DISSERTATIO 15C REVISITED: CONCEPT AND BODY IN KANT'S PROBLEMS OF DIRECTIONALITY

Introduction

In the way Kant treats the problems posed by directionality (i.e. spatial incongruence and orientation), absolute space –in a sense often considered 'Newtonian'– and our own body appear as key elements of the solution in different forms and levels[1]. However, in order to understand exactly the role played by absolute space and/or body, and why they can furnish a solution at all, it is first necessary to make sufficiently clear what exactly the problem is for which these elements are a solution. In particular, it is necessary to sufficiently clarify a principle that almost always explicitly accompanies Kant's discussion of these problems, and certainly always hovers over it, namely, the principle explicitly formulated for the first time in *Dissertatio* 15C which states that two incongruent counterparts cannot be distinguished "notis mentis per sermonem intelligibilibus" (AA 04: 403)[2].

Because of this, instead of proceeding directly to the analysis of Kant's different solutions to the problems of directionality, this article focus on the logically previous stage in which those problems are themselves shaped, and in which the thesis of the *Dissertatio* must be properly assessed. By doing so, it intends to help clear up some pre-conditions for subsequent discussions to take place, from those on Kant's conception of space to those related to the role of the body. In particular, and this will be its main

[1]The joint treatment of the questions of incongruent counterparts and orientation under the common heading of "Kantian problems of directionality" is not obvious, and definitely not accepted by everyone. See *infra*, pp. 12-13.

[2] Although Kant's main thesis in *Dissertatio* 15C is certainly the intuitive character of space (AA 02, 302), in what follows I will deal directly only with this claim, so in order to avoid overloading the text with repeated rephrasing, I will from now on refer to it as "the" thesis of *Dissertatio* 15C.

target, it purports to shed light on the relation between that previous stage and the apparition of the body. For, indeed, although there already exists a certain tradition of interpretation that understands Kant's perspective of transcendental subjectivity as the perspective of a bodily or "embodied" subject[3], a sufficiently clear and generally-accepted analysis of the type of resources necessary to distinguish left and right, and of the relation in Kant of these resources to our own body, is still missing. In this sense, and in spite of important suggestions, a systematic, vigorous and clear connection between the thesis of *Dissertatio 15C* about the incapacity of *notae* [*Merkmale*] to solve problems of directionality and the participation of the body in such solutions remains to be established. This article intends to contribute to establishing that connection, and it aims to do so by putting the notion of the "coordinate system" under the spotlight[4]. The article is meant as a systematic reconstruction of logical connections, not as a historical study of the evolution of Kant's own opinions on the matter (although I believe that a better understanding of the logical connections between these ideas can greatly contribute to a precise reconstruction of that evolution).

[3] See, despite their very different perspectives: Kaulbach, Friedrich: "Leibbewusstsein und Welterfahrung beim frühen und späten Kant". In:*Kant Studien* 54, 1963, 464-90, and *Immanuel Kant*, Berlín 1982, 119ff; Robinson, Hoke: "Incongruent Counterparts and the Refutation of Idealism". In: *Kant Studien*72, 391–397; Svare, Helga: *Body and practice in Kant*, Dordrecht 2006; Nuzzo, Angelica: *Ideal Embodiment: Kant's theory of sensibility*. Bloomington (IN) 2008; Rukgaber, Mathhew: "'The Key to Transcendental Philosophy': Space, Time and the Body in Kant". In: *KantStudien* 100(2) 2009, 166-186; Oroño, Matías: "Las contrapartidas incongruentes y el cuerpo propio en el idealismo trascendental de Kant". In: *Ideas y Valores* 66 (163). 2017, 153-176.

[4] Although it appears in numerous works (e.g. Kaulbach 1963, Robinson 1992, but also Bennett, Jonathan: *Kant's Dialectic*, Cambridge, 1974, Gloy, Karen: "Die Kantische Differenz von Begriff und Anschauung und ihre Begründung". In: *KantStudien*75, 1984, 1-37; Ferrarin, Alfredo: "Lived Space, Geometric Space in Kant". In: *Studi Kantiani* 19, 2006, 11-30, or Friebe, Cord: "Substanz/Akzidens-Ontologie in kongruenter Gegenstücke", *Kant Studien* 97 (1), 2006, 33-49) this notion has not yet been taken as the fundamental vector in the analysis of directionality. Friebe 2006, for example, incorporated it in his discussion of the intrinsic or extrinsic character of the spatial coordinates of empirical objects (see especially 34-35 and 39), but not precisely where it was most needed: to illuminate directionality itself. Similarly, Bennett also made use of it (1974, 144) but restricted its scope to the construction of toponymic labels. Although it may seem paradoxical, because of the way he has been situated in this discussion, Reidemeister's indications in the last paragraphs of his 1947 paper constitute one of the clearest formulations of the dependence of directions on a given coordinate system (Reidermeister Kurt: "Über den Unterschied der Gegenden im Raum". In: *Zeitschrift für philosophiche Forschung* 2, 1947, 151).

Indeed, the thesis of *Dissertatio* 15C has been considered mistaken by the prevailing reading for decades (from Herman Weyl through Mühlhölzer to Rusnock and George), and only a handful of interpreters (like Bennett, Severo and, in his footsteps, Bernecker) have tried to rehabilitate it, although in a soft or 'weakened' version. Against the standard interpretation, I will maintain that the thesis is both internally coherent in Kant and correct in itself. But in contrast to the usual line of defence, I will try to rehabilitate this claim in its strongest version. In particular, I will dispute the widespread idea that incongruent counterparts can be mathematically distinguished, and will try to show that, when they apparently succeed, the alleged 'mathematical descriptions' of incongruent counterparts are not purely mathematical and, when they are purely mathematical (in Kant's strict sense), they do not (and cannot) succeed.

The article is divided into two main parts. Part I provides a minimal historical and conceptual framework and examines the thesis of *Dissertatio*15C, trying to clarify its meaning and to defend its soundness in the light of the repeated objections (and also the few exonerations) to which it has been exposed in the literature. Part II reveals the systematic role played by the body in Kant's treatment of incongruent counterparts and orientation as dependent on the situation described in *Dissertatio*15C.

I. *Dissertatio* 15C

Before starting the analysis of the claim in the *Dissertatio*, let me briefly give an adequate context for it by recalling a characteristic state of affairs in modern science.

The question of the directions of space is intrinsically connected to conceptions of the material world, and its study can undoubtedly be traced back at least to the Greeks. Nonetheless, the question of establishing directions in space received a strong thrust, new vitality, and an utterly different framework in the 17th and 18th centuries, in the context of the process of mathematisation of natural science. Indeed, if the modern approach imposed the mathematisation of all natural processes, it did so in first place with respect to motion. In this sense, Descartes' attempt at the 'geometrisation' of motion[5]set the epistemological standard for the whole of modern physics. But Descartes did not only mark the modern view of nature; he also made radically explicit one of its fundamental assumptions, namely, that magnitudes (extensional magnitude or space, in this case) are to become now the benchmark for the intelligibility of the natural world.

[5] See: Descartes, René: *The World*, or *Treatise on the Light*, chapter 7, AT XI, 39-40.

