

Twisted extreme trans-Neptunian orbital parameter space: statistically significant asymmetries confirmed

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ABSTRACT

Asymmetric debris discs have been found around stars other than the Sun; asymmetries are sometimes attributed to perturbations induced by unseen planets. The presence or absence of asymmetries in our own trans-Neptunian belt remains controversial. The study of sensitive tracers in a sample of objects relatively free from the perturbations exerted by the four known giant planets and most stellar flybys may put an end to this debate. The analysis of the distribution of the mutual nodal distances of the known extreme trans-Neptunian objects (ETNOs) that measure how close two orbits may get to each other could be such a game changer. Here, we use a sample of 51 ETNOs together with random shufflings of this sample and two unbiased scattered-disc orbital models to confirm a statistically significant (62σ) asymmetry between the shortest mutual ascending and descending nodal distances as well as the existence of multiple highly improbably ($p < 0.0002$) correlated pairs of orbits with mutual nodal distances as low as 0.2 au at 152 au from the Solar system’s barycentre or 1.3 au at 339 au. We conclude that these findings fit best with the notion that trans-Plutonian planets exist.

Key words: methods: statistical – celestial mechanics – minor planets, asteroids: general – Kuiper belt: general – Oort Cloud.

1 INTRODUCTION

Asymmetric debris discs or exoKuiper belts are sometimes the result of perturbations induced by companions (see e.g. Marino et al. 2020; Faramaz et al. 2021; Lovell et al. 2021). When no stellar or substellar companions are detected, the observed asymmetries are often attributed to secular interactions with unseen planets (see e.g. Sende & Löhne 2019, but see also Zderic et al. (2021)). In this respect, our local trans-Neptunian belt is not too different from the so-called exoKuiper belts (see e.g. Hughes, Duchêne & Matthews 2018; Wyatt 2020), but our vantage point does not help in confirming or rejecting the presence of asymmetries out there. The existence of asymmetries or gaps in our local trans-Neptunian belt has been suggested (see e.g. Volk & Malhotra 2017; Li & Jeff Xia 2020; Oldroyd & Trujillo 2021) or rejected (see e.g. Van Laerhoven et al. 2019; Gladman & Volk 2021) based on nearly the same input data and similar diagnostic tools, particularly the statistical analysis of the orbital inclination distributions. In order to produce robust results, the identification of asymmetries in the trans-Neptunian or Kuiper belt requires the study of sensitive tracers in a sample of objects the least possible affected by perturbations from the four known giant planets and most stellar flybys.

A promising instance of tracer and object sample is in the distribution of mutual nodal distances of the extreme trans-Neptunian

objects (ETNOs) —TNOs with semimajor axis, $a > 150$ au and perihelion distance, $q > 30$ au— as discussed by de la Fuente Marcos & de la Fuente Marcos (2021). Using data for 39 ETNOs, de la Fuente Marcos & de la Fuente Marcos (2021) found that the distribution of mutual nodal distances of the studied sample showed a statistically significant asymmetry between the shortest mutual ascending and descending nodal distances, hereinafter the Δ_+ – Δ_- asymmetry. In addition, a highly improbable pair of ETNOs made of (505478) 2013 UT₁₅ and 2016 SG₅₈ with a mutual ascending nodal distance of 1.35 au at 339 au from the barycentre of the Solar system was uncovered by the analysis; the probability of finding such a pair by chance was estimated to be 0.000205 ± 0.000005 .

The Dark Energy Survey (DES) has recently announced the discovery of 815 new TNOs (Bernardinelli et al. 2022). After this data release, the ETNO data set now includes 51 objects (an increase of nearly 31 per cent, from 39) and the Δ_+ – Δ_- distribution has 1275 pairs instead of 741 (a 72 per cent increase). Here, we use this larger data set to further investigate the Δ_+ – Δ_- asymmetry and the presence of improbable orbit pairs. This Letter is organized as follows. In Section 2, we discuss data and methods. Theoretical expectations from two different models are reviewed in Section 3. The updated distribution of mutual nodal distances is presented and assessed in Section 4. Section 5 shows that the statistically significant asymmetries found by de la Fuente Marcos & de la Fuente Marcos (2021) are confirmed. Our results are discussed in Section 6 and our conclusions are summarized in Section 7.

