.152 40	K_2	1.2118 5	1.2112 8	1.216 25	1.208 39
S_1	1.3160 112	1.3218 494	-4.203 498	-5.406 2.141	η_2	1.1868 89	1.1806 139	0.424 430	0.967 672
K_1	1.1505 06	1.1505 3	0.147 2	0.142 12	$2K_2$	1.2348 224	1.2420 350	-2.993 1036	-2.165 1.614
ψ_1	1.2509 76	1.2740 332	0.596 345	-0.125 1.491	M_3	1.0751 64	1.0781 59	0.620 342	0.824 312
φ_1	1.1928 44	1.1848 193	0.675 210	0.494 933					
θ_1	1.1470 51	1.1540 226	-1.133 254	-0.988 1.124					
J_1	1.1671 10	1.1651 44	0.557 47	0.504 214					
SO_1	1.1220 57	1.1309 257	0.974 290	0.917 1.304					
OO_1	1.1748 13	1.1805 60	-0.033 65	0.133 294					
ν_1	1.1666 66	1.1755 311	0.314 326	0.253 1.515					

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