The possibility of mathematical expression on the basis of what is given in space would thus become the standard of objectivity for physical knowledge, and precisely for this reason the pure science of space (geometry) was destined to play a decisive role in that type of knowledge. But if the methods of analytical geometry presuppose the use of coordinate systems fixed at a certain centre (even if it is only in the imagination), their application to concrete physical phenomena will soon raise the question of where and how these systems should be anchored to the material world[6]. This last question has numerous facets and consequences that reach as far as Einstein's theory of relativity, but it has been brought to the fore only because it helps bring out the fact –yet to be properly illuminated–that the questions of directionality have their seat precisely in that interstice between mathematical constructions and the material world[7]. But let us now turn to the principle in question.

As already mentioned, the first complete formulation of the principle of the incapacity of *notae* [*Merkmale*] to solve problems of directionality is the one appearing in *Dissertatio*15C. The full passage reads like this:

> Quae iaceant in spatio dato unam plagam versus, quae in oppositam vergant, discursive describe s. ad notas intellectuals revocari nulla mentis acie possunt, ideoque, cum in solidis perfecte similibus atque aequalibus, sed discongruentibus, cuius generis sunt manus sinistra et dextra (quatenus solum secundum extensionem concipiuntur) aut triangular sphaerica e duobus hemisphaeriis oppositis, sit diversitas, per quam impossibile est, ut termini extensionis coincident, quanquam per omnia, quae notis menti per sermonem intelligibilibus eferre licet, sibi substitui possint...[8]

Replicas of the same idea appear on all the subsequent occasions in which Kant explicitly deals with spatial incongruence: *Prolegomena* ("Wir können daher auch den Unterschied ähnlicher und gleicher, aber doch incongruenter Dinge [....] durch keinen

[6] 'Anchoring' is precisely the term used by Bennett (1974, 145) to refer to the conceptual scheme by which we 'name' locations in space.

[7] By saying this I am, in principle, only joining the agreement among some interpreters that the problem of incongruence is not a mathematical problem as such, but a problem of how geometry can be applied to nature. However, this essentially correct statement still needs to be qualified in the appropriate direction.

[8] *Dissertatio*, AA 02: 403. This formulation of *Dissertatio*15C is the formulation of choice in the authors facing the issue (see: Bennett, Jonathan: "The difference between right and left". In: *American Philosophical Quarterly* 7 (3) 1970, 175; Rusnock, Paul & George, Rolf: "A Last Shot at Kant and Incongruent Counterparts". In: *Kant Studien* 86 (3) 1995, 272-273; Severo, Rogério Passos: "Three remarks on the interpretation of Kant on incongruent counterparts". In: *Kantian Review* 9, 2005, 39).

einzigen Begriff verständlich machen", AA 04: 286) and *Metaphysische Anfangsgrunde der Naturwissenschaft* ("dieser Unterschied zwar in der Anschauung geben, aber gar nicht auf deutliche Begriffe bringen, mithin nicht verständlich erklären (*dari, non intelligi*) läßt", AA 04: 484). And it is even possible to find in 1768 –when Kant does not yet have the resources and distinctions he will have later– a forerunner of this claim, which will almost literally be reproduced also in the *Prolegomena*: if descriptions are a matter of concepts, as it seems, then "eine vollstandige Beschreibung der einen [Hand] in allen Stücken auch von der andern gelten [muss]" (AA 02: 377), and therefore cannot express the difference between the one and the other[9].

The main claim behind all these passages seems to establish that the understanding (*Verstand*), through its form of representation, the concept (*Begriff*), is unable to account for the difference between two incongruous counterparts. As already mentioned, this claim has been classically deemed as mistaken in the literature[10]. And even the commentators most compassionate towards Kant have considered that the thesis of *Dissertatio 15C* does not mean what it literally seems to mean, i.e. that two incongruous counterparts cannot be distinguished using conceptual resources alone. For "taken literally", wrote Severo, "this is indeed false"[11]. Against the literal or "strong" interpretation, these commentators have emphasised the distinction between *distinguishing* two incongruous counterparts and *identifying* each of them as such, and have restricted Kant's thesis only to the latter operation[12]. In what follows I will try to show that, in Kant's sense, the thesis of the inability of the concept is essentially correct

[9] *Prol*, AA 04: 286. The recognition of this passage as a forerunner of the thesis of *Dissertatio 15C* is, I believe, crucial for the full reconstruction of the meaning of the problem. But, on the other hand, it places my reading in direct opposition to the one defended, for example, by Zerbudis (see, Zerbudis, Ezequiel, "Incongruent Counterparts and the Origin of Kant's Distinction between Sensibility and Understanding". In: *Archivfür Geschichte der Philosophie* 94 (3), 2012, 326-352, especially 332-335).

[10] From Rusnock& George ("this statement is one which refutes itself as soon as it is uttered", Rusnock & George1995, 273), through Mühlhölzer (Mühlhölzer, Felix: "Das Phänomen der inkongruenten Gegenstücke aus Kantischer und heutiger Sicht", *KantStudien*, 83, 1992, 449) to Friebe 2006, who, in addition to criticising Kant's thesis, found the passage "enormously irritating".

[11] Severo 2005, 39.

[12]Severo 2005 43f. This was also how Bennett proceeded, quickly dismissing the strong (and, in his words, more 'vulnerable') version as simply false. The less vulnerable version (what he called the "Kantian hypothesis") was "almost true", but ended up being also false on account of Wu's experiment (Bennett 1970, 175-176). Bernecker, Sven: "Kant on Spatial Orientation". In: *European Journal of Philosophy* 20 (4) 2010, 525, follows the Bennet-Severo line, but does not add anything new.

even in its stronger version, and that this inability is precisely the fundamental underlying reason that explains the existence of incongruent counterparts and the problem of orientation. To this end, I will begin by re-examining some of the classical objections (and, allegedly, counterexamples) raised against the *Dissertatio* thesis. I will restrict myself, for the moment, to the question of spatial incongruence.

The idea that there exist descriptions of incongruent counterparts that do not involve intuition and can therefore be regarded as 'purely conceptual' is certainly not a recent one[13], but it seems to recur in the philosophical agenda. Almost all attempts in this direction have been based on the assumption that it is possible to describe *mathematically* two incongruent counterparts as distinct objects, and so also the difference between them. Thus, in a tradition going back at least to Hermman Weyl[14], it has been claimed that mathematics –especially contemporary mathematics– has enough resources to describe the difference between two incongruent counterparts and to provide, in fact, an adequate explanation of it. Along these lines, and in an article that was considered quasi-definitive at the time[15], Felix Mühlhölzer set out in 1992 to precisely explain the "kombinatorische Grundlage des Recht-Links-Phänomens" to which Weyl had already pointed, and in doing so he claimed to have shown that today's logic and mathematics already possess sufficient "begrifflichen Resourcen" to explain this phenomenon without resorting to intuition[16] (therefore dismissing the thesis of the indispensability of intuition). In the first place, Mühlhölzer used the notion of isometry in R^3, and the classification of all such isometries as 'direct' or 'inverse', to (supposedly) capture and characterise in conceptual terms the left-right orientation. In a second step, he enabled a procedure of the same type but using permutations, and showed that the method of isometries could be reduced to this one, so that –in the manner proposed by Weyl– the difference between direct or inverse permutations would be revealed as the true 'first ground' of the phenomenon of incongruent counterparts.