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2 DATA AND METHODS

The distribution of the mutual nodal distances of the known ETNOs provides information on how close two orbits may get to each other. The shortest mutual nodal distance is the minimum orbit intersection distance or MOID that, in absence of protective mechanisms such as mean-motion or secular resonances, could be close to the minimum approach distance between two ETNOs; in other words, a sufficiently short nodal distance might be signalling recurrent close flybys and, perhaps, even a past physical connection between two ETNOs. Here, we compute the distribution of the mutual nodal distances of the known ETNOs (51 as of 2-February-2022) as described in appendix A of [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#), using eqs. 16 and 17 in [Saillenfest et al. \(2017\)](#).

For a given pair of objects, the computation of the mutual nodal distances requires the values of semimajor axis, a , eccentricity, e , inclination, i , longitude of the ascending node, Ω , and argument of perihelion, ω . As in [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#), we have used barycentric values accessed from Jet Propulsion Laboratory's (JPL) Horizons on-line solar system data and ephemeris computation service¹ that uses the new DE440/441 general-purpose planetary solution ([Park et al. 2021](#)). Data were queried using tools provided by the PYTHON package ASTROQUERY ([Ginsburg et al. 2019](#)). Statistical analyses were performed using NUMPY ([van der Walt, Colbert & Varoquaux 2011](#); [Harris et al. 2020](#)) and visualized using the MATPLOTLIB library ([Hunter 2007](#)).

3 THEORETICAL EXPECTATIONS

Synthetic ETNOs can be randomly generated in accordance with a nominal scattered-disc model. The study of the distribution of nodal distances of sets of simulated objects can provide an unbiased reference to evaluate the statistical significance of any features identified in the nodal distances distribution of real ETNOs. Following the analysis in [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#), we have considered the scattered-disc models discussed by [Brown \(2001, 2017\)](#) and [Napier et al. \(2021\)](#). The single difference between both models is in the (a, e, q) set of parameters. While [Napier et al. \(2021\)](#) favours a model in which a follows the distribution $N(a) \propto a^{0.7}$ and e is uniformly distributed in the interval (0.69, 0.999), [Brown \(2001, 2017\)](#) argues for a uniform distribution in both a and q . As described in appendix C of [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) and for the latter scattered disc model, a follows a uniform distribution in the interval (150, 1000) au, q is also uniform in the interval (30, 100) au, the angular elements Ω and ω are drawn from a uniform distribution in the interval $(0^\circ, 360^\circ)$, and i follows the so-called Brown distribution of inclinations ([Brown 2001, 2017](#)) in the interval $(0^\circ, 60^\circ)$.

Figure 1 shows representative distributions of both scattered-disc models corresponding to 51 synthetic ETNOs. In [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) and after analyzing 10 instances of 10^4 synthetic ETNOs each (or 49995000 pairs per instance) randomly generated in accordance with [Brown \(2001, 2017\)](#), it was found that the first percentile (that signals extreme outliers) of Δ_+ was 1.460 ± 0.009 au and the one for Δ_- was 1.450 ± 0.009 au, which implies that, under the assumptions in this model, the median and 16th and 84th percentiles of the first percentiles of the mutual ascending and descending nodal distances must be identical. A similar result was found for the model in