The first thing to note in this respect is that none of these attempts is, as their authors have claimed, 'merely conceptual' as opposed to 'intuitive' in the sense that

[13]Vaihinger already offered a considerable list of authors who believed they were in possession of purely conceptual (and not intuitive) descriptions of the difference between incongruent counterparts (see Vaihinger 1892, 527f).

[14] Weyl, Hermann: *Philosophy of Mathematics and Natural Science*. Princeton 1949, 78f.

[15]See Rusnock-George 1995, 257.

[16]Mühlhölzer 1992, 449-452.

these terms have in Kant from 1770 (when he first enunciates the thesis) onwards[17]. Indeed, if up to 1770 Kant can be considered to be following the Leibnitian use of the terms, assuming that mathematics are a part of conceptual knowledge, from at least 1770 Kant is already ascribing mathematical knowledge to intuition[18]. From that moment on, and for the whole of the critical period, mathematics will always be *cognition sensitiva*, and if 'concept' and 'intuition' are to be separated and opposed as they are understood by Kant in this period, what remains on the side of the concept is not mathematics, but rather the 'mere concept' linked to the analytical procedure. Therefore, the resources offered by Weyl-Mühlhölzer and his followers cannot at all be 'merely conceptual' in the sense that this expression has in Kant, since in the very construction of the order of permutations, intuition is already involved[19]. The consideration that these mathematical resources are not intuitive can only rest on a misunderstanding about the meaning of (mere) 'concept' and (mere) 'logic' in Kant on the one hand, and in the later development of logic and mathematics on the other (a misunderstanding whose historical roots probably go back as far as Frege). At any rate, from the point of view of the meaning of the terms used by Kant, the "conceptual means of contemporary logic and set theory" to which Mühlhölzer referred are perfectly intuitive[20].

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[17] On this point Bernecker (2010, 523) is correct but, I believe, insufficient. Oroño (2017, 167) has also recognized this fact, although he concentrates in the temporal succession underlying the use of numbers.

[18] See *Dissertatio*, §12, §15C. In contradistinction to Kant, 'concept' in Leibniz is not opposed to 'intuition': both refer to the construction in the pure proceeding of the mind. In fact, intuitive knowledge can be for Leibniz the supreme form of knowledge (*Discourse on Metaphysics,* 24) without ceasing to be knowledge of *notions* at all. Leibniz's notion of '*notio*' does not refer only to a universal mode of representation, for there exist for him (and it is essential that there exist) *singular notiones (*cf. L. Couturat, *Opuscules et fragments inédits de Leibniz*, Paris 1903, 519). That the concept was not opposed to intuition in Descartes either can already be seen in Rule III (AT X, 368) and in Rule XII (AT X, 420: the faculty that intuits is precisely the understanding).

[19]Cfr. Torretti, Roberto: *Kant. Estudio sobre los fundamentos de la filosofía crítica*, Santiago de Chile 1967, 128.

[20] Against Rusnock-George, who went so far as to say: "If anything is based in concepts alone, it is the modern combinatorial solution to Kant's incongruence problem" (Rusnock-George 1995, 264). But also against Passos Severo: "It is mathematically possible to distinguish incongruent counterparts by means of concepts alone" (Passos Severo 2005, 39), and Bernecker: "the intuition thesis is fully compatible with incongruent counterparts being distinguishable mathematically" (Bernecker2010, 523). Mühlhölzer's statement belongs to Mühlhölzer 1992, 452.

Perhaps, however, Kant was not claiming that the difference between incongruent counterparts could not be mathematically defined, but only that such a difference cannot be expressed by (discursive) 'mere concepts'. In this sense, it has been said, Kant would not be denying "that an analytic description of incongruent objects is possible in purely mathematical terms"[21].The thesis from *Dissertatio*15C would then be "fully compatible with incongruent counterparts being distinguishable mathematically"[22], and its target would be considerably reduced. As a matter of fact, this interpretation could seemingly gain further support from a passage from MAN (AA 04: 483-4) in which Kant explicitly states that the concept of this internal difference "can be constructed" (although it cannot be expressed discursively) and with another passage from *Dissertatio*15C (AA 02: 403) stating that "the difference [between counterparts] […] can be characterised by a pure intuition". However, if incongruent counterparts have made their way into the critical period unscathed and Kant is being consistent on this point, this line of interpretation cannot be correct. Let me first pause and describe a global consideration.

When Kant presents the 'paradox' involving the existence of similar and equal but incongruous objects in §13 of the *Prolegomena*, he does so by looking at all the aspects that can be known "für sich" in each of these objects[23]. These aspects of each object taken in isolation are then identified with the determinations that pertain to quantity and quality[24]. Now, since these determinations are by definition "vollig einerlei" in the two objects under consideration, it is clear that the determinations pertaining to quantity and quality *cannot* themselves distinguish one from the other. But in Kant's mature conception, mathematics is concerned only with quantity, since mathematics is based on the construction of concepts, and "only the concept of

[21] Nuzzo 2008, 330f, footnote 65. It is true that in the same quoted note Nuzzo points in a different direction, which qualifies his statement but, as we shall see, if the qualification is correct (and I believe it is), then the description would no longer be "in purely mathematical terms".

[22] Bernecker 2010, 523.

[23] *Prol*, AA 04: 285; a little later (AA 04: 286) Kant repeats again that it is a question of what in each "allein" can be described.

[24] *Prol*, AA 04: 285. In KrV A 281 / B 337, discussing the principle of the identity of indiscernibles, Kant will again make it explicit that the conceptual identity in question is equality "according to quantity and quality".

quantities can be constructed"[25]. This means that none of the determinations that are amenable to mathematics can distinguish two incongruent objects. Therefore, all the resources that are purely and strictly mathematical must also be excluded from what can distinguish one counterpart from the other. This consideration does not depend on any decision about the usage that Kant makes of spatial incongruence in each of the stages of his philosophy; it only relies on the fact that this phenomenon is still entertained by Kant in 1783 (and 1786) and is still introduced as pointing to a real difference that, nonetheless, cannot be described by using certain resources (among which mathematical ones would be now clearly included).

One could of course still try to stick to incongruent counterparts being distinguishable mathematically by saying that 'distinguishing' in this context does not merely mean the act of telling two objects apart (like a right and a left hand). What 'to distinguish' means in this context, so it is said, is the ability to identify each of the objects as such, for example a right hand as a right hand. In other words, it refers to the ability to solve the which-is-which problem. By tightening the requirement that concepts cannot fulfil, the range of *Dissertatio*15C is thus weakened, and by using this weakening procedure we can be –it seems–at ease again: mathematical descriptions would then be able to tell two incongruent counterparts apart, but not to establish which is which (as mentioned above, this is the route followed by Bennett, Severo and Bernecker). But this cannot be correct either, and this takes us to the core question.

In fact, if the difference between two incongruent objects (in the stronger sense of telling apart) could be expressed mathematically, then the whole phenomenon –and Kant's analysis of it– would become unintelligible. For what Kant insistently points out is that the identity of the relative distances between the constituent parts of a body (hence the equality in the 'quantity of space' between any two of its points) does not prevent them from being differently oriented and, as such, different objects. The identity of counterparts arises, Kant writes, "wenn man bloss [...] auf die Proportion und Lage der Theile unter einander und auf die Grösse des Ganzen...[sieht]" (AA 02: 381), i.e. when one attends only to the metrical properties of the object. These metrical properties are the *only ones* that mathematics, understood in Kant's strict sense, can describe, for

[25] KrV A 714 / B 742. As far as quality is concerned, Kant has established in the *Anticipations of Perception* (A 176 / B 218) that "in every quality we can know *a priori* only the intensive *quantity* of it", confirming the connection between quantity and constructability. See also reflection 5593 dated 1778-1779: "die Mathematik [handelt] so weit von Qvalitäten, als die bloße Anschauungreicht" (AA 18, 244).

mathematics deals with quantity; quantity is the result of a "synthesis of the homogeneous"[26], and this composition of the homogeneous is precisely what generates the representation of these properties. The concepts of mathematics, understood as concepts constructed in pure intuition, must on this point be considered as impotent as any other concepts[27]. In other words, even if the notion of 'concept' is taken in a sense wide enough to include mathematical concepts, 'concept' remains powerless on this point. And this powerlessness essentially means the inability to distinguish (not merely identify) two similar but incongruent objects.

As a matter of fact, it is precisely this impotence of all quantitative-qualitative determinations that lies at the core of the phenomenon of incongruent counterparts. For even if an object is perfectly determined in terms of its quantity and quality, it is not for that reason already determined how it is to occupy space, i.e. how it is to be situated in relation to space as a totality. In this sense, what is decisive for Kant is that objects in space are *conceptually* (and this includes mathematically) *under-determined*. It is precisely here that the condition of possibility of the phenomenon of spatial incongruence lies, for it is precisely the powerlessness of the metrical-conceptual to characterise, alone and completely, the presence of a body in space that allows for incongruent pairs of objects to exist[28]. In this sense, the thesis of *Dissertatio*15C, together with its replicas and forerunners, must be considered inherent to the problem of incongruent counterparts, and inseparable from it[29]. This is precisely what the example of the lone primordial hand, to which Kant refers at the end of the '68 essays, makes

[26] KrV B 202, footnote.

[27] This phenomenon of conceptual under-determination has already been pointed out by several commentators. Walford made particularly clear the metrical character of *Lage* (see Walford, David: "Towards and Interpretation of Kant's 1768 *Gegenden im Raume* Essay". In: *KantStudien*92, 2001, 409f.) and, at the same time, what happens if the perspective adopted only takes into account metrical notions: the result, then, is "an impoverished and *merely mathematical* definition of space" (411, my emphasis). In the same vein, Scaravelli had already indicated that the space of Euclidean geometry is a "spazio depotenziato"" (*Scaravelli*, Luigi: "Gli incongruenti e la genesi dello spazio kantiano". In: *Giornale critic della filosofia italiana*, 31, 1952; reprinted in –from which I cite– *Scritti kantiani*, vol. 2. Firenze 1968, 328).

[28] Walford was also perfectly right on this point, as he considered this to be a "crucially important feature of *Lage*" (Walford 2001, 410).

[29] I disagree, hence, with Torretti (1967, 127), who claimed that the argument of *Prol*. §13 does *not* "depend at all" on the impossibility of conceptually distinguishing two counterparts.

clear: examined from a strictly conceptual point of view, as the Leibnitians want, this hand would be "gänzlich unbestimmt" with respect to the property of being a left or a right hand, since the space it occupies would be in both cases the same[30]. Merely conceptual descriptions, even if they are metrical, are ambiguous here. And it is the 'clearance' left open by the metrical characterisation of a body that allows that body, without ever ceasing to satisfy such characterisation, to occupy space in different ways. Since the metrical properties of a body do not univocally determine its mode of presence in space, a 'double' of the same body (Scaravelli very correctly called it "*duplicato"*) can arise that is nevertheless *not* super imposable with the 'original'[31]: it is precisely these pairs of 'doubles' that are corporeal extensions completely equal and similar, and yet so different in themselves that "die Grenzen der einen nicht zugleich die Grenzen der andern sein können"[32]. And it is they that possess the "very remarkable characteristic" of responding to the same metrical determination. It is, in short, the *same concept* of one and the same object that allows that object to exist in two different ways in space (and, not in vain, when Kant explicitly introduces incongruent counterparts in the sixth paragraph of GUGR (AA 02: 382), he constructs them on the basis of a *single* concept of a single object).

So far I have tried to present Kant's phenomenon of spatial incongruence as having its roots in the indeterminacy (and hence, ambiguity) of all metrical descriptions of directional aspects of bodies in space. By taking it back to this root, I think, several features of the phenomenon can be made sense of with considerable ease (among others, its relation to Kant's critique of the Leibnitian *principium individuationis*, as apparent in the "Amphiboly of Concepts of Reflection"). But here I am going to focus only on one of those features, namely: why and how is it possible that the body has been considered (to some extent by Kant himself explicitly, and certainly by several interpreters in his footsteps) to provide a solution to the problem of spatial incongruence. This, I hope, will also shed additional light on the habit of reducing such a problem to a mathematical

[30] GUGR, AA 02: 381.

[31] It is precisely this possibility of identical doubles or replicates of the same configuration of spatial relations that Leibniz denies again and again in the face of Clarke. See Leibniz's letters III (paragraph 5) and IV (paragraphs 6 and 13) in *The Leibniz-Clarke Correspondence*, edited by H. G. Alexander, Manchester 1956.

[32] GUGR AA 02: 381. See also AA 02: 382.

problem, solvable by means of mathematical descriptions. In particular, it will shed light on a state of affairs that seem to contradict what I have been claiming so far: mathematical descriptions have been indeed very often used to (supposedly) describe that crucial difference, so they have been able to obtain at least the appearance of describing two different incongruent counterparts. But if so far my claim is correct, and the thesis of *Dissertatio* 15C is to be taken literally, two incongruent counterparts are mathematically undistinguishable: how then have all the mathematical analyses proposed in the literature been able to attain, at least apparently, their goal?

II. Body

If the problem of spatial incongruence is one of (conceptual) indeterminacy and ambiguity, as I am claiming, then its solution could only come from a device capable of determining what otherwise remains ambiguous. Kant explicitly says in the second and third paragraphs of GUGR (AA 03: 378f) that our own body is such a device:

> [es] ist kein Wunder, daß wir von dem Verhältniß dieser Durchschnittsflächen zu unserem Körper den ersten Grund hernehmen, den Begriff der Gegenden im Raume zu erzeugen

> Sogar sind unsere Urtheile von den Weltgegenden dem Begriffe untergeordnet, den wir von Gegenden überhaupt haben, in so fern sie in Verhältniß auf die Seiten unseres Körpers bestimmt sind

With this device, those aspects of spatiality that were ambiguous and "unbestimmt" when conceptually described become now "bestimmt". Without it, they remain mere metrical relations, useless to deal with real space, as Kant brilliantly shows in his analysis of celestial maps (AA 02: 379f, AA 08: 135) but also of orientation in general. For this indeterminacy also pervades our most common "Kenntniß der Lage der Örter":