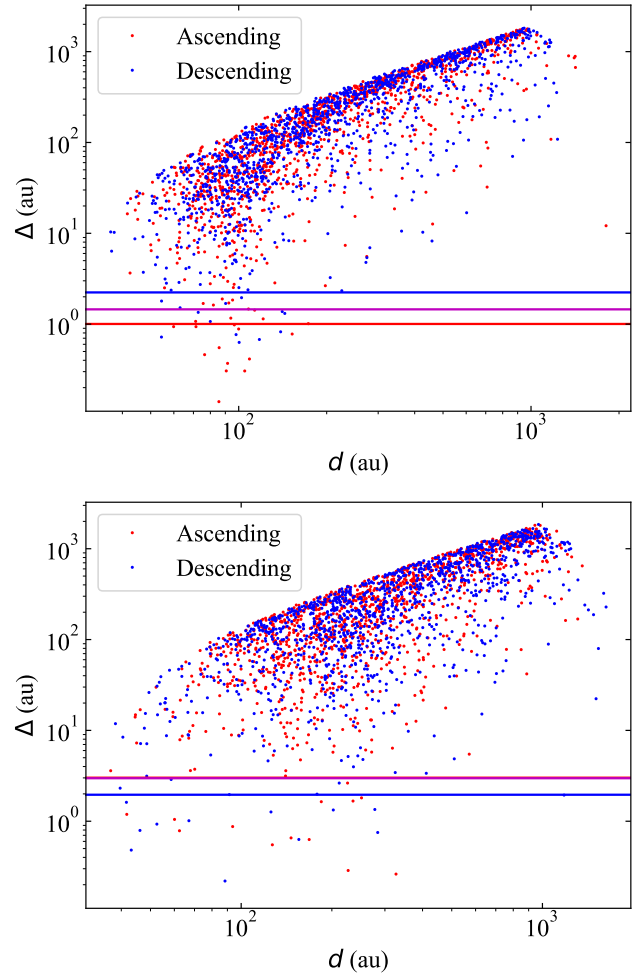


Figure 1. Mutual nodal distances as a function of the barycentric distance to the node for a representative sample of 51 synthetic ETNOs or 1275 pairs. The top panel shows results compatible with the scattered-disc model discussed by [Brown \(2001, 2017\)](#); the bottom panel displays results consistent with the model in [Napier et al. \(2021\)](#). The values of mutual nodal distances of ascending nodes, Δ_+ , are shown in red and those of descending nodes, Δ_- , in blue. The horizontal lines show the relevant first percentiles. In purple, we show 1.455 au (top) and 2.975 au (bottom). See the text for details.

[Napier et al. \(2021\)](#), but in this case the first percentile of Δ_+ is 2.98 au and the one for Δ_- is 2.97 au. Using a steeper slope for the semimajor axis distribution (e.g. $\propto a^{1.3}$) further increases the value of the extreme outliers threshold. In the following, we will consider 1.455 au as a reference for the first percentile value of Δ_{\pm} .

4 UPDATED NODAL DISTANCES DISTRIBUTION

The nodal distances distribution corresponding to the 51 known ETNOs or 1275 pairs was computed as described in appendix A of [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#), taking into account both the nominal (mean) values of the barycentric orbital elements and their uncertainties. Figure 2 shows the median values of the mutual nodal distances as a function of the average barycentric distance to the node. The values of mutual nodal distances of ascending nodes, Δ_+ , are displayed in red and those of descending

¹ <https://ssd.jpl.nasa.gov/?horizons>

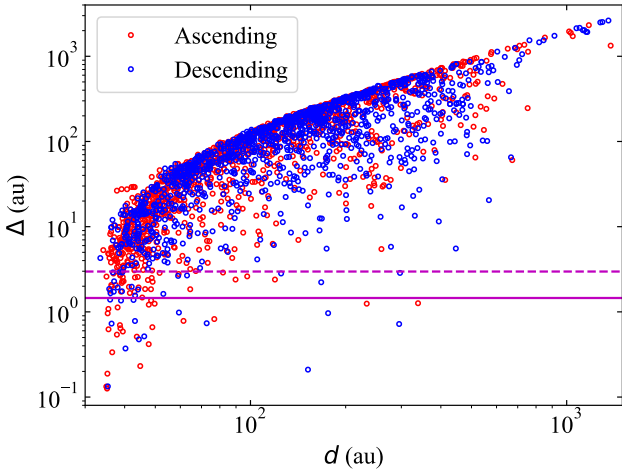


Figure 2. Mutual nodal distances as a function of the barycentric distance to the node for the sample of 51 ETNOs or 1275 pairs. The values of mutual nodal distances of ascending nodes, Δ_+ , are shown in red and those of descending nodes, Δ_- , in blue. The solid purple line corresponds to 1.455 au and the dashed one to 2.975 au. See the text for details.

nodes, Δ_- , in blue. As in [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) and in order to identify severe outliers, we use the first percentile of the distribution (see e.g. [Wall & Jenkins 2012](#)) that is 0.768 ± 0.004 au for Δ_+ and 1.392 ± 0.010 au for Δ_- (means and standard deviations from 10 experiments).