> [sie] uns zu nichts hilft, wenn wir die so geordnete Dinge und das ganze System der wechselseitigen Lagen nicht durch die Beziehung auf die Seiten unseres Körpers nach den Gegenden stellen können

The orientation problem, indeed, displays the same under-determination/determination dialectic as the problem of spatial incongruence, and must therefore be considered systematically along with this, despite some interpreters' protests (see *supra*, footnote 1). Although in the 1768 paper Kant explicitly relates the question of the directionality of space to the problem of orientation (AA 02: 379-380), and in 1786 the problem of

orientation is described in terms of the difference between left and right (AA 08: 135-136), in fact the specialized literature has traditionally paid much more attention to the phenomenon of incongruent counterparts than to the problem of orientation, and in doing so has often separated and isolated each of them in different compartments. Actually Vaihinger, already in 1892, considered this to be the only correct way of approaching the question: although Kant had wrongly amalgamated these "only apparently related" problems in 1768, later (in 1783 and 1786) he would correct himself and –accordingly– would clearly separate one from the other[33]. In contrast to Vaihinger, the interpretation pursued here suggests that both questions form a unitary systematic plexus whose unity must be firmly maintained. First and foremost, because both questions spring from the very same systematic condition, as I have tried to show, but also because of the textual evidence supporting it: beside the passages just mentioned, Kant seems to refer in both cases to one and the same point: both the problem of orientation and the problem of incongruent counterparts refer, in effect, to an apparently "subjective" basis of distinction ("*Unserer* Körper" in AA 02: 378, "*meine* Hände" in AA 03: 379, emphasis is mine) in 1768, "einer subjective Unterscheidungsgrund" (AA 08, 134-135) in 1786[34].

It is in fact only this under-determination phenomenon common to incongruence and orientation problems, and only in the stronger version offered here, that can make the resource to the body comprehensible. Albeit many interpreters have assigned the body a central role in Kant's directionality problems and even have fixed their attention on the notion of a 'coordinate system' (see among others, those cited *supra*, in footnotes 2 and 3), in my opinion it is only now, when one has reached this point, that the intervention of the body in Kant's analysis can be considered properly justified. For only now can it be properly seen that, in order to determine directions in space, and consequently to distinguish two incongruent counterparts, an action is required that cannot be 'conceptual', neither in the narrow sense of discursive representations *nor* in the wider sense that includes mathematical (i. e., pure sensible) concepts. Only now can it be seen that the notion of a coordinate system includes two dimensions, one of which is certainly metrical but the other is irreducibly directional.

[33]Vaihinger, Hans: *Kommentar zu Kants Kritik der reinen Vernunft*, vol. 2. Stuttgart 1892, 523-534, footnote.

[34]Opposing Vaihinger is also Nuzzo, who from the very beginning correctly maintains the unity of these two problems (Nuzzo, 2008, 21).

The decisive state of affairs lying at the centre of Kant's directionality problems is, in effect, one of representational ambiguity or under-determination: if metrically described (and mathematics in this sense can only proceed metrically) two incongruent counterparts, or the indications for two opposite directions, do not appear as two, but as one, identical, reality. There is just one (and the same) concept in place, but two real objects that respond to it. If this was not the case and the thesis of *Dissertatio*15C was not correct, or if it was correct only in the weakened form that merciful interpreters have supposed, then the action required to distinguish two objects like two human hands *could* still be of a conceptual kind, and the intervention of an action of a different kind could not be considered as absolutely necessary. In what follows I will argue that this situation would make Kant's (and Kant interpreters') reference to the body unnecessary and, to that extent, it could not be considered properly justified.

Since the under-determination at issue is a directional under-determination, the required determination could only take place if the ambiguity of spatial *directions* could somehow be broken (and, to be sure, broken in each of the three spatial dimensions); in its turn, this disambiguation can seemingly only be achieved by fixing a certain point in space as the origin of three perpendicular lines or axes, and then distinguishing the resulting six half-lines as different directions; in other words, by instituting an *oriented reference system*. But this is exactly what our own body does *inasmuch as* it is an asymmetrical body existing in space. Because it exists in space it already fixes a certain point in it, and because it is asymmetrical, it is capable of breaking through the ambiguity that afflicts purely metrical determinations, thereby producing orientation.

Kant's claims go further than this, as our own body does not only establish directions by determining what was earlier undetermined; it does not only produce orientation. It produces the *original* orientation, the *original* system of directions, and must therefore be viewed as the "erste Grund" of all representations of directionality. The bodily perspective is thus constituted as the first and last system of reference, the one beyond which it is not possible to go back, and therefore as the primary matrix for the interpretation of all orientations: all possible fixations of other references of systems of coordinates and directional indications are comprehensible only to the extent that they are connected with this centre. The oriented system of reference linked to one's own body would thus be the ultimate framework in which all indications of directions acquire their meaning. According to this, the body should play an indispensable role in

establishing and distinguishing directions *at large*, and its contribution could be presupposed *whenever* directions are in play (centrally, whenever instructions for orientation are given or received, and whenever two incongruent counterparts are distinguished).

Against this, attempts have been made to reduce the relevance of the hands and the body in Kant's argument to mere illustrations or examples (probably induced by Kant himself, as he introduced the asymmetry of our body as "das gemeinste und klärste Beispiel", AA02: 381). Nevertheless, if the body truly is the "erste Grund" of directionality, as Kant seems to suggest, then its relevance must be far greater than that of a mere illustration[35].The extent to which one's own body must be, in this argument, indispensable can perhaps be most clearly illustrated by resorting to what Bennett tried to do in his classic work of 1970. Bennett denied that the distinction between two incongruent objects had to refer to the observer's point of view understood in terms of the "observer's body", and even denied that human bodies were in any way necessary to establish such a distinction[36]. Bennett's basic thesis was that even if our body did not have the asymmetries it has (even, in the most extreme possibility, we had a spherical shape) we would still be able to distinguish "left" and "right" as long as we could use a pair of incongruent objects as a criterion. But this claim of Bennett's ignores the fundamental problem and leaves unanswered precisely the decisive questions: how do spherical beings relate to oriented or incongruent objects? Are they in front of them? Behind them? Can it be said of an absolutely spherical being that it has, in any sense, a 'behind'? If we can in any sense say that a spherical being has a front or a back, is it not rather because we do have a front and a back and project these onto it? But if we assume that a spherical being can properly have a 'behind', are we not thereby already introducing asymmetries that destroy the spherical character we had assigned to it in the first place?[37] In order for a reference system to enable the distinction between "left" and

[35] Nuzzo has been particularly clear and forceful in this claim (see Nuzzo 2008, 26). See also Walford, 2001, 428.

[36] Bennett 1970, reprinted in (from where I cite) Van Cleve, James & Frederick, Robert E. (eds.): *The Philosophy of Right and Left: Incongruent Counterparts and the Nature of Space*, Dordrecht 1991, 107f.

[37] This seems to be the middle way Bennett follows in imagining spherical humans who nevertheless still have something like 'face' and 'back', and who face the world as something in front of them (Bennett, 1970, 108). But this is as much as to deny, properly speaking, their spherical character. Walford also

"right", it must settle on the spatial ambiguity that exists between two opposite "Gegenden" (it must itself be oriented; see, *supra*, p. 14); but for this distinction to be meaningful to us, the reference system in question must be somehow (but spatially!) related to the original orientation system, and it is this last condition that Bennett seems to have overlooked in his classic paper: unless they had already organized space into "Gegenden" (which seems highly unlikely given their perfectly symmetrical shape), spherical beings would have no systems of orientation, or, for that matter, directions as such.

If the oriented reference system linked to our own body is established as the original system of reference, and as such as all-pervading and indispensable in every directionality problem, then there are several important systematic consequences. One of them has to do with mathematics, and with the way mathematical descriptions seemingly solve the problem of distinguishing two incongruent counterparts. But before moving to it, I will for a moment examine another of those systematic consequences: if the body (as the original reference system) must be present whenever orientation instructions are given or received, then the significance of Wu's experiment is radically diminished, and the arguments based on it are largely neutralised. Let me pause to develop this briefly.

It had been traditionally assumed, indeed, that nature did not show any preference between the two ends of any of the directional axes and, consequently, that these ends could always be switched without measurable change in the experimental situation. The resulting belief was that no phenomenon of nature intrinsically pointed to one direction rather than its opposite, so that no such phenomenon could be used to unambiguously define "left" and "right". In other words, there were no physical experiment that we could use to convey our use of the terms "right" or "left" to an imagined inhabitant of a very distant planet (Planet X, in Gardner's Ozma Problem) or to a likewise imagined individual that has switched the meanings of our two terms (an "Alphan", in Bennet's example).

All we could do to communicate our use of "left" and "right" to such inhabitants and speakers is present them with an asymmetric object or structure and the explain to them how we, in our convention, label its different directions. Unless we and they can

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analysed the objection of the spheres and correctly pointed out that those who so object "have smuggled themselves into the picture" (Walford 2001, 429, footnote 32).

observe in common such an object or structure, there is no way to convey that our meaning of "left" and "right". But this, so claimed Bennett, would be tantamount to recognize that Kant was right: if we are to explain the difference between "left" and "right" [in the sense of which is which] we must resort to "sensorily presented instances", i. e., we must resort to "showing".

Allegedly this all changed in 1956, when the Chinese-American physicist Chien-Shiung Wu experimentally proved by using nuclei of the highly radioactive isotope cobalt-60 that parity was *not* conserved in all interactions present in nature, contrary to what had been previously expected. To test whether the principle of parity conservation also held in weak interactions, Wu experimental set-up forced the cobalt-60 nuclei to line up in one and the same axis (N-S), so that the particles emitted in their decay could only follow one this two directions: either from the north end of the nucleus or from its south end. According to the traditional belief, the particles emitted by these nuclei should be flung out equally in the two directions. However, Wu proved that this was not the case: more particles were emitted from the south end than from the north end. Martin Gardner commented on the discovery: "Madam Wu's experiment provided for the first time in the history of science a method of labeling the ends of a magnetic axis in a way that is not at all conventional"[38]. Thanks to the intrinsic handedness of this phenomenon, it was said, nature herself provides us with an operational definition of left and right, thereby solving the "Ozma problem" as well as the "Alphan problem".

But in the very same moment these problems were solved, Kant's hypothesis would have been proven wrong: the thesis of the indispensability of intuition would have been refuted on the basis of Wu's 1956 experiment. Indeed, from their work onwards, the fall of parity was considered a decisive element for the question of directionality, and was echoed in many different analyses, under the idea that a way had been found to operationally define right and left without any reference to "sensorily presented instances", i. e., without "showing".

However, as already noted[39], for this definition to be effective it is necessary to perform the experiment itself, take a datum from experience and relate it to the original

[38] Gardner, M: *The Ambidextrous Universe. Left, Right and the Fall of Parity*. Second edition, revised and updated, Middlesex 1982 (first edition 1964), p. 205

39 Because Curd had already pointed out that there was still "ostension at a distance" (Curd, Martin: "Showing and telling: Can the difference between right and left be explained in words?" In: Van Cleve &

directions defined on the body itself. In fact, it could be said that, on this point (although not so in physics in general) Wu's experiment does not change anything essential in the existing state of affairs. If anything, the experiment makes up an analogue of the operational definitions of basic units of measurement that have recently been attained. In that sense, and following Curd's hint, it could be said that the fall of parity changes the state of affairs only in a (very) limited way: instead of having to resort to asymmetric objects to point to, the situation is now one in which that function is performed by phenomena that can be recreated anywhere. In this way, and following the historical fate of the platinum-iridium bar in Paris, it would seem that the basic operation of defining spatial directions would thus have been "de-localised", and thus universalised and de-anthropomorphised. And indeed, something of this kind has taken place, but the decisive role of one's own body has not been affected in the slightest. For, just as in order to fix the value of a metre it is still necessary to carry out the corresponding experiment and take note of it in the context of immediate experience, so, in the fixing of directions by means of Wu's experiment, it is still necessary to translate the results into the system of reference associated with the body[40].

But another important conclusion can be derived from the recognition of the body as the original system on which directionality is ultimately determined, and this brings me to a last essential point. If my reconstruction is so far correct, then the definition of directions –reduced to its most schematic form– always involves an act of fixing a centre of reference and the three corresponding asymmetries, and this act is essentially distinct from the production of homogeneous units in the sense of quantity or any other conceptual device. But if the description of incongruent counterparts involves the use of directions, then it would seem that anyone who has obtained a description of incongruent counterparts as two distinct objects would have had to produce these acts in some way, explicitly or implicitly. What about, then, all of the 'mathematical' (allegedly conceptual) descriptions?

That this is indeed the case in the allegedly 'mathematical' descriptions of the phenomenon of spatial incongruence adduced in the literature can be suspected, first of all, from the fact that all the adduced descriptions describe two incongruent objects in

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Frederick (eds.), 1991, 200); Walford correctly echoed Curd in his review of Van Cleve & Frederick (Walford, David: *Kant Studien*85 (1), 1994, 102).

[40] On this, though in a very different sense, see Rusnock& George 1995, 276. In their footsteps, see also Oroño 2017, 168.

opposition to each other by means of additional elements. However, with this an important slippage has already occurred, for what in principle was at issue was not the simultaneous description of two different objects, but only the possibility or impossibility of (conceptually) determining *all* the *real* characteristics (including orientation) of a single object, i.e. the possibility or impossibility of such "vollstandige Beschreibung" (AA 02: 377). In this sense, it seems obligatory to say that all those commentators (from Kirchmann and Thiele, to Weyl and Mühlhölzer) who have sought to refute the thesis of *Dissertatio*15C by 'mathematically' describing a spatial configuration that includes the two incongruent counterparts in their relation to a third element external to both, are missing the point, since it is precisely the need for this spatial relation to a third element (the oriented system of reference) that is in question[41]. In describing that combined spatial configuration, these authors are performing operations that go beyond what Kant here considers 'conceptual' (even this expression includes pure sensible concepts), and in particular they are somehow fixing a point external to the two objects to be used as a point of reference. This is especially evident in some of the older interpreters (for example, Kirchmann and Thiele), who sought to describe conceptually the difference between two incongruent counterparts by selecting a point between them, or a line between them, but there is nothing else that lies at the heart of the more modern attempts[42]. Because however abstract the mathematical means of describing two incongruent counterparts may be (linear spaces and linear applications, automorphisms, etc.), and however much one tries –through them and the use of coordinates– to seek refuge in the merely numerical, if these structures and numbers are to describe figures in real space, then they have to be interpreted spatially, but to do so it is necessary that the numbers designate distances with respect to certain lines, etc., and that cannot happen without fixing a system of coordinates with a given centre and directions. That this is the nodal question, and the truly decisive operation, was noticed by Milhaud (with a certain naiveté) as early as the beginning of the 20th

[41] For an insuperable awareness of this, see Reidemeister 1947, 68.