We have tested the significance of the correlations found above by shuffling the orbital elements: the Fisher–Yates shuffle ([Fisher & Yates 1938](#); [Knuth 1969](#)) in the form implemented in PYTHON’s `RANDOM.SHUFFLE` was applied to each orbital element independently to produce a random permutation of the original sequence. Each randomized data set (sampling 51 orbits 10^6 times) was processed as the original one was. When considering the nominal values of the orbital elements $\Delta_{\pm} = 1.353$ au; if both mean values and uncertainties are included in the shuffling process, then $\Delta_{\pm} = 1.454$ au. This randomization test shows that by removing any existing correlations in the data set, the $\Delta_+ - \Delta_-$ asymmetry is also removed. We interpret this result as signalling that the asymmetry is intrinsic to the data set and not an artifact or due to chance. On the other hand, $\Delta_{\pm} = 1.454$ au is close to the value $\Delta_- = 1.392 \pm 0.010$ au (but statistically inconsistent, 6.2σ); we interpret this as evidence in favour of the mutual descending nodes being rather randomly distributed while the distribution of mutual ascending nodes not being statistically compatible (171.5σ) with a random one.

On the other hand, [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) found one highly improbably correlated pair of orbits; with the new data, we found five pairs of ETNOs with mutual nodal distances below the first percentile of the distribution and barycentric distance above 150 au. The five pairs are (Δ_{\pm} as median value and 16th and 84th percentiles): (527603) 2007 VJ₃₀₅ and 2015 VQ₂₀₇ with $\Delta_- = 0.21^{+0.21}_{-0.15}$ au at 152 au, (594337) 2016 QU₈₉ and 2016 SA₅₉ with $\Delta_- = 0.72^{+0.72}_{-0.51}$ au at 295 au, (496315) 2013 GP₁₃₆ and 2014 SX₄₀₃ with $\Delta_- = 0.97^{+1.03}_{-0.68}$ au at 176 au, 527603 and 2013 RC₁₅₆ with $\Delta_+ = 1.25^{+1.36}_{-0.88}$ au at 233 au, and (505478) 2013 UT₁₅ and 2016 SG₅₈ with $\Delta_+ = 1.26^{+1.31}_{-0.89}$ au at 339 au. As for the statistical significance of these highly improbably correlated pairs within the context of the randomized data set, the probability of having mutual nodal distances below the first per-

centile of the distribution at a barycentric distance above 300 au is $0.00014507 \pm 0.00000013$ and that of having mutual nodes closer than 0.25 au beyond 150 au is $0.00013444 \pm 0.00000004$. It is therefore confirmed that within the known ETNOs data set, the five pairs pointed out above are indeed remarkable.

5 STATISTICALLY SIGNIFICANT ASYMMETRIES CONFIRMED

In Section 3, we have shown that $\Delta_+ = 0.768 \pm 0.004$ au and $\Delta_- = 1.392 \pm 0.010$ au. The respective values obtained by [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) were 1.450 ± 0.010 au and 2.335 ± 0.014 au (for 39 ETNOs or 741 pairs and also from 10 experiments). Therefore, the new data confirm that the distribution of mutual nodal distances has a statistically significant asymmetry between the shortest mutual ascending and descending nodal distances, this time at the 62σ level (63σ in [de la Fuente Marcos & de la Fuente Marcos 2021](#)).

On the other hand, [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#) found that the peculiar pair of ETNOs made of (505478) 2013 UT₁₅ and 2016 SG₅₈ had a mutual ascending nodal distance of 1.35 au at 339 au from the Sun. With the new data, 505478 and 2016 SG₅₈ have $\Delta_+ = 1.26^{+1.31}_{-0.89}$ au at 339 au and we found four new pairs of ETNOs with mutual nodal distances within the first percentile of the distribution and barycentric distance above 150 au. Within the context of the randomized data set discussed above, these five pairs of ETNOs are highly improbably correlated as the probability of having mutual nodal distances below the first percentile of the distribution at a barycentric distance above 300 au is as low as $0.00014507 \pm 0.00000013$.