[42] Kirchmann and Thiele were cited by Vaihinger (Vaihinger 1892, 528). More modern attempts are essentially resumed in Mühlhölzer (who, ironically, also speaks of the possibility of putting a "third body" in play, 1992, 440), but see also Falkenburg, Brigitte: "Incongruent Counterparts: Kant's 1768 Argument Against Relationalism". In: Ralph Schumacher, Rolf-Peter Horstmann& Volker Gerhardt (eds.), *Kant Und Die Berliner Aufklärung: Akten des Ix. Internationalen Kant-Kongresses*. New York 2001, vol. 2, 15), and Friebe 2006, 40.

century, when he pointed out that, given a system of rectangular coordinates, "il suffira de changer z en –z pour quoi soit représentée quantitativament la figure symétrique de la précédente" (i.e. an incongruent counterpart)[43]. But this approach (as well as Mühlhölzer's and all the rest) can only work by introducing the notion of specular symmetry, and with it that of the axis of symmetry, as well as its arithmetical counterpart: the minus sign '–' in front of the y-component. But neither one nor the other is possible without the fixation of an absolute point as the centre of coordinates and the fixation of the 'left' half-line as the half-line of the negative numbers[44].

It should be noted, furthermore, that this need to fix an oriented coordinate system affects not only mathematics when applied to the physical world, but also 'pure' mathematical tools as long as they remain mathematics used and usable for humans. Earlier it was said (see *supra*, footnote 6) that the problem of directionality pertains to the interstice (which becomes decisive after the scientific revolution) in which mathematical concepts are applied to the objects of the material world, and not directly tothe realm of the purely mathematical. This essentially correct statement must now be complemented by the recognition that also the realm of so-called 'pure' mathematics (as long as it is a realm explored by humans for humans) has already incorporated this dimension of directionality, and goes beyond the strict notion of quantity as synthesis of

[43] Milhaud, Gaston: "La connaissance mathématique et l'idéalisme transcendental". In: *Revue de Métaphysiqueet de Morale* 12 (3), 1904, 394. Less naively, and discussing Milhaud, Scaravelli also indicated that the fundamental question was hidden in the passage from z to –z (see Sacaravelli 1952/1968, 326, footnote).

[44] That the 'purely mathematical' descriptions somehow concealed mechanisms of directionality or orientation has already been pointed out several times in the literature (see Torretti 1967, 129; Gloy 1984, 24, but also Lange, Heinrich: "Über den Unterschied der Gegenden im Raume". In: *Kant Studien* 50, 1958, 1483f) although, I believe, not systematically enough. That the use of the minus sign '–' in this context already *presupposes* a decision on the question of orientation, and cannot serve as founding ground in any way, can be seen especially well in Kant's treatment of the notion of 'negative magnitude' in his 1763 essay. Although from a purely logical point of view they cannot be distinguished at all (and therefore there is no room for their logical opposition), two forces or two movements exerted in opposite directions can nevertheless oppose each other in reality and even cancel each other out (**N**G, AA 02: 176). Indeed, in that case they would differ only in the *direction in which they are* exerted or produced, and so we can call "falling a negative rising, receding a negative advancing"; but for this, as is always the case when one wants to establish a scale of measurement in which there are "positive" values and "negative" values, it is necessary to fix a centre of coordinates or "0" from which the half-line of the positive values and the half-line of the negative values are determined.

the homogeneous. As a matter of fact, problems of orientation arise (and are solved) routinely in the purely mathematical realm, but the way in which these questions are resolved is a further argument, I believe, in favour of the interpretation that I am putting forward, for in so-called 'pure' mathematics, problems of orientation indeed arise, but they are all solved through mechanisms of disambiguation that refer, ultimately, to the body itself. This is what happens in fact, in a central place in vector calculus, when it becomes necessary to convey the direction of the vector resulting from a vector (or cross) product; at that moment, and in a move of enormous significance but usually overlooked, a 'right-hand rule' is stipulated whereby the direction of the vector $\vec{u} \times \vec{v}$ resulting from vectorially multiplying the vectors $\vec{u}$ and $\vec{v}$ will be the direction indicated by the thumb of an imaginary right hand that 'takes' the vector $\vec{u}$ on the vector $\vec{v}$ through the shortest path[45]. But even before that move, and at the very heart of basic coordinate geometry, it occurs in the conventional identification of the observer's left-hand side –as positioned facing the line– with the half-line of negative numbers[46]. In other words, pure geometry also 'anchors', even if only in the imagination, coordinate systems, and makes continuous use of the original directions associated with the body (which should perhaps not come as a surprise, because every time we picture a given spatial configuration to ourselves, we are already picturing it in relation to our original system of coordinates: we represent it in front of us, more or less close to us, etc.)[47].

Conclusions

This article has tried to reveal a logical connection between the thesis of *Dissertatio*15C and the intervention of the body in Kant's problems of directionality. It has attempted to show that, if that thesis was not correct (or it was correct only in a weak sense), then the presence of the body in that context could not be properly justified. In particular, I have argued that neither 'merely conceptual' (discursive) determinations nor mathematical (conceptual but purely sensible) determinations are capable, on their own, of

[45] See, for example, J. E. Marsden and A. Tromba, *Vector Calculus* (5th ed.), New York 2004, 43.

[46] See Scaravelli 1952/1968, 314, footnote 15, and Robinson 1981, 395, footnote. 21.

[47] In this sense, but not in others, we agree with Ferrarin: "the absolute system of reference is necessary not only for movement, but also for geometry" (Ferrarin, 2006, 23). Actually Weyl also made this state of affairs very clear: every spatial configuration is arranged "*before our eyes*" (1949, 75).

differentiating between two incongruent counterparts, thereby departing not only from the usual objections to *Dissertatio* 15C, but also from its defenders. In effect, I have tried to specify and clarify the meaning for this discussion of such terms as 'mathematical', 'conceptual' or 'intuitive' and, on that basis, I have stated that the thesis of *Dissertatio* 15C (i.e. the thesis of conceptual powerlessness regarding directions) is basically correct. Indeed, I have claimed that the mathematical indistinguishability of two incongruent counterparts (in the strongest sense of the word "distinguish", and in the strict Kantian core conception of mathematics) is inseparable from the phenomenon of spatial incongruence. In this sense, I have tried to present the metrical under-determination of bodies in space as the true core of spatial incongruence (but also of orientation problems). The sheer possibility of two incongruent objects existing in space must be considered to rely, in the end, upon the fact that both of them are under-determined from a quantitative-metrical viewpoint, for, "aus diesen [merely quantitative-metrical] *datis*", as Kant states in the important reflection 3606, "sind *zwey* Körper möglich, nemlich nach zwey opponirten Gegenden"[48]. If this is correct, then it is not enough to say, as several interpreters have, that the *Dissertatio* thesis refers only to the identification of concrete directions, but not to the operation of distinction as such: if the operation of distinction as such were indeed, as they claim, mathematically feasible in the strict sense of this word, then Kant's analysis of spatial incongruence would become meaningless. In particular, the need for reference systems fixating mechanisms, and the subsequent intervention of the body, would become impossible to grasp. In this sense, I have also argued that the 'counterexamples' usually used against *Dissertatio*15Cin the literature are not really such counterexamples, because they conceal mechanisms of fixation of oriented reference systems in a veiled way. This reconstruction can help, I believe, both to systematise previous suggestions made by