In Fig. 1, both models fail to reproduce the high density of large nodal distances observed at barycentric distances below 100 au in Fig. 2, although the model in [Brown \(2001, 2017\)](#) appears to produce better results. The actual value of Δ_- for the sample of 51 known ETNOs is 1.392 ± 0.010 au that is somewhat consistent with the one from the scattered-disc model discussed by [Brown \(2001, 2017\)](#), but incompatible with that of [Napier et al. \(2021\)](#).

As for the statistical significance of ETNO pairs like the ones pointed out above within the context of the scattered-disc model discussed by [Brown \(2001, 2017\)](#), the analysis of 10^5 experiments sampling 51 synthetic orbits each indicates that the probability of finding five pairs with $\Delta_{\pm} < 1.455$ au at a barycentric distance above 150 au is 0.0577 ± 0.0013 that suggests that the five ETNO pairs could indeed be unusual, although the evidence may not be fully conclusive as there is nearly 6 per cent chance of obtaining a result at least as extreme as the one observed using the scattered-disc model discussed by [Brown \(2001, 2017\)](#).

6 DISCUSSION

The larger ETNO data set assembled thanks to the recent DES data release ([Bernardinelli et al. 2022](#)) has led to significant improvements in the results initially reported by [de la Fuente Marcos & de la Fuente Marcos \(2021\)](#). Although it is difficult to argue against the reality of the $\Delta_+ - \Delta_-$ asymmetry and the number of strongly correlated orbits seems to defy predictions from two scattered-disc models, the source of these features is still unclear.

Following, e.g., [Napier et al. \(2021\)](#) one may argue that observational bias could be the source of the features observed here; in particular, if the components of some or all the peculiar pairs

Table 1. Discovery circumstances of the members of the pairs discussed in the text. Data include: discovery date, right ascension (α) and declination (δ) at discovery time, heliocentric distance (r_d) and apparent motion (μ_d) at discovery time, and discovery observatory —O means OSSOS discovery, A means Apache Point, D means DECam, and M means Mauna Kea. Data sources: JPL’s SSDG SBDB and MPC.

| Object | disc. date | α (h:m:s) | δ (°:′:″) | r_d (au) | μ_d (″ h ⁻¹) | obs. |
|------------------------|-------------|---------------------|---------------------|---------------|---------------------------------|------|
| 496315 | 2013-Apr-04 | 14:07:32.88 | −11:09:38.8 | 45.4 | 2.63 | O |
| 505478 | 2013-Oct-31 | 00:54:20.28 | 05:12:29.1 | 61.2 | 1.91 | O |
| 527603 | 2007-Nov-04 | 00:29:31.74 | −00:45:45.0 | 35.2 | 2.60 | A |
| 594337 | 2016-Aug-25 | 02:44:19.76 | −00:38:53.5 | 35.5 | 1.20 | D |
| 2013 RC ₁₅₆ | 2013-Sep-04 | 20:02:16.73 | −51:57:29.2 | 38.6 | 2.46 | D |
| 2014 SX ₄₀₃ | 2014-Sep-25 | 00:44:30.34 | −23:49:43.1 | 41.1 | 3.12 | D |
| 2015 VQ ₂₀₇ | 2015-Nov-05 | 00:40:57.35 | −24:29:07.9 | 31.4 | 2.70 | D |
| 2016 SA ₅₉ | 2016-Sep-28 | 02:41:32.23 | −00:17:00.2 | 40.7 | 2.52 | M |
| 2016 SG ₅₈ | 2016-Sep-27 | 02:35:41.45 | 01:05:18.7 | 36.1 | 2.70 | M |

of ETNOs pointed out above were discovered by the same survey or at similar locations in the sky, observational bias could not be easily rejected as irrelevant. Table 1 shows the discovery circumstances of the members of the pairs discussed above. No components of a given pair were found by the same survey and only one pair had both components appearing closely projected at discovery time, (594337) 2016 QU₈₉ and 2016 SA₅₉. Observational bias may not be the source of the highly improbably correlated pairs.