[48] AA 08: 89, my emphasis. The texts goes on and explains: "Dieses will so viel sagen. Wenn ein Punkt gegeben ist, so kann aus einer gegebenen Weite ein anderer Punkt in unendlich viel Stellen gegeben werden (d .i. er liegt in einer Kugelfläche). Sind zwey Punkte gegeben, so liegt der dritte Punkt in einem Zirkel . Sind drey gegeben, so liegt der vierte Punkt in zwey Stellen auf entgegengesetzten seiten. Alle Lage der Punkte läßt sich geometrisch bestimmen aus gewissen datis, die letzte Lage aber des vierten Punktes nur in Ansehung unseres Körpers".

several scholars[49] and to explain the recurrence of mathematical descriptions in the discussion: it is precisely the body, as a direction-fixing mechanism, that invisibly underlies and completes the "purely mathematical" descriptions and gives them the appearance of possessing a capacity that mathematics, in Kant's strict sense of the word, does not possess.

Of course, many aspects and dimensions of Kant's analysis of spatial incongruence and orientation have been neglected or, directly ignored, in bringing this concept-body link into the light. Many of these aspects and dimensions (especially those related to the progressive development of Kant's views) should be taken into account and included again if a coherent global interpretation is to result from this. The same goes for a certain logical gap that should be filled before getting to any global interpretation. But once this link has been unearthed and established, I believe, it will be easier to detect the field lines that might have taken Kant from one conception to the other, and assess whether, or to what extent, he really was shifted by them (in particular, whether a unitary line of thought can be found running through all versions of the argument, or, on the contrary, there are interruptions and discontinuities). Let me lastly illustrate this by briefly examining the logical gap just mentioned and one of the aspects that have neglected so far, and pointing to their usage as field line detectors.

Firstly, on one hand, it has been repeatedly claimed that the body could justifiably appear as a solution to Kant's directionality problems only if the thesis of *Dissertatio*15C was true, because only then could a coordinate fixing mechanism appear as justifiably required. In fact, it is clear that any other mechanism (and not necessarily our own body) would also do, as long it was able to fix a centre-point and certain asymmetries in space. But the question that should fill this logical gap (why, then, *our own* body?) can be extremely efficient when it comes to understanding why and how Kant moved from his 1768 'absolutist' or 'pseudo-absolutist' stance to the ideal-subjectivist position of 1770 and then into the mature conception of the first *Critique*. Because if, on one hand, the determination of directions requires the fixation of a system of coordinates, and, on the other hand, directionality necessarily and universally pertains to the nature of space, then it seems natural to assume that the ability to

[49] For example, Severo's suggestion that "three indexicals of direction—one for each spatial dimension […] must also be regarded essential", along with 'I' and 'now' (Severo, Passos, Rogerio: "A Note on Essential Indexicals of Direction". In: *Thought*, vol.1 (1), 2012, 10).

recognise (and efficiently deal with) directions, i.e., the ability to fix a system of coordinates ,must also be universally and necessarily shared (and not in vain, in 1786 will Kant explicitly speak of the need of a "Vermögen" capable to "[Richtungen] a priori bestimmen"[50]. This seems to lead to the idea that directionality should be somehow connected to subjectivity itself as the ultimate root of all universality and necessity and, in this sense, it does not seem unnatural that Kant could have concluded that space must be "etwas Subjektives und Ideales, was aus der Natur der Seele nach einem festen Gesetz hervorgeht, wie ein Schema, um alles äußerlich Wahrgenommene zuordnen" (AA 02: 403). At the same time, this 'universalisation' of directionality by means of its 'subjectivisation' could only be accomplished if subjectivity itself somehow incorporates the ability to fix coordinates systems in space. Since our own body does have this ability and, at the same, it is something each human being has, a line of evidence emerges that takes (and maybe took Kant) from here to the notion of an 'embodied subjectivity' (and, in the end, in *Transcendental Aesthetic* Kant still says that "nur aus dem Standpunkte eines Menschen [können wir] von Raum […] reden" (KrV A 26 / 42).

But, on the other hand, if the phenomena of spatial incongruence and orientation ultimately actually refer to the fixation of systems of coordinates in real space, as has been insistently repeated in previous paragraphs, and this fixation can only take place in and through real existing objects, then the problems of directionality should have to do with existence, and this can shed light on a second line of evidence. Because if this was all correct, and directionality was indeed connected to existence, then nothing less could be expected than the *Verstand* and its products (concepts) being absolutely powerless in this respect: existence for Kant is, as Hegel rightly pointed out, "something absolutely other than a concept"[51]. In this sense, the question of directionality would be touching on an essential strand of Kant's and, insofar as it is linked to the Kantian concept of existence, the so-called 'Kant's thesis about being' (both in 1763 and in 1781) should also be taken into account and put to use for a complete treatment of that question. In particular, it could be used to further illuminate the fact that direction cannot be brought into a concept, i.e., that there cannot be a concept of 'left-handedness' that applies to all

[50] WDO, AA 08: 135.

[51] Hegel, G.W.F: *Sämtliche Werke*(vol. 19). Stuttgart-Bad Canstatt 1965, 584.

left-handed hands (and only to them) and that can be subsumed under the concept of ‘hand’ in the way that ‘human’ as ‘rational animal’ can under ‘animal’[52] .

[52] On this point Kaulbach was already perfectly clear: considerations that have to do with experiences of real space "have to be based on intuition, and their contents cannot be understood as particular cases to be subsumed under a universal concept" (Kaulbach, Friedrich, “Zum Problem des Realraumes“. In: *Zeitschrift für philosophische Forschung*, Bd. 10, H. 3, 1956, 399). In this sense the mere talking about directionality in the vicinity of, and in connection with, other notions can be profoundly misleading, and should be avoided as far as possible. Kant himself, it is true, was not always careful enough and sometimes slipped the term ‘Begriff’ while referring to directionality (in AA 04: 484, for example). At any rate, the literature has repeatedly indulged in this vice, most conspicuously in the line of interpretation that stems from Rusnock& George. Zerbudis, for example, writes that according to Kant's 1768 essay, it is necessary "to add a new item to the list of the notions required for the complete characterisation of a body's shape, namely, its orientation or direction" (Zerbudis 2012, 329). And a little later he even speaks of “the concepts of *right* and *left*” (334). But if the interpretation suggested here is correct directionality is not, and cannot be, a notion (*notio*) nor a concept in the restricted sense.