On the other hand, objects with $q > 30$ au may still experience close interactions with Neptune that perhaps could induce the observed features. A more selective criterion to avoid this issue is to make the cut at larger q (e.g. 37 au or 40 au). Considering $q > 40$ au, there are 21 ETNOs or 210 pairs and a similar analysis gives $\Delta_+ = 0.578 \pm 0.004$ au and $\Delta_- = 3.29 \pm 0.03$ au, which confirms a statistically significant (90σ) asymmetry. On the other hand, four out of nine objects (44 per cent) in Table 1 have perihelion distance $q > 37$ au; two out of nine (22 per cent) have $q > 40$ au. However, none of the pairs have both members with $q > 37$ au.

Other than bias and the influence of Neptune, perhaps the simplest conjecture with which one may try to account for the observed features is in assuming that they are linked to the presence of families, analogues of the well-studied asteroid families present in the main asteroid belt. The subject of finding collisional families of TNOs has been explored by Chiang et al. (2003) and Marcus et al. (2011). Collisional asteroid families have already been found in the regular trans-Neptunian space: the first confirmed family found in the outer Solar system was the one associated with dwarf planet Haumea (Brown et al. 2007) although a candidate collisional family had previously been proposed by Chiang (2002) and later confirmed by de la Fuente Marcos & de la Fuente Marcos (2018), who found four new candidate collisional families of TNOs and a number of unbound TNOs that may have a common origin. However, arguments against the collisional ETNO family conjecture come from different lines of reasoning. On the one hand, there is only one object in common between the five pairs of ETNOs with highly improbably correlated orbits, (527603) 2007 VJ₃₀₅, that is a member of two pairs but with nodes at rather different barycentric distances. On the other hand, Fig. 3 shows that the unusual pairs have values of the angular separation between orbital poles, α_p , that are generally above 10° with the one of pair (505478) 2013 UT₁₅–2016 SG₅₈

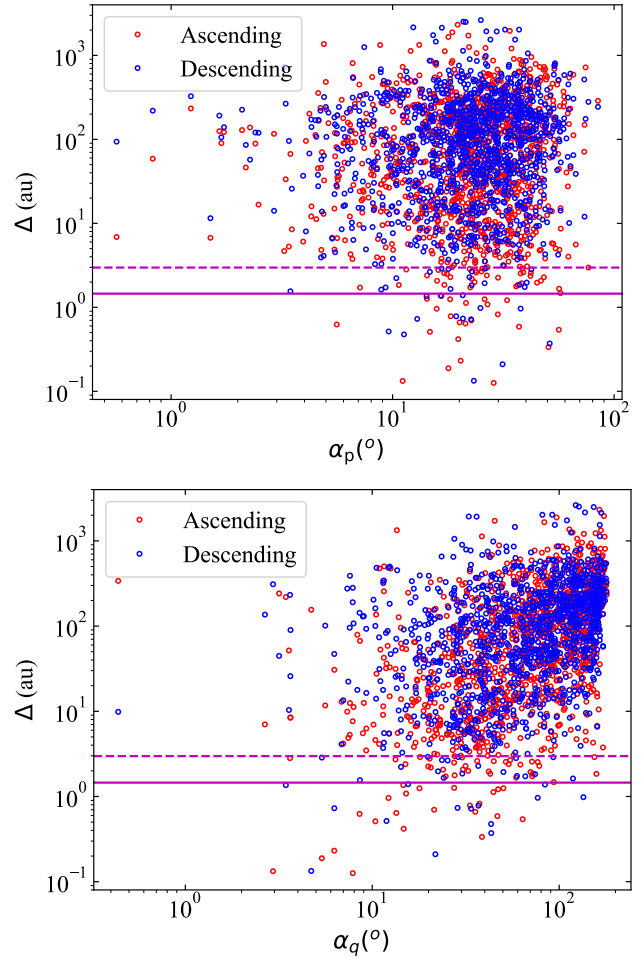


Figure 3. Mutual nodal distances as a function of the angular separations between orbital poles, α_p (top panel), and perihelia, α_q (bottom panel), for the sample of 51 ETNOs or 1275 pairs. The values of mutual nodal distances of ascending nodes, Δ_+ , are shown in red and those of descending nodes, Δ_- , in blue. The solid purple line corresponds to 1.455 au and the dashed one to 2.975 au. See the text for details.

slightly above 14° and that of pair (594337) 2016 QU₈₉–2016 SA₅₉ being about 23° , the others are higher; the values of the angular separations between perihelia, α_q , are also relatively large. In sharp contrast, the values of α_p and α_q are expected to be small when objects come from a recent fragmentation event (see e.g. de la Fuente Marcos & de la Fuente Marcos 2018). The values of α_p and α_q were computed as described in appendix B of de la Fuente Marcos & de la Fuente Marcos (2021).

The orbital pole is the intersection between the celestial sphere and a hypothetical axis perpendicular to the plane of the orbit under study. The direction of perihelia is the intersection between the celestial sphere and a hypothetical line that goes from the focus of the orbit towards the point where the orbit under study reaches perihelion. Using observational data, de la Fuente Marcos & de la Fuente Marcos (2018) showed that any fragments produced by a relatively recent disruption event must have low values of α_p and α_q , probably under $\sim 2^\circ$; fragments from an old disruption episode may have values of α_q uniformly distributed in the interval (0° , 180°) and a wide range in the values of α_p . The distribution of α_q (histogram not shown) is far from uniform. The pairs with low values of α_p

and α_q in Fig. 3 all have mutual nodal distances well above the first percentile of the distribution. All this evidence points away from a collisional origin for the highly correlated ETNOs.

The fact remains that some pairs of ETNOs have one improbably close mutual nodal distance and that a mechanism must exist to help keeping them that close for a period of time long enough to make them identifiable even in relatively small samples as the one studied here. Close encounters with a massive planetary perturber may play that role. Small mutual nodal distances have been found for a number of members of the 29P/Schwassmann-Wachmann 1 comet complex; in this case, nodes of Centaurs are kept close by Jupiter (de la Fuente Marcos et al. 2021). Variations of this scenario within the context of the ETNOs have been explored by de León, de la Fuente Marcos & de la Fuente Marcos (2017) and de la Fuente Marcos, de la Fuente Marcos & Aarseth (2017). However, the barycentric distances of the five relevant pairs singled out above show a significant dispersion, so either one planetary perturber moves in an eccentric orbit (e.g. $a \sim 250$ au and $e \sim 0.4$) or multiple perturbers might be necessary to produce the observed effects. The unlikely presence (due to its intrinsically low probability) of a triplet among the outlier pairs might also be signalling the influence of trans-Plutonian perturber(s). It is unclear if the mechanism proposed by Zderic et al. (2021) could be able to produce the observed features; although it leads to a lopsided outer Solar system, it does it via apsidal clustering, which may not be present in the real data (see e.g. Napier et al. 2021).

7 CONCLUSIONS

Kuiper belt analogues or exoKuiper belts exhibit a diversity of asymmetries and some of them could be caused by unseen planets (see e.g. Hughes, Duchêne & Matthews 2018; Wyatt 2020). The question of whether or not our Solar system hosts an asymmetric trans-Neptunian belt has been asked for decades. This Letter argues that, thanks to the new data, we might have finally got a definite answer to this important conundrum. The main conclusions of our study are:

- (i) We confirm the presence of a statistically significant (62σ) asymmetry between the shortest mutual ascending and descending nodal distances in a sample of 51 ETNOs.
- (ii) We confirm the existence of five highly improbably correlated pairs of orbits with mutual nodal distances as low as 0.2 au at 152 au from the Solar system's barycentre or 1.26 au at 339 au.

We consider that it is improbable that the features discussed here could result from the presence of collisional families, our findings are more supportive of the existence of trans-Plutonian planets.

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DATA AVAILABILITY

The data underlying this paper were accessed from JPL's Horizons (<https://ssd.jpl.nasa.gov/?horizons>). The derived data generated in this research will be shared on reasonable request to the corresponding author.